# Robust Estimators in Modern Power System Networks

BY

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# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# THESIS ABSTRACT

**NAME:** Xia Yuanhai
**TITLE OF STUDY:** ROBUST ESTIMATORS IN MODERN POWER SYSTEM NETWORKS
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*State Estimation has become more important since the development of Smart Grid. More data are available to monitor the system's states in real time, due to the use of variable meters such as Phasor measurement unit. The requirements of estimation accuracy and efficiency have also been raised.*

*The research conducted in this thesis is mainly focus on the accuracy improvement of the estimated state variables when facing different types of bad data from available measurements. This thesis presents a new robust estimator based on least-trimmed-square and least-absolute-value, whose results are more accurate than existing robust estimators.*

*6-bus, IEEE 14-bus and IEEE 30-bus systems are used to evaluate the performance of the proposed estimator.*

# ملخص الرسالة

**الاسم الكامل**: **اكسيا يوانهاي**

**عنوان الرسالة:** التخمينات القوية لحالة أنظمة القوى الكهربائية الحديثة

**التخصص:** الهندسة الكهربائية

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**الخلاصة**

إن تخمين وتقدير حالة أنظمة القوى الكهربائية أصبحت أكثر أهمية منذ تطوير الشبكات الكهربائية الذكية. حيث أن الكثير من المعلومات أصبحت متوفرة لمراقبة حالة الأنظمة الكهربائية في الوقت الآني مع وجود واستخدام أنواع كثيرة من أجهزة القياس مثل وحدات قياس الطور (PMU)مما يحتم زيادة الدقة والكفائة لتقدير حالة أنظمة القوى الكهربائية.

وهذا البحث الذي تم في هذه الأطروحة يركز على تحسين الدقة في تقدير المتغيرات المراد قياسها في ظل وجود أنواع معينة من المعلومات السيئة عن هذه المتغيرات من خلال أجهزة القياس المتوفرة. هذه الأطروحة تقدم طريقة جديدة ومتينة لتقدير وتخمين المتغيرات معتمدةً على مربع أقل تقليص و أقل قيمة مطلقة والتي نتائجها أكثر دقة من الطرق التقديرية الأخرى المستخدمة حاليا.

ولفحص أداء الطريقة المستخدمة في التقدير في هذا البحث تم استخدام نظام 6-bus و نظام ال 14-bus المعتمد من ال (IEEE) و النظام ال 30-bus المعتمد أيضا من ال(IEEE).

# CHAPTER 1

# INTRODUCTION

Power systems consist of generation, transmission, and distribution systems. Large generations are always far away from the consumers. Conventionally, transmission systems are needed to deliver the generated power to substations and then distributed to consumers. To ensure the security and stability of the power system, state variables need to be monitored in order to take proper action in case of any emergency occurs.

However, since power systems are always large and there are too many variables such as voltage, current, and power flow to be monitored, it's not economic to place multimeter everywhere. Moreover, the measurements are not always correct considering environment noise, communication failure and other type of meter errors. State estimation are needed to filter out all these defects.

## 1.1 State Estimation Principle

State estimation (SE) is the process of estimating unknown system state variables based on measurements from that system according to some criteria. Usually, the process involves redundant imperfect measurements. The process of estimating the system states is based on a statistical criterion that estimates the true value of the state variables to minimize or maximize a selected objective function.

According to the criterion stated above, minimizing the summation of the squares of the differences between the estimated and measured values, i.e, Least-Square (LS) is widely used. The major developments in this area have taken place in the twentieth century in applications from the aerospace field, which involves the location and trajectory of an aerospace vehicle given redundant and imperfect measurements of its position and velocity vector. it was then introduced to many fields related to control system and played more important roles.

Since the pioneering work of F.C. Schweppe in 1970 [1, 2, 3], SE has become a key function in supervisory control and planning of electric power grids. It serves to monitor the state of grid and enables energy management systems (EMS) to perform various important control and planning tasks such as power flow optimization and bad data detection.

Regarding Power System State Estimation (PSSE), before any security assessment can be made or control actions taken, reliable estimation of the existing states in the system must be determined. Nowadays, state estimation is considered

to be the heart of a modern electric control center. Its basic role is to generate a coherent and reliable real-time database from the information provided by remote signaling and measurement meters.

Up to date, many estimation algorithms have been proposed to improve the accuracy and efficiency of the estimation process. Least-Square estimator is the most famous and widely used method because of its simplicity and simulation velocity, it has been used since earlier nineteenth century and still in service in several complex systems. Other estimation algorithms such as Least-Absolute-value (LAV) and Least-Mean-Square (LMS) are also used as basic algorithms relating to state estimation. Large amount of improved methods have been introduced on the basis of these algorithms.

Comprehensive treatment of modern power system state estimation can be found in books first by Monticelli [4] in 1999 and then by Abur and AG Exposito in 2004 [5]. Beginning with the role of the state estimator in a security framework as one of the key modern Energy Management System (EMS) applications, they covers all parts of the state estimation process starting with power flow, problem formulation, basic solution techniques, observability analysis, detection and identification of bad data, and robust state estimation procedures.

## 1.2 Challengers

State estimation is used to monitor the states of power systems for many years A lot of challengers evolve as more requirements on power quality and increased

complexity of power systems.

### 1.2.1 Conventional Challenges

Conventional challengers mostly rely on the accuracy. Since the computer are not powerful enough to perform real-time estimation at that time, state estimation is mainly used to detect measurement errors and estimate unknown variables.

Regarding measurement errors, there many come from three types of sources:

- **Random errors**: imprecise of measurement instruments
- **Intermittent errors**: large noise or temporary communication failures
- **Systematic errors**: mainly from the nonlinearity of transformers and Deterioration of instrument because of environment issues.

When coming to the collected data themselves, they can be divided into two types from the view of results. The first type is small departures from the real value mainly introduced by random errors stated above, almost no data can avoid it. The other one is large departure that are grossly in error with actual value, which are called bad data. Bad data may be induced by many reasons, such as broken instruments, large noise, or bad connection between control center and measurement facilities. These bad data affect the estimation in a large scale that must be detected and filtered out.

Bad data can be categorized into two categories: single bad data and multi bad data. Single bad data often occurs in small systems. For large systems,

multiple bad data are dominant. For multiple bad data cases, correlation plays an important role. Those measurements with strong correlation may influence other state variables and ruin the estimation of original good data, while those weak correlated measurements may be influenced by other errors. When multiple bad data are consistent with each other, they are said to be conforming. We can divide multiple bad data into four groups according to above discussion [5]:

1. **Multiple non-interacting and non-conforming bad data**: Non-Consistent bad data in measurements with no or weakly correlation with each other;

2. **Multiple non-interacting but conforming bad data**: Consistent bad data in measurements with no or weakly correlation with each other;

3. **Multiple interacting but non-conforming bad data**: Non-Consistent bad data in measurements that are strongly correlated

4. **Multiple interacting and conforming bad data**: Consistent bad data in measurements that are strongly correlated

Some bad data are obvious and can be detected and eliminated prior to state estimation, by simple plausibility checks, such as negative voltage magnitudes and value of several order small or large. Unfortunately, not all types of bad data are easily detectable by such means. How to detect and eliminate these bad data is the main challenging for conventional state estimation.

### 1.2.2 Ongoing Challenges

As the development of technology is going on, more energy consumption is in the form of electrical power in developed and fast growing developing countries such as China and India. Besides traditional large generators, more distributed generation systems such as wind and solar power generators are located everywhere. Moreover, interconnection between large Regional Transmission Organization (RTO) and Independent System Operators (ISO) also increases the complexity of power system, making distributed state estimation more important.

Real-time monitoring is another challenge that matters much. The use of large digital computers in control centers has made it possible to track the changing conditions in the power system with a mathematical model in the computer. The real-time model can be used to assess the security of the present system as well as to check out possible control strategies. An overview paper "Real-Time Modeling of Power Networks" by Bose and Clements [6] covers the overall role of the SE in real-time power system state estimation.

To solve the remote and real time monitoring stated above, Phasor Measurement Unit (PMU) is introduced to measure power grid data on the basis of the same time for synchronization regarding extreme large modern power system. Usually, for the measured signal from remote location, those data cannot be exactly collected at the same time, which may induce estimation error for abnormal grid situations such as power failure. So all measurements must be synchronized before estimation by PMU.

As the demands for reliable electric power become greater, and as labor becomes a more significant part of the cost of providing electric power, much more automated technologies need to be developed that would allow remote monitoring and even decrease the need of personnel to be on hand at substations. Supervisory Control and Data Acquisition (SCADA) systems are one of possible solution.

In all, ongoing challengers can be concluded to be complex real-time and automated state estimators.

## 1.3 State of Art Research

Considering the above conventional and ongoing challenges, many researchers have proposed improved algorithms and novel ideas about it.

Currently, the scope of State Estimation is mostly limited to the transmission level, where each Transmission System Operator (TSO) continuously tracks its own grid from a centralized EMS. As the concept of smart grid are proposed and developed recently, Distributed State Estimation (DSE) have been mentioned more frequently, in spite of that, the SE notion for distribution systems was considered long ago [7, 8].

In the past, when small consumers takes little part in the electricity market and their influence on the smart grid is small in the whole view. As demands on electricity increase, environment concern become more urgent. Power system is transferring to more clean and reliable energy such as wind and solar power, which are often known as Distributed Generations (DG). Consumers are active

to take part in the usage of electrical with implementation of smart grid. DSE becomes a must to acquire variable and main system robustness and reliableness [9].

Though hardware/software have been well developed, reliable methods should be used to ensure real-time or near real-time data processing, considering the huge amount of data in large systems. Many papers have proposed methods in this aspect, Xie and Choi proposed a fully distributed state estimation algorithm for wide-area monitoring in power systems in 2012 [10], Gomez-Quiles and Gomez-Exposito introduced a two-stage DSE based on regional or multi-TSO case [11]. These papers give several methods to improve the efficiency of data processing in minimizing the input and estimated parameters in each estimation process.

Some other researchers focus on the estimation algorithm itself with fixed input parameters to save computation time. In reference [12], Gao and Wang introduced unscented transformation to state estimation, and in reference [13], symbolic optimal ordering and signed Cholesky factorization is used to avoid the ill-condition caused by transformation of large sparse matrix.

The papers cited above mainly deal with computation efficiency for large systems. The basic LS (WLS) algorithms are used in most of them, which lead to bad results when there are large outliers. Reference [14] gives the convergence of WLS estimation estimations. Like those estimator such as LAV, LMS, LMR, Wu and others introduced Maximum-Exponential-Square (MES) in reference [15], which has a very strong ability to suppress bad data without any extra bad data

identification loop, while at the same time it is fast in computation. Based on the alternating direction method of multipliers, reference [16] considers local and overall robustness of power system in DSE.

Moreover, the popular genetic evolution algorithms, especially neural networks, are used in very large and complex power system [17, 18, 19], but the results still need improvement.

## 1.4 Thesis Objective

In this thesis, it is intended to improve the accuracy of estimation results considering complex situation induced by multiple types of bad data. Besides conventional state estimators such as Weight-Least-Square and Least-Absolute-Value, other robust estimators are used to detect and filter out the bad data, such as Least-Median-Square and Least-Trimmed-Square. The main objective of this thesis is to introduce a new robust estimator known as Least-Trimmed-Absolute-Value estimator.

DC estimation are conducted on 6-bus system and IEEE 14-bus system first, then these two systems, plus IEEE 30-bus system, are used to conduct AC estimation. Various types of bad data are simulated to evaluate the performance of the proposed robust estimator.

MATLAB is used as the implementation platform.

## 1.5 Contents in the Thesis

In this thesis, chapter one gives an introduction of power system state estimation, including general background. It also presents existing challenges and state of art research in this field. In chapter two, the basic of power system state estimation is presented in detail, with DC and AC state estimation respectively. Theoretical equations and calculation methods are discussed step by step. Chapter three discusses the observability of power system considering both the selection of measurements before estimation and after bad-data elimination. Chapter 4 gives the basic method for bad data detection and demonstrates various kinds of robust estimators. It also introduces the Least trimmed absolute value algorithm. Chapter five illustrates the performance test of different estimators in several systems with various bad data scenarios. Chapter 6 is conclusion and future work.

# CHAPTER 2

# POWER SYSTEM STATE ESTIMATION

This chapter discusses the main characteristic of generalized state estimation problems using weighted-Least-Square (WLS) algorithm. Firstly, the concept of state variable is introduced, then states and parameters for estimation are presented. Power system models and power flow are discussed at last.

## 2.1 State estimation

The state of a system specifies its behavior, many control schemes such as inverse dynamics and feedback linearization rely on the availability of the system states. However, in many practical systems, only input and output of a system are measurable. Therefore, estimating the states of systems plays a key role in monitoring the system, detecting and diagnosing faults, and achieving better performance. In such cases, unmeasured states have to be inferred from a measurable

subset of the states. Furthermore, imperfect measurements, noise and communication disturbance introduce large amount of bad data, which may induce errors in the system states. Researchers have proposed many robust estimators to detect and correct these bad data, especially in automated systems, which are widely used today.

State estimation is the process of estimating unknown system state variable based on measurements from that system according to some criteria. Usually, the process involves imperfect measurements that are redundant and the estimation is based on a statistical criterion that estimates the true value of the state variables by minimizing or maximizing the selected criterion.

Due to the cost of monitoring and telecommunication equipments, it is unfeasible to fully monitor the entire system state of modern power grids. So state estimation techniques are introduced into power system in the 1970 [1, 2, 3]. State estimation is now a well-established and crucial part of Energy Management System.

Many algorithms have been proposed. Least-Squares (LS) estimator has been known and used since the early part of the nineteenth century. It provides optimal solution if the measurements have normal distribution errors. Since the low breakdown point of this method, extreme outliers with arbitrarily large errors can have an infinitely large influence on the resulting estimation. Various robust estimators have been proposed to combat these problems so that the outliers have little or no effect on the estimated states. Robust estimators are discussed in later

chapters.

### 2.1.1 Weight-Least-Square (WLS) algorithm

For a network with $N$ buses, the state vector contains $N$ bus voltage magnitudes and $N-1$ bus voltage angle. One of the buses is chosen as the reference bus and is assigned a voltage angle of zero degrees. Thus, the state vector, $x$, has $n = 2N-1$ dimensions. Computation of the state provides complete information on the current condition of the electric power network.

Assume that there are $m$ measurements. They can be written as an $m$ dimensional vector $z$, which is related to the state vector through the measurement equation.

In this case, the system equation is:

$$z = h(x) + r \tag{2.1}$$

where

- z: $m$ vector of measurements
- h(x): function relating measurements with state vectors
- r: $m$ vector measurements of residue
- x: n vector of true states
- m: number of measurements

- n: number of state variables

The WLS estimator is defined as:

$$\min_{x\in\mathbb{R}} J(x) = \frac{1}{2}(z - h(x))^T R^{-1}(z - h(x)) = \frac{1}{2} r^T R^{-1} r \tag{2.2}$$

where $R^{-1}$ is a diagonal matrix whose elements are the measurement weights. Depending on the application, these weights may represent entities such as meter accuracy, reliability, or an engineering judgement expressing the relative importance one wishes to allocate to each individual measurement. Mostly, $R^{-1}$ is based on the reciprocals of variance of measurement error

$$R^{-1} = diag\{\sigma_1^{-2}, \sigma_2^{-2}, \cdots, \sigma_m^{-2}\} \tag{2.3}$$

The performance index $J(x)$ can be differentiated to obtain the first-order optimal conditions:

$$\frac{\partial J(x)}{\partial x} = H^T R^{-1} H x - H^T R^{-1} z = 0 \tag{2.4}$$

where $H = \partial h(x)/\partial x$ is an $m \times n$ Jacobian matrix. Solving the equation:

$$x = (H^T R^{-1} H)^{-1} H^T R^{-1} z = G^{-1} H^T R^{-1} z \tag{2.5}$$

where $x$ is the state variable, $G = H^T R^{-1} H$ is *gain matrix*.

For non-linear state estimation such as the AC in power system, the accuracy of single time WLS is low, iterative WLS is needed. For a non-linear equation,

$$h(x + \Delta x) = h(x) + H\Delta x \tag{2.6}$$

where $H = \partial h/\partial x$, $\Delta x$ is sufficient small.

From the standard WLS,

$$(H^T R^{-1} H)\Delta x = H^T R^{-1}[z - h(x)] \tag{2.7}$$

$$\Delta x = (H^T R^{-1} H)^{-1} H^T R^{-1}[z - h(x)] \tag{2.8}$$

Considering $x^{k+1} = x^k + \Delta x$ and the gain matrix $G(x) = H^T W H$,

$$x^{k+1} = x^k + \Delta x = x^k + G(x)^{-1} H^T R^{-1}[z - h(x)] \tag{2.9}$$

## 2.2 Model Implementation

In power system, the model to perform the state estimation are DC and AC models. For a $n$ bus system, the system function $h(x)$ and Jacobian Matrix $H$ can be calculated as follows.

### 2.2.1 AC State Estimation

For the $n$ bus system, according to Kirchhoff Circuit Laws, the power injection at arbitrary bus $k$ is equal to all the power flow from bus $k$ to other buses as following equation:

$$P_k = \sum_{j=1}^{n} |V_k||V_j|(G_{kj}\cos(\delta_k - \delta_j) + B_{kj}\sin(\delta_k - \delta_j)) \tag{2.10}$$

$$Q_k = \sum_{j=1}^{n} |V_k||V_j|(G_{kj}\sin(\delta_k - \delta_j) - B_{kj}\cos(\delta_k - \delta_j)) \tag{2.11}$$

where $G_{jk}$ is the conductance between bus $j$ and bus $k$, $B_{jk}$ is the susceptance between bus $j$ and bus $k$.

The power flow between bus $k$ and bus $j$ is:

$$P_{kj} = -|V_k|^2 G_{kj} - |V_k||V_j|(-G_{kj}\cos(\delta_k - \delta_j) - B_{kj}\sin(\delta_k - \delta_j)) \tag{2.12}$$

$$Q_{kj} = -|V_k|^2(-B_{kj} + SA_{kj}) - |V_k||V_j|(-G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j)) \tag{2.13}$$

where $SA_{kj}$ is the shunt admittance between bus $k$ and bus $j$.

Then the Jacobian function $H$ can be calculated as follows:

$$H = \frac{\partial h}{\partial x} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ H_{41} & H_{42} \\ H_{51} & H_{52} \end{bmatrix} = \begin{bmatrix} \frac{\partial V}{\partial \delta} & \frac{\partial V}{\partial V} \\ \frac{\partial P_b}{\partial \delta} & \frac{\partial P_b}{\partial V} \\ \frac{\partial Q_b}{\partial \delta} & \frac{\partial Q_b}{\partial V} \\ \frac{\partial P_l}{\partial \delta} & \frac{\partial P_l}{\partial V} \\ \frac{\partial Q_l}{\partial \delta} & \frac{\partial Q_l}{\partial V} \end{bmatrix} \tag{2.14}$$

where $P_b$ and $Q_b$ are the bus injection power, $P_l$ and $Q_l$ are the line flow power. in terms of:

$$z = H\left\{\delta_2, \delta_3, \cdots, \delta_n, V_1, V_2, \cdots, V_n\right\}^T + r \tag{2.15}$$

For the calculation of Jacobian function, just as shown above, $H$ is divided into 10 separate sub-matrix.

Part 1: $H_{11}$, Derivative of voltage with respect to angle:

$$\frac{\partial V}{\partial \delta} = 0 \tag{2.16}$$

Part 2: $H_{12}$, Derivative of voltage with respect to voltage:

$$\frac{\partial V_k}{\partial V_j} = \begin{cases} 1, & \text{if k = j;} \\ 0, & \text{else.} \end{cases} \tag{2.17}$$

Part 3: $H_{21}$, Derivative of real power injection with respect to angle:

$$\frac{\partial P_k}{\partial \delta_j} = \begin{cases} \sum_{j=1}^{n-1}(|V_k||V_j|(-G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j))) - V_k^2 B_{kk}, & \text{if k=j;} \\ |V_k||V_j|(G_{kj}\sin(\delta_k - \delta_j) - B_{kj}\cos(\delta_k - \delta_j)), & \text{else.} \end{cases} \tag{2.18}$$

Part 4: $H_{22}$, Derivative of real power injection with respect to voltage:

$$\frac{\partial P_k}{\partial V_j} = \begin{cases} \sum_{j=1}^{n}(|V_k|(G_{kj}\cos(\delta_k - \delta_j) + B_{kj}\sin(\delta_k - \delta_j))) + V_k G_{kk}, & \text{if k=j;} \\ |V_k|(G_{kj}\cos(\delta_k - \delta_j) + B_{kj}\sin(\delta_k - \delta_j)), & \text{else.} \end{cases} \tag{2.19}$$

Part 5: $H_{31}$, Derivative of reactive power injection with respect to angle:

$$\frac{\partial Q_k}{\partial \delta_j} = \begin{cases} \sum_{j=1}^{n-1}(|V_k||V_j|(G_{kj}\cos(\delta_k - \delta_j) + B_{kj}\sin(\delta_k - \delta_j))) - V_k^2 G_{kk}, & \text{if k=j;} \\ |V_k||V_j|(-G_{kj}\cos(\delta_k - \delta_j) - B_{kj}\sin(\delta_k - \delta_j)), & \text{else.} \end{cases} \tag{2.20}$$

Part 6: $H_{32}$, Derivative of reactive power injection with respect to voltage:

$$\frac{\partial Q_k}{\partial V_j} = \begin{cases} \sum_{j=1}^{n}(|V_k|(G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j))) - V_k G_{kk}, & \text{if k=j;} \\ |V_k|(G_{kj}\sin(\delta_k - \delta_j) - B_{kj}\cos(\delta_k - \delta_j)), & \text{else.} \end{cases} \tag{2.21}$$

Part 7: $H_{41}$, Derivative of real power flow with respect to angle:

$$\frac{\partial P_{kj}}{\partial \delta_i} = \begin{cases} |V_k||V_j|(-G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j)), & \text{if k=i;} \\ -|V_k||V_j|(-G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j)), & \text{if j=i;} \\ 0, & \text{else.} \end{cases} \tag{2.22}$$

Part 8: $H_{42}$, Derivative of real power flow with respect to voltage:

$$\frac{\partial P_{kj}}{\partial \delta_i} = \begin{cases} -|V_j|(-G_{kj}\cos(\delta_k - \delta_j) - B_{kj}\sin(\delta_k - \delta_j)) - 2V_k G_{kj}, & \text{if k=i;} \\ -|V_k|(-G_{kj}\cos(\delta_k - \delta_j) - B_{kj}\sin(\delta_k - \delta_j)), & \text{if j=i;} \\ 0, & \text{else.} \end{cases} \tag{2.23}$$

Part 9: $H_{51}$, Derivative of reactive power flow with respect to angle:

$$\frac{\partial P_{kj}}{\partial \delta_i} = \begin{cases} -|V_k||V_j|(-G_{kj}\cos(\delta_k - \delta_j) - B_{kj}\sin(\delta_k - \delta_j)), & \text{if k=i;} \\ |V_k||V_j|(-G_{kj}\cos(\delta_k - \delta_j) - B_{kj}\sin(\delta_k - \delta_j)), & \text{if j=i;} \\ 0, & \text{else.} \end{cases} \tag{2.24}$$

Part 10: $H_{52}$, Derivative of reactive power flow with respect to voltage:

$$\frac{\partial P_{kj}}{\partial \delta_i} = \begin{cases} -|V_j|(-G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j)) - 2V_k(B_{kj} + SA_{kj}), & \text{if k=i;} \\ -|V_k|(-G_{kj}\sin(\delta_k - \delta_j) + B_{kj}\cos(\delta_k - \delta_j)), & \text{if j=i;} \\ 0, & \text{else.} \end{cases} \tag{2.25}$$

With the above equations, we can calculate out the Jacobian Matrix with assumed initial values and later updates. Put them into the functions in Section 2.1 and perform the state estimation algorithms in latter chapter, we will get the state variables and estimation results.

### 2.2.2 DC State Estimation

DC state estimation is the simplicity of AC state estimation by assuming that all bus magnitudes $V_i = 1$ and and line resistance $r_{ij} = 0$. So there are no reactive power flow and real power flow only relates to bus angle:

$$P_{ij} = \frac{\delta_i - \delta_j}{x_{ij}} \tag{2.26}$$

So DC state estimation forms a linear problem. We can derive the DC state estimation according to the AC equations by regarding the line resistance as 0,

bus magnitude as 1, $\sin(\delta_i - \delta_j)$ as $(\delta_j - \delta_j)$ and $\cos(\delta_i - \delta_j)$ as 1.

Derivative of Real Power Flows with Angles:

$$\frac{\partial P_{ij}}{\partial \delta_k} = \begin{cases} B_{kj}, & \text{if k = i;} \\ -B_{kj}, & \text{if k = j;} \\ 0, & \text{else.} \end{cases} \tag{2.27}$$

Derivative of Real Power Injections with Angles:

$$\frac{\partial P_i}{\partial \delta_k} = \begin{cases} \sum B_{ij}, & \text{if k = i;} \\ -B_{kj}, & \text{else.} \end{cases} \tag{2.28}$$

where $j$ means all the other buses connected with bus $i$.

Then we can form the Jacobian function $H$ of DC power flow:

$$H = \frac{\partial h}{\partial x} = \begin{Bmatrix} \frac{\partial P_b}{\partial \delta} \\ \frac{\partial P_l}{\partial \delta} \end{Bmatrix} \tag{2.29}$$

where $P_b$ is the bus injection power, $P_l$ is the line flow power.

For DC estimation, the $H$ in Section 2.1 will be determined, others parts are the same as in AC estimation.

## 2.3 Summary

In this chapter, the equations for both AC and DC estimation are given and the concept of basic state estimation estimator, Weighted-Least-Square, is introduced in details. They act as the basis for later improvement and comparison.

# CHAPTER 3

# OBSERVABILITY ANALYSIS

Observability, introduced by American-Hungarian scientist Rudolf E. Kalman for linear dynamic system, is "a measure for how well internal states of a system can be inferred by knowledge of its external outputs". A system is said to be observable "if, for any possible sequence of state and control vectors, the current state can be determined in finite time using only the outputs". Otherwise, the system is said to be unobservable.

For power system state estimation, the first requirements for the measurement data is observable. In the planning step, regarding the places to put measurement instruments and which measurements should be collected and used, system observability must be considered ahead.

## 3.1 Power System State Estimation Observability

For power system, state estimator are used to estimate the unknown system state vector given a set of measurement vector. When the measurements are sufficient to calculate the system state vector, it is regarded as an observable system [20, 21, 22]. This analysis should be carried out both off-line and on-line.

Before the installation of measurement facilities, the location should be planned in order to ensure both the overall observability and local redundancy. Since the installation of hardware is time consuming and costly for additional individuals, considering the possible bad data that will eliminated for further calculation, a suitable redundancy rate should be determined in case of emergency situations in the planning step.

For real-time or near real-time estimation, observability analysis should also be done on-line, prior to running the state estimator. If the system is observable, the state estimation can be conducted straight away. Otherwise, the system can be divided them into observable region and unobservable islands. For the states in unobservable islands, they can be solved by adding pseudo-measurements, which are usually from historical data or short-term forecast. Another solution is merging the unobservable buses with their observable neighborhoods, but it lose the data of these buses though the overall observability is guaranteed. The last but most efficient solution is just add more meters to unobservable buses, though it's very costly.

Three parts are divided in this chapter. First part is topology planning, which involves the locations to put meters in the planning step; The second part deals with observability checking algorithms before conducting estimation; The last part present the algorithm regarding where to add additional pseudo-measurements or to put additional meters.

## 3.2 Topology planning

Before collecting the values from measurement instruments, where to place these facilities is an important issue. This part deals with topology planning of measurement instruments placement.

A physical island is defined as "a connected part of a network, with branches representing transmission lines, transformers and series capacitors", similarly, an observable island is an island where all branch flows can be calculated from the measurements. In the planning step, an observable island is build first and some other values are added as redundancy according to some criteria.

Topological analysis is different from conventional numerical methods, which determine the observability of a system by precise calculation. Topological algorithm analyzes the observability only based on the connection between each bus, making it suitable for the planning step, when no measurement are available. The steps to execute the algorithm are given for a $n$ bus system:

1. Find those buses with only one connected branch with others, put meters on these branches;

2. Eliminate the buses connected with branches that already put meters, all branches connected with these buses are also removed;

3. Perform step 1 and 2 iteratively until no bus need to be eliminated. Minimum number of meters for an observable system forms;

4. Considering required global redundancy, local redundancy and history bad data type, put additional meters for power flow and injection.

For median redundancy cases, since the meters are enough to guarantee global redundancy, meters can be put at each bus injection first for better coverage, then performs step 4 for local redundancy consideration.

## 3.3 Observability Algorithms

The static state estimation for on-line applications in power system control centers processes a set of redundant measurements to obtain the best estimation of the system. This algorithm tests the observability of a network and identifies unobservable islands. It is based on the results derived in the theory of network observability developed in [20].

Formally the observability algorithm should be applied first to the $P-\theta$ model and then to the $Q-V$ model separately. But since the real and reactive power measurements usually come in pairs practically, $Q-V$ analysis is not that necessary, it's acceptable to just regard them as DC model and deal with $P-\theta$ model only and $Q$, $V$ share the same locations with $P$ and $\theta$.

In state estimation, if a branch neither has a power flow nor an power injection measurement at one of its terminal buses, it does not appear in the matrix $H$, which means that the branch does not play any role both in observability analysis and state estimation. Such a branch will be discarded from further consideration.

Since the observability analysis is usually performed before state estimation, for real time consideration, conventional numerical method is better for fast calculation with various data type automated. To perform computation analysis, triangular factorization is needed.

For a linearized measurement model $z = Hx + r$ as stated in Chapter 2, the following three statements are equivalent regarding observability:

1. The system is observable.

2. $\bar{H}$ has full rank no matter which column is eliminated from the original $H$.

3. Gain matrix $G = H'H$ can be transformed to upper triangular factorization form.

For $H = (\bar{H}, h)$ where $\bar{H}$ is observable, the gain matrix is:

$$G = H'H = \begin{bmatrix} \bar{H}'\bar{H} & \bar{H}'h \\ h'\bar{H} & h'h \end{bmatrix} \tag{3.1}$$

With all the consideration and background stated above, the algorithm can be conducted in the following steps [21]:

1. Initialize all available measurement data.

2. Update the observable system by removing all branches with no power flow and injection measurements at their terminal buses.

3. Generate the Gain matrix $G$.

4. Perform triangular factorization of $G$. Adding pseudo-measurements when a zero pivot appears. If only one zero pivot occurs, stop; else continue the algorithm.

5. Solve the DC estimation $G\hat{\theta} = H'Wz$ for $\theta$. Regarding all the measurements equal to zero except for the $\theta$-pseudo-measurements.

6. Evaluate power flows $\hat{P_k m} = (\hat{\theta_k} - \hat{\theta_m})/x_{km}$, where $\hat{\theta_k}$ and $\hat{\theta_m}$ are the $k-th$ and $m-th$ parts of $\hat{\theta}$. All branches $k-m$ are within the observable system obtained in Step 2. If there are no power flow $P_{km} \neq 0$, stop; else, go on.

7. Update the system by removing all unobservable branches $k-m$ with $P_{km} \neq 0$.

8. Update the measurement data by removing power injection measurements from buses next to at least one of the branches removed in Step 7, which are irrelevant measurements.

9. Go to Step 2.

## 3.4 Measurement Placement Algorithms

During the on-line observability analysis, if the unobservable island are found during the steps performed before, additional pseudo-measurements should be added to the system, making the observable islands unable to ruin the state estimation. The selection is performed iteratively by adding one pseudo-measurement each time.

To make the system observable, new meters are usually placed for power injection monitoring at unobservable buses. In order to guarantee that the new injection measurement is not redundant, the place should be carefully selected at which the estimated power injection of the observable islands is non-zero. The algorithm that are designed for measurement placement are presented in details as follows [21]:

1. Generate gain matrix $G$.

2. Perform triangular factorization of $G$. Adding pseudo-measurements when a zero pivot appears. If only one zero pivot occurs (usually at the end), stop; else continue the algorithm.

3. Solve the DC estimation $G\hat{\theta} = H'Wz$ for $\theta$. Regarding all the measurements equal to zero except for the $\theta$-pseudo-measurements.

4. Determine the buses with no power flow and injection measurements. These buses are candidates to have pseudo-measurements. If there is no candidate buses, stop; else, go on.

5. Add injection pseudo-measurement to one of the candidate buses and update the $LDL'$ factors of $G$.

6. Solve the DC state estimation for $\hat{\theta}$ in step 3. Calculate the residuals $\theta_k^m - \hat{\theta_k}$ for all pseudo-measurements with non-zero residual. Update the $LDL'$ factors of $G$.

7. Return to Step 3.

After performing the algorithm, the unobservable system can be transfered to be observable by adding sufficient additional pseudo-measurements, which will be helpful for later estimation.

## 3.5 Summary

This chapter presents the algorithms for network observability analysis. From the off-line planning stage to on-line observable/unobservable island detection and placing additional pseudo-measurements to restore observability are all discussed in details.

# CHAPTER 4

# BAD DATA DETECTION AND ROBUST ESTIMATORS

## 4.1 Bad Data Detection

In any state estimator, detection of bad-data and minimize the measurement error are essential functions. Considering a practical raw data set for estimation, many scenarios may lead to both small and large errors as stated in section 1.2.1. Generally there are three different interpretations: small departures for all data points, large departures for a small number of data points, or both situation occurs simultaneously, which is more common in practice. Fig. 4.1 shows the distribution. Situations with large errors lead to the notion of outlier points, which are generally the most stressful part for statistical procedures. In fact, a single outlier can completely spoil the Least Squares estimation, causing it to break down Fig. 4.2.

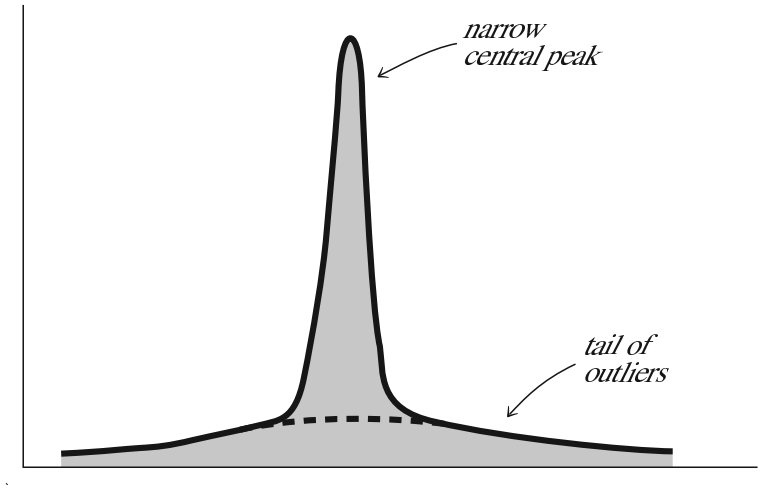


Figure 4.1: Small departures for most data points and large departures for a small number of data points.

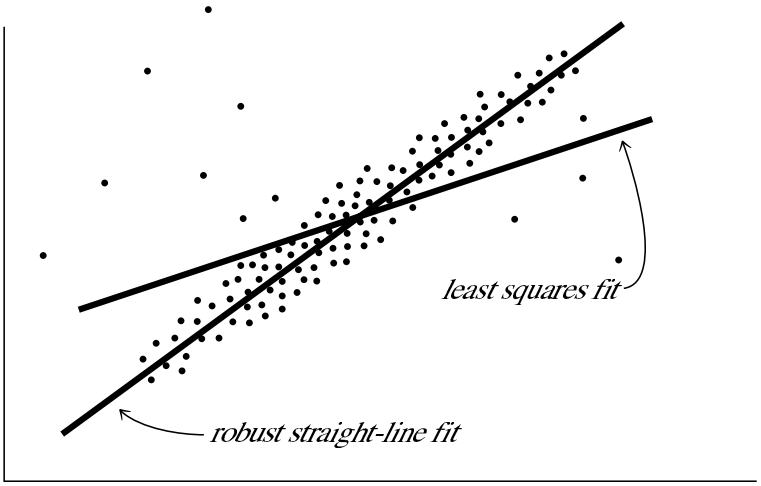


Figure 4.2: Outliers ruin the result.

Some obvious bad data, such as error with several order difference, negative voltage magnitude, and large disparity between in/out power flow, are easy to detect by simple plausible check. But most of time those outliers are far away from the true value, so estimators must have the ability to correct those bad data themselves. Regarding the combination of bad data, section 1.2.1 has list some of the most popular ones.

The general idea of detecting bad data is that removal of these corresponding measurements should not interfere the observability of the system. One of the most widely used methods for detecting bad data is the $Chi - Squares\chi_2$ test.

### 4.1.1 Chi-Square $\chi_2$ distribution

For a data set $x = \{x_1, x_2, \ldots, x_m\}$, where each $x_i$ is from standard normal distribution $x_i \sim N(0, 1)$, a new variable $Y$ is defined by:

$$Y = \sum_{i=1}^{m} x_i^2 \tag{4.1}$$

Then $Y$ is said to have a $\chi^2$ distribution with $m$ degrees of freedom, i.e. $Y \sim \chi_m^2$. Where $m$ is the number of independent variables in the summation of square. It can be proved that the $\chi_m^2$ distribution has mean $m$ and variance $2m$.

When the $m$ random variables are constrained by $n$ independent equations, $Y$ will have a $\chi_2$ distribution with at most $m - n$ degrees of freedom. This is the $m$ measurements and $n$ state variables in power system state estimation.

According to Central Limit Theorem, the sum of a large number of random variables following any distribution with bounded variance approximates a normal distribution. Figure. 4.3 illustrates $\chi_2$ Probability Density Function with 8 degrees of freedom. Since $\chi^2_{8,0.95} = 15.5$, i.e. $Prob(\chi_2 > 15.5) = 0.05$, the region on the right of the vertical dash line represents the probability of $x$ is larger than a certain threshold $x_t = 15.5$ with probability error 0.05. If the measured value of $x$ exceeds this threshold, then with 0.95 probability, the measured $x$ will not

have a $\chi_2$ distribution, i.e. presence of bad data will be suspected.

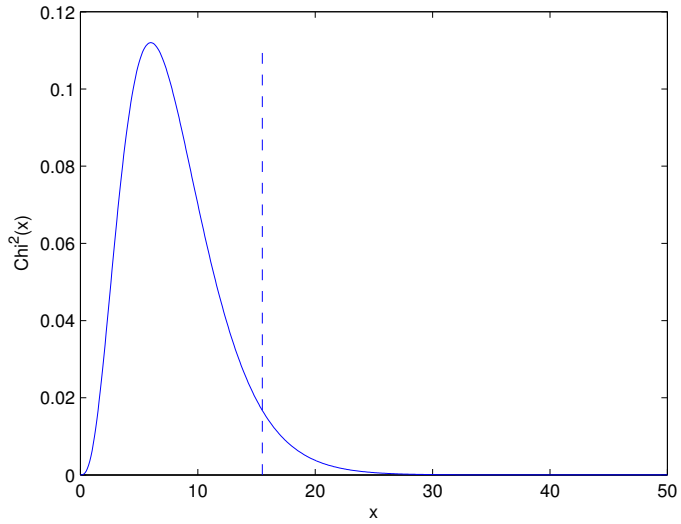


Figure 4.3: $\chi_2$ Probability Density Function.

### 4.1.2 $\chi_2$ test for bad-data detection in WLS estimator

Given the WLS discussed in Chapter 2.1.1, the objective function $J(x)$ can be used as $f(x)$ for bad-data detection. The detail are given as follows:

- Perform WLS estimator and calculate the objective function:

$$J(\hat{x}) = \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x}_i))^2}{\sigma_i^2} \tag{4.2}$$

  where $\hat{x}$ is the estimated state vector.

- Look up the value from the $\chi^2$ distribution table corresponding to a detection confidence with probability $p$ and $(m-n)$ degrees of freedom, let

the value be $\chi^2_{(m-n),p}$.

- Test if $J(\hat{x}) > \chi^2_{(m-n),p}$. If yes, bad data will be suspected and trimmed (LTS); else, the measurements will be assumed to be free of bad data.

## 4.2 Robust Estimators

The term "robust" was coined in statistics by G.E.P. Box in 1953. When referring to a statistical estimator, it means "insensitive to small departures from the idealized assumptions for which the estimator is optimized." [23]

There are two parameters to evaluate the robustness of an estimator:

- **Breakdown point**: The breakdown point of an estimator for finite sample is the fraction of data that can be given arbitrarily large or small extreme values without yielding an bad result.

- **Leverage point**: The leverage point is a small amount of change causes a small or large change in system behavior.

Statisticians have developed various sorts of robust statistical estimators, most of them can be grouped in one of three categories:

- **M-estimator**: follow from maximum-likelihood arguments, are usually the most relevant class for model-fitting, i.e., estimation of parameters.

- **L-estimator**: "linear combinations of order statistics." These are most applicable to estimations of central value and tendency, though they can

occasionally be applied to some problems in estimation of parameters. Two "typical" L-estimates are the median and Tukey's trimean.

- **R-estimator**: each residual is weighted by a score based on selected criteria.

Besides these three main categories, there are some other kinds of robust techniques from the fields of optimal control and filtering rather than from the field of mathematical statistics. Here, the basic Weighted-Least-Square (WLS) algorithm and the three main categories are reviewed.

### 4.2.1 M-estimator

M-estimators, introduced by Huber [23], are a broad class of estimators. M-estimation is maximum likelihood estimation obtained by minimizing an objective function, which is expressed as a function of the measurement residuals $\rho(r)$, subject to the constraints given by the measurement equations:

$$Minimize \quad J(r) = \sum_{i=1}^{m} \rho(r_i) \tag{4.3}$$

$$Subject \quad to \quad z = h(x) + r \tag{4.4}$$

where

- $\rho(r_i)$ is a function of the measurement residual $r_i$
- $z$ is the measurement vector

- $x$ is the state vector
- $h(x)$ is the measurement function relating to state vector

The function $\rho(r_i)$ plays an important role in generating desirable results, it has to be chosen with the following properties:

- $\rho(r) > 0$ for $r = 0$
- $\rho(r) \geq 0$ for any $r$
- $\rho(r)$ is monotonically increasing in both $+r$ and $-r$ directions
- $\rho(r) = \rho(-r)$, i.e. symmetric with respect to $r = 0$

According to [24] and [25], the expression of $\rho(r)$ can be:

- Quadratic-Constant (QC) [24]

$$\rho(r) = \begin{cases} r_i^2/\sigma_i^2, & |r_i/\sigma_i| \leq a \\ a^2/\sigma_i^2, & otherwise \end{cases} \tag{4.5}$$

- Quadratic-Linear (QL) [24]

$$\rho(r) = \begin{cases} r_i^2/\sigma_i^2, & |r_i/\sigma_i| \leq a \\ 2a\sigma_i|r_i| - a^2\sigma_i^2, & otherwise \end{cases} \tag{4.6}$$

- Square Root (SR) [24]

$$\rho(r) = \begin{cases} r_i^2/\sigma_i^2, & |r_i/\sigma_i| \leq a \\ 4a^{3/2}\sqrt{r_i/\sigma_i} - 3a^2, & otherwise \end{cases} \tag{4.7}$$

- Schweppe-Huber Generalized-M (SHGM) [25]

$$\rho(r) = \begin{cases} \frac{1}{2} r_i^2/\sigma_i^2, & |r_i/\sigma_i| \leq a \\ a\omega_i |r_i/\sigma_i| - \frac{1}{2} a^2 \omega_i^2, & otherwise \end{cases} \tag{4.8}$$

where $a$ is a turning parameter specified according to different situations, $\omega_i$ is a iteratively modified weighting factor.

With all the parameters settled, there are many methods to solve this problem, such as Newton's method. These methods requires computation of the first and second derivatives of $\rho$.

### 4.2.2 L-estimator

L-estimator [26], a linear combination of order statistics of the measurements. The main benefit of L-estimators is that they are often extremely simple and robust. With sorted data, they are very easy to calculate and interpret, and are often resistant to outliers. Though L-estimators are inefficient for whole estimation, in many circumstances they are reasonably efficient and adequate for initial estimation.

L-estimators are based on a definition of quantiles as follows:

$$\psi(r_j) = \begin{cases} p - 1, & r_j < 0, \\ p, & otherwise. \end{cases} \tag{4.9}$$

where $r_j = x_j - z$, the signed residual from the $j^{th}$ estimated data, $x_j$, to the measurement value $z$.

The objective function is:

$$\min J = \sum_{j=1}^{N} \rho(r_j) \tag{4.10}$$

where

$$\rho(r_j) = r_j\psi(r_j) = \begin{cases} r_j(p-1), & r_j < 0, \\ r_j p, & otherwise. \end{cases} \tag{4.11}$$

It's easy to show that when $p = \frac{1}{2}$, the half-quantile, $\psi_p(r_j) = \frac{1}{2}sgn(r_j)$ corresponds to the sample median.

### 4.2.3 R-estimator

R-estimator [27], involve ranking residuals, each residual is weighted by a score based on its rank with following objective:

$$\min J = \sum_{j=1}^{m} a_m(R_j) r_j \tag{4.12}$$

where $R_j$ is the rank of the $j^{th}$ residuals in $\{r_1, \cdots, r_m\}$ and $a_m$ is a nondecreasing score function following $\sum_k a_m(k) = 0$.

As proposed in the same paper, Jaeckel's estimator focuses on finding the coefficient that minimizes $J$:

$$\min J = \sum_{j=1}^{m} (R_j - \frac{m+1}{2}) r_j \tag{4.13}$$

Like L-estimators, the major inconvenience in R-estimators is that they are low efficient and not easy for optimization, and the definition of the score function

implicitly needs prior information about the noise contamination rate.

As for the three main categorizes stated above, M-estimators are preferred in modern robust statistics though they are more complex for computation. But some L-estimator and R-estimator features are used in combination with M-estimator for better robustness.

## 4.3 Existing Robust Estimator

In this section, major robust estimation algorithms will be presented. Besides the original Weighted-Least-Square (WLS), Least-Absolute-Value (LAV) [28], which belong to M-estimator, Least-Median-Square (LMS) [29, 30], belongs to L-estimator, Least-Trimmed-Square (LTS) [29] and Least-Measurements-Rejected (LMR) [31], belong to R-estimator, will also be implemented.

For a general expression, the objective function is setup upon the classical linear regression model discussed above:

$$minimize\ J(x) = \sum \rho(r_i) \tag{4.14}$$

Regarding the function $\rho(r)$, Figure 4.4 give the $\rho(r)$ of the basic least square algorithm.

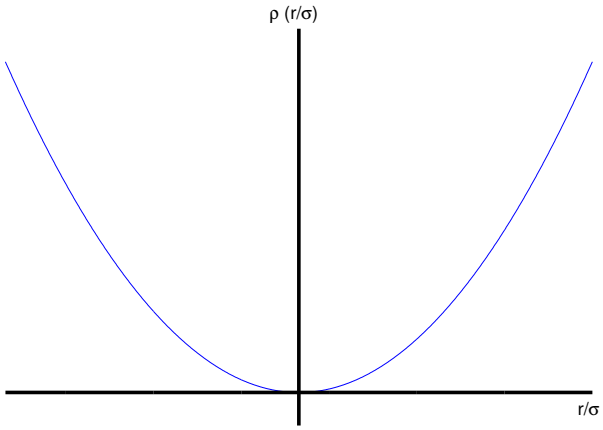


Figure 4.4: Least Square objective function.

### 4.3.1 Least-Absolute-Value (LAV)

Least-Absolute-Value (LAV) is a mathematical optimization technique similar to the popular least squares algorithm that attempts to find a function which closely approximates a set of data, the method minimizes the sum of absolute errors. Regarding the function $\rho(r)$ talked above, Figure 4.5 give the $\rho(r)$ of least absolute value algorithm.

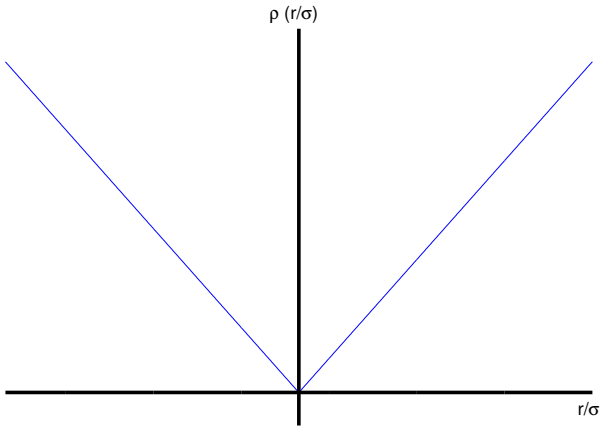


Figure 4.5: Least Absolute Value objective function.

For the same system described in Equ. 2.1, LAV estimator minimizes the objective function:

$$\min J = \sum_{j=1}^{m} |r_j| \tag{4.15}$$

where $r_j = z - h_j(x)$ is the $j^{th}$ residue.

Assuming an initial solution $x_0$ for the state and using the first-order approximation of $h_j(x)$ around $x_0$, the above equation can be written around the operating point $x_0$ as the following:

$$\Delta z = H(x^0 \Delta x + r) \tag{4.16}$$

where $\Delta z = z - h(x^0)$, $\Delta x = x - x^0$ and $H(x^0) = \frac{\partial h(x^0)}{\partial x}$.

The problem can be transformed into a successive set of linear programming (LP) problems, each minimizes the objective function:

$$J(x^k) = \sum_{j=1}^{m} (u_i^k + v_i^k) \tag{4.17}$$

where $u_i^k$ and $v_i^k$ are both positive, $u_i^k - v_i^k = z - h(x^k) - H(x^k)\Delta x = \Delta z^k - H(x^k)\Delta x$ is the measurement residual vector at the $k^{th}$ iteration.

For the $k^{th}$ iteration, the optimization problem to be solved can be formulated as ($k$ are dropped for simplicity):

$$\min \sum_{j=1}^{m} (u_i + v_i) \tag{4.18}$$

Subject to

$$H\Delta x_u - H\Delta x_v + u - v = \Delta z \tag{4.19}$$

$$\Delta x_u, \Delta x_v, u, v \geq 0 \tag{4.20}$$

where $\Delta x = \Delta x_u - \Delta x_v$.

Then it can be transformed into a basic LP problem:

$$\min c^T \cdot Y \tag{4.21}$$

Subject to

$$A \cdot Y = b \tag{4.22}$$

$$Y \geq 0 \tag{4.23}$$

where:

$c^T = [0_n, 0_n, 1_m, 1_m]$; $0_n = [0, \cdots, 0]$ , a zero vector of order $n$;

$1_m = [1, \cdots, 1]$, an all 1 vector of order $m$;

$b = \Delta z$; $Y_T = [\Delta x_u^T, \Delta x_v^T, u^T, v^T]$;

$A = [H, -H, I_m, -I_m]$; $I_m$: identity matrix of order $m$.

This LP problem can now be solved using the simplex method. The overall state estimation solution will be obtained by successively solving these LP problems until $|\Delta x|$ is less than a chosen threshold.

### 4.3.2 Least-Median-Square (LMS)

Unlike WLS and LAV, which have a breakdown point of zero, Least-Median-Square is an estimator with high breakdown point. They may yield the correct solution even if up to half the redundant measurements are bad leverage points.

The LMS estimator minimizes the median of squared residues instead of the regular summation, as its name hints. The objective function for minimization is

$$\min J = r_{k(\nu)} \tag{4.24}$$

where $\nu = [\frac{m}{2}]$, the largest integer that is smaller or equal to $\frac{m}{2}$, and $r^2_{k(\nu)}$ means the $\nu^{th}$ ordered squared residue at the $k^{th}$ iteration, $r^2_{k(i)}$ are ordered increasingly: $r_{k(1)^2} \leq \cdots \leq r_{k(m)^2}$

The Objective function of LMS estimator is that it seeks a regression that minimizes the value of a tolerance $t$ whereby the majority of the measurements fall within tolerance. The implementation can be formed using the following equation:

$$\min_{x,k,t} t \tag{4.25}$$

subject to:

$$b - t - Mk \leq Ax \quad \leq \quad b + t + Mk \tag{4.26}$$

$$k_1 + k_2 + \cdots + k_m \quad \leq \quad K \tag{4.27}$$

where: $k$ is an unknown binary integer vector (either 0 or 1); $t$ is an unknown scalar tolerance; $M$ is a specified arbitrary large positive scalar; $K$ is $[\frac{m}{2}]$.

In this case, if $M$ is sufficient large, we can eliminate those large outliers and keep other measurements within the smallest tolerance. We can transformed it into a Mixed-Integer-Linear-Programming (MILP) as follows:

$$\min c^T \cdot Y \tag{4.28}$$

Subject to

$$A \cdot Y \quad \leq \quad B \tag{4.29}$$

where: $c^T = [0_n, 0_m, 1]$

$$A = \begin{bmatrix} H & -M & -1 \\ -H & -M & -1 \\ 0_n & 1_m & 0 \end{bmatrix} \tag{4.30}$$

$B^T = [b, -b, t]$, $Y^T = [x, k, t]$

$M = diag(m)$, $t > 0$ and $k = 0$ or 1.

### 4.3.3 Least-Measurement-Rejected (LMR)

Least-Measurement-Rejected algorithm is an variant of LMS, compared with LMS, LMR minimize the number of rejected value with fixed tolerance, given by:

$$\min_{x,k} \sum k_i \tag{4.31}$$

subject to:

$$b - t - Mk \leq Ax \quad \leq \quad b + t + Mk \tag{4.32}$$

$$k_1 + k_2 + \cdots + k_m \quad \leq \quad K \tag{4.33}$$

where $t$ is given fixed tolerance and other parameters are the same as in LMS algorithm.

Transformed to MILP form:

$$\min c^T \cdot Y \tag{4.34}$$

Subject to

$$A \cdot Y \;\; \leq \;\; B \tag{4.35}$$

where: $c^T = [0_n, 1_m]$

$$A = \begin{bmatrix} H & -M \\ -H & -M \end{bmatrix} \tag{4.36}$$

$B^T = [b + t, -b + t]$, $Y^T = [\Delta x, k]$

$M = diag(m)$ and $k = 0$ or 1.

### 4.3.4 Least-Trimmed-Square (LTS)

Least-Trimmed-Square is an approach that simply ignores or trims the largest residuals by applicable analysis and computation. Figure 4.6 give the $\rho(r)$ of least absolute value algorithm.

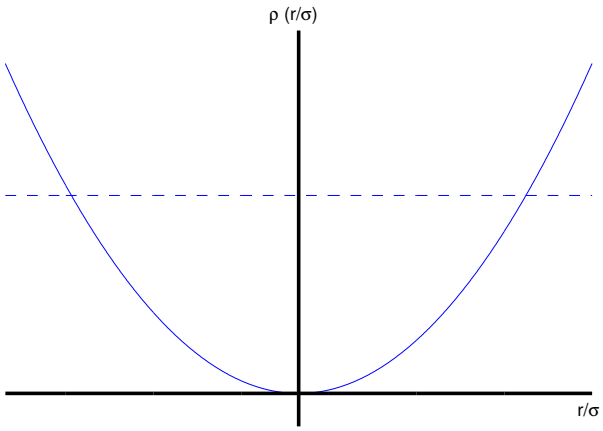


Figure 4.6: Least Trimmed Square objective function.

Assume there are $n$ points, and $r_1, \cdots, r_n$ represent the residuals corresponding to some choices for the slope and intercept. Then, order the squared residuals and label them as $r_1^2 \leq \cdots \leq r_n^2$. So $r_1^2$ is the smallest squared residual, and $r_n^2$ is the largest. Finally, let $h = kn$, $k$ is the number within $[0, 1]$, we judge it based on the sum of the $h$ smallest squared residuals instead. If $n = 100$ and $k = 0.75$, then $h = 75$, we would use the sum of the 75 smallest squared residuals, $r_1^2 + \cdots + r_{75}^2$ to judge how well $Y$ provides a good fit to data.

We can form the problem as follows:

$$\min_{x,k,r} \sum r^T r \tag{4.37}$$

subject to:

$$b - Mk \leq Ax - r \quad \leq \quad b + Mk \tag{4.38}$$

$$k_1 + k_2 + \cdots + k_m \quad \leq \quad K \tag{4.39}$$

where $K$ is specified number of measurements that should be rejected, $r$ is residues and others are the same as in LMS.

## 4.4 Proposed LTAV Robust Estimators

Among robust estimators are proposed above, LAV and LTS are two excellent representatives. LAV, a variant of WLS by changing the objective function $\rho(r)$ in section 4.2.1, is proved to be good at estimating value. LTS, by adding a trimming strategy to WLS estimator, performs well in correcting large outliers.

Inspired by these two algorithms, Least-Trimmed-Absolute-Value (LTAV) estimator uses the fitting function of LAV and trims the given largest residues for each iteration in the estimation process. Regarding the function $\rho(r)$ talked above, Figure 4.7 give the $\rho(r)$ of least trimmed absolute value algorithm.

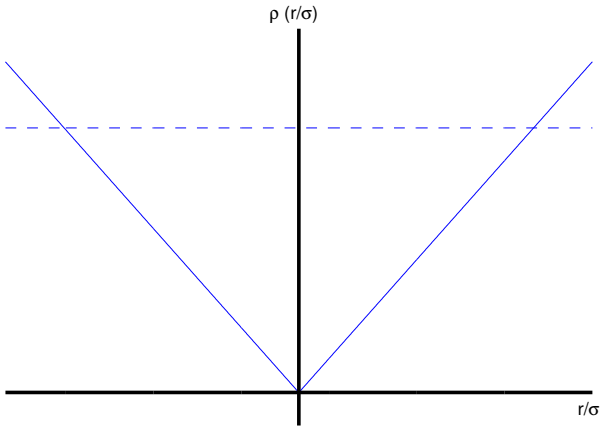


Figure 4.7: Least Trimmed Absolute Value objective function.

The minimization problem can be written as:

$$minimize \sum_{i=1}^{m-K} |\hat{r_i}| \tag{4.40}$$

where $K$ is the number of trimmed data and $|\hat{r_i}|$ is sorted $|r_i|$ from smallest to largest.

### 4.4.1 General Steps

For LTAV estimator, the general steps to perform it can be:

- **Sampling of measurements:** Randomly select specified $k$ sample sets of $75\% \times m$ measurements with observability consideration. For each selected sample set, estimate the sate by LAV.

- **Summation of Least Trimmed Absolute Value Error:** Find the state

candidates having the least summation of trimmed absolute value among all candidates in step 1.

- **Bad Data Removal:** Based on the residual generated by state candidate, eliminate the data according to a certain threshold.

- **Re-estimation:** Re-estimate the states using the remaining data.

With the standard steps described above, some constraints appears. As stated in step 1, system observability is the key point to be considered after sampling, usually a redundancy of 4 is enough for a random selection observable rate acceptable. Besides, LAV depends on linear programming, which is more likely to be trapped compared with matrix computation used in WLS, the algorithm are more like to be trapped for lower redundancy cases.

To solve the above problems, increasing the sample set rate and decrease the number of sample sets are reasonable for most cases that do not have very high accuracy requirements. Moreover, lower sample set rate and large number does not guarantee good results. If only 1 sample set of $100\% \times m$ are used, it means that the bad data removal relays on LAV estimation, which is proved to be very effective for most case.

### 4.4.2 Mixed Integer Linear Programming Implementation

To make it simple and relay less on the sampling process, LTAV are formulated as a mixed integer linear programming (MILP) problem, which in turn can be

solved by applying one of the well-developed MILP solution methods. The mixed integer linear programming model is built as:

$$\min_{x,k,u,v} \sum (u+v) \tag{4.41}$$

subject to:

$$b - Mk \leq Ax - u + v \quad \leq \quad b + Mk \tag{4.42}$$

$$k_1 + k_2 + \cdots + k_m \quad \leq \quad K \tag{4.43}$$

It can be transformed into the basic MILP problem format:

$$\min c^T \cdot Y \tag{4.44}$$

Subject to

$$A \cdot Y \quad \leq \quad B \tag{4.45}$$

where: $c^T = [0_m, 0_n, 1_n, 1_n]$

$$A = \begin{bmatrix} H & -M & -I & I \\ -H & -M & I & -I \\ 0_n & 1_m & 0_m & 0_m \end{bmatrix} \tag{4.46}$$

$B^T = [b, -b, K]$, $Y^T = [x, k, u, v]$

$M = diag(m)$, $I = diag(1)$, $t > 0$ and $k = 0$ or 1.

The $K$ here is different from LTS estimator, whose $K$ is a fixed number, copied from the idea of LMS and LMR, $k$ in LTAV is changeable with the maximum integer smaller than $m/2$, where $m$ is the number of measurements.

## 4.5 Summary

In this chapter, an introduction of robust estimator classification are presented at the beginning, then several proposed algorithms are illustrated, then, a new state estimator, Least-Trimmed-Absolute-Value (LTAV), is proposed. These algorithms are used for performance evaluation in the later chapter.

# CHAPTER 5

# RESULTS AND DISCUSSION

In this chapter, the proposed Least-Trimmed-Absolute-Value is presented and compared with other estimators. Three power systems: 6-bus [Fig. 5.1], IEEE 14-bus [Fig. 5.2] and IEEE 30-bus [Fig. 5.3]. DC and AC systems are used to evaluate the performance of the proposed estimator. Various kinds of bad data are simulated for testing. Lpsolve [32] is used to solve the mixed integer linear programming. All the codes are written in MATLAB platform.

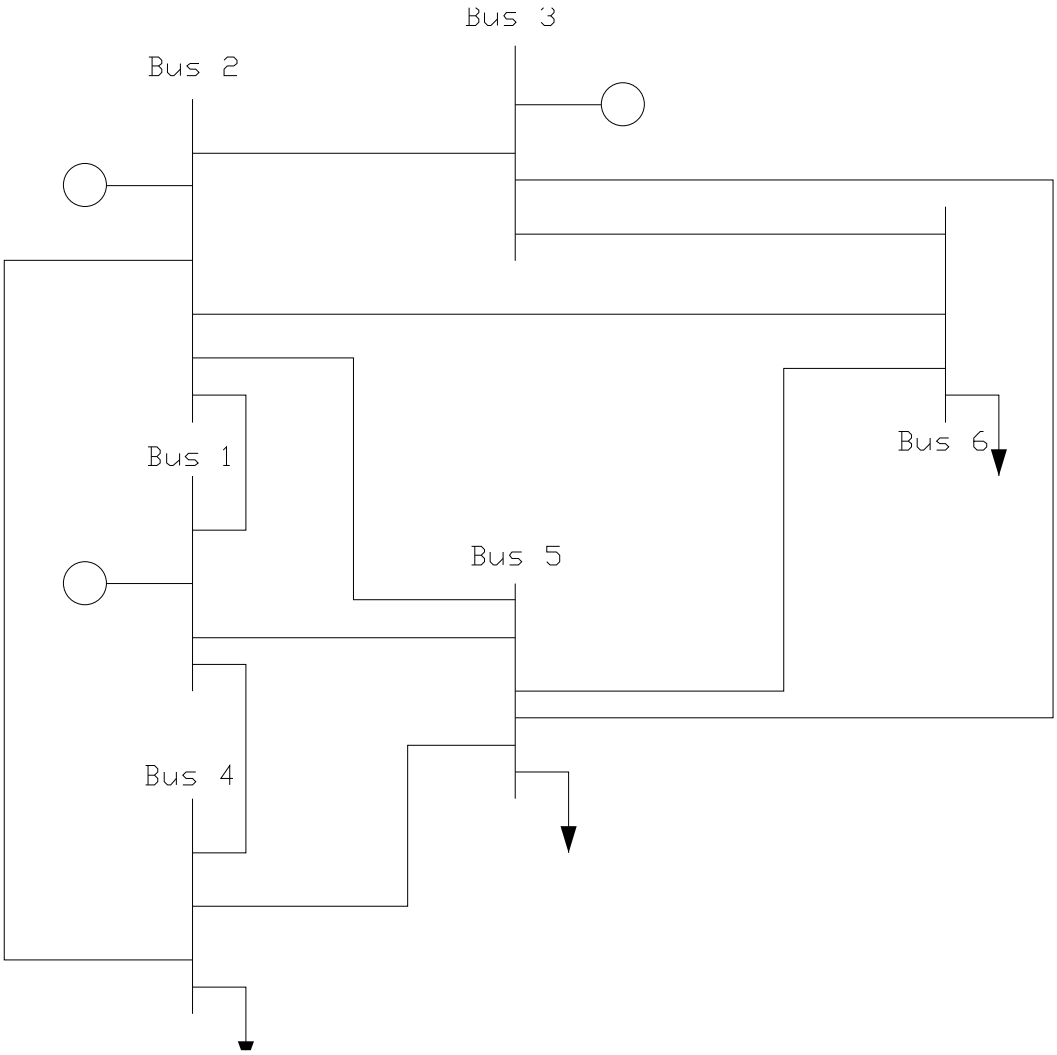


Figure 5.1: 6-bus system.

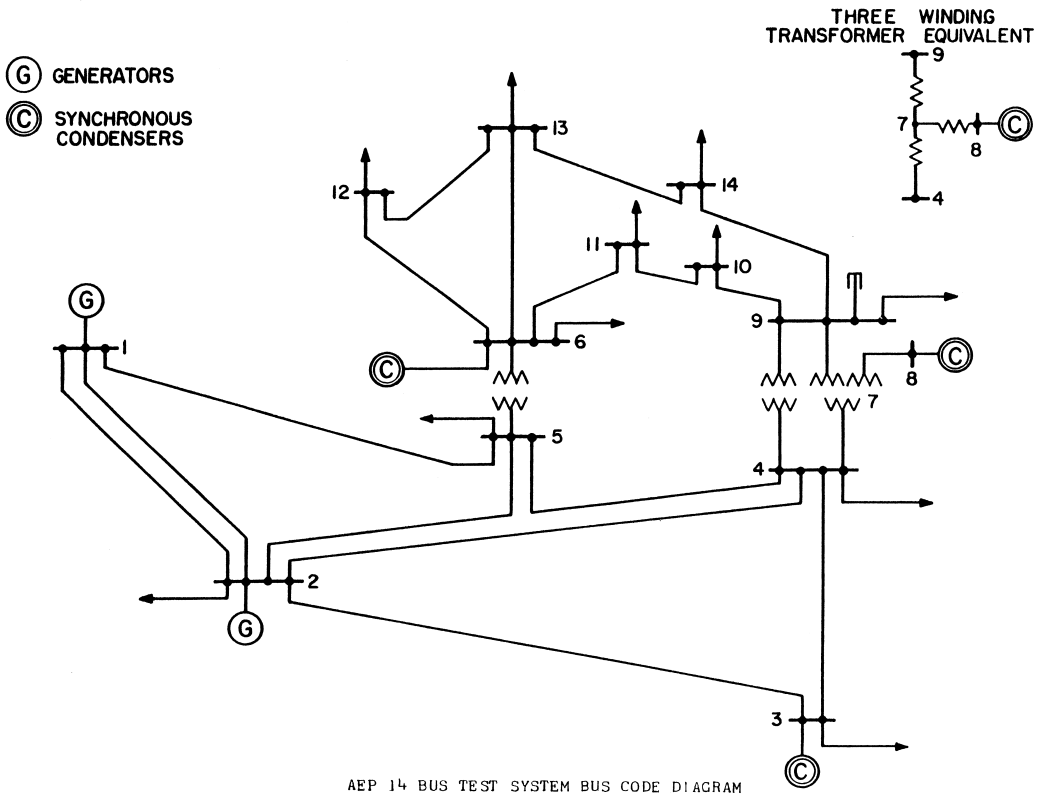


Figure 5.2: IEEE 14-bus system.

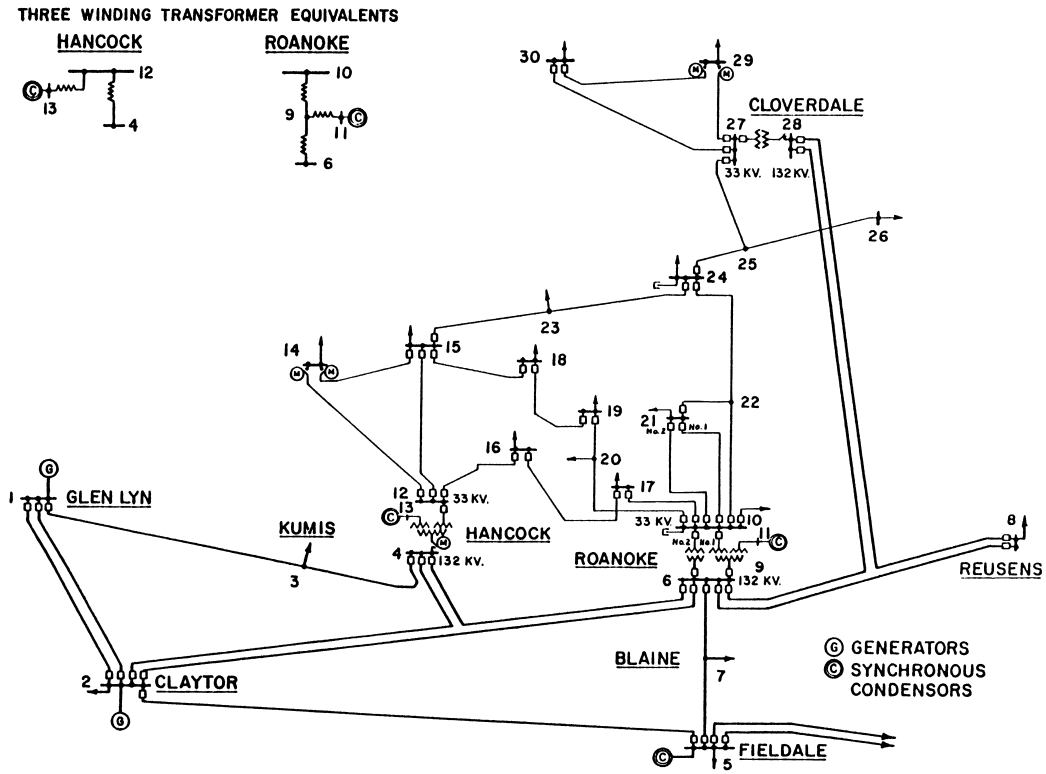


Figure 5.3: 30-bus system.

## 5.1 6-Bus System

For 6-bus system, both DC and AC state estimation are carried out to test the performance of the proposed LTAV compared with other existing robust estimators. Measurements data with full and median redundancy are used for estimation.

### 5.1.1 6-bus DC State Estimation

As stated in section 2.2.2, all bus magnitude are assumed to be 1 and line resistance $r_{ij} = 0$. Besides, the reactance of each branch is chosen to be 1 for

simplicity. All the system data are presented in the Appendix 6.2.

Four major cases are conducted to demonstrate the efficiency of estimators, including base case with only small random error, single bad data case, multiple non-interacting case and multiple interacting case. For cases with bad data, conforming condition is considered with reverse and large error. For each case, full and median redundancy estimation are presented for comparison with the same bad data.

The parameters used in the estimation are:

1. Tolerance of iterative loop for all estimators: 0.0001;

2. Fixed trimmed number for LTS: $K = 2$;

3. $M$ for LMS: 3; $M$ for LMR: 500; $M$ for LTAV: 3;

For many case, the results have share the same trends though with different number, in those cases, comments will be omitted, only cases with new type of results are carefully analyzed.

#### 5.1.1.1 Case 0: basic case with only random noise

In this case, only random small noise are added to the actual value to simulate the raw measurements. Full and median redundancy are used for state estimation.

**Full redundancy**

For full redundancy cases, all the power injection and power flow are used for estimation. So meters are put on every generators, loads and branches. In this scenario, there is no need to consider the system observability ahead.

Table 5.1 shows the actual values of real power injection and flow in the system from load flow. Random noise are generated using normal distribution of $\sigma = 0.02$ and added to the actual value to simulate the measurements, which are also in Table 5.1. As for full redundancy cases, all the data are used for estimation.

Table 5.1: Raw data for 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1 | 1.0108 | P1-2 | 0.1875 | 0.1788 | P2-6 | 0.2875 | 0.302 |
| P2 | 0.5 | 0.5367 | P1-4 | 0.425 | 0.4319 | P3-5 | 0.2375 | 0.2362 |
| P3 | 0.6 | 0.5548 | P1-5 | 0.3875 | 0.4591 | P3-6 | 0.325 | 0.3393 |
| P4 | -0.7 | -0.6828 | P2-3 | -0.0375 | 0.0179 | P4-5 | -0.0375 | -0.0416 |
| P5 | -0.7 | -0.6936 | P2-4 | 0.2375 | 0.2105 | P5-6 | 0.0875 | 0.085 |
| P6 | -0.7 | -0.7262 | P2-5 | 0.2 | 0.2607 | | | |

Table 5.2 shows the state variables, Table 5.3 presents the estimation results of each estimator and original actual value, measured value. Table 5.4 shows the error between each estimator and actual value.

Table 5.2: State Variable of case 0, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1936 | -0.1912 | -0.1963 | -0.1850 | -0.2145 | -0.1912 |
| $\theta 3$ | -0.1824 | -0.1764 | -0.1929 | -0.1486 | -0.2093 | -0.1764 |
| $\theta 4$ | -0.4264 | -0.4234 | -0.4280 | -0.4234 | -0.4424 | -0.4234 |
| $\theta 5$ | -0.4036 | -0.3962 | -0.4078 | -0.4019 | -0.4450 | -0.3962 |
| $\theta 6$ | -0.5028 | -0.4966 | -0.5109 | -0.4874 | -0.5297 | -0.4966 |

Table 5.3: Estimation Results of case 0, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0236 | 1.0108 | 1.0321 | 1.0103 | 1.1020 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.5475 | 0.5367 | 0.5580 | 0.5362 | 0.5539 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5527 | 0.5549 | 0.5363 | 0.6285 | 0.5613 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.6820 | -0.6828 | -0.6801 | -0.6833 | -0.6678 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7130 | -0.6936 | -0.7107 | -0.7651 | -0.8290 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7289 | -0.7260 | -0.7356 | -0.7267 | -0.7203 | -0.7259 |
| P1-2 | 0.1875 | 0.1788 | 0.1936 | 0.1912 | 0.1963 | 0.1850 | 0.2145 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4264 | 0.4234 | 0.4280 | 0.4234 | 0.4424 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4036 | 0.3962 | 0.4078 | 0.4019 | 0.4450 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0111 | -0.0148 | -0.0034 | -0.0364 | -0.0052 | -0.0148 |
| P2-4 | 0.2375 | 0.2105 | 0.2328 | 0.2322 | 0.2317 | 0.2384 | 0.2279 | 0.2322 |
| P2-5 | 0.2000 | 0.2607 | 0.2101 | 0.2051 | 0.2115 | 0.2169 | 0.2305 | 0.2051 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P2-6 | 0.2875 | 0.3020 | 0.3093 | 0.3054 | 0.3146 | 0.3024 | 0.3152 | 0.3054 |
| P3-5 | 0.2375 | 0.2362 | 0.2212 | 0.2199 | 0.2149 | 0.2533 | 0.2357 | 0.2198 |
| P3-6 | 0.3250 | 0.3393 | 0.3204 | 0.3202 | 0.3180 | 0.3388 | 0.3204 | 0.3202 |
| P4-5 | -0.0375 | -0.0416 | -0.0228 | -0.0272 | -0.0203 | -0.0215 | 0.0026 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.0992 | 0.1004 | 0.1031 | 0.0855 | 0.0847 | 0.1003 |

Table 5.4: Estimation errors of case 0, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0236 | 0.0108 | 0.0321 | 0.0103 | 0.1020 | 0.0108 |
| P2 | 0.0367 | 0.0475 | 0.0367 | 0.0580 | 0.0362 | 0.0539 | 0.0367 |
| P3 | 0.0452 | 0.0473 | 0.0451 | 0.0637 | 0.0285 | 0.0387 | 0.0452 |
| P4 | 0.0172 | 0.0180 | 0.0172 | 0.0199 | 0.0167 | 0.0322 | 0.0172 |
| P5 | 0.0064 | 0.0130 | 0.0064 | 0.0107 | 0.0651 | 0.1290 | 0.0064 |
| P6 | 0.0262 | 0.0289 | 0.0260 | 0.0356 | 0.0267 | 0.0203 | 0.0259 |
| P1-2 | 0.0087 | 0.0061 | 0.0037 | 0.0088 | 0.0025 | 0.0270 | 0.0037 |
| P1-4 | 0.0069 | 0.0014 | 0.0016 | 0.0030 | 0.0016 | 0.0174 | 0.0016 |
| P1-5 | 0.0716 | 0.0161 | 0.0087 | 0.0203 | 0.0144 | 0.0575 | 0.0087 |
| P2-3 | 0.0554 | 0.0264 | 0.0227 | 0.0341 | 0.0011 | 0.0323 | 0.0227 |
| P2-4 | 0.0270 | 0.0047 | 0.0053 | 0.0058 | 0.0009 | 0.0096 | 0.0053 |
| P2-5 | 0.0607 | 0.0101 | 0.0051 | 0.0115 | 0.0169 | 0.0305 | 0.0051 |
| P2-6 | 0.0145 | 0.0218 | 0.0179 | 0.0271 | 0.0149 | 0.0277 | 0.0179 |
| P3-5 | 0.0013 | 0.0163 | 0.0176 | 0.0226 | 0.0158 | 0.0018 | 0.0177 |
| P3-6 | 0.0143 | 0.0046 | 0.0048 | 0.0070 | 0.0138 | 0.0046 | 0.0048 |
| P4-5 | 0.0041 | 0.0147 | 0.0103 | 0.0172 | 0.0160 | 0.0401 | 0.0103 |
| P5-6 | 0.0025 | 0.0117 | 0.0129 | 0.0156 | 0.0020 | 0.0028 | 0.0128 |
| **SUM** | **0.4095** | **0.3121** | **0.2527** | **0.3931** | **0.2832** | **0.6276** | **0.2528** |

As illustrated by the results, for case with only small random noise, most estimators generate good estimation. The results of LTS is worse than WLS, because there are no outliers, the data with the largest error lay on the same bus, making the bus has less coverage by true value, and induce large estimation error.

### Median redundancy

Compared with full redundancy, median case are more common in actual use considering the large scale of modern power system. The coefficient $K$ of median redundancy is usually $1.8 - 2.0$, i.e. number of measurement $= K\times$ number of state variable. For the 6-bus system DC estimation with 5 state variables, 10 measurements are picked out as raw data for estimation.

Observability is the key point that should be considered regarding median redundancy. According to section 3.2 in Chapter 3, the location to put measurement facilities should be conducted following the topology analysis.

For 6-bus cases, there are more branches connecting each bus compared with large system, means that each bus has more branch value to estimate, selection of all power injection ensure a better influence on branches flow estimation, so all injection measurement are selected at first. Other 4 measurements are placed step by step according to algorithm and well distributed to guarantee local redundancy as shown in figure 5.4. The raw data are presented in Table. 5.5.

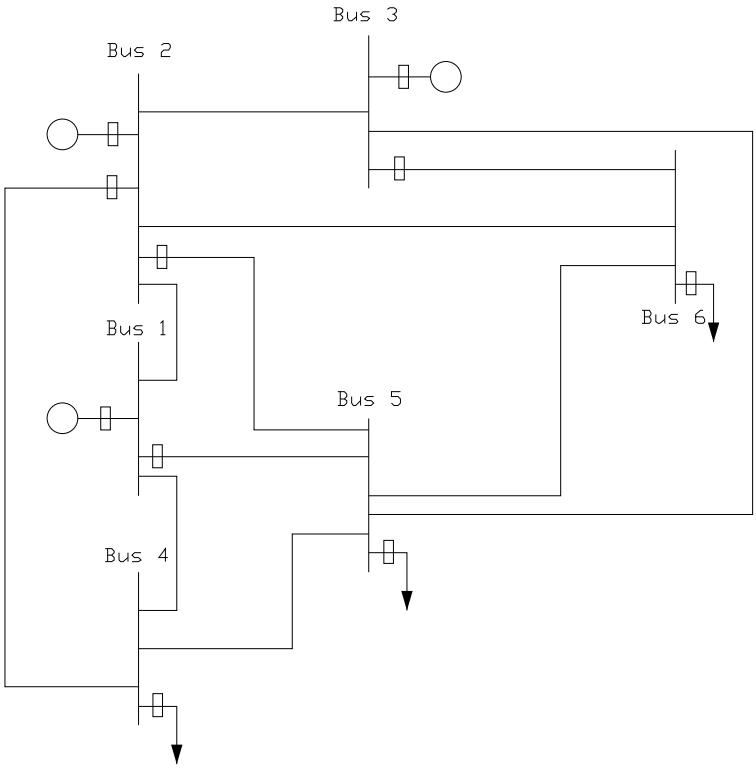


Figure 5.4: 6-bus system with meters for median redundancy.

Table 5.5: raw data for 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | P5 | -0.7000 | -0.6936 | P2-5 | 0.2000 | 0.2607 |
| P2 | 0.5000 | 0.5367 | P6 | -0.7000 | -0.7262 | P3-6 | 0.3250 | 0.3393 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P3 | 0.6000 | 0.5548 | P1-5 | 0.3875 | 0.4591 | | | |
| P4 | -0.7000 | -0.6828 | P2-4 | 0.2375 | 0.2105 | | | |

Table 5.6 shows the system state variables generated by each estimator; Table 5.7 presents the estimation results and Table 5.8 gives the summation of the absolute value of errors between estimation results, the summation of raw measurements are also presented for comparison. $N/A$ means that the data are not used in the estimation.

Table 5.6: State Variable of case 0, small noise, 6-bus DC system, meidan redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1964 | -0.1913 | -0.1989 | -0.1911 | -0.2327 | -0.1912 |
| $\theta 3$ | -0.1855 | -0.1765 | -0.1829 | -0.1763 | -0.2289 | -0.1765 |
| $\theta 4$ | -0.4256 | -0.4234 | -0.4234 | -0.4232 | -0.4574 | -0.4233 |
| $\theta 5$ | -0.4053 | -0.3963 | -0.4026 | -0.3963 | -0.4604 | -0.3963 |
| $\theta 6$ | -0.5079 | -0.4968 | -0.5082 | -0.4968 | -0.5512 | -0.4967 |

Table 5.7: Estimation Results of case 0, small noise, 6-bus DC system, meidan redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0273 | 1.0110 | 1.0248 | 1.0106 | 1.1505 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.5423 | 0.5367 | 0.5227 | 0.5372 | 0.5341 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5530 | 0.5548 | 0.5609 | 0.5553 | 0.5578 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.6751 | -0.6827 | -0.6688 | -0.6823 | -0.6790 | -0.6825 |
| P5 | -0.7000 | -0.6936 | -0.7111 | -0.6936 | -0.6994 | -0.6941 | -0.8317 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7364 | -0.7262 | -0.7402 | -0.7267 | -0.7317 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.1964 | 0.1913 | 0.1989 | 0.1911 | 0.2327 | 0.1912 |
| P1-4 | 0.4250 | N/A | 0.4256 | 0.4234 | 0.4234 | 0.4232 | 0.4574 | 0.4233 |
| P1-5 | 0.3875 | 0.4591 | 0.4053 | 0.3963 | 0.4026 | 0.3963 | 0.4604 | 0.3963 |
| P2-3 | -0.0375 | N/A | -0.0109 | -0.0147 | -0.0160 | -0.0148 | -0.0039 | -0.0148 |
| P2-4 | 0.2375 | 0.2105 | 0.2292 | 0.2322 | 0.2245 | 0.2321 | 0.2246 | 0.2321 |
| P2-5 | 0.2000 | 0.2607 | 0.2089 | 0.2051 | 0.2037 | 0.2052 | 0.2276 | 0.2051 |
| P2-6 | 0.2875 | N/A | 0.3115 | 0.3055 | 0.3093 | 0.3057 | 0.3185 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.2198 | 0.2198 | 0.2196 | 0.2200 | 0.2315 | 0.2198 |
| P3-6 | 0.3250 | 0.3393 | 0.3224 | 0.3203 | 0.3253 | 0.3205 | 0.3224 | 0.3203 |
| P4-5 | -0.0375 | N/A | -0.0203 | -0.0271 | -0.0208 | -0.0269 | 0.0030 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.1026 | 0.1005 | 0.1056 | 0.1005 | 0.0909 | 0.1004 |

Table 5.8: Estimation errors of case 0, small noise, 6-bus DC system, meidan redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0273 | 0.0110 | 0.0248 | 0.0106 | 0.1505 | 0.0108 |
| P2 | 0.0423 | 0.0367 | 0.0227 | 0.0372 | 0.0341 | 0.0367 |
| P3 | 0.0470 | 0.0452 | 0.0391 | 0.0447 | 0.0422 | 0.0452 |
| P4 | 0.0249 | 0.0173 | 0.0312 | 0.0177 | 0.0210 | 0.0175 |
| P5 | 0.0111 | 0.0064 | 0.0006 | 0.0059 | 0.1317 | 0.0064 |

| | | | | | | |
|---|---|---|---|---|---|---|
| P6 | 0.0364 | 0.0262 | 0.0402 | 0.0267 | 0.0317 | 0.0262 |
| P1-2 | 0.0089 | 0.0038 | 0.0114 | 0.0036 | 0.0452 | 0.0037 |
| P1-4 | 0.0006 | 0.0016 | 0.0016 | 0.0018 | 0.0324 | 0.0017 |
| P1-5 | 0.0178 | 0.0088 | 0.0150 | 0.0088 | 0.0729 | 0.0088 |
| P2-3 | 0.0266 | 0.0228 | 0.0215 | 0.0227 | 0.0336 | 0.0227 |
| P2-4 | 0.0083 | 0.0053 | 0.0130 | 0.0054 | 0.0129 | 0.0054 |
| P2-5 | 0.0089 | 0.0051 | 0.0037 | 0.0052 | 0.0276 | 0.0051 |
| P2-6 | 0.0240 | 0.0180 | 0.0218 | 0.0182 | 0.0310 | 0.0180 |
| P3-5 | 0.0177 | 0.0177 | 0.0179 | 0.0175 | 0.0060 | 0.0177 |
| P3-6 | 0.0026 | 0.0047 | 0.0003 | 0.0045 | 0.0026 | 0.0047 |
| P4-5 | 0.0172 | 0.0104 | 0.0167 | 0.0106 | 0.0405 | 0.0104 |
| P5-6 | 0.0151 | 0.0130 | 0.0181 | 0.0130 | 0.0034 | 0.0129 |
| **SUM** | **0.3368** | **0.2539** | **0.2997** | **0.2540** | **0.7195** | **0.2540** |

**Comparison**

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.9. From the results, it is very easy to find that full redundancy case has better estimation for all estimators.

Table 5.9: Estimation errors of case 0, small noise, 6-bus DC system

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.4095** | **0.3121** | **0.2527** | **0.3931** | **0.2832** | **0.6276** | **0.2528** |
| **SUM** | **N/A** | **0.3368** | **0.2539** | **0.2997** | **0.2540** | **0.7195** | **0.2540** |

#### 5.1.1.2 Case 1: Single bad data

In this case, two types of single bad data are presented: reverse and large bad data. Reverse means that the measurement has negative value compared with actual value with small noise, while for large error, just as it shows, the measurement has a large difference with actual value. These two types of error are considered in the thesis.

**Reverse bad data**

The same as in case 0, full and median redundancy are considered separately.

**Full redundancy**

Based on Table 5.1, the power flow between bus 2 and bus 4 are changed to $-0.2105$ to simulate reverse bad data. Table 5.10 presents the state variables, Table 5.11 shows the estimated value and Table 5.12 shows the errors between each estimator and actual value.

Table 5.10: State Variable of case 1, reverse, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2131 | -0.1912 | -0.1900 | -0.1850 | -0.1878 | -0.1912 |
| $\theta 3$ | -0.2070 | -0.1764 | -0.1732 | -0.1486 | -0.1742 | -0.1763 |
| $\theta 4$ | -0.4054 | -0.4234 | -0.4312 | -0.4234 | -0.4246 | -0.4234 |
| $\theta 5$ | -0.4132 | -0.3962 | -0.4243 | -0.4019 | -0.3971 | -0.3962 |
| $\theta 6$ | -0.5274 | -0.4966 | -0.4930 | -0.4874 | -0.4946 | -0.4966 |

Table 5.11: Estimation Results of case 1, reverse, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0316 | 1.0108 | 1.0456 | 1.0103 | 1.0095 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.4874 | 0.5367 | 0.5715 | 0.5362 | 0.5513 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5327 | 0.5549 | 0.5878 | 0.6285 | 0.5569 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.5898 | -0.6828 | -0.6791 | -0.6833 | -0.6890 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7130 | -0.6936 | -0.8343 | -0.7651 | -0.7041 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7489 | -0.7260 | -0.6914 | -0.7267 | -0.7247 | -0.7262 |
| P1-2 | 0.1875 | 0.1788 | 0.2131 | 0.1912 | 0.1900 | 0.1850 | 0.1878 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4054 | 0.4234 | 0.4312 | 0.4234 | 0.4246 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4132 | 0.3962 | 0.4243 | 0.4019 | 0.3971 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0061 | -0.0148 | -0.0169 | -0.0364 | -0.0136 | -0.0149 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1923** | **0.2322** | **0.2411** | **0.2384** | **0.2368** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.2001 | 0.2051 | 0.2343 | 0.2169 | 0.2092 | 0.2051 |
| P2-6 | 0.2875 | 0.3020 | 0.3143 | 0.3054 | 0.3030 | 0.3024 | 0.3068 | 0.3055 |
| P3-5 | 0.2375 | 0.2362 | 0.2062 | 0.2199 | 0.2512 | 0.2533 | 0.2229 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3204 | 0.3202 | 0.3198 | 0.3388 | 0.3204 | 0.3203 |
| P4-5 | -0.0375 | -0.0416 | 0.0078 | -0.0272 | -0.0068 | -0.0215 | -0.0276 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.1142 | 0.1004 | 0.0687 | 0.0855 | 0.0975 | 0.1004 |

Table 5.12: Estimation errors of case 1, reverse, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0316 | 0.0108 | 0.0456 | 0.0103 | 0.0095 | 0.0108 |
| P2 | 0.0367 | 0.0126 | 0.0367 | 0.0715 | 0.0362 | 0.0513 | 0.0367 |
| P3 | 0.0452 | 0.0673 | 0.0451 | 0.0122 | 0.0285 | 0.0431 | 0.0449 |
| P4 | 0.0172 | 0.1102 | 0.0172 | 0.0209 | 0.0167 | 0.0110 | 0.0172 |
| P5 | 0.0064 | 0.0130 | 0.0064 | 0.1343 | 0.0651 | 0.0041 | 0.0064 |
| P6 | 0.0262 | 0.0489 | 0.0260 | 0.0086 | 0.0267 | 0.0247 | 0.0262 |
| P1-2 | 0.0087 | 0.0256 | 0.0037 | 0.0025 | 0.0025 | 0.0003 | 0.0037 |
| P1-4 | 0.0069 | 0.0196 | 0.0016 | 0.0062 | 0.0016 | 0.0004 | 0.0016 |
| P1-5 | 0.0716 | 0.0257 | 0.0087 | 0.0368 | 0.0144 | 0.0096 | 0.0087 |
| P2-3 | 0.0554 | 0.0314 | 0.0227 | 0.0206 | 0.0011 | 0.0239 | 0.0226 |
| **P2-4** | **0.4480** | **0.0452** | **0.0053** | **0.0036** | **0.0009** | **0.0007** | **0.0053** |
| P2-5 | 0.0607 | 0.0001 | 0.0051 | 0.0343 | 0.0169 | 0.0092 | 0.0051 |
| P2-6 | 0.0145 | 0.0268 | 0.0179 | 0.0155 | 0.0149 | 0.0193 | 0.0180 |
| P3-5 | 0.0013 | 0.0313 | 0.0176 | 0.0137 | 0.0158 | 0.0146 | 0.0176 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P3-6 | 0.0143 | 0.0046 | 0.0048 | 0.0052 | 0.0138 | 0.0046 | 0.0047 |
| P4-5 | 0.0041 | 0.0453 | 0.0103 | 0.0307 | 0.0160 | 0.0099 | 0.0103 |
| P5-6 | 0.0025 | 0.0267 | 0.0129 | 0.0188 | 0.0020 | 0.0100 | 0.0129 |
| **SUM** | **0.8305** | **0.5661** | **0.2527** | **0.4808** | **0.2832** | **0.2463** | **0.2526** |

As presented by the tables, all estimators succeeding in correct the bad data. WLS is a little worse than others with relative large error in $P_4$ estimation. For LTS, since there exist large error compared with random ones, it generates good results compared with the base case.

**Median redundancy**

To ensure a valid comparison with full redundancy case, the same reverse bad data $P_{2-4} = -0.2105$ is used on the base of Table 5.5. Table 5.13 shows the state variables, Table 5.14 presents the estimated value and Table 5.15 contains the errors between each estimator and actual value.

Table 5.13: State Variable of case 1, reverse, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2195 | -0.1912 | -0.1912 | -0.1911 | -0.1897 | -0.1912 |
| $\theta 3$ | -0.2185 | -0.1765 | -0.1764 | -0.1763 | -0.1754 | -0.1763 |
| $\theta 4$ | -0.4014 | -0.4234 | -0.4234 | -0.4232 | -0.4234 | -0.4234 |
| $\theta 5$ | -0.4182 | -0.3963 | -0.4277 | -0.3963 | -0.3977 | -0.3962 |
| $\theta 6$ | -0.5409 | -0.4967 | -0.4967 | -0.4968 | -0.4977 | -0.4966 |

Table 5.14: Estimation Results of case 1, reverse, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0391 | 1.0109 | 1.0423 | 1.0106 | 1.0109 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.4817 | 0.5367 | 0.5681 | 0.5372 | 0.5455 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5230 | 0.5548 | 0.5863 | 0.5553 | 0.5591 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.5666 | -0.6826 | -0.6513 | -0.6823 | -0.6828 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7107 | -0.6936 | -0.8506 | -0.6941 | -0.7023 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7665 | -0.7262 | -0.6947 | -0.7267 | -0.7304 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.2195 | 0.1912 | 0.1912 | 0.1911 | 0.1897 | 0.1912 |
| P1-4 | 0.4250 | N/A | 0.4014 | 0.4234 | 0.4234 | 0.4232 | 0.4234 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4182 | 0.3963 | 0.4277 | 0.3963 | 0.3977 | 0.3962 |
| P2-3 | -0.0375 | N/A | -0.0009 | -0.0148 | -0.0148 | -0.0148 | -0.0144 | -0.0149 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1819** | **0.2321** | **0.2322** | **0.2321** | **0.2337** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.1987 | 0.2050 | 0.2364 | 0.2052 | 0.2080 | 0.2050 |
| P2-6 | 0.2875 | N/A | 0.3214 | 0.3055 | 0.3055 | 0.3057 | 0.3080 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.1997 | 0.2198 | 0.2512 | 0.2200 | 0.2223 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3224 | 0.3203 | 0.3202 | 0.3205 | 0.3224 | 0.3203 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P4-5 | -0.0375 | N/A | 0.0168 | -0.0271 | 0.0043 | -0.0269 | -0.0257 | -0.0272 |
| P5-6 | 0.0875 | N/A | 0.1227 | 0.1005 | 0.0690 | 0.1005 | 0.1000 | 0.1004 |

Table 5.15: Estimation errors of case 1, reverse, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0391 | 0.0109 | 0.0423 | 0.0106 | 0.0109 | 0.0108 |
| P2 | 0.0183 | 0.0367 | 0.0681 | 0.0372 | 0.0455 | 0.0367 |
| P3 | 0.0770 | 0.0452 | 0.0137 | 0.0447 | 0.0409 | 0.0449 |
| P4 | 0.1334 | 0.0174 | 0.0487 | 0.0177 | 0.0173 | 0.0172 |
| P5 | 0.0107 | 0.0064 | 0.1506 | 0.0059 | 0.0023 | 0.0064 |
| P6 | 0.0665 | 0.0262 | 0.0053 | 0.0267 | 0.0304 | 0.0262 |
| P1-2 | 0.0320 | 0.0037 | 0.0037 | 0.0036 | 0.0022 | 0.0037 |
| P1-4 | 0.0236 | 0.0016 | 0.0016 | 0.0018 | 0.0016 | 0.0016 |
| P1-5 | 0.0307 | 0.0088 | 0.0401 | 0.0088 | 0.0102 | 0.0087 |
| P2-3 | 0.0366 | 0.0227 | 0.0227 | 0.0227 | 0.0231 | 0.0226 |
| **P2-4** | **0.0556** | **0.0054** | **0.0053** | **0.0054** | **0.0038** | **0.0053** |
| P2-5 | 0.0013 | 0.0050 | 0.0364 | 0.0052 | 0.0080 | 0.0050 |
| P2-6 | 0.0339 | 0.0180 | 0.0180 | 0.0182 | 0.0205 | 0.0180 |
| P3-5 | 0.0378 | 0.0177 | 0.0137 | 0.0175 | 0.0152 | 0.0176 |
| P3-6 | 0.0026 | 0.0047 | 0.0048 | 0.0045 | 0.0026 | 0.0047 |
| P4-5 | 0.0543 | 0.0104 | 0.0418 | 0.0106 | 0.0118 | 0.0103 |
| P5-6 | 0.0352 | 0.0130 | 0.0185 | 0.0130 | 0.0125 | 0.0129 |
| **SUM** | **0.6886** | **0.2539** | **0.5352** | **0.2540** | **0.2589** | **0.2526** |

For these case, the estimation results of WLS for the bad data $P_{2-4}$ is acceptable, though it's still large compared with other estimators. But it induce relative large error in $P_4$. Other estimators generate good estimation.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.16. From the results, it is very easy to find that full redundancy case has better estimation results for most estimators.

Table 5.16: Estimation errors of case 1, single reverse, 6-bus DC system

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.8305** | **0.5661** | **0.2527** | **0.4808** | **0.2832** | **0.2463** | **0.2526** |
| **SUM** | **N/A** | **0.6886** | **0.2539** | **0.5352** | **0.2540** | **0.2589** | **0.2526** |

### Large bad data

Compared with reverse bad data case, large bad data is induced with the same flow. Full and median redundancy are considered.

**Full redundancy**

Compared with the base data shown in Table 5.1, a large error $P_{2-4} = 0.9105$ instead of the original one. State variables are presented in Table 5.17, Table 5.18 shows the estimated value and Table 5.19 presents the errors between each estimator and actual value.

Table 5.17: State Variable of case 1, large, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1611 | -0.1912 | -0.1900 | -0.1783 | -0.1891 | -0.1912 |
| $\theta 3$ | -0.1416 | -0.1764 | -0.1732 | -0.1408 | -0.1741 | -0.1763 |
| $\theta 4$ | -0.4614 | -0.4234 | -0.4312 | -0.4370 | -0.4245 | -0.4234 |
| $\theta 5$ | -0.3878 | -0.3962 | -0.4243 | -0.3960 | -0.3957 | -0.3962 |
| $\theta 6$ | -0.4620 | -0.4966 | -0.4930 | -0.4806 | -0.4945 | -0.4966 |

Table 5.18: Estimation Results of case 1, large, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0103 | 1.0108 | 1.0456 | 1.0113 | 1.0092 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.6475 | 0.5367 | 0.5715 | 0.5628 | 0.5434 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5860 | 0.5549 | 0.5878 | 0.6325 | 0.5569 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.8354 | -0.6828 | -0.6791 | -0.7369 | -0.6887 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7130 | -0.6936 | -0.8343 | -0.7431 | -0.6962 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.6955 | -0.7260 | -0.6914 | -0.7267 | -0.7247 | -0.7262 |
| P1-2 | 0.1875 | 0.1788 | 0.1611 | 0.1912 | 0.1900 | 0.1783 | 0.1891 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4614 | 0.4234 | 0.4312 | 0.4370 | 0.4245 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.3878 | 0.3962 | 0.4243 | 0.3960 | 0.3957 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0194 | -0.0148 | -0.0169 | -0.0375 | -0.0150 | -0.0149 |
| **P2-4** | **0.2375** | **0.9105** | **0.3003** | **0.2322** | **0.2411** | **0.2587** | **0.2354** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.2268 | 0.2051 | 0.2343 | 0.2177 | 0.2066 | 0.2051 |
| P2-6 | 0.2875 | 0.3020 | 0.3009 | 0.3054 | 0.3030 | 0.3023 | 0.3054 | 0.3055 |
| P3-5 | 0.2375 | 0.2362 | 0.2462 | 0.2199 | 0.2512 | 0.2552 | 0.2216 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3204 | 0.3202 | 0.3198 | 0.3398 | 0.3204 | 0.3203 |
| P4-5 | -0.0375 | -0.0416 | -0.0736 | -0.0272 | -0.0068 | -0.0411 | -0.0288 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.0742 | 0.1004 | 0.0687 | 0.0846 | 0.0988 | 0.1004 |

Table 5.19: Estimation errors of case 1, large, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0103 | 0.0108 | 0.0456 | 0.0113 | 0.0092 | 0.0108 |
| P2 | 0.0367 | 0.1475 | 0.0367 | 0.0715 | 0.0628 | 0.0434 | 0.0367 |
| P3 | 0.0452 | 0.0140 | 0.0451 | 0.0122 | 0.0325 | 0.0431 | 0.0449 |
| P4 | 0.0172 | 0.1354 | 0.0172 | 0.0209 | 0.0369 | 0.0113 | 0.0172 |
| P5 | 0.0064 | 0.0130 | 0.0064 | 0.1343 | 0.0431 | 0.0038 | 0.0064 |
| P6 | 0.0262 | 0.0045 | 0.0260 | 0.0086 | 0.0267 | 0.0247 | 0.0262 |
| P1-2 | 0.0087 | 0.0264 | 0.0037 | 0.0025 | 0.0092 | 0.0016 | 0.0037 |
| P1-4 | 0.0069 | 0.0364 | 0.0016 | 0.0062 | 0.0120 | 0.0005 | 0.0016 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P1-5 | 0.0716 | 0.0003 | 0.0087 | 0.0368 | 0.0085 | 0.0082 | 0.0087 |
| P2-3 | 0.0554 | 0.0181 | 0.0227 | 0.0206 | 0.0000 | 0.0225 | 0.0226 |
| **P2-4** | **0.6730** | **0.0628** | **0.0053** | **0.0036** | **0.0212** | **0.0021** | **0.0053** |
| P2-5 | 0.0607 | 0.0268 | 0.0051 | 0.0343 | 0.0177 | 0.0066 | 0.0051 |
| P2-6 | 0.0145 | 0.0134 | 0.0179 | 0.0155 | 0.0148 | 0.0179 | 0.0180 |
| P3-5 | 0.0013 | 0.0087 | 0.0176 | 0.0137 | 0.0177 | 0.0159 | 0.0176 |
| P3-6 | 0.0143 | 0.0046 | 0.0048 | 0.0052 | 0.0148 | 0.0046 | 0.0047 |
| P4-5 | 0.0041 | 0.0361 | 0.0103 | 0.0307 | 0.0036 | 0.0087 | 0.0103 |
| P5-6 | 0.0025 | 0.0133 | 0.0129 | 0.0188 | 0.0029 | 0.0113 | 0.0129 |
| **SUM** | **1.0555** | **0.5715** | **0.2527** | **0.4808** | **0.3357** | **0.2356** | **0.2526** |

For large bad data case, LAV and LTAV generates best results among all estimation. WLS is still bad compared with others though it generate better results when compared with measurements. Other robust estimators performs well with satisfied results.

**Median redundancy**

Compared with the base data shown in Table 5.5, a large error $P_{2-4} = 0.9105$ instead of the original one corresponding to full redundancy case. State variables are presented in Table 5.20, Table 5.21 shows the estimated value and Table 5.22 shows the errors between each estimator and actual value.

Table 5.20: State Variable of case 1, large, 6-bus DC system, median redundancy

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1580 | -0.1912 | -0.1912 | -0.1913 | -0.2127 | -0.1912 |
| $\theta 3$ | -0.1306 | -0.1764 | -0.1764 | -0.1765 | -0.2059 | -0.1763 |
| $\theta 4$ | -0.4658 | -0.4234 | -0.4234 | -0.4236 | -0.4422 | -0.4234 |
| $\theta 5$ | -0.3839 | -0.3962 | -0.4277 | -0.3965 | -0.4217 | -0.3962 |
| $\theta 6$ | -0.4530 | -0.4967 | -0.4967 | -0.4970 | -0.5282 | -0.4966 |

Table 5.21: Estimation Results of case 1, large, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0077 | 1.0108 | 1.0423 | 1.0113 | 1.0766 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.6430 | 0.5370 | 0.5681 | 0.5372 | 0.5344 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.6030 | 0.5548 | 0.5863 | 0.5553 | 0.5450 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.8554 | -0.6828 | -0.6513 | -0.6830 | -0.6922 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7119 | -0.6936 | -0.8506 | -0.6941 | -0.7194 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.6864 | -0.7262 | -0.6947 | -0.7267 | -0.7445 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.1580 | 0.1912 | 0.1912 | 0.1913 | 0.2127 | 0.1912 |
| P1-4 | 0.4250 | N/A | 0.4658 | 0.4234 | 0.4234 | 0.4236 | 0.4422 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.3839 | 0.3962 | 0.4277 | 0.3965 | 0.4217 | 0.3962 |
| P2-3 | -0.0375 | N/A | -0.0274 | -0.0147 | -0.0148 | -0.0148 | -0.0068 | -0.0149 |

| **P2-4** | **0.2375** | **0.9105** | **0.3077** | **0.2322** | **0.2322** | **0.2323** | **0.2295** | **0.2322** |
|---|---|---|---|---|---|---|---|---|
| P2-5 | 0.2000 | 0.2607 | 0.2258 | 0.2051 | 0.2364 | 0.2052 | 0.2090 | 0.2051 |
| P2-6 | 0.2875 | N/A | 0.2949 | 0.3055 | 0.3055 | 0.3057 | 0.3155 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.2532 | 0.2198 | 0.2512 | 0.2200 | 0.2158 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3224 | 0.3203 | 0.3202 | 0.3205 | 0.3224 | 0.3203 |
| P4-5 | -0.0375 | N/A | -0.0819 | -0.0272 | 0.0043 | -0.0271 | -0.0205 | -0.0272 |
| P5-6 | 0.0875 | N/A | 0.0691 | 0.1004 | 0.0690 | 0.1005 | 0.1066 | 0.1004 |

Table 5.22: Estimation errors of case 1, large, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0077 | 0.0108 | 0.0423 | 0.0113 | 0.0766 | 0.0108 |
| P2 | 0.1430 | 0.0370 | 0.0681 | 0.0372 | 0.0344 | 0.0367 |
| P3 | 0.0030 | 0.0452 | 0.0137 | 0.0447 | 0.0550 | 0.0449 |
| P4 | 0.1554 | 0.0172 | 0.0487 | 0.0170 | 0.0078 | 0.0172 |
| P5 | 0.0119 | 0.0064 | 0.1506 | 0.0059 | 0.0194 | 0.0064 |
| P6 | 0.0136 | 0.0262 | 0.0053 | 0.0267 | 0.0445 | 0.0262 |
| P1-2 | 0.0295 | 0.0037 | 0.0037 | 0.0038 | 0.0252 | 0.0037 |
| P1-4 | 0.0408 | 0.0016 | 0.0016 | 0.0014 | 0.0172 | 0.0016 |
| P1-5 | 0.0036 | 0.0087 | 0.0401 | 0.0090 | 0.0342 | 0.0087 |
| P2-3 | 0.0101 | 0.0228 | 0.0227 | 0.0227 | 0.0307 | 0.0226 |
| **P2-4** | **0.0702** | **0.0053** | **0.0053** | **0.0052** | **0.0080** | **0.0053** |
| P2-5 | 0.0258 | 0.0051 | 0.0364 | 0.0052 | 0.0090 | 0.0051 |
| P2-6 | 0.0074 | 0.0180 | 0.0180 | 0.0182 | 0.0280 | 0.0180 |
| P3-5 | 0.0157 | 0.0177 | 0.0137 | 0.0175 | 0.0217 | 0.0176 |
| P3-6 | 0.0026 | 0.0047 | 0.0048 | 0.0045 | 0.0026 | 0.0047 |
| P4-5 | 0.0444 | 0.0103 | 0.0418 | 0.0104 | 0.0170 | 0.0103 |
| P5-6 | 0.0184 | 0.0129 | 0.0185 | 0.0130 | 0.0191 | 0.0129 |
| **SUM** | **0.6033** | **0.2536** | **0.5352** | **0.2536** | **0.4503** | **0.2526** |

**Comparison**

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.23. From the results, it's very easy to find that full redundancy case has better estimation results for all estimators.

Table 5.23: Estimation errors of case 1, single large, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.0555** | **0.5715** | **0.2527** | **0.4808** | **0.3357** | **0.2356** | **0.2526** |
| **SUM** | **N/A** | **0.6033** | **0.2536** | **0.5352** | **0.2536** | **0.4503** | **0.2526** |

#### 5.1.1.3 Case 2: Multiple non-interacting bad data

For actual large system, multiple bad data case are more common. In this part, two non-interacting bad data are generated to simulate the situation. Since

the connection between each bus of 6-bus system are very redundant, it very hard to pick up buses with no connection with each other, measurements with weak connection are selected.

Considering that only ten available measurements for median case, two bad data are selected to simulate the multiple bad data case. Two reverse bad data, two large bad data and one reverse one large bad data are the three cases for performance evaluation.

**Two reverse bad data**

Base on the raw data used in the single reverse bad data case, a weakly connected power flow $P_{3-6}$ is changed to $-0.3393$ to simulate the two non-interacting bad data case. Other measurements are the same as Table 5.1.

**Full Redundancy**

Table 5.24 presents the state variables generated by the estimation of each estimator; The estimated system results and the error between them and the actual value are illustrated in Table 5.25 and Table 5.26 respectively.

Table 5.24: State Variable of case 2, multi reverse, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2131 | -0.1912 | -0.2049 | -0.1897 | -0.1924 | -0.1912 |
| $\theta 3$ | -0.2409 | -0.1764 | -0.2174 | -0.1720 | -0.1821 | -0.1764 |
| $\theta 4$ | -0.4054 | -0.4234 | -0.4237 | -0.4314 | -0.4276 | -0.4234 |
| $\theta 5$ | -0.4132 | -0.3962 | -0.4239 | -0.3893 | -0.4031 | -0.3962 |
| $\theta 6$ | -0.4935 | -0.4966 | -0.5380 | -0.4922 | -0.5004 | -0.4966 |

Table 5.25: Estimation Results of case 2, multi reverse, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0316 | 1.0108 | 1.0525 | 1.0104 | 1.0231 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.4874 | 0.5367 | 0.5784 | 0.5362 | 0.5511 | 0.5367 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P3 | 0.6000 | 0.5548 | 0.3969 | 0.5548 | 0.5147 | 0.5553 | 0.5497 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.5898 | -0.6828 | -0.6424 | -0.7152 | -0.6874 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7130 | -0.6936 | -0.7353 | -0.6611 | -0.7130 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.6132 | -0.7259 | -0.7679 | -0.7257 | -0.7235 | -0.7259 |
| P1-2 | 0.1875 | 0.1788 | 0.2131 | 0.1912 | 0.2049 | 0.1897 | 0.1924 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4054 | 0.4234 | 0.4237 | 0.4314 | 0.4276 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4132 | 0.3962 | 0.4239 | 0.3893 | 0.4031 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | 0.0278 | -0.0148 | 0.0124 | -0.0178 | -0.0104 | -0.0148 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1923** | **0.2322** | **0.2188** | **0.2417** | **0.2352** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.2001 | 0.2051 | 0.2190 | 0.1996 | 0.2107 | 0.2051 |
| P2-6 | 0.2875 | 0.3020 | 0.2804 | 0.3054 | 0.3331 | 0.3025 | 0.3079 | 0.3054 |
| P3-5 | 0.2375 | 0.2362 | 0.1722 | 0.2198 | 0.2065 | 0.2173 | 0.2210 | 0.2198 |
| **P3-6** | **0.3250** | **-0.3393** | **0.2525** | **0.3202** | **0.3207** | **0.3202** | **0.3183** | **0.3202** |
| P4-5 | -0.0375 | -0.0416 | 0.0078 | -0.0272 | 0.0001 | -0.0421 | -0.0246 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.0803 | 0.1003 | 0.1142 | 0.1029 | 0.0973 | 0.1003 |

Table 5.26: Estimation errors of case 2, multi reverse, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0316 | 0.0108 | 0.0525 | 0.0104 | 0.0231 | 0.0108 |
| P2 | 0.0367 | 0.0126 | 0.0367 | 0.0784 | 0.0362 | 0.0511 | 0.0367 |
| P3 | 0.0452 | 0.2031 | 0.0452 | 0.0853 | 0.0447 | 0.0503 | 0.0452 |
| P4 | 0.0172 | 0.1102 | 0.0172 | 0.0576 | 0.0152 | 0.0126 | 0.0172 |
| P5 | 0.0064 | 0.0130 | 0.0064 | 0.0353 | 0.0389 | 0.0130 | 0.0064 |
| P6 | 0.0262 | 0.0868 | 0.0259 | 0.0679 | 0.0257 | 0.0235 | 0.0259 |
| P1-2 | 0.0087 | 0.0256 | 0.0037 | 0.0174 | 0.0022 | 0.0049 | 0.0037 |
| P1-4 | 0.0069 | 0.0196 | 0.0016 | 0.0013 | 0.0064 | 0.0026 | 0.0016 |
| P1-5 | 0.0716 | 0.0257 | 0.0087 | 0.0364 | 0.0018 | 0.0156 | 0.0087 |
| P2-3 | 0.0554 | 0.0653 | 0.0227 | 0.0499 | 0.0197 | 0.0271 | 0.0227 |
| **P2-4** | **0.4480** | **0.0452** | **0.0053** | **0.0187** | **0.0042** | **0.0023** | **0.0053** |
| P2-5 | 0.0607 | 0.0001 | 0.0051 | 0.0190 | 0.0004 | 0.0107 | 0.0051 |
| P2-6 | 0.0145 | 0.0071 | 0.0179 | 0.0456 | 0.0150 | 0.0204 | 0.0179 |
| P3-5 | 0.0013 | 0.0653 | 0.0177 | 0.0310 | 0.0202 | 0.0165 | 0.0177 |
| **P3-6** | **0.6643** | **0.0725** | **0.0048** | **0.0043** | **0.0048** | **0.0067** | **0.0048** |
| P4-5 | 0.0041 | 0.0453 | 0.0103 | 0.0376 | 0.0046 | 0.0129 | 0.0103 |
| P5-6 | 0.0025 | 0.0072 | 0.0128 | 0.0267 | 0.0154 | 0.0098 | 0.0128 |
| **SUM** | **1.4805** | **0.8362** | **0.2528** | **0.6650** | **0.2659** | **0.3031** | **0.2528** |

All estimators generate better results than measurements, but WLS fails to estimate $P_3$ and $P_4$, LMS's results on these two values are also not satisfied. Other estimators generate good estimation. LAV and LTAV share the same best results, actually they have the same estimation, the reason is that LTAV does not have fixed number of trimmed data as LTS, for LTAV, $K$ is changeable. Since LAV generates very good results in this case, LTAV does not trim any data since it's the best results.

**Median Redundancy**

For median redundancy case, the same value $P_{2-4} = -0.2105$ and $P_{3-6} = -0.3393$ act as reverse bad data. The state variables, estimation results and error with actual value are presented in the following three tables.

Table 5.27: State Variable of case 2, multi reverse, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2195 | -0.1912 | -0.2051 | -0.1911 | -0.1953 | -0.1913 |
| $\theta 3$ | -0.2562 | -0.1765 | -0.2180 | -0.1765 | -0.1849 | -0.1765 |
| $\theta 4$ | -0.4014 | -0.4234 | -0.4234 | -0.4232 | -0.4268 | -0.4235 |
| $\theta 5$ | -0.4182 | -0.3963 | -0.4240 | -0.3963 | -0.4047 | -0.3963 |
| $\theta 6$ | -0.5032 | -0.4967 | -0.5384 | -0.4965 | -0.5052 | -0.4968 |

Table 5.28: Estimation Results of case 2, multi reverse, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0391 | 1.0109 | 1.0525 | 1.0106 | 1.0267 | 1.0111 |
| P2 | 0.5000 | 0.5367 | 0.4817 | 0.5367 | 0.5784 | 0.5372 | 0.5453 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.3722 | 0.5548 | 0.5134 | 0.5543 | 0.5503 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.5666 | -0.6826 | -0.6411 | -0.6823 | -0.6805 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7107 | -0.6936 | -0.7353 | -0.6941 | -0.7112 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.6157 | -0.7262 | -0.7679 | -0.7257 | -0.7307 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.2195 | 0.1912 | 0.2051 | 0.1911 | 0.1953 | 0.1913 |
| P1-4 | 0.4250 | N/A | 0.4014 | 0.4234 | 0.4234 | 0.4232 | 0.4268 | 0.4235 |
| P1-5 | 0.3875 | 0.4591 | 0.4182 | 0.3963 | 0.4240 | 0.3963 | 0.4047 | 0.3963 |
| P2-3 | -0.0375 | N/A | 0.0368 | -0.0148 | 0.0130 | -0.0145 | -0.0103 | -0.0148 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1819** | **0.2321** | **0.2183** | **0.2321** | **0.2315** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.1987 | 0.2051 | 0.2190 | 0.2052 | 0.2094 | 0.2051 |
| P2-6 | 0.2875 | N/A | 0.2837 | 0.3055 | 0.3333 | 0.3055 | 0.3099 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.1620 | 0.2198 | 0.2060 | 0.2198 | 0.2197 | 0.2198 |
| **P3-6** | **0.3250** | **-0.3393** | **0.2470** | **0.3202** | **0.3203** | **0.3200** | **0.3203** | **0.3203** |
| P4-5 | -0.0375 | N/A | 0.0168 | -0.0271 | 0.0007 | -0.0269 | -0.0221 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.0850 | 0.1004 | 0.1143 | 0.1002 | 0.1005 | 0.1005 |

Table 5.29: Estimation errors of case 2, multi reverse, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0391 | 0.0109 | 0.0525 | 0.0106 | 0.0267 | 0.0111 |
| P2 | 0.0183 | 0.0367 | 0.0784 | 0.0372 | 0.0453 | 0.0367 |
| P3 | 0.2278 | 0.0452 | 0.0866 | 0.0457 | 0.0497 | 0.0452 |
| P4 | 0.1334 | 0.0174 | 0.0589 | 0.0177 | 0.0195 | 0.0172 |
| P5 | 0.0107 | 0.0064 | 0.0353 | 0.0059 | 0.0112 | 0.0064 |
| P6 | 0.0843 | 0.0262 | 0.0679 | 0.0257 | 0.0307 | 0.0262 |
| P1-2 | 0.0320 | 0.0037 | 0.0176 | 0.0036 | 0.0078 | 0.0038 |
| P1-4 | 0.0236 | 0.0016 | 0.0016 | 0.0018 | 0.0018 | 0.0015 |
| P1-5 | 0.0307 | 0.0088 | 0.0365 | 0.0088 | 0.0172 | 0.0088 |
| P2-3 | 0.0743 | 0.0227 | 0.0505 | 0.0230 | 0.0272 | 0.0227 |
| **P2-4** | **0.0556** | **0.0054** | **0.0192** | **0.0054** | **0.0060** | **0.0053** |
| P2-5 | 0.0013 | 0.0051 | 0.0190 | 0.0052 | 0.0094 | 0.0051 |
| P2-6 | 0.0038 | 0.0180 | 0.0458 | 0.0180 | 0.0224 | 0.0180 |

| | | | | | | |
|---|---|---|---|---|---|---|
| P3-5 | 0.0755 | 0.0177 | 0.0315 | 0.0177 | 0.0178 | 0.0177 |
| **P3-6** | **0.0780** | **0.0048** | **0.0047** | **0.0050** | **0.0048** | **0.0047** |
| P4-5 | 0.0543 | 0.0104 | 0.0382 | 0.0106 | 0.0154 | 0.0104 |
| P5-6 | 0.0025 | 0.0129 | 0.0268 | 0.0127 | 0.0130 | 0.0130 |
| **SUM** | **0.9452** | **0.2539** | **0.6711** | **0.2545** | **0.3257** | **0.2538** |

In this case, the analysis of results are almost the same as in full redundancy case, WLS and LMS generate relative bad results compared with other estimators, since they can not successfully estimate $P_3$ and $P_4$. Other estimators have good estimation.

### Comparison

The comparison between full and median redundancy are presented in Table 5.30, only summation of errors are shown here. Except LTS, other estimators generate better results in full redundancy case, the reason is that the incorrect trimmed data in full redundancy case is interacting with the bad-data and fails to detect the other bad data.

Table 5.30: Estimation errors of case 2, multi reverse, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.0555** | **0.6033** | **0.2536** | **0.5352** | **0.2536** | **0.4503** | **0.2526** |
| **SUM** | **N/A** | **0.9452** | **0.2539** | **0.6711** | **0.2545** | **0.3257** | **0.2538** |

## Two non-conforming bad data

Non-conforming bad data means that the two bad data are not the same type: one is reverse and the other is large bad data. Full and median redundancy case are used to verify the performance of proposed estimators. Both case have the same bad data: $P_{2-4} = -0.2105$ and $P_{3-6} = 0.9393$.

**Full redundancy**

The five system state variables are presented in Table 5.31, Table 5.32 shows the results and Table 5.33 illustrates the error between estimation and actual value.

Table 5.31: State Variable of case 2, non-conforning, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2131 | -0.1912 | -0.2049 | -0.1898 | -0.1924 | -0.1912 |
| $\theta 3$ | -0.1770 | -0.1764 | -0.2178 | -0.1720 | -0.1821 | -0.1764 |
| $\theta 4$ | -0.4054 | -0.4234 | -0.4237 | -0.4314 | -0.4276 | -0.4234 |
| $\theta 5$ | -0.4132 | -0.3962 | -0.4239 | -0.3893 | -0.4031 | -0.3962 |
| $\theta 6$ | -0.5574 | -0.4966 | -0.5376 | -0.4923 | -0.5004 | -0.4966 |

Table 5.32: Estimation Results of case 2, non-conforning, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0316 | 1.0108 | 1.0525 | 1.0105 | 1.0231 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.4874 | 0.5367 | 0.5784 | 0.5362 | 0.5511 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.6527 | 0.5549 | 0.5131 | 0.5553 | 0.5497 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.5898 | -0.6828 | -0.6424 | -0.7152 | -0.6874 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7130 | -0.6936 | -0.7353 | -0.6611 | -0.7130 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.8689 | -0.7260 | -0.7663 | -0.7257 | -0.7235 | -0.7259 |
| P1-2 | 0.1875 | 0.1788 | 0.2131 | 0.1912 | 0.2049 | 0.1898 | 0.1924 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4054 | 0.4234 | 0.4237 | 0.4314 | 0.4276 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4132 | 0.3962 | 0.4239 | 0.3893 | 0.4031 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0361 | -0.0148 | 0.0129 | -0.0177 | -0.0104 | -0.0148 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1923** | **0.2322** | **0.2188** | **0.2417** | **0.2352** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.2001 | 0.2051 | 0.2190 | 0.1996 | 0.2107 | 0.2051 |
| P2-6 | 0.2875 | 0.3020 | 0.3443 | 0.3054 | 0.3327 | 0.3025 | 0.3079 | 0.3054 |
| P3-5 | 0.2375 | 0.2362 | 0.2362 | 0.2199 | 0.2061 | 0.2173 | 0.2210 | 0.2198 |
| **P3-6** | **0.3250** | **0.9393** | **0.3804** | **0.3202** | **0.3198** | **0.3203** | **0.3183** | **0.3202** |
| P4-5 | -0.0375 | -0.0416 | 0.0078 | -0.0272 | 0.0001 | -0.0421 | -0.0246 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.1442 | 0.1004 | 0.1137 | 0.1030 | 0.0973 | 0.1003 |

Table 5.33: Estimation errors of case 2, non-conforming, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0316 | 0.0108 | 0.0525 | 0.0105 | 0.0231 | 0.0108 |
| P2 | 0.0367 | 0.0126 | 0.0367 | 0.0784 | 0.0362 | 0.0511 | 0.0367 |
| P3 | 0.0452 | 0.0527 | 0.0451 | 0.0869 | 0.0447 | 0.0503 | 0.0452 |
| P4 | 0.0172 | 0.1102 | 0.0172 | 0.0576 | 0.0152 | 0.0126 | 0.0172 |
| P5 | 0.0064 | 0.0130 | 0.0064 | 0.0353 | 0.0389 | 0.0130 | 0.0064 |
| P6 | 0.0262 | 0.1689 | 0.0260 | 0.0663 | 0.0257 | 0.0235 | 0.0259 |
| P1-2 | 0.0087 | 0.0256 | 0.0037 | 0.0174 | 0.0023 | 0.0049 | 0.0037 |
| P1-4 | 0.0069 | 0.0196 | 0.0016 | 0.0013 | 0.0064 | 0.0026 | 0.0016 |
| P1-5 | 0.0716 | 0.0257 | 0.0087 | 0.0364 | 0.0018 | 0.0156 | 0.0087 |
| P2-3 | 0.0554 | 0.0014 | 0.0227 | 0.0504 | 0.0198 | 0.0271 | 0.0227 |
| **P2-4** | **0.4480** | **0.0452** | **0.0053** | **0.0187** | **0.0042** | **0.0023** | **0.0053** |
| P2-5 | 0.0607 | 0.0001 | 0.0051 | 0.0190 | 0.0004 | 0.0107 | 0.0051 |
| P2-6 | 0.0145 | 0.0568 | 0.0179 | 0.0452 | 0.0150 | 0.0204 | 0.0179 |
| P3-5 | 0.0013 | 0.0013 | 0.0176 | 0.0314 | 0.0202 | 0.0165 | 0.0177 |

| **P3-6** | **0.6143** | **0.0554** | **0.0048** | **0.0052** | **0.0047** | **0.0067** | **0.0048** |
|---|---|---|---|---|---|---|---|
| P4-5 | 0.0041 | 0.0453 | 0.0103 | 0.0376 | 0.0046 | 0.0129 | 0.0103 |
| P5-6 | 0.0025 | 0.0567 | 0.0129 | 0.0262 | 0.0155 | 0.0098 | 0.0128 |
| **SUM** | **1.4305** | **0.7222** | **0.2527** | **0.6658** | **0.2659** | **0.3031** | **0.2528** |

WLS and LMS generate relative bad results compared with others though they are much better than measurements. The estimation of $P_4$ and $P_6$ are not satisfied for WLS, and LMS's estimations are relative large for power injection.

### Median redundancy

The same bad data as in full redundancy case, the following three tables present the results and error with actual value of the system.

Table 5.34: State Variable of case 2, non-conforming, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2195 | -0.1912 | -0.2051 | -0.1911 | -0.1953 | -0.1913 |
| $\theta 3$ | -0.1852 | -0.1765 | -0.2181 | -0.1763 | -0.1849 | -0.1765 |
| $\theta 4$ | -0.4014 | -0.4234 | -0.4234 | -0.4232 | -0.4268 | -0.4235 |
| $\theta 5$ | -0.4182 | -0.3963 | -0.4240 | -0.3963 | -0.4047 | -0.3963 |
| $\theta 6$ | -0.5742 | -0.4967 | -0.5383 | -0.4968 | -0.5052 | -0.4968 |

Table 5.35: Estimation Results of case 2, non-conforming, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0391 | 1.0109 | 1.0525 | 1.0106 | 1.0267 | 1.0111 |
| P2 | 0.5000 | 0.5367 | 0.4817 | 0.5367 | 0.5784 | 0.5372 | 0.5453 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.6563 | 0.5548 | 0.5131 | 0.5553 | 0.5503 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.5666 | -0.6826 | -0.6411 | -0.6823 | -0.6805 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7107 | -0.6936 | -0.7353 | -0.6941 | -0.7112 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.8998 | -0.7262 | -0.7676 | -0.7267 | -0.7307 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.2195 | 0.1912 | 0.2051 | 0.1911 | 0.1953 | 0.1913 |
| P1-4 | 0.4250 | N/A | 0.4014 | 0.4234 | 0.4234 | 0.4232 | 0.4268 | 0.4235 |
| P1-5 | 0.3875 | 0.4591 | 0.4182 | 0.3963 | 0.4240 | 0.3963 | 0.4047 | 0.3963 |
| P2-3 | -0.0375 | N/A | -0.0343 | -0.0147 | 0.0130 | -0.0148 | -0.0103 | -0.0148 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1819** | **0.2321** | **0.2183** | **0.2321** | **0.2315** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.1987 | 0.2050 | 0.2190 | 0.2052 | 0.2094 | 0.2051 |
| P2-6 | 0.2875 | N/A | 0.3548 | 0.3055 | 0.3332 | 0.3057 | 0.3099 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.2330 | 0.2198 | 0.2059 | 0.2200 | 0.2197 | 0.2198 |
| **P3-6** | **0.3250** | **0.9393** | **0.3890** | **0.3203** | **0.3202** | **0.3205** | **0.3203** | **0.3203** |
| P4-5 | -0.0375 | N/A | 0.0168 | -0.0271 | 0.0007 | -0.0269 | -0.0221 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.1560 | 0.1005 | 0.1142 | 0.1005 | 0.1005 | 0.1005 |

Table 5.36: Estimation errors of case 2, non-conforming, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0391 | 0.0109 | 0.0525 | 0.0106 | 0.0267 | 0.0111 |
| P2 | 0.0183 | 0.0367 | 0.0784 | 0.0372 | 0.0453 | 0.0367 |
| P3 | 0.0563 | 0.0452 | 0.0869 | 0.0447 | 0.0497 | 0.0452 |
| P4 | 0.1334 | 0.0174 | 0.0589 | 0.0177 | 0.0195 | 0.0172 |
| P5 | 0.0107 | 0.0064 | 0.0353 | 0.0059 | 0.0112 | 0.0064 |
| P6 | 0.1998 | 0.0262 | 0.0676 | 0.0267 | 0.0307 | 0.0262 |
| P1-2 | 0.0320 | 0.0037 | 0.0176 | 0.0036 | 0.0078 | 0.0038 |
| P1-4 | 0.0236 | 0.0016 | 0.0016 | 0.0018 | 0.0018 | 0.0015 |
| P1-5 | 0.0307 | 0.0088 | 0.0365 | 0.0088 | 0.0172 | 0.0088 |
| P2-3 | 0.0032 | 0.0228 | 0.0505 | 0.0227 | 0.0272 | 0.0227 |
| **P2-4** | **0.0556** | **0.0054** | **0.0192** | **0.0054** | **0.0060** | **0.0053** |
| P2-5 | 0.0013 | 0.0050 | 0.0190 | 0.0052 | 0.0094 | 0.0051 |
| P2-6 | 0.0673 | 0.0180 | 0.0457 | 0.0182 | 0.0224 | 0.0180 |
| P3-5 | 0.0045 | 0.0177 | 0.0316 | 0.0175 | 0.0178 | 0.0177 |
| **P3-6** | **0.0640** | **0.0047** | **0.0048** | **0.0045** | **0.0048** | **0.0047** |
| P4-5 | 0.0543 | 0.0104 | 0.0382 | 0.0106 | 0.0154 | 0.0104 |
| P5-6 | 0.0685 | 0.0130 | 0.0267 | 0.0130 | 0.0130 | 0.0129 |
| **SUM** | **0.8626** | **0.2539** | **0.6713** | **0.2540** | **0.3257** | **0.2538** |

The same situation with full redundancy case, LAV, LMR and LTAV generate better estimation compared with other estimators, WLS fails to estimate $P_4$ and $P_6$ though the overall summation is better than measurements.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.37.Full redundancy case has better estimation results for all estimators. The reason why LTAV has the same results with LAV has been explained in Section 5.1.1.3.

Table 5.37: Estimation errors of case 2, non-conforming, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.4305** | **0.7222** | **0.2527** | **0.6658** | **0.2659** | **0.3031** | **0.2528** |
| **SUM** | **N/A** | **0.8626** | **0.2539** | **0.6713** | **0.2540** | **0.3257** | **0.2538** |

### Two large bad data

Compared with the previous two cases, $P_{2-4} = 0.9105$ and $P_{3-6} = 0.9393$ are introduced to simulate the two large bad data in both full and median redundancy case. The results are presented below.

**Full redundancy**

Based on the original raw data and large bad data shown above, Table 5.38 gives the state variables, and the later two tables shows the estimation results and error with actual values.

Table 5.38: State variable of case 2, multi large, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1611 | -0.1912 | -0.2049 | -0.1898 | -0.1924 | -0.1912 |
| $\theta 3$ | -0.1116 | -0.1764 | -0.2178 | -0.1720 | -0.1821 | -0.1764 |
| $\theta 4$ | -0.4614 | -0.4234 | -0.4237 | -0.4314 | -0.4276 | -0.4234 |
| $\theta 5$ | -0.3878 | -0.3962 | -0.4239 | -0.3893 | -0.4031 | -0.3962 |
| $\theta 6$ | -0.4920 | -0.4966 | -0.5376 | -0.4923 | -0.5004 | -0.4966 |

Table 5.39: Estimation Results of case 2, multi large, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0103 | 1.0108 | 1.0525 | 1.0104 | 1.0231 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.6475 | 0.5367 | 0.5784 | 0.5362 | 0.5511 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.7060 | 0.5549 | 0.5131 | 0.5553 | 0.5497 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.8354 | -0.6828 | -0.6424 | -0.7151 | -0.6874 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7130 | -0.6936 | -0.7353 | -0.6611 | -0.7130 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.8155 | -0.7260 | -0.7663 | -0.7257 | -0.7235 | -0.7259 |
| P1-2 | 0.1875 | 0.1788 | 0.1611 | 0.1912 | 0.2049 | 0.1898 | 0.1924 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4614 | 0.4234 | 0.4237 | 0.4314 | 0.4276 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.3878 | 0.3962 | 0.4239 | 0.3893 | 0.4031 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0494 | -0.0148 | 0.0129 | -0.0177 | -0.0104 | -0.0148 |
| **P2-4** | **0.2375** | **0.9105** | **0.3003** | **0.2322** | **0.2188** | **0.2416** | **0.2352** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.2268 | 0.2051 | 0.2190 | 0.1995 | 0.2107 | 0.2051 |
| P2-6 | 0.2875 | 0.3020 | 0.3309 | 0.3054 | 0.3327 | 0.3025 | 0.3079 | 0.3054 |
| P3-5 | 0.2375 | 0.2362 | 0.2762 | 0.2199 | 0.2061 | 0.2173 | 0.2210 | 0.2198 |
| **P3-6** | **0.3250** | **0.9393** | **0.3804** | **0.3202** | **0.3198** | **0.3203** | **0.3183** | **0.3202** |
| P4-5 | -0.0375 | -0.0416 | -0.0736 | -0.0272 | 0.0001 | -0.0421 | -0.0246 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.1042 | 0.1004 | 0.1137 | 0.1030 | 0.0973 | 0.1003 |

Table 5.40: Estimation errors of case 2, multi large, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0103 | 0.0108 | 0.0525 | 0.0104 | 0.0231 | 0.0108 |
| P2 | 0.0367 | 0.1475 | 0.0367 | 0.0784 | 0.0362 | 0.0511 | 0.0367 |
| P3 | 0.0452 | 0.1060 | 0.0451 | 0.0869 | 0.0447 | 0.0503 | 0.0452 |
| P4 | 0.0172 | 0.1354 | 0.0172 | 0.0576 | 0.0151 | 0.0126 | 0.0172 |
| P5 | 0.0064 | 0.0130 | 0.0064 | 0.0353 | 0.0389 | 0.0130 | 0.0064 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P6 | 0.0262 | 0.1155 | 0.0260 | 0.0663 | 0.0257 | 0.0235 | 0.0259 |
| P1-2 | 0.0087 | 0.0264 | 0.0037 | 0.0174 | 0.0023 | 0.0049 | 0.0037 |
| P1-4 | 0.0069 | 0.0364 | 0.0016 | 0.0013 | 0.0064 | 0.0026 | 0.0016 |
| P1-5 | 0.0716 | 0.0003 | 0.0087 | 0.0364 | 0.0018 | 0.0156 | 0.0087 |
| P2-3 | 0.0554 | 0.0119 | 0.0227 | 0.0504 | 0.0198 | 0.0271 | 0.0227 |
| **P2-4** | **0.6730** | **0.0628** | **0.0053** | **0.0187** | **0.0041** | **0.0023** | **0.0053** |
| P2-5 | 0.0607 | 0.0268 | 0.0051 | 0.0190 | 0.0005 | 0.0107 | 0.0051 |
| P2-6 | 0.0145 | 0.0434 | 0.0179 | 0.0452 | 0.0150 | 0.0204 | 0.0179 |
| P3-5 | 0.0013 | 0.0387 | 0.0176 | 0.0314 | 0.0202 | 0.0165 | 0.0177 |
| **P3-6** | **0.6143** | **0.0554** | **0.0048** | **0.0052** | **0.0047** | **0.0067** | **0.0048** |
| P4-5 | 0.0041 | 0.0361 | 0.0103 | 0.0376 | 0.0046 | 0.0129 | 0.0103 |
| P5-6 | 0.0025 | 0.0167 | 0.0129 | 0.0262 | 0.0155 | 0.0098 | 0.0128 |
| **SUM** | **1.6555** | **0.8828** | **0.2527** | **0.6658** | **0.2659** | **0.3031** | **0.2528** |

Though WLS successfully correct the bad data, it fails in power injection estimation of bus $2, 3$ and 4, LMS's estimation of all bus injection are also not satisfied. Other estimators generate good results in both bad data and other values.

### Median redundancy

The following three tables show the results and errors in detail.

Table 5.41: State Variable of case 2, multi large, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1580 | -0.1911 | -0.2051 | -0.1913 | -0.1953 | -0.1913 |
| $\theta 3$ | -0.0973 | -0.1764 | -0.2181 | -0.1765 | -0.1849 | -0.1765 |
| $\theta 4$ | -0.4658 | -0.4234 | -0.4234 | -0.4236 | -0.4268 | -0.4235 |
| $\theta 5$ | -0.3839 | -0.3963 | -0.4240 | -0.3965 | -0.4047 | -0.3963 |
| $\theta 6$ | -0.4863 | -0.4967 | -0.5383 | -0.4970 | -0.5052 | -0.4968 |

Table 5.42: Estimation Results of case 2, multi large, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0077 | 1.0108 | 1.0525 | 1.0113 | 1.0267 | 1.0111 |
| P2 | 0.5000 | 0.5367 | 0.6430 | 0.5370 | 0.5784 | 0.5372 | 0.5453 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.7364 | 0.5548 | 0.5131 | 0.5553 | 0.5503 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.8554 | -0.6828 | -0.6411 | -0.6830 | -0.6805 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.7119 | -0.6936 | -0.7353 | -0.6941 | -0.7112 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.8198 | -0.7262 | -0.7676 | -0.7267 | -0.7307 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.1580 | 0.1911 | 0.2051 | 0.1913 | 0.1953 | 0.1913 |
| P1-4 | 0.4250 | N/A | 0.4658 | 0.4234 | 0.4234 | 0.4236 | 0.4268 | 0.4235 |
| P1-5 | 0.3875 | 0.4591 | 0.3839 | 0.3963 | 0.4240 | 0.3965 | 0.4047 | 0.3963 |
| P2-3 | -0.0375 | N/A | -0.0608 | -0.0147 | 0.0130 | -0.0148 | -0.0103 | -0.0148 |
| **P2-4** | **0.2375** | **0.9105** | **0.3077** | **0.2323** | **0.2183** | **0.2323** | **0.2315** | **0.2322** |
| P2-5 | 0.2000 | 0.2607 | 0.2258 | 0.2051 | 0.2190 | 0.2052 | 0.2094 | 0.2051 |
| P2-6 | 0.2875 | N/A | 0.3283 | 0.3055 | 0.3332 | 0.3057 | 0.3099 | 0.3055 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P3-5 | 0.2375 | N/A | 0.2866 | 0.2198 | 0.2059 | 0.2200 | 0.2197 | 0.2198 |
| **P3-6** | **0.3250** | **0.9393** | **0.3890** | **0.3202** | **0.3202** | **0.3205** | **0.3203** | **0.3203** |
| P4-5 | -0.0375 | N/A | -0.0819 | -0.0271 | 0.0007 | -0.0271 | -0.0221 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.1025 | 0.1004 | 0.1142 | 0.1005 | 0.1005 | 0.1005 |

Table 5.43: Estimation errors of case 2, multi large, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0077 | 0.0108 | 0.0525 | 0.0113 | 0.0267 | 0.0111 |
| P2 | 0.1430 | 0.0370 | 0.0784 | 0.0372 | 0.0453 | 0.0367 |
| P3 | 0.1364 | 0.0452 | 0.0869 | 0.0447 | 0.0497 | 0.0452 |
| P4 | 0.1554 | 0.0172 | 0.0589 | 0.0170 | 0.0195 | 0.0172 |
| P5 | 0.0119 | 0.0064 | 0.0353 | 0.0059 | 0.0112 | 0.0064 |
| P6 | 0.1198 | 0.0262 | 0.0676 | 0.0267 | 0.0307 | 0.0262 |
| P1-2 | 0.0295 | 0.0036 | 0.0176 | 0.0038 | 0.0078 | 0.0038 |
| P1-4 | 0.0408 | 0.0016 | 0.0016 | 0.0014 | 0.0018 | 0.0015 |
| P1-5 | 0.0036 | 0.0088 | 0.0365 | 0.0090 | 0.0172 | 0.0088 |
| P2-3 | 0.0233 | 0.0228 | 0.0505 | 0.0227 | 0.0272 | 0.0227 |
| **P2-4** | **0.0702** | **0.0052** | **0.0192** | **0.0052** | **0.0060** | **0.0053** |
| P2-5 | 0.0258 | 0.0051 | 0.0190 | 0.0052 | 0.0094 | 0.0051 |
| P2-6 | 0.0408 | 0.0180 | 0.0457 | 0.0182 | 0.0224 | 0.0180 |
| P3-5 | 0.0491 | 0.0177 | 0.0316 | 0.0175 | 0.0178 | 0.0177 |
| **P3-6** | **0.0640** | **0.0048** | **0.0048** | **0.0045** | **0.0048** | **0.0047** |
| P4-5 | 0.0444 | 0.0104 | 0.0382 | 0.0104 | 0.0154 | 0.0104 |
| P5-6 | 0.0150 | 0.0129 | 0.0267 | 0.0130 | 0.0130 | 0.0129 |
| **SUM** | **0.9806** | **0.2537** | **0.6713** | **0.2536** | **0.3257** | **0.2538** |

The results have almost the same situation as full redundancy case, LTAV generates good results.

### Comparison

Full redundancy and median redundancy estimation with the same condition of error are presented in Table 5.44. From the results, it is very easy to find that full redundancy case has better estimation results for most estimators. For LTS, median redundancy have better results.

Table 5.44: Estimation errors of case 2, multi large, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.6555** | **0.8828** | **0.2527** | **0.6658** | **0.2659** | **0.3031** | **0.2528** |
| **SUM** | **N/A** | **0.9806** | **0.2537** | **0.6713** | **0.2536** | **0.3257** | **0.2538** |

#### 5.1.1.4 Case 3: Multiple interacting bad data

In case 2, the two bad data are not interacting with each other. In this part, two bad data that are interacting with each other: $P_{2-4}$ and $P_{2-5}$. Thus for bus 2, only the power injection are available for estimation, it should be harder than non-interacting case. Three types of bad data cases including two conforming and one non-conforming used here, the same as in multiple non-interacting case.

**Two reverse bad data**

The two reverse bad data are $P_{2-4} = -0.2105$ and $P_{2-5} = -0.2607$ by add a negative sign to original measurements. Full and median redundancy are used for performance evaluation.

**Full redundancy**

Full redundancy take all measurements as available value, the system state variables are presented in Table 5.45, Table 5.46 and Table 5.47 gives the estimation results and error with actual value.

Table 5.45: State Variable of case 3, multi reverse, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2255 | -0.1912 | -0.2033 | -0.1850 | -0.1937 | -0.1912 |
| $\theta 3$ | -0.2070 | -0.1764 | -0.2121 | -0.1486 | -0.1811 | -0.1763 |
| $\theta 4$ | -0.4054 | -0.4234 | -0.4246 | -0.4234 | -0.4276 | -0.4234 |
| $\theta 5$ | -0.4008 | -0.3962 | -0.4210 | -0.4019 | -0.4019 | -0.3962 |
| $\theta 6$ | -0.5274 | -0.4966 | -0.5336 | -0.4874 | -0.5015 | -0.4966 |

Table 5.46: Estimation Results of case 3, multi reverse, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0316 | 1.0108 | 1.0489 | 1.0103 | 1.0232 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.4129 | 0.5367 | 0.5748 | 0.5362 | 0.5434 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5327 | 0.5549 | 0.5217 | 0.6285 | 0.5538 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.5898 | -0.6828 | -0.6494 | -0.6833 | -0.6871 | -0.6828 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P5 | -0.7000 | -0.6936 | -0.6385 | -0.6936 | -0.7317 | -0.7651 | -0.7054 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7489 | -0.7260 | -0.7643 | -0.7267 | -0.7278 | -0.7262 |
| P1-2 | 0.1875 | 0.1788 | 0.2255 | 0.1912 | 0.2033 | 0.1850 | 0.1937 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4054 | 0.4234 | 0.4246 | 0.4234 | 0.4276 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4008 | 0.3962 | 0.4210 | 0.4019 | 0.4019 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0185 | -0.0148 | 0.0088 | -0.0364 | -0.0126 | -0.0149 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1798** | **0.2322** | **0.2213** | **0.2384** | **0.2338** | **0.2322** |
| **P2-5** | **0.2000** | **-0.2607** | **0.1752** | **0.2050** | **0.2177** | **0.2169** | **0.2081** | **0.2050** |
| P2-6 | 0.2875 | 0.3020 | 0.3019 | 0.3054 | 0.3303 | 0.3024 | 0.3078 | 0.3055 |
| P3-5 | 0.2375 | 0.2362 | 0.1937 | 0.2199 | 0.2090 | 0.2533 | 0.2208 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3204 | 0.3202 | 0.3215 | 0.3388 | 0.3204 | 0.3203 |
| P4-5 | -0.0375 | -0.0416 | -0.0046 | -0.0272 | -0.0035 | -0.0215 | -0.0257 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.1266 | 0.1004 | 0.1125 | 0.0855 | 0.0996 | 0.1004 |

Table 5.47: Estimation errors of case 3, multi reverse, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0316 | 0.0108 | 0.0489 | 0.0103 | 0.0232 | 0.0108 |
| P2 | 0.0367 | 0.0871 | 0.0367 | 0.0748 | 0.0362 | 0.0434 | 0.0367 |
| P3 | 0.0452 | 0.0673 | 0.0451 | 0.0783 | 0.0285 | 0.0462 | 0.0449 |
| P4 | 0.0172 | 0.1102 | 0.0172 | 0.0506 | 0.0167 | 0.0129 | 0.0172 |
| P5 | 0.0064 | 0.0615 | 0.0064 | 0.0317 | 0.0651 | 0.0054 | 0.0064 |
| P6 | 0.0262 | 0.0489 | 0.0260 | 0.0643 | 0.0267 | 0.0278 | 0.0262 |
| P1-2 | 0.0087 | 0.0380 | 0.0037 | 0.0158 | 0.0025 | 0.0062 | 0.0037 |
| P1-4 | 0.0069 | 0.0196 | 0.0016 | 0.0004 | 0.0016 | 0.0026 | 0.0016 |
| P1-5 | 0.0716 | 0.0133 | 0.0087 | 0.0335 | 0.0144 | 0.0144 | 0.0087 |
| P2-3 | 0.0554 | 0.0190 | 0.0227 | 0.0463 | 0.0011 | 0.0249 | 0.0226 |
| **P2-4** | **0.4480** | **0.0577** | **0.0053** | **0.0162** | **0.0009** | **0.0037** | **0.0053** |
| **P2-5** | **0.4607** | **0.0248** | **0.0050** | **0.0177** | **0.0169** | **0.0081** | **0.0050** |
| P2-6 | 0.0145 | 0.0144 | 0.0179 | 0.0428 | 0.0149 | 0.0203 | 0.0180 |
| P3-5 | 0.0013 | 0.0438 | 0.0176 | 0.0285 | 0.0158 | 0.0167 | 0.0176 |
| P3-6 | 0.0143 | 0.0046 | 0.0048 | 0.0035 | 0.0138 | 0.0046 | 0.0047 |
| P4-5 | 0.0041 | 0.0329 | 0.0103 | 0.0340 | 0.0160 | 0.0118 | 0.0103 |
| P5-6 | 0.0025 | 0.0391 | 0.0129 | 0.0250 | 0.0020 | 0.0121 | 0.0129 |
| **SUM** | **1.2305** | **0.7138** | **0.2527** | **0.6124** | **0.2832** | **0.2841** | **0.2526** |

All estimators generate better overall results than measurements, WLS and LMS have relative bad results compared with others since the estimation of $P_2$ and $P_4$ are not perfect.

### Median redundancy

System state variables, estimation results and error are presented in the following three tables:

Table 5.48: State Variable of case 3, multi reverse, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2337 | -0.1912 | -0.2037 | -0.1912 | -0.1968 | -0.1913 |
| $\theta 3$ | -0.2193 | -0.1765 | -0.2140 | -0.1763 | -0.1841 | -0.1765 |
| $\theta 4$ | -0.4030 | -0.4234 | -0.4234 | -0.4232 | -0.4269 | -0.4235 |
| $\theta 5$ | -0.4056 | -0.3963 | -0.4214 | -0.3961 | -0.4034 | -0.3963 |
| $\theta 6$ | -0.5417 | -0.4967 | -0.5344 | -0.4968 | -0.5064 | -0.4968 |

Table 5.49: Estimation Results of case 3, multi reverse, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0423 | 1.0109 | 1.0485 | 1.0106 | 1.0271 | 1.0111 |
| P2 | 0.5000 | 0.5367 | 0.4013 | 0.5367 | 0.5744 | 0.5362 | 0.5368 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5230 | 0.5548 | 0.5174 | 0.5553 | 0.5544 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.5697 | -0.6826 | -0.6451 | -0.6823 | -0.6804 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.6304 | -0.6936 | -0.7313 | -0.6931 | -0.7028 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7665 | -0.7262 | -0.7639 | -0.7267 | -0.7350 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.2337 | 0.1912 | 0.2037 | 0.1912 | 0.1968 | 0.1913 |
| P1-4 | 0.4250 | N/A | 0.4030 | 0.4234 | 0.4234 | 0.4232 | 0.4269 | 0.4235 |
| P1-5 | 0.3875 | 0.4591 | 0.4056 | 0.3963 | 0.4214 | 0.3961 | 0.4034 | 0.3963 |
| P2-3 | -0.0375 | N/A | -0.0143 | -0.0148 | 0.0103 | -0.0150 | -0.0127 | -0.0148 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1693** | **0.2321** | **0.2197** | **0.2320** | **0.2301** | **0.2322** |
| **P2-5** | **0.2000** | **-0.2607** | **0.1719** | **0.2050** | **0.2176** | **0.2049** | **0.2066** | **0.2051** |
| P2-6 | 0.2875 | N/A | 0.3080 | 0.3055 | 0.3306 | 0.3055 | 0.3096 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.1863 | 0.2198 | 0.2073 | 0.2198 | 0.2193 | 0.2198 |
| P3-6 | 0.3250 | 0.3393 | 0.3224 | 0.3203 | 0.3203 | 0.3205 | 0.3224 | 0.3202 |
| P4-5 | -0.0375 | N/A | 0.0026 | -0.0271 | -0.0020 | -0.0271 | -0.0235 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.1361 | 0.1004 | 0.1130 | 0.1007 | 0.1030 | 0.1004 |

Table 5.50: Estimation errors of case 3, multi reverse, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0423 | 0.0109 | 0.0485 | 0.0106 | 0.0271 | 0.0111 |
| P2 | 0.0987 | 0.0367 | 0.0744 | 0.0362 | 0.0368 | 0.0367 |
| P3 | 0.0770 | 0.0452 | 0.0826 | 0.0447 | 0.0456 | 0.0452 |
| P4 | 0.1303 | 0.0174 | 0.0549 | 0.0177 | 0.0196 | 0.0172 |
| P5 | 0.0696 | 0.0064 | 0.0313 | 0.0069 | 0.0028 | 0.0064 |
| P6 | 0.0665 | 0.0262 | 0.0639 | 0.0267 | 0.0350 | 0.0262 |
| P1-2 | 0.0462 | 0.0037 | 0.0162 | 0.0037 | 0.0093 | 0.0038 |
| P1-4 | 0.0220 | 0.0016 | 0.0016 | 0.0018 | 0.0019 | 0.0015 |
| P1-5 | 0.0181 | 0.0088 | 0.0339 | 0.0086 | 0.0159 | 0.0088 |
| P2-3 | 0.0232 | 0.0227 | 0.0478 | 0.0225 | 0.0248 | 0.0227 |
| **P2-4** | **0.0682** | **0.0054** | **0.0178** | **0.0055** | **0.0074** | **0.0053** |
| **P2-5** | **0.0281** | **0.0050** | **0.0176** | **0.0049** | **0.0066** | **0.0051** |
| P2-6 | 0.0205 | 0.0180 | 0.0431 | 0.0180 | 0.0221 | 0.0180 |
| P3-5 | 0.0512 | 0.0177 | 0.0302 | 0.0177 | 0.0182 | 0.0177 |
| P3-6 | 0.0026 | 0.0047 | 0.0047 | 0.0045 | 0.0026 | 0.0048 |
| P4-5 | 0.0401 | 0.0104 | 0.0355 | 0.0104 | 0.0140 | 0.0104 |
| P5-6 | 0.0486 | 0.0129 | 0.0255 | 0.0132 | 0.0155 | 0.0129 |
| **SUM** | **0.8531** | **0.2539** | **0.6296** | **0.2537** | **0.3052** | **0.2538** |

The results are similar to full redundancy case, WLS and LMS perform bad in estimation of power flow relating to the bad data. Other estimators generate satisfied results.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.51. Full redundancy cases have better estimation results for most estimators except LTS.

Table 5.51: Estimation errors of case 3, multi reverse, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.2305** | **0.7138** | **0.2527** | **0.6124** | **0.2832** | **0.2841** | **0.2526** |
| **SUM** | **N/A** | **0.8531** | **0.2539** | **0.6296** | **0.2537** | **0.3052** | **0.2538** |

## Two non-conforming bad data

One large and one reverse bad data consist the non-conforming case, full and median redundancy case are considered.

### Full redundancy

The following three tables present state variables, estimation results and error respectively in detail.

Table 5.52: State Variable of case 3, non-conforming, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1964 | -0.1912 | -0.2033 | -0.1850 | -0.1937 | -0.1912 |
| $\theta 3$ | -0.2070 | -0.1764 | -0.2121 | -0.1486 | -0.1811 | -0.1763 |
| $\theta 4$ | -0.4054 | -0.4234 | -0.4246 | -0.4234 | -0.4276 | -0.4234 |
| $\theta 5$ | -0.4298 | -0.3962 | -0.4210 | -0.4019 | -0.4019 | -0.3962 |
| $\theta 6$ | -0.5274 | -0.4966 | -0.5336 | -0.4874 | -0.5015 | -0.4966 |

Table 5.53: Estimation Results of case 3, non-conforming, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0316 | 1.0108 | 1.0489 | 1.0103 | 1.0232 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.5874 | 0.5367 | 0.5748 | 0.5362 | 0.5434 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5327 | 0.5549 | 0.5217 | 0.6285 | 0.5538 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.5898 | -0.6828 | -0.6494 | -0.6833 | -0.6871 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.8130 | -0.6936 | -0.7317 | -0.7651 | -0.7054 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7489 | -0.7260 | -0.7643 | -0.7267 | -0.7278 | -0.7262 |
| P1-2 | 0.1875 | 0.1788 | 0.1964 | 0.1912 | 0.2033 | 0.1850 | 0.1937 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4054 | 0.4234 | 0.4246 | 0.4234 | 0.4276 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4298 | 0.3962 | 0.4210 | 0.4019 | 0.4019 | 0.3962 |

| P2-3 | -0.0375 | 0.0179 | 0.0106 | -0.0148 | 0.0088 | -0.0364 | -0.0126 | -0.0149 |
|---|---|---|---|---|---|---|---|---|
| **P2-4** | **0.2375** | **-0.2105** | **0.2089** | **0.2322** | **0.2213** | **0.2384** | **0.2338** | **0.2322** |
| **P2-5** | **0.2000** | **0.9607** | **0.2334** | **0.2050** | **0.2177** | **0.2169** | **0.2081** | **0.2050** |
| P2-6 | 0.2875 | 0.3020 | 0.3310 | 0.3054 | 0.3303 | 0.3024 | 0.3078 | 0.3055 |
| P3-5 | 0.2375 | 0.2362 | 0.2228 | 0.2199 | 0.2090 | 0.2533 | 0.2208 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3204 | 0.3202 | 0.3215 | 0.3388 | 0.3204 | 0.3203 |
| P4-5 | -0.0375 | -0.0416 | 0.0245 | -0.0272 | -0.0035 | -0.0215 | -0.0257 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.0976 | 0.1004 | 0.1125 | 0.0855 | 0.0996 | 0.1004 |

Table 5.54: Estimation errors of case 3, non-conforming, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0316 | 0.0108 | 0.0489 | 0.0103 | 0.0232 | 0.0108 |
| P2 | 0.0367 | 0.0874 | 0.0367 | 0.0748 | 0.0362 | 0.0434 | 0.0367 |
| P3 | 0.0452 | 0.0673 | 0.0451 | 0.0783 | 0.0285 | 0.0462 | 0.0449 |
| P4 | 0.0172 | 0.1102 | 0.0172 | 0.0506 | 0.0167 | 0.0129 | 0.0172 |
| P5 | 0.0064 | 0.1130 | 0.0064 | 0.0317 | 0.0651 | 0.0054 | 0.0064 |
| P6 | 0.0262 | 0.0489 | 0.0260 | 0.0643 | 0.0267 | 0.0278 | 0.0262 |
| P1-2 | 0.0087 | 0.0089 | 0.0037 | 0.0158 | 0.0025 | 0.0062 | 0.0037 |
| P1-4 | 0.0069 | 0.0196 | 0.0016 | 0.0004 | 0.0016 | 0.0026 | 0.0016 |
| P1-5 | 0.0716 | 0.0423 | 0.0087 | 0.0335 | 0.0144 | 0.0144 | 0.0087 |
| P2-3 | 0.0554 | 0.0481 | 0.0227 | 0.0463 | 0.0011 | 0.0249 | 0.0226 |
| **P2-4** | **0.4480** | **0.0286** | **0.0053** | **0.0162** | **0.0009** | **0.0037** | **0.0053** |
| **P2-5** | **0.7607** | **0.0334** | **0.0050** | **0.0177** | **0.0169** | **0.0081** | **0.0050** |
| P2-6 | 0.0145 | 0.0435 | 0.0179 | 0.0428 | 0.0149 | 0.0203 | 0.0180 |
| P3-5 | 0.0013 | 0.0147 | 0.0176 | 0.0285 | 0.0158 | 0.0167 | 0.0176 |
| P3-6 | 0.0143 | 0.0046 | 0.0048 | 0.0035 | 0.0138 | 0.0046 | 0.0047 |
| P4-5 | 0.0041 | 0.0620 | 0.0103 | 0.0340 | 0.0160 | 0.0118 | 0.0103 |
| P5-6 | 0.0025 | 0.0101 | 0.0129 | 0.0250 | 0.0020 | 0.0121 | 0.0129 |
| **SUM** | **1.5305** | **0.7742** | **0.2527** | **0.6124** | **0.2832** | **0.2841** | **0.2526** |

All estimator generate better than measurements, WLS and LMS have relative bad results in estimation of power injection.

### Median redundancy

Same as previous cases, all results and comparison are shown in the following three tables:

Table 5.55: State Variable of case 3, non-conforming, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.2005 | -0.1912 | -0.2037 | -0.1911 | -0.1968 | -0.1913 |
| $\theta 3$ | -0.2175 | -0.1765 | -0.2140 | -0.1763 | -0.1841 | -0.1765 |
| $\theta 4$ | -0.3993 | -0.4234 | -0.4234 | -0.4232 | -0.4269 | -0.4235 |
| $\theta 5$ | -0.4351 | -0.3963 | -0.4214 | -0.3963 | -0.4034 | -0.3963 |
| $\theta 6$ | -0.5399 | -0.4967 | -0.5344 | -0.4968 | -0.5064 | -0.4968 |

Table 5.56: Estimation Results of case 3, non-conforming, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0349 | 1.0109 | 1.0485 | 1.0106 | 1.0271 | 1.0111 |
| P2 | 0.5000 | 0.5367 | 0.5895 | 0.5367 | 0.5744 | 0.5372 | 0.5368 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5230 | 0.5548 | 0.5174 | 0.5553 | 0.5544 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.5624 | -0.6826 | -0.6451 | -0.6823 | -0.6804 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.8185 | -0.6936 | -0.7313 | -0.6941 | -0.7028 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.7665 | -0.7262 | -0.7639 | -0.7267 | -0.7350 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.2005 | 0.1912 | 0.2037 | 0.1911 | 0.1968 | 0.1913 |
| P1-4 | 0.4250 | N/A | 0.3993 | 0.4234 | 0.4234 | 0.4232 | 0.4269 | 0.4235 |
| P1-5 | 0.3875 | 0.4591 | 0.4351 | 0.3963 | 0.4214 | 0.3963 | 0.4034 | 0.3963 |
| P2-3 | -0.0375 | N/A | 0.0170 | -0.0148 | 0.0103 | -0.0148 | -0.0127 | -0.0148 |
| **P2-4** | **0.2375** | **-0.2105** | **0.1989** | **0.2321** | **0.2197** | **0.2321** | **0.2301** | **0.2322** |
| **P2-5** | **0.2000** | **0.9607** | **0.2347** | **0.2051** | **0.2176** | **0.2052** | **0.2066** | **0.2051** |
| P2-6 | 0.2875 | N/A | 0.3394 | 0.3055 | 0.3306 | 0.3057 | 0.3096 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.2176 | 0.2198 | 0.2073 | 0.2200 | 0.2193 | 0.2198 |
| P3-6 | 0.3250 | 0.3393 | 0.3224 | 0.3203 | 0.3203 | 0.3205 | 0.3224 | 0.3202 |
| P4-5 | -0.0375 | N/A | 0.0358 | -0.0271 | -0.0020 | -0.0269 | -0.0235 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.1047 | 0.1004 | 0.1130 | 0.1005 | 0.1030 | 0.1004 |

Table 5.57: Estimation errors of case 3, non-conforming, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0349 | 0.0109 | 0.0485 | 0.0106 | 0.0271 | 0.0111 |
| P2 | 0.0895 | 0.0367 | 0.0744 | 0.0372 | 0.0367 | 0.0367 |
| P3 | 0.0770 | 0.0452 | 0.0826 | 0.0447 | 0.0456 | 0.0452 |
| P4 | 0.1376 | 0.0174 | 0.0549 | 0.0177 | 0.0196 | 0.0172 |
| P5 | 0.1185 | 0.0064 | 0.0313 | 0.0059 | 0.0028 | 0.0064 |
| P6 | 0.0665 | 0.0262 | 0.0639 | 0.0267 | 0.0350 | 0.0262 |
| P1-2 | 0.0130 | 0.0037 | 0.0162 | 0.0036 | 0.0093 | 0.0038 |
| P1-4 | 0.0257 | 0.0016 | 0.0016 | 0.0018 | 0.0019 | 0.0015 |
| P1-5 | 0.0476 | 0.0088 | 0.0339 | 0.0088 | 0.0159 | 0.0088 |
| P2-3 | 0.0545 | 0.0227 | 0.0478 | 0.0227 | 0.0248 | 0.0227 |
| **P2-4** | **0.0386** | **0.0054** | **0.0178** | **0.0054** | **0.0074** | **0.0053** |
| **P2-5** | **0.0347** | **0.0051** | **0.0176** | **0.0052** | **0.0066** | **0.0051** |
| P2-6 | 0.0519 | 0.0180 | 0.0431 | 0.0182 | 0.0221 | 0.0180 |
| P3-5 | 0.0199 | 0.0177 | 0.0302 | 0.0175 | 0.0182 | 0.0177 |
| P3-6 | 0.0026 | 0.0047 | 0.0047 | 0.0045 | 0.0026 | 0.0048 |
| P4-5 | 0.0733 | 0.0104 | 0.0355 | 0.0106 | 0.0140 | 0.0104 |
| P5-6 | 0.0172 | 0.0129 | 0.0255 | 0.0130 | 0.0155 | 0.0129 |
| **SUM** | **0.9032** | **0.2539** | **0.6296** | **0.2540** | **0.3052** | **0.2538** |

Similar results as in full redundancy case, WLS and LMS generate relative bad results with unsatisfied power injection estimation connected with bad data.

### Comparison

The results are compared in Table 5.58, median redundancy case have better results with LMR, full redundancy have advantage with all other estimators.

Table 5.58: Estimation errors of case 3, non-conforming, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.5305** | **0.7742** | **0.2527** | **0.6124** | **0.2832** | **0.2841** | **0.2526** |
| **SUM** | **N/A** | **0.9032** | **0.2539** | **0.6296** | **0.2540** | **0.3052** | **0.2538** |

### Two large bad data

The two large bad data are simulated just by changing the first number to 9, with $P_{2-4} = 0.9105$ and $P_{2-5} = 0.9607$. Two cases are considered.

#### Full redundancy

Detail results of estimation are presented in the following three tables:

Table 5.59: State Variable of case 3, multi large, 6-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1444 | -0.1912 | -0.2033 | -0.1898 | -0.1937 | -0.1912 |
| $\theta 3$ | -0.1416 | -0.1764 | -0.2121 | -0.1720 | -0.1811 | -0.1763 |
| $\theta 4$ | -0.4614 | -0.4234 | -0.4246 | -0.4314 | -0.4276 | -0.4234 |
| $\theta 5$ | -0.4045 | -0.3962 | -0.4210 | -0.3893 | -0.4019 | -0.3962 |
| $\theta 6$ | -0.4620 | -0.4966 | -0.5336 | -0.4923 | -0.5015 | -0.4966 |

Table 5.60: Estimation Results of case 3, multi large, 6-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0103 | 1.0108 | 1.0489 | 1.0105 | 1.0232 | 1.0108 |
| P2 | 0.5000 | 0.5367 | 0.7475 | 0.5368 | 0.5748 | 0.5362 | 0.5434 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.5860 | 0.5549 | 0.5217 | 0.5553 | 0.5538 | 0.5551 |
| P4 | -0.7000 | -0.6828 | -0.8354 | -0.6828 | -0.6494 | -0.7151 | -0.6871 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.8130 | -0.6936 | -0.7317 | -0.6611 | -0.7054 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.6955 | -0.7261 | -0.7643 | -0.7257 | -0.7278 | -0.7262 |
| P1-2 | 0.1875 | 0.1788 | 0.1444 | 0.1912 | 0.2033 | 0.1898 | 0.1937 | 0.1912 |
| P1-4 | 0.4250 | 0.4319 | 0.4614 | 0.4234 | 0.4246 | 0.4314 | 0.4276 | 0.4234 |
| P1-5 | 0.3875 | 0.4591 | 0.4045 | 0.3962 | 0.4210 | 0.3893 | 0.4019 | 0.3962 |
| P2-3 | -0.0375 | 0.0179 | -0.0028 | -0.0148 | 0.0088 | -0.0177 | -0.0126 | -0.0149 |
| **P2-4** | **0.2375** | **0.9105** | **0.3170** | **0.2322** | **0.2213** | **0.2416** | **0.2338** | **0.2322** |
| **P2-5** | **0.2000** | **0.9607** | **0.2601** | **0.2051** | **0.2177** | **0.1996** | **0.2081** | **0.2050** |
| P2-6 | 0.2875 | 0.3020 | 0.3176 | 0.3055 | 0.3303 | 0.3025 | 0.3078 | 0.3055 |
| P3-5 | 0.2375 | 0.2362 | 0.2629 | 0.2199 | 0.2090 | 0.2173 | 0.2208 | 0.2199 |
| P3-6 | 0.3250 | 0.3393 | 0.3204 | 0.3202 | 0.3215 | 0.3202 | 0.3204 | 0.3203 |
| P4-5 | -0.0375 | -0.0416 | -0.0569 | -0.0272 | -0.0035 | -0.0421 | -0.0257 | -0.0272 |
| P5-6 | 0.0875 | 0.0850 | 0.0575 | 0.1004 | 0.1125 | 0.1030 | 0.0996 | 0.1004 |

Table 5.61: Estimation errors of case 3, multi large, 6-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0103 | 0.0108 | 0.0489 | 0.0105 | 0.0232 | 0.0108 |
| P2 | 0.0367 | 0.2475 | 0.0368 | 0.0748 | 0.0362 | 0.0434 | 0.0367 |
| P3 | 0.0452 | 0.0140 | 0.0451 | 0.0783 | 0.0447 | 0.0462 | 0.0449 |
| P4 | 0.0172 | 0.1354 | 0.0172 | 0.0506 | 0.0151 | 0.0129 | 0.0172 |
| P5 | 0.0064 | 0.1130 | 0.0064 | 0.0317 | 0.0389 | 0.0054 | 0.0064 |
| P6 | 0.0262 | 0.0045 | 0.0261 | 0.0643 | 0.0257 | 0.0278 | 0.0262 |
| P1-2 | 0.0087 | 0.0431 | 0.0037 | 0.0158 | 0.0023 | 0.0062 | 0.0037 |
| P1-4 | 0.0069 | 0.0364 | 0.0016 | 0.0004 | 0.0064 | 0.0026 | 0.0016 |
| P1-5 | 0.0716 | 0.0170 | 0.0087 | 0.0335 | 0.0018 | 0.0144 | 0.0087 |
| P2-3 | 0.0554 | 0.0347 | 0.0227 | 0.0463 | 0.0198 | 0.0249 | 0.0226 |
| **P2-4** | **0.6730** | **0.0795** | **0.0053** | **0.0162** | **0.0041** | **0.0037** | **0.0053** |
| **P2-5** | **0.7607** | **0.0601** | **0.0051** | **0.0177** | **0.0004** | **0.0081** | **0.0050** |
| P2-6 | 0.0145 | 0.0301 | 0.0180 | 0.0428 | 0.0150 | 0.0203 | 0.0180 |
| P3-5 | 0.0013 | 0.0254 | 0.0176 | 0.0285 | 0.0202 | 0.0167 | 0.0176 |
| P3-6 | 0.0143 | 0.0046 | 0.0048 | 0.0035 | 0.0048 | 0.0046 | 0.0047 |
| P4-5 | 0.0041 | 0.0194 | 0.0103 | 0.0340 | 0.0046 | 0.0118 | 0.0103 |
| P5-6 | 0.0025 | 0.0300 | 0.0129 | 0.0250 | 0.0155 | 0.0121 | 0.0129 |
| **SUM** | **1.7555** | **0.9049** | **0.2530** | **0.6124** | **0.2659** | **0.2841** | **0.2526** |

Robust estimators generate much better results than measurements by correct the bad data, WLS's performs bad both at bad data and connected power injection estimation.

### Median redundancy

Table 5.62 gives system state variables, Table 5.63 presents the estimation results, the comparison results with actual value are shown in Table 5.64.

Table 5.62: State variable of case 3, multi large, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1390 | -0.1911 | -0.2037 | -0.1913 | -0.1968 | -0.1913 |
| $\theta 3$ | -0.1296 | -0.1764 | -0.2141 | -0.1765 | -0.1841 | -0.1765 |
| $\theta 4$ | -0.4637 | -0.4234 | -0.4234 | -0.4236 | -0.4269 | -0.4235 |
| $\theta 5$ | -0.4008 | -0.3962 | -0.4214 | -0.3965 | -0.4034 | -0.3963 |
| $\theta 6$ | -0.4519 | -0.4967 | -0.5343 | -0.4970 | -0.5064 | -0.4968 |

Table 5.63: Estimation Results of case 3, multi large, 6-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0000 | 1.0108 | 1.0034 | 1.0108 | 1.0485 | 1.0113 | 1.0271 | 1.0111 |
| P2 | 0.5000 | 0.5367 | 0.7509 | 0.5370 | 0.5744 | 0.5372 | 0.5368 | 0.5367 |
| P3 | 0.6000 | 0.5548 | 0.6030 | 0.5548 | 0.5171 | 0.5553 | 0.5544 | 0.5548 |
| P4 | -0.7000 | -0.6828 | -0.8512 | -0.6828 | -0.6451 | -0.6830 | -0.6804 | -0.6828 |
| P5 | -0.7000 | -0.6936 | -0.8198 | -0.6936 | -0.7313 | -0.6941 | -0.7028 | -0.6936 |
| P6 | -0.7000 | -0.7262 | -0.6864 | -0.7262 | -0.7636 | -0.7267 | -0.7350 | -0.7262 |
| P1-2 | 0.1875 | N/A | 0.1390 | 0.1911 | 0.2037 | 0.1913 | 0.1968 | 0.1913 |
| P1-4 | 0.4250 | N/A | 0.4637 | 0.4234 | 0.4234 | 0.4236 | 0.4269 | 0.4235 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P1-5 | 0.3875 | 0.4591 | 0.4008 | 0.3962 | 0.4214 | 0.3965 | 0.4034 | 0.3963 |
| P2-3 | -0.0375 | N/A | -0.0095 | -0.0147 | 0.0104 | -0.0148 | -0.0127 | -0.0148 |
| **P2-4** | **0.2375** | **0.9105** | **0.3247** | **0.2323** | **0.2197** | **0.2323** | **0.2301** | **0.2322** |
| **P2-5** | **0.2000** | **0.9607** | **0.2618** | **0.2051** | **0.2176** | **0.2052** | **0.2066** | **0.2051** |
| P2-6 | 0.2875 | N/A | 0.3129 | 0.3055 | 0.3305 | 0.3057 | 0.3096 | 0.3055 |
| P3-5 | 0.2375 | N/A | 0.2712 | 0.2198 | 0.2073 | 0.2200 | 0.2193 | 0.2198 |
| P3-6 | 0.3250 | 0.3393 | 0.3224 | 0.3203 | 0.3202 | 0.3205 | 0.3224 | 0.3202 |
| P4-5 | -0.0375 | N/A | -0.0629 | -0.0271 | -0.0020 | -0.0271 | -0.0235 | -0.0271 |
| P5-6 | 0.0875 | N/A | 0.0511 | 0.1004 | 0.1129 | 0.1005 | 0.1030 | 0.1004 |

Table 5.64: Estimation errors of case 3, multi large, 6-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0034 | 0.0108 | 0.0485 | 0.0113 | 0.0271 | 0.0111 |
| P2 | 0.2509 | 0.0370 | 0.0744 | 0.0372 | 0.0368 | 0.0367 |
| P3 | 0.0030 | 0.0452 | 0.0829 | 0.0447 | 0.0456 | 0.0452 |
| P4 | 0.1512 | 0.0172 | 0.0549 | 0.0170 | 0.0196 | 0.0172 |
| P5 | 0.1198 | 0.0064 | 0.0313 | 0.0059 | 0.0028 | 0.0064 |
| P6 | 0.0136 | 0.0262 | 0.0636 | 0.0267 | 0.0350 | 0.0262 |
| P1-2 | 0.0485 | 0.0036 | 0.0162 | 0.0038 | 0.0093 | 0.0038 |
| P1-4 | 0.0387 | 0.0016 | 0.0016 | 0.0014 | 0.0019 | 0.0015 |
| P1-5 | 0.0133 | 0.0087 | 0.0339 | 0.0090 | 0.0159 | 0.0088 |
| P2-3 | 0.0280 | 0.0228 | 0.0479 | 0.0227 | 0.0248 | 0.0227 |
| **P2-4** | **0.0872** | **0.0052** | **0.0178** | **0.0052** | **0.0074** | **0.0053** |
| **P2-5** | **0.0618** | **0.0051** | **0.0176** | **0.0052** | **0.0066** | **0.0051** |
| P2-6 | 0.0254 | 0.0180 | 0.0430 | 0.0182 | 0.0221 | 0.0180 |
| P3-5 | 0.0337 | 0.0177 | 0.0302 | 0.0175 | 0.0182 | 0.0177 |
| P3-6 | 0.0026 | 0.0047 | 0.0048 | 0.0045 | 0.0026 | 0.0048 |
| P4-5 | 0.0254 | 0.0104 | 0.0355 | 0.0104 | 0.0140 | 0.0104 |
| P5-6 | 0.0364 | 0.0129 | 0.0254 | 0.0130 | 0.0155 | 0.0129 |
| **SUM** | **0.9428** | **0.2536** | **0.6298** | **0.2536** | **0.3052** | **0.2538** |

The results are similar with full redundancy case.

### Comparison

The comparison are presented in Table 5.65. From the results, it is very easy to find that full redundancy cases have better estimation results for all estimators except LMR.

Table 5.65: Estimation errors of case 3, multi large, 6-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.7555** | **0.9049** | **0.2530** | **0.6124** | **0.2659** | **0.2841** | **0.2526** |
| **SUM** | **N/A** | **0.9428** | **0.2536** | **0.6298** | **0.2536** | **0.3052** | **0.2538** |

#### 5.1.1.5 Summary

Table 5.66 and Table 5.67 presents the comparison between each estimator for full and median redundancy cases respectively.

Table 5.66: Comparison of full redundancy 6-bus estimation between estimators

| N.o | Type | Meas. | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|---|---|
| **Case 0** | **Random** | 0.4095 | 0.3121 | 0.2527 | 0.3931 | 0.2832 | 0.6276 | 0.2528 |
| **Case 1** | **Reverse** | 0.8305 | 0.5661 | 0.2527 | 0.4808 | 0.2832 | 0.2463 | 0.2526 |
| | **Large** | 1.0555 | 0.5715 | 0.2527 | 0.4808 | 0.3357 | 0.2356 | 0.2526 |
| **Case 2** | **Reverse** | 1.4805 | 0.8362 | 0.2528 | 0.6650 | 0.2659 | 0.3031 | 0.2528 |
| | **Non-conf** | 1.4305 | 0.7222 | 0.2527 | 0.6658 | 0.2659 | 0.3031 | 0.2528 |
| | **Large** | 1.6555 | 0.8828 | 0.2527 | 0.6658 | 0.2659 | 0.3031 | 0.2528 |
| **Case 3** | **Reverse** | 1.2305 | 0.7138 | 0.2527 | 0.6124 | 0.2832 | 0.2841 | 0.2526 |
| | **Non-conf** | 1.5305 | 0.7742 | 0.2527 | 0.6124 | 0.2832 | 0.2841 | 0.2526 |
| | **Large** | 1.7555 | 0.9049 | 0.2530 | 0.6124 | 0.2659 | 0.2841 | 0.2526 |

Table 5.67: Comparison of median redundancy 6-bus estimation between estimators

| N.o | Type | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|---|
| **Case 0** | **Random** | 0.3368 | 0.2539 | 0.2997 | 0.2540 | 0.7195 | 0.2540 |
| **Case 1** | **Reverse** | 0.6886 | 0.2539 | 0.5352 | 0.2540 | 0.2589 | 0.2526 |
| | **Large** | 0.6033 | 0.2536 | 0.5352 | 0.2536 | 0.4503 | 0.2526 |
| **Case 2** | **Reverse** | 0.9452 | 0.2539 | 0.6711 | 0.2545 | 0.3257 | 0.2538 |
| | **Non-conf** | 0.8626 | 0.2539 | 0.6713 | 0.2540 | 0.3257 | 0.2538 |
| | **Large** | 0.9806 | 0.2537 | 0.6713 | 0.2536 | 0.3257 | 0.2538 |
| **Case 3** | **Reverse** | 0.8531 | 0.2539 | 0.6296 | 0.2537 | 0.3052 | 0.2538 |
| | **Non-conf** | 0.9032 | 0.2539 | 0.6296 | 0.2540 | 0.3052 | 0.2538 |
| | **Large** | 0.9428 | 0.2536 | 0.6298 | 0.2536 | 0.3052 | 0.2538 |

As illustrated by the two tables, in all the 18 cases, LAV, LMR and LTAV generate best estimation, LTS is a little fall behind but very few difference for cases with bad data, LMS and WLS's results are not that satisfied. Comparing full and median redundancy case, the former one generate better results in most cases.

WLS, as the basic algorithm with few robust feature, generate relative bad results as expected. The estimation from LMS often cannot generate power flow relating to bad data precisely. LTS is unusual, it generate bad results on the base case while for other cases with bad data, the estimation are quite satisfied. The

reason is that for the base case, measurements related with bus 5 have large error compared with others, leading to more trimming and generate bad estimation. For cases with bad data, the errors are much smaller compared with bad data, so it can have good estimation results. Some cases share the same results, especially LTS and LTAV, the reason is that they trim the same data and then have the same raw data for estimation.

As for LTAV, it's shown to be the best estimator in many cases among the test conducted above, for those LTAV does not generate best results, they are only very little difference with the best results.

### 5.1.2 6-bus AC state estimation

Base on the theory in section 2.2.1, all actual value are used for the power flow compared with the simplification on DC power flow. All the data used in this section are presented in Appendix 6.2.

For consistency, the same four different cases are conducted as in DC part, also both median and full redundancy are considered with the same kind of error at the same location in all the four major categories: DC & AC full & median redundancy.

For estimators, only four are used in the comparison: WLS, LAV, LTS and LTAV. The reason is that both LMS and LMR relay on mixed linear programming in each iteration but the convergence speed is far beyond satisfaction even it's just a very small system. LTS and LTAV can be revised to used the conven-

tional methods rather than mixed linear programming, the method for LTAV are presented in section 4.4.1 in detail.

The parameters used in the estimation are:

1. Tolerance of iterative loop for all estimators: 0.0001;

2. Fixed trimmed number for LTS in the sample precess: $nt = 0$; for LTAV: $nt = 0$;

3. Sample set for LTS: $nsample = 1$; for LTAV: $nsample = 1$;

#### 5.1.2.1 Case 0: basic case with only random noise

The base case only have random small noise rather than large error. Full and median redundancy scenarios are presented.

**Full redundancy**

Table 5.68 shows the actual values of real power injection and flow from load flow, the data are taken from [33]. Random noised with normal distribution with $\sigma = 0.01665$ for voltage measurements and $\sigma = 0.05$ for power measurements, which are also taken form [33], are generated and added to the actual value to simulate the measurements in the same table. As for full redundancy case, all the data are used for estimation.

Table 5.68: Raw data for 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | P2-1 | -0.2780 | -0.3490 | Q1-5 | 0.1130 | 0.0940 |
| V2 | 1.0500 | 1.0339 | P2-4 | 0.3310 | 0.3280 | Q2-1 | 0.1280 | 0.0970 |
| V3 | 1.0700 | 1.0900 | P2-5 | 0.1550 | 0.1740 | Q2-4 | 0.4610 | 0.3830 |
| V4 | 0.9896 | 0.9813 | P2-6 | 0.2620 | 0.2230 | Q2-5 | 0.1540 | 0.2200 |
| V5 | 0.9857 | 0.9791 | P2-3 | 0.0290 | 0.0860 | Q2-6 | 0.1240 | 0.1500 |
| V6 | 1.0043 | 0.9952 | P3-2 | -0.0290 | -0.0210 | Q2-3 | -0.1230 | -0.1190 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P1 | 1.0790 | 1.1310 | P3-5 | 0.1910 | 0.1770 | Q3-2 | 0.0570 | 0.1020 |
| P2 | 0.5000 | 0.4840 | P3-6 | 0.4380 | 0.4330 | Q3-5 | 0.2320 | 0.2390 |
| P3 | 0.6000 | 0.5510 | P4-1 | -0.4250 | -0.4010 | Q3-6 | 0.6070 | 0.5830 |
| P4 | -0.7000 | -0.7180 | P4-2 | -0.3160 | -0.2980 | Q4-1 | -0.1990 | -0.1430 |
| P5 | -0.7000 | -0.7200 | P4-5 | 0.0410 | 0.0070 | Q4-2 | -0.4510 | -0.4430 |
| P6 | -0.7000 | -0.7230 | P5-4 | -0.0400 | -0.0210 | Q4-5 | -0.0490 | -0.1740 |
| Q1 | 0.1600 | 0.2020 | P5-1 | -0.3450 | -0.3660 | Q5-4 | -0.0280 | -0.0150 |
| Q2 | 0.7440 | 0.7190 | P5-2 | -0.1500 | -0.1170 | Q5-1 | -0.1350 | -0.1750 |
| Q3 | 0.8960 | 0.9060 | P5-3 | -0.1800 | -0.2510 | Q5-2 | -0.1800 | -0.2220 |
| Q4 | -0.7000 | -0.7190 | P5-6 | 0.0160 | -0.0210 | Q5-3 | -0.2610 | -0.1990 |
| Q5 | -0.7000 | -0.6770 | P6-5 | -0.0160 | 0.0100 | Q5-6 | -0.0970 | -0.0080 |
| Q6 | -0.7000 | -0.6090 | P6-2 | 0.2570 | -0.1960 | Q6-5 | 0.0390 | 0.0290 |
| P1-2 | 0.2870 | 0.3150 | P6-3 | -0.4280 | -0.4680 | Q6-2 | -0.1600 | -0.2230 |
| P1-4 | 0.4360 | 0.3890 | Q1-2 | -0.1540 | -0.1320 | Q6-3 | -0.5790 | -0.5110 |
| P1-5 | 0.3560 | 0.3570 | Q1-4 | 0.2010 | 0.2120 | | | |

Table 5.69 shows the state variables. The first columns of Table 5.70 present the original actual value and measurements, later four columns show estimation results of each estimator, and the last four columns give the error between each estimator and actual value. At the last row, the summation of estimation error are presented for overall comparison. All tables in AC estimation follow this format.

Table 5.69: State Variable of case 0, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0666 | -0.0675 | -0.0666 | -0.0666 | **V1** | 1.0462 | 1.0442 | 1.0462 | 1.0462 |
| $\theta 3$ | -0.0773 | -0.0788 | -0.0773 | -0.0773 | **V2** | 1.0423 | 1.0397 | 1.0423 | 1.0423 |
| $\theta 4$ | -0.0757 | -0.0766 | -0.0757 | -0.0757 | **V3** | 1.0626 | 1.0591 | 1.0626 | 1.0626 |
| $\theta 5$ | -0.0962 | -0.0975 | -0.0962 | -0.0962 | **V4** | 0.9827 | 0.9789 | 0.9827 | 0.9827 |
| $\theta 6$ | -0.1070 | -0.1086 | -0.1070 | -0.1070 | **V5** | 0.9794 | 0.9764 | 0.9794 | 0.9794 |
| | | | | | **V6** | 0.9991 | 0.9952 | 0.9991 | 0.9991 |

Table 5.70: Estimation Results and Errors of case 0, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0462 | 1.0442 | 1.0462 | 1.0462 | 0.0135 | 0.0038 | 0.0058 | 0.0038 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0423 | 1.0397 | 1.0423 | 1.0423 | 0.0161 | 0.0077 | 0.0103 | 0.0077 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0626 | 1.0591 | 1.0626 | 1.0626 | 0.0200 | 0.0074 | 0.0109 | 0.0074 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9827 | 0.9789 | 0.9827 | 0.9827 | 0.0083 | 0.0069 | 0.0107 | 0.0069 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9794 | 0.9764 | 0.9794 | 0.9794 | 0.0065 | 0.0062 | 0.0092 | 0.0062 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 0.9991 | 0.9952 | 0.9991 | 0.9991 | 0.0091 | 0.0053 | 0.0091 | 0.0053 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1189 | 1.1310 | 1.1189 | 1.1189 | 0.0520 | 0.0399 | 0.0520 | 0.0399 | 0.0399 |
| P2 | 0.5000 | 0.4840 | 0.4750 | 0.4840 | 0.4750 | 0.4750 | 0.0160 | 0.0250 | 0.0160 | 0.0250 | 0.0250 |
| P3 | 0.6000 | 0.5510 | 0.5970 | 0.5907 | 0.5970 | 0.5970 | 0.0490 | 0.0030 | 0.0093 | 0.0030 | 0.0030 |
| P4 | -0.7000 | -0.7180 | -0.7021 | -0.7104 | -0.7021 | -0.7021 | 0.0180 | 0.0021 | 0.0104 | 0.0021 | 0.0021 |
| P5 | -0.7000 | -0.7200 | -0.7196 | -0.7200 | -0.7196 | -0.7196 | 0.0200 | 0.0196 | 0.0200 | 0.0196 | 0.0196 |
| P6 | -0.7000 | -0.7230 | -0.6896 | -0.6940 | -0.6896 | -0.6896 | 0.0230 | 0.0104 | 0.0060 | 0.0104 | 0.0104 |
| Q1 | 0.1600 | 0.2020 | 0.1894 | 0.2020 | 0.1894 | 0.1894 | 0.0420 | 0.0294 | 0.0420 | 0.0294 | 0.0294 |
| Q2 | 0.7440 | 0.7190 | 0.7017 | 0.7190 | 0.7017 | 0.7017 | 0.0250 | 0.0423 | 0.0250 | 0.0423 | 0.0423 |
| Q3 | 0.8960 | 0.9060 | 0.8647 | 0.8595 | 0.8647 | 0.8647 | 0.0100 | 0.0313 | 0.0365 | 0.0313 | 0.0313 |
| Q4 | -0.7000 | -0.7190 | -0.7001 | -0.7158 | -0.7001 | -0.7001 | 0.0190 | 0.0001 | 0.0158 | 0.0001 | 0.0001 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q5 | -0.7000 | -0.6770 | -0.6828 | -0.6770 | -0.6828 | -0.6828 | 0.0230 | 0.0172 | 0.0230 | 0.0172 | 0.0172 |
| Q6 | -0.7000 | -0.6090 | -0.6641 | -0.6706 | -0.6641 | -0.6641 | 0.0910 | 0.0359 | 0.0294 | 0.0359 | 0.0359 |
| P1-2 | 0.2870 | 0.3150 | 0.3032 | 0.3071 | 0.3032 | 0.3032 | 0.0280 | 0.0162 | 0.0201 | 0.0162 | 0.0162 |
| P1-4 | 0.4360 | 0.3890 | 0.4476 | 0.4521 | 0.4476 | 0.4476 | 0.0470 | 0.0116 | 0.0161 | 0.0116 | 0.0116 |
| P1-5 | 0.3560 | 0.3570 | 0.3681 | 0.3717 | 0.3681 | 0.3681 | 0.0010 | 0.0121 | 0.0157 | 0.0121 | 0.0121 |
| P2-1 | -0.2780 | -0.3490 | -0.2935 | -0.2972 | -0.2935 | -0.2935 | 0.0710 | 0.0155 | 0.0192 | 0.0155 | 0.0155 |
| P2-4 | 0.3310 | 0.3280 | 0.3236 | 0.3280 | 0.3236 | 0.3236 | 0.0030 | 0.0074 | 0.0030 | 0.0074 | 0.0074 |
| P2-5 | 0.1550 | 0.1740 | 0.1566 | 0.1578 | 0.1566 | 0.1566 | 0.0190 | 0.0016 | 0.0028 | 0.0016 | 0.0016 |
| P2-6 | 0.2620 | 0.2230 | 0.2589 | 0.2630 | 0.2589 | 0.2589 | 0.0390 | 0.0031 | 0.0010 | 0.0031 | 0.0031 |
| P2-3 | 0.0290 | 0.0860 | 0.0293 | 0.0325 | 0.0293 | 0.0293 | 0.0570 | 0.0003 | 0.0035 | 0.0003 | 0.0003 |
| P3-2 | -0.0290 | -0.0210 | -0.0289 | -0.0321 | -0.0289 | -0.0289 | 0.0080 | 0.0001 | 0.0031 | 0.0001 | 0.0001 |
| P3-5 | 0.1910 | 0.1770 | 0.1920 | 0.1898 | 0.1920 | 0.1920 | 0.0140 | 0.0010 | 0.0012 | 0.0010 | 0.0010 |
| P3-6 | 0.4380 | 0.4330 | 0.4338 | 0.4330 | 0.4338 | 0.4338 | 0.0050 | 0.0042 | 0.0050 | 0.0042 | 0.0042 |
| P4-1 | -0.4250 | -0.4010 | -0.4359 | -0.4401 | -0.4359 | -0.4359 | 0.0240 | 0.0109 | 0.0151 | 0.0109 | 0.0109 |
| P4-2 | -0.3160 | -0.2980 | -0.3090 | -0.3128 | -0.3090 | -0.3090 | 0.0180 | 0.0070 | 0.0032 | 0.0070 | 0.0070 |
| P4-5 | 0.0410 | 0.0070 | 0.0428 | 0.0425 | 0.0428 | 0.0428 | 0.0340 | 0.0018 | 0.0015 | 0.0018 | 0.0018 |
| P5-4 | -0.0400 | -0.0210 | -0.0424 | -0.0421 | -0.0424 | -0.0424 | 0.0190 | 0.0024 | 0.0021 | 0.0024 | 0.0024 |
| P5-1 | -0.3450 | -0.3660 | -0.3566 | -0.3599 | -0.3566 | -0.3566 | 0.0210 | 0.0116 | 0.0149 | 0.0116 | 0.0116 |
| P5-2 | -0.1500 | -0.1170 | -0.1518 | -0.1528 | -0.1518 | -0.1518 | 0.0330 | 0.0018 | 0.0028 | 0.0018 | 0.0018 |
| P5-3 | -0.1800 | -0.2510 | -0.1814 | -0.1792 | -0.1814 | -0.1814 | 0.0710 | 0.0014 | 0.0008 | 0.0014 | 0.0014 |
| P5-6 | 0.0160 | -0.0210 | 0.0125 | 0.0140 | 0.0125 | 0.0125 | 0.0370 | 0.0035 | 0.0020 | 0.0035 | 0.0035 |
| P6-5 | -0.0160 | 0.0100 | -0.0120 | -0.0135 | -0.0120 | -0.0120 | 0.0260 | 0.0040 | 0.0025 | 0.0040 | 0.0040 |
| P6-2 | 0.2570 | -0.1960 | -0.2533 | -0.2572 | -0.2533 | -0.2533 | 0.4530 | 0.5103 | 0.5142 | 0.5103 | 0.5103 |
| P6-3 | -0.4280 | -0.4680 | -0.4243 | -0.4234 | -0.4243 | -0.4243 | 0.0400 | 0.0037 | 0.0046 | 0.0037 | 0.0037 |
| Q1-2 | -0.1540 | -0.1320 | -0.1413 | -0.1396 | -0.1413 | -0.1413 | 0.0220 | 0.0127 | 0.0144 | 0.0127 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2131 | 0.2214 | 0.2131 | 0.2131 | 0.0110 | 0.0121 | 0.0204 | 0.0121 | 0.0121 |
| Q1-5 | 0.1130 | 0.0940 | 0.1175 | 0.1202 | 0.1175 | 0.1175 | 0.0190 | 0.0045 | 0.0072 | 0.0045 | 0.0045 |
| Q2-1 | 0.1280 | 0.0970 | 0.1171 | 0.1160 | 0.1171 | 0.1171 | 0.0310 | 0.0109 | 0.0120 | 0.0109 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4496 | 0.4585 | 0.4496 | 0.4496 | 0.0780 | 0.0114 | 0.0025 | 0.0114 | 0.0114 |
| Q2-5 | 0.1540 | 0.2200 | 0.1461 | 0.1467 | 0.1461 | 0.1461 | 0.0660 | 0.0079 | 0.0073 | 0.0079 | 0.0079 |
| Q2-6 | 0.1240 | 0.1500 | 0.1119 | 0.1168 | 0.1119 | 0.1119 | 0.0260 | 0.0121 | 0.0072 | 0.0121 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1229 | -0.1190 | -0.1229 | -0.1229 | 0.0040 | 0.0001 | 0.0040 | 0.0001 | 0.0001 |
| Q3-2 | 0.0570 | 0.1020 | 0.0585 | 0.0549 | 0.0585 | 0.0585 | 0.0450 | 0.0015 | 0.0021 | 0.0015 | 0.0015 |
| Q3-5 | 0.2320 | 0.2390 | 0.2240 | 0.2216 | 0.2240 | 0.2240 | 0.0070 | 0.0080 | 0.0104 | 0.0080 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5822 | 0.5830 | 0.5822 | 0.5822 | 0.0240 | 0.0248 | 0.0240 | 0.0248 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2076 | -0.2140 | -0.2076 | -0.2076 | 0.0560 | 0.0086 | 0.0150 | 0.0086 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4410 | -0.4486 | -0.4410 | -0.4410 | 0.0080 | 0.0100 | 0.0024 | 0.0100 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0515 | -0.0531 | -0.0515 | -0.0515 | 0.1250 | 0.0025 | 0.0041 | 0.0025 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0246 | -0.0225 | -0.0246 | -0.0246 | 0.0130 | 0.0034 | 0.0055 | 0.0034 | 0.0034 |
| Q5-1 | -0.1350 | -0.1750 | -0.1358 | -0.1371 | -0.1358 | -0.1358 | 0.0400 | 0.0008 | 0.0021 | 0.0008 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1725 | -0.1726 | -0.1725 | -0.1725 | 0.0420 | 0.0075 | 0.0074 | 0.0075 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2531 | -0.2507 | -0.2531 | -0.2531 | 0.0620 | 0.0079 | 0.0103 | 0.0079 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0969 | -0.0942 | -0.0969 | -0.0969 | 0.0890 | 0.0001 | 0.0028 | 0.0001 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0397 | 0.0373 | 0.0397 | 0.0397 | 0.0100 | 0.0007 | 0.0017 | 0.0007 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1481 | -0.1519 | -0.1481 | -0.1481 | 0.0630 | 0.0119 | 0.0081 | 0.0119 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5556 | -0.5559 | -0.5556 | -0.5556 | 0.0680 | 0.0234 | 0.0231 | 0.0234 | 0.0234 |
| **SUM** | | | | | | | **2.4385** | **1.1079** | **1.1898** | **1.1079** | **1.1079** |

As illustrated above, all estimators generate better overall results than measurements, and the difference between each estimator are really small and all generate good results. Since there is no bad data in these case, no bad data are detected and eliminated, estimation of WLS, LTS and LTAV are the same.

**Median redundancy**

To keep consistency, the reduced part is the same as in the DC estimation. i.e, the location of real power measurements in DC part are copied here and add the measurements of reactive power at the same location as real power. Table. 5.71 shows the raw data for median redundancy estimation.

Table 5.71: raw data for 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | Q1 | 0.1600 | 0.2020 | P2-4 | 0.3310 | 0.3280 |
| P1 | 1.0790 | 1.1310 | Q2 | 0.7440 | 0.7190 | P2-5 | 0.1550 | 0.1740 |
| P2 | 0.5000 | 0.4840 | Q3 | 0.8960 | 0.9060 | P3-6 | 0.4380 | 0.4330 |
| P3 | 0.6000 | 0.5510 | Q4 | -0.7000 | -0.7190 | Q1-5 | 0.1130 | 0.0940 |
| P4 | -0.7000 | -0.7180 | Q5 | -0.7000 | -0.6770 | Q2-4 | 0.4610 | 0.3830 |
| P5 | -0.7000 | -0.7200 | Q6 | -0.7000 | -0.6090 | Q2-5 | 0.1540 | 0.2200 |
| P6 | -0.7000 | -0.7230 | P1-5 | 0.3560 | 0.3570 | Q3-6 | 0.6070 | 0.5830 |

Table 5.72 shows the system state variables generated by each estimator; Table 5.73 presents the estimation results and summation of the absolute value of errors between estimation results, the summation of raw measurements are also presented for comparison.

Table 5.72: State Variable of case 0, small noise, 6-bus AC system, meidan redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0700 | -0.0766 | -0.0700 | -0.0700 | **V1** | 1.0342 | 0.9829 | 1.0342 | 1.0342 |
| $\theta 3$ | -0.0852 | -0.0968 | -0.0852 | -0.0852 | **V2** | 1.0303 | 0.9802 | 1.0303 | 1.0303 |
| $\theta 4$ | -0.0792 | -0.0861 | -0.0792 | -0.0792 | **V3** | 1.0519 | 1.0039 | 1.0519 | 1.0519 |
| $\theta 5$ | -0.1005 | -0.1133 | -0.1005 | -0.1005 | **V4** | 0.9697 | 0.9138 | 0.9697 | 0.9697 |
| $\theta 6$ | -0.1153 | -0.1302 | -0.1153 | -0.1153 | **V5** | 0.9658 | 0.9120 | 0.9658 | 0.9658 |
| | | | | | **V6** | 0.9879 | 0.9367 | 0.9879 | 0.9879 |

Table 5.73: Estimation Results and Errors of case 0, small noise, 6-bus AC system, meidan redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0342 | 0.9829 | 1.0342 | 1.0342 | 0.0158 | 0.0671 | 0.0158 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0303 | 0.9802 | 1.0303 | 1.0303 | 0.0197 | 0.0698 | 0.0197 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0519 | 1.0039 | 1.0519 | 1.0519 | 0.0181 | 0.0661 | 0.0181 | 0.0181 |
| V4 | 0.9896 | N/A | 0.9697 | 0.9138 | 0.9697 | 0.9697 | 0.0199 | 0.0758 | 0.0199 | 0.0199 |
| V5 | 0.9857 | N/A | 0.9658 | 0.9120 | 0.9658 | 0.9658 | 0.0199 | 0.0737 | 0.0199 | 0.0199 |
| V6 | 1.0043 | N/A | 0.9879 | 0.9367 | 0.9879 | 0.9879 | 0.0164 | 0.0677 | 0.0164 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1414 | 1.1310 | 1.1414 | 1.1414 | 0.0624 | 0.0520 | 0.0624 | 0.0624 |
| P2 | 0.5000 | 0.4840 | 0.5009 | 0.5452 | 0.5009 | 0.5009 | 0.0009 | 0.0452 | 0.0009 | 0.0009 |
| P3 | 0.6000 | 0.5510 | 0.5670 | 0.5582 | 0.5670 | 0.5670 | 0.0330 | 0.0418 | 0.0330 | 0.0330 |
| P4 | -0.7000 | -0.7180 | -0.7070 | -0.6964 | -0.7070 | -0.7070 | 0.0070 | 0.0036 | 0.0070 | 0.0070 |
| P5 | -0.7000 | -0.7200 | -0.7081 | -0.7200 | -0.7081 | -0.7081 | 0.0081 | 0.0200 | 0.0081 | 0.0081 |
| P6 | -0.7000 | -0.7230 | -0.7105 | -0.7230 | -0.7105 | -0.7105 | 0.0105 | 0.0230 | 0.0105 | 0.0105 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q1 | 0.1600 | 0.2020 | 0.1903 | 0.2020 | 0.1903 | 0.1903 | 0.0303 | 0.0420 | 0.0303 | 0.0303 |
| Q2 | 0.7440 | 0.7190 | 0.6962 | 0.7190 | 0.6962 | 0.6962 | 0.0478 | 0.0250 | 0.0478 | 0.0478 |
| Q3 | 0.8960 | 0.9060 | 0.8840 | 0.9060 | 0.8840 | 0.8840 | 0.0120 | 0.0100 | 0.0120 | 0.0120 |
| Q4 | -0.7000 | -0.7190 | -0.6973 | -0.7190 | -0.6973 | -0.6973 | 0.0027 | 0.0190 | 0.0027 | 0.0027 |
| Q5 | -0.7000 | -0.6770 | -0.7034 | -0.6770 | -0.7034 | -0.7034 | 0.0034 | 0.0230 | 0.0034 | 0.0034 |
| Q6 | -0.7000 | -0.6090 | -0.6356 | -0.6111 | -0.6356 | -0.6356 | 0.0644 | 0.0889 | 0.0644 | 0.0644 |
| P1-2 | 0.2870 | N/A | 0.3114 | 0.3060 | 0.3114 | 0.3114 | 0.0244 | 0.0190 | 0.0244 | 0.0244 |
| P1-4 | 0.4360 | N/A | 0.4553 | 0.4471 | 0.4553 | 0.4553 | 0.0193 | 0.0111 | 0.0193 | 0.0193 |
| P1-5 | 0.3560 | 0.3570 | 0.3746 | 0.3779 | 0.3746 | 0.3746 | 0.0186 | 0.0219 | 0.0186 | 0.0186 |
| P2-1 | -0.2780 | N/A | -0.3010 | -0.2947 | -0.3010 | -0.3010 | 0.0230 | 0.0167 | 0.0230 | 0.0230 |
| P2-4 | 0.3310 | 0.3280 | 0.3229 | 0.3280 | 0.3229 | 0.3229 | 0.0081 | 0.0030 | 0.0081 | 0.0081 |
| P2-5 | 0.1550 | 0.1740 | 0.1578 | 0.1657 | 0.1578 | 0.1578 | 0.0028 | 0.0107 | 0.0028 | 0.0028 |
| P2-6 | 0.2620 | N/A | 0.2749 | 0.2876 | 0.2749 | 0.2749 | 0.0129 | 0.0256 | 0.0129 | 0.0129 |
| P2-3 | 0.0290 | N/A | 0.0463 | 0.0586 | 0.0463 | 0.0463 | 0.0173 | 0.0296 | 0.0173 | 0.0173 |
| P3-2 | -0.0290 | N/A | -0.0457 | -0.0578 | -0.0457 | -0.0457 | 0.0167 | 0.0288 | 0.0167 | 0.0167 |
| P3-5 | 0.1910 | N/A | 0.1818 | 0.1830 | 0.1818 | 0.1818 | 0.0092 | 0.0080 | 0.0092 | 0.0092 |
| P3-6 | 0.4380 | 0.4330 | 0.4309 | 0.4330 | 0.4309 | 0.4309 | 0.0071 | 0.0050 | 0.0071 | 0.0071 |
| P4-1 | -0.4250 | N/A | -0.4430 | -0.4337 | -0.4430 | -0.4430 | 0.0180 | 0.0087 | 0.0180 | 0.0180 |
| P4-2 | -0.3160 | N/A | -0.3079 | -0.3101 | -0.3079 | -0.3079 | 0.0081 | 0.0059 | 0.0081 | 0.0081 |
| P4-5 | 0.0410 | N/A | 0.0439 | 0.0473 | 0.0439 | 0.0439 | 0.0029 | 0.0063 | 0.0029 | 0.0029 |
| P5-4 | -0.0400 | N/A | -0.0435 | -0.0467 | -0.0435 | -0.0435 | 0.0035 | 0.0067 | 0.0035 | 0.0035 |
| P5-1 | -0.3450 | N/A | -0.3624 | -0.3642 | -0.3624 | -0.3624 | 0.0174 | 0.0192 | 0.0174 | 0.0174 |
| P5-2 | -0.1500 | N/A | -0.1527 | -0.1598 | -0.1527 | -0.1527 | 0.0027 | 0.0098 | 0.0027 | 0.0027 |
| P5-3 | -0.1800 | N/A | -0.1706 | -0.1703 | -0.1706 | -0.1706 | 0.0094 | 0.0097 | 0.0094 | 0.0094 |
| P5-6 | 0.0160 | N/A | 0.0212 | 0.0210 | 0.0212 | 0.0212 | 0.0052 | 0.0050 | 0.0052 | 0.0052 |
| P6-5 | -0.0160 | N/A | -0.0205 | -0.0202 | -0.0205 | -0.0205 | 0.0045 | 0.0042 | 0.0045 | 0.0045 |
| P6-2 | 0.2570 | N/A | -0.2688 | -0.2805 | -0.2688 | -0.2688 | 0.5258 | 0.5375 | 0.5258 | 0.5258 |
| P6-3 | -0.4280 | N/A | -0.4212 | -0.4223 | -0.4212 | -0.4212 | 0.0068 | 0.0057 | 0.0068 | 0.0068 |
| Q1-2 | -0.1540 | N/A | -0.1442 | -0.1448 | -0.1442 | -0.1442 | 0.0098 | 0.0092 | 0.0098 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.2138 | 0.2251 | 0.2138 | 0.2138 | 0.0128 | 0.0241 | 0.0128 | 0.0128 |
| Q1-5 | 0.1130 | 0.0940 | 0.1206 | 0.1217 | 0.1206 | 0.1206 | 0.0076 | 0.0087 | 0.0076 | 0.0076 |
| Q2-1 | 0.1280 | N/A | 0.1225 | 0.1289 | 0.1225 | 0.1225 | 0.0055 | 0.0009 | 0.0055 | 0.0055 |
| Q2-4 | 0.4610 | 0.3830 | 0.4529 | 0.4774 | 0.4529 | 0.4529 | 0.0081 | 0.0164 | 0.0081 | 0.0081 |
| Q2-5 | 0.1540 | 0.2200 | 0.1494 | 0.1504 | 0.1494 | 0.1494 | 0.0046 | 0.0036 | 0.0046 | 0.0046 |
| Q2-6 | 0.1240 | N/A | 0.1007 | 0.0950 | 0.1007 | 0.1007 | 0.0233 | 0.0290 | 0.0233 | 0.0233 |
| Q2-3 | -0.1230 | N/A | -0.1294 | -0.1327 | -0.1294 | -0.1294 | 0.0064 | 0.0097 | 0.0064 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0671 | 0.0773 | 0.0671 | 0.0671 | 0.0101 | 0.0203 | 0.0101 | 0.0101 |
| Q3-5 | 0.2320 | N/A | 0.2372 | 0.2457 | 0.2372 | 0.2372 | 0.0052 | 0.0137 | 0.0052 | 0.0052 |
| Q3-6 | 0.6070 | 0.5830 | 0.5797 | 0.5830 | 0.5797 | 0.5797 | 0.0273 | 0.0240 | 0.0273 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.2049 | -0.2074 | -0.2049 | -0.2049 | 0.0059 | 0.0084 | 0.0059 | 0.0059 |
| Q4-2 | -0.4510 | N/A | -0.4429 | -0.4595 | -0.4429 | -0.4429 | 0.0081 | 0.0085 | 0.0081 | 0.0081 |
| Q4-5 | -0.0490 | N/A | -0.0495 | -0.0521 | -0.0495 | -0.0495 | 0.0005 | 0.0031 | 0.0005 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0245 | -0.0133 | -0.0245 | -0.0245 | 0.0035 | 0.0147 | 0.0035 | 0.0035 |
| Q5-1 | -0.1350 | N/A | -0.1348 | -0.1243 | -0.1348 | -0.1348 | 0.0002 | 0.0107 | 0.0002 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1741 | -0.1687 | -0.1741 | -0.1741 | 0.0059 | 0.0113 | 0.0059 | 0.0059 |
| Q5-3 | -0.2610 | N/A | -0.2639 | -0.2641 | -0.2639 | -0.2639 | 0.0029 | 0.0031 | 0.0029 | 0.0029 |
| Q5-6 | -0.0970 | N/A | -0.1061 | -0.1067 | -0.1061 | -0.1061 | 0.0091 | 0.0097 | 0.0091 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0510 | 0.0580 | 0.0510 | 0.0510 | 0.0120 | 0.0190 | 0.0120 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1344 | -0.1208 | -0.1344 | -0.1344 | 0.0256 | 0.0392 | 0.0256 | 0.0256 |
| Q6-3 | -0.5790 | N/A | -0.5522 | -0.5483 | -0.5522 | -0.5522 | 0.0268 | 0.0307 | 0.0268 | 0.0268 |
| **SUM** | | | | | | | **1.3777** | **1.9299** | **1.3777** | **1.3777** |

WLS, LTS and LTAV still generate the same estimation as full redundancy case, and all estimators generate worse results than full redundancy case since there are few raw data for reference.

**Comparison**

The comparison between full and median redundancy with the same condition of error are presented in Table 5.74. From the results, it is very easy to find that full redundancy case has better estimation results for all estimators.

Table 5.74: Estimation errors of case 0, small noise, 6-bus AC system

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **2.4385** | **1.1079** | **1.1898** | **1.1079** | **1.1079** |
| **SUM** | **N/A** | **1.3777** | **1.9299** | **1.3777** | **1.3777** |

#### 5.1.2.2 Case 1: Single bad data

In order to keep consistence for better comparison, the same type of error at the same location with DC median cases are used for evaluation.

**Reverse bad data**

Based on the raw data in Table 5.68, a reverse bad data is used to replace the original one $P_{2-4} = -0.3280$, others remain the same. For median redundancy case, the same bad data are applied on the basis of Table. 5.71.

**Full redundancy**

With the reverse bad data presented above, the following two tables show the state variables, estimation and error between estimation and actual value.

Table 5.75: State Variable of case 1, reverse, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0717 | -0.0679 | -0.0717 | -0.0666 | **V1** | 1.0476 | 1.0448 | 1.0476 | 1.0462 |
| $\theta 3$ | -0.0861 | -0.0809 | -0.0861 | -0.0774 | **V2** | 1.0413 | 1.0401 | 1.0413 | 1.0423 |
| $\theta 4$ | -0.0700 | -0.0755 | -0.0700 | -0.0757 | **V3** | 1.0608 | 1.0591 | 1.0608 | 1.0626 |
| $\theta 5$ | -0.1009 | -0.0983 | -0.1009 | -0.0962 | **V4** | 0.9876 | 0.9798 | 0.9876 | 0.9827 |
| $\theta 6$ | -0.1160 | -0.1107 | -0.1160 | -0.1071 | **V5** | 0.9790 | 0.9768 | 0.9790 | 0.9794 |
| | | | | | **V6** | 0.9969 | 0.9952 | 0.9969 | 0.9991 |

Table 5.76: Estimation Results and Errors of case 1, reverse, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0476 | 1.0448 | 1.0476 | 1.0462 | 0.0135 | 0.0024 | 0.0052 | 0.0024 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0413 | 1.0401 | 1.0413 | 1.0423 | 0.0161 | 0.0087 | 0.0099 | 0.0087 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0608 | 1.0591 | 1.0608 | 1.0626 | 0.0200 | 0.0092 | 0.0109 | 0.0092 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9876 | 0.9798 | 0.9876 | 0.9827 | 0.0083 | 0.0020 | 0.0097 | 0.0020 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9790 | 0.9768 | 0.9790 | 0.9794 | 0.0065 | 0.0067 | 0.0089 | 0.0067 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 0.9969 | 0.9952 | 0.9969 | 0.9991 | 0.0091 | 0.0074 | 0.0091 | 0.0074 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1345 | 1.1310 | 1.1345 | 1.1190 | 0.0520 | 0.0555 | 0.0520 | 0.0555 | 0.0400 |
| P2 | 0.5000 | 0.4840 | 0.3686 | 0.4840 | 0.3686 | 0.4741 | 0.0160 | 0.1314 | 0.0160 | 0.1314 | 0.0259 |
| P3 | 0.6000 | 0.5510 | 0.5648 | 0.5783 | 0.5648 | 0.5967 | 0.0490 | 0.0352 | 0.0217 | 0.0352 | 0.0033 |
| P4 | -0.7000 | -0.7180 | -0.5370 | -0.6857 | -0.5370 | -0.7007 | 0.0180 | 0.1630 | 0.0143 | 0.1630 | 0.0007 |
| P5 | -0.7000 | -0.7200 | -0.7298 | -0.7200 | -0.7298 | -0.7197 | 0.0200 | 0.0298 | 0.0200 | 0.0298 | 0.0197 |
| P6 | -0.7000 | -0.7230 | -0.7228 | -0.7066 | -0.7228 | -0.6899 | 0.0230 | 0.0228 | 0.0066 | 0.0228 | 0.0101 |
| Q1 | 0.1600 | 0.2020 | 0.1816 | 0.2020 | 0.1816 | 0.1893 | 0.0420 | 0.0216 | 0.0420 | 0.0216 | 0.0293 |
| Q2 | 0.7440 | 0.7190 | 0.6958 | 0.7190 | 0.6958 | 0.7017 | 0.0250 | 0.0482 | 0.0250 | 0.0482 | 0.0423 |
| Q3 | 0.8960 | 0.9060 | 0.8669 | 0.8603 | 0.8669 | 0.8647 | 0.0100 | 0.0291 | 0.0357 | 0.0291 | 0.0313 |
| Q4 | -0.7000 | -0.7190 | -0.6926 | -0.7188 | -0.6926 | -0.7000 | 0.0190 | 0.0074 | 0.0188 | 0.0074 | 0.0000 |
| Q5 | -0.7000 | -0.6770 | -0.6824 | -0.6770 | -0.6824 | -0.6828 | 0.0230 | 0.0176 | 0.0230 | 0.0176 | 0.0172 |
| Q6 | -0.7000 | -0.6090 | -0.6632 | -0.6688 | -0.6632 | -0.6641 | 0.0910 | 0.0368 | 0.0312 | 0.0368 | 0.0359 |
| P1-2 | 0.2870 | 0.3150 | 0.3314 | 0.3096 | 0.3314 | 0.3034 | 0.0280 | 0.0444 | 0.0226 | 0.0444 | 0.0164 |
| P1-4 | 0.4360 | 0.3890 | 0.4176 | 0.4465 | 0.4176 | 0.4473 | 0.0470 | 0.0184 | 0.0105 | 0.0184 | 0.0113 |
| P1-5 | 0.3560 | 0.3570 | 0.3855 | 0.3749 | 0.3855 | 0.3683 | 0.0010 | 0.0295 | 0.0189 | 0.0295 | 0.0123 |
| P2-1 | -0.2780 | -0.3490 | -0.3201 | -0.2996 | -0.3201 | -0.2937 | 0.0710 | 0.0421 | 0.0216 | 0.0421 | 0.0157 |
| **P2-4** | **0.3310** | **-0.3280** | **0.2099** | **0.3128** | **0.2099** | **0.3226** | **0.6590** | **0.1211** | **0.0182** | **0.1211** | **0.0084** |
| P2-5 | 0.1550 | 0.1740 | 0.1547 | 0.1592 | 0.1547 | 0.1566 | 0.0190 | 0.0003 | 0.0042 | 0.0003 | 0.0016 |
| P2-6 | 0.2620 | 0.2230 | 0.2782 | 0.2717 | 0.2782 | 0.2591 | 0.0390 | 0.0162 | 0.0097 | 0.0162 | 0.0029 |
| P2-3 | 0.0290 | 0.0860 | 0.0459 | 0.0400 | 0.0459 | 0.0295 | 0.0570 | 0.0169 | 0.0110 | 0.0169 | 0.0005 |
| P3-2 | -0.0290 | -0.0210 | -0.0454 | -0.0396 | -0.0454 | -0.0291 | 0.0080 | 0.0164 | 0.0106 | 0.0164 | 0.0001 |
| P3-5 | 0.1910 | 0.1770 | 0.1758 | 0.1850 | 0.1758 | 0.1919 | 0.0140 | 0.0152 | 0.0060 | 0.0152 | 0.0009 |
| P3-6 | 0.4380 | 0.4330 | 0.4344 | 0.4330 | 0.4344 | 0.4338 | 0.0050 | 0.0036 | 0.0050 | 0.0036 | 0.0042 |
| P4-1 | -0.4250 | -0.4010 | -0.4074 | -0.4347 | -0.4074 | -0.4357 | 0.0240 | 0.0176 | 0.0097 | 0.0176 | 0.0107 |
| P4-2 | -0.3160 | -0.2980 | -0.1984 | -0.2980 | -0.1984 | -0.3081 | 0.0180 | 0.1176 | 0.0180 | 0.1176 | 0.0079 |
| P4-5 | 0.0410 | 0.0070 | 0.0687 | 0.0470 | 0.0687 | 0.0430 | 0.0340 | 0.0277 | 0.0060 | 0.0277 | 0.0020 |
| P5-4 | -0.0400 | -0.0210 | -0.0677 | -0.0465 | -0.0677 | -0.0426 | 0.0190 | 0.0277 | 0.0065 | 0.0277 | 0.0026 |
| P5-1 | -0.3450 | -0.3660 | -0.3730 | -0.3628 | -0.3730 | -0.3567 | 0.0210 | 0.0280 | 0.0178 | 0.0280 | 0.0117 |
| P5-2 | -0.1500 | -0.1170 | -0.1499 | -0.1542 | -0.1499 | -0.1518 | 0.0330 | 0.0001 | 0.0042 | 0.0001 | 0.0018 |
| P5-3 | -0.1800 | -0.2510 | -0.1657 | -0.1746 | -0.1657 | -0.1812 | 0.0710 | 0.0143 | 0.0054 | 0.0143 | 0.0012 |
| P5-6 | 0.0160 | -0.0210 | 0.0266 | 0.0181 | 0.0266 | 0.0126 | 0.0370 | 0.0106 | 0.0021 | 0.0106 | 0.0034 |
| P6-5 | -0.0160 | 0.0100 | -0.0260 | -0.0176 | -0.0260 | -0.0121 | 0.0260 | 0.0100 | 0.0016 | 0.0100 | 0.0039 |
| P6-2 | 0.2570 | -0.1960 | -0.2720 | -0.2656 | -0.2720 | -0.2535 | 0.4530 | 0.5290 | 0.5226 | 0.5290 | 0.5105 |
| P6-3 | -0.4280 | -0.4680 | -0.4248 | -0.4234 | -0.4248 | -0.4243 | 0.0400 | 0.0032 | 0.0046 | 0.0032 | 0.0037 |
| Q1-2 | -0.1540 | -0.1320 | -0.1404 | -0.1393 | -0.1404 | -0.1413 | 0.0220 | 0.0136 | 0.0147 | 0.0136 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2006 | 0.2207 | 0.2006 | 0.2130 | 0.0110 | 0.0004 | 0.0197 | 0.0004 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1214 | 0.1207 | 0.1214 | 0.1176 | 0.0190 | 0.0084 | 0.0077 | 0.0084 | 0.0046 |
| Q2-1 | 0.1280 | 0.0970 | 0.1193 | 0.1160 | 0.1193 | 0.1171 | 0.0310 | 0.0087 | 0.0120 | 0.0087 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4429 | 0.4597 | 0.4429 | 0.4495 | 0.0780 | 0.0181 | 0.0013 | 0.0181 | 0.0115 |
| Q2-5 | 0.1540 | 0.2200 | 0.1445 | 0.1463 | 0.1445 | 0.1461 | 0.0660 | 0.0095 | 0.0077 | 0.0095 | 0.0079 |
| Q2-6 | 0.1240 | 0.1500 | 0.1114 | 0.1160 | 0.1114 | 0.1119 | 0.0260 | 0.0126 | 0.0080 | 0.0126 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1224 | -0.1190 | -0.1224 | -0.1229 | 0.0040 | 0.0006 | 0.0040 | 0.0006 | 0.0001 |
| Q3-2 | 0.0570 | 0.1020 | 0.0585 | 0.0550 | 0.0585 | 0.0585 | 0.0450 | 0.0015 | 0.0020 | 0.0015 | 0.0015 |
| Q3-5 | 0.2320 | 0.2390 | 0.2249 | 0.2223 | 0.2249 | 0.2240 | 0.0070 | 0.0071 | 0.0097 | 0.0071 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5836 | 0.5830 | 0.5836 | 0.5822 | 0.0240 | 0.0234 | 0.0240 | 0.0234 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2013 | -0.2144 | -0.2013 | -0.2076 | 0.0560 | 0.0023 | 0.0154 | 0.0023 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4405 | -0.4506 | -0.4405 | -0.4410 | 0.0080 | 0.0105 | 0.0004 | 0.0105 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0509 | -0.0538 | -0.0509 | -0.0515 | 0.1250 | 0.0019 | 0.0048 | 0.0019 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0245 | -0.0217 | -0.0245 | -0.0246 | 0.0130 | 0.0035 | 0.0063 | 0.0035 | 0.0034 |
| Q5-1 | -0.1350 | -0.1750 | -0.1359 | -0.1370 | -0.1359 | -0.1358 | 0.0400 | 0.0009 | 0.0020 | 0.0009 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1711 | -0.1722 | -0.1711 | -0.1725 | 0.0420 | 0.0089 | 0.0078 | 0.0089 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2550 | -0.2517 | -0.2550 | -0.2531 | 0.0620 | 0.0060 | 0.0093 | 0.0060 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0959 | -0.0944 | -0.0959 | -0.0969 | 0.0890 | 0.0011 | 0.0026 | 0.0011 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0390 | 0.0375 | 0.0390 | 0.0397 | 0.0100 | 0.0000 | 0.0015 | 0.0000 | 0.0007 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q6-2 | -0.1600 | -0.2230 | -0.1456 | -0.1504 | -0.1456 | -0.1481 | 0.0630 | 0.0144 | 0.0096 | 0.0144 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5566 | -0.5559 | -0.5566 | -0.5556 | 0.0680 | 0.0224 | 0.0231 | 0.0224 | 0.0234 |
| **SUM** | | | | | | | **3.0945** | **1.9207** | **1.2904** | **1.9207** | **1.1096** |

As shown in the third table, for the bad data, all estimators generate much closer results to actual value compared with measurement, and the estimation from LAV and LTAV are much better. Since LAV can successfully detect the bad data, LTAV generates the best estimation after the bad data are removed.

### Median redundancy

The used raw data are stated in section 5.1.2.2, Table 5.77 shows the state variables, Table 5.78 contains the estimated value and errors between each estimator with actual value.

Table 5.77: State Variable of case 1, reverse, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0782 | -0.0791 | -0.0782 | -0.0701 | **V1** | 1.0349 | 0.9809 | 1.0349 | 1.0342 |
| $\theta 3$ | -0.1025 | -0.0995 | -0.1025 | -0.0854 | **V2** | 1.0274 | 0.9770 | 1.0274 | 1.0303 |
| $\theta 4$ | -0.0720 | -0.0834 | -0.0720 | -0.0791 | **V3** | 1.0467 | 1.0007 | 1.0467 | 1.0518 |
| $\theta 5$ | -0.1094 | -0.1148 | -0.1094 | -0.1006 | **V4** | 0.9753 | 0.9129 | 0.9753 | 0.9698 |
| $\theta 6$ | -0.1329 | -0.1329 | -0.1329 | -0.1155 | **V5** | 0.9632 | 0.9090 | 0.9632 | 0.9657 |
| | | | | | **V6** | 0.9822 | 0.9333 | 0.9822 | 0.9879 |

Table 5.78: Estimation Results and Errors of case 1, reverse, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0349 | 0.9809 | 1.0349 | 1.0342 | 0.0151 | 0.0691 | 0.0151 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0274 | 0.9770 | 1.0274 | 1.0303 | 0.0226 | 0.0730 | 0.0226 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0467 | 1.0007 | 1.0467 | 1.0518 | 0.0233 | 0.0693 | 0.0233 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9753 | 0.9129 | 0.9753 | 0.9698 | 0.0142 | 0.0767 | 0.0142 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9632 | 0.9090 | 0.9632 | 0.9657 | 0.0225 | 0.0766 | 0.0225 | 0.0199 |
| V6 | 1.0043 | N/A | 0.9822 | 0.9333 | 0.9822 | 0.9879 | 0.0222 | 0.0710 | 0.0222 | 0.0165 |
| P1 | 1.0790 | 1.1310 | 1.1767 | 1.1310 | 1.1767 | 1.1418 | 0.0977 | 0.0520 | 0.0977 | 0.0628 |
| P2 | 0.5000 | 0.4840 | 0.3875 | 0.4840 | 0.3875 | 0.4998 | 0.1125 | 0.0160 | 0.1125 | 0.0002 |
| P3 | 0.6000 | 0.5510 | 0.4973 | 0.5510 | 0.4973 | 0.5663 | 0.1027 | 0.0490 | 0.1027 | 0.0337 |
| P4 | -0.7000 | -0.7180 | -0.4773 | -0.6291 | -0.4773 | -0.7046 | 0.2227 | 0.0709 | 0.2227 | 0.0046 |
| P5 | -0.7000 | -0.7200 | -0.7169 | -0.7200 | -0.7169 | -0.7082 | 0.0169 | 0.0200 | 0.0169 | 0.0082 |
| P6 | -0.7000 | -0.7230 | -0.7825 | -0.7230 | -0.7825 | -0.7113 | 0.0825 | 0.0230 | 0.0825 | 0.0113 |
| Q1 | 0.1600 | 0.2020 | 0.1790 | 0.2020 | 0.1790 | 0.1902 | 0.0190 | 0.0420 | 0.0190 | 0.0302 |
| Q2 | 0.7440 | 0.7190 | 0.6935 | 0.7190 | 0.6935 | 0.6961 | 0.0505 | 0.0250 | 0.0505 | 0.0479 |
| Q3 | 0.8960 | 0.9060 | 0.8878 | 0.9060 | 0.8878 | 0.8840 | 0.0082 | 0.0100 | 0.0082 | 0.0120 |
| Q4 | -0.7000 | -0.7190 | -0.6839 | -0.7190 | -0.6839 | -0.6972 | 0.0161 | 0.0190 | 0.0161 | 0.0028 |
| Q5 | -0.7000 | -0.6770 | -0.7026 | -0.6770 | -0.7026 | -0.7034 | 0.0026 | 0.0230 | 0.0026 | 0.0034 |
| Q6 | -0.7000 | -0.6090 | -0.6333 | -0.6113 | -0.6333 | -0.6355 | 0.0667 | 0.0887 | 0.0667 | 0.0645 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P1-2 | 0.2870 | N/A | 0.3544 | 0.3167 | 0.3544 | 0.3119 | 0.0674 | 0.0297 | 0.0674 | 0.0249 |
| P1-4 | 0.4360 | N/A | 0.4172 | 0.4332 | 0.4172 | 0.4549 | 0.0188 | 0.0028 | 0.0188 | 0.0189 |
| P1-5 | 0.3560 | 0.3570 | 0.4051 | 0.3811 | 0.4051 | 0.3749 | 0.0491 | 0.0251 | 0.0491 | 0.0189 |
| P2-1 | -0.2780 | N/A | -0.3412 | -0.3046 | -0.3412 | -0.3014 | 0.0632 | 0.0266 | 0.0632 | 0.0234 |
| **P2-4** | **0.3310** | **-0.3280** | **0.1641** | **0.2811** | **0.1641** | **0.3213** | **0.1669** | **0.0499** | **0.1669** | **0.0097** |
| P2-5 | 0.1550 | 0.1740 | 0.1589 | 0.1619 | 0.1589 | 0.1578 | 0.0039 | 0.0069 | 0.0039 | 0.0028 |
| P2-6 | 0.2620 | N/A | 0.3205 | 0.2868 | 0.3205 | 0.2754 | 0.0585 | 0.0248 | 0.0585 | 0.0134 |
| P2-3 | 0.0290 | N/A | 0.0852 | 0.0589 | 0.0852 | 0.0467 | 0.0562 | 0.0299 | 0.0562 | 0.0177 |
| P3-2 | -0.0290 | N/A | -0.0844 | -0.0581 | -0.0844 | -0.0461 | 0.0554 | 0.0291 | 0.0554 | 0.0171 |
| P3-5 | 0.1910 | N/A | 0.1500 | 0.1785 | 0.1500 | 0.1815 | 0.0410 | 0.0125 | 0.0410 | 0.0095 |
| P3-6 | 0.4380 | 0.4330 | 0.4318 | 0.4307 | 0.4318 | 0.4309 | 0.0062 | 0.0073 | 0.0062 | 0.0071 |
| P4-1 | -0.4250 | N/A | -0.4069 | -0.4205 | -0.4069 | -0.4427 | 0.0181 | 0.0045 | 0.0181 | 0.0177 |
| P4-2 | -0.3160 | N/A | -0.1531 | -0.2646 | -0.1531 | -0.3063 | 0.1629 | 0.0514 | 0.1629 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0827 | 0.0560 | 0.0827 | 0.0443 | 0.0417 | 0.0150 | 0.0417 | 0.0033 |
| P5-4 | -0.0400 | N/A | -0.0813 | -0.0551 | -0.0813 | -0.0439 | 0.0413 | 0.0151 | 0.0413 | 0.0039 |
| P5-1 | -0.3450 | N/A | -0.3909 | -0.3671 | -0.3909 | -0.3627 | 0.0459 | 0.0221 | 0.0459 | 0.0177 |
| P5-2 | -0.1500 | N/A | -0.1538 | -0.1561 | -0.1538 | -0.1527 | 0.0038 | 0.0061 | 0.0038 | 0.0027 |
| P5-3 | -0.1800 | N/A | -0.1397 | -0.1659 | -0.1397 | -0.1703 | 0.0403 | 0.0141 | 0.0403 | 0.0097 |
| P5-6 | 0.0160 | N/A | 0.0488 | 0.0242 | 0.0488 | 0.0214 | 0.0328 | 0.0082 | 0.0328 | 0.0054 |
| P6-5 | -0.0160 | N/A | -0.0479 | -0.0233 | -0.0479 | -0.0207 | 0.0319 | 0.0073 | 0.0319 | 0.0047 |
| P6-2 | 0.2570 | N/A | -0.3126 | -0.2797 | -0.3126 | -0.2693 | 0.5696 | 0.5367 | 0.5696 | 0.5263 |
| P6-3 | -0.4280 | N/A | -0.4220 | -0.4199 | -0.4220 | -0.4212 | 0.0060 | 0.0081 | 0.0060 | 0.0068 |
| Q1-2 | -0.1540 | N/A | -0.1438 | -0.1435 | -0.1438 | -0.1442 | 0.0102 | 0.0105 | 0.0102 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.1956 | 0.2215 | 0.1956 | 0.2136 | 0.0054 | 0.0205 | 0.0054 | 0.0126 |
| Q1-5 | 0.1130 | 0.0940 | 0.1272 | 0.1240 | 0.1272 | 0.1207 | 0.0142 | 0.0110 | 0.0142 | 0.0077 |
| Q2-1 | 0.1280 | N/A | 0.1275 | 0.1292 | 0.1275 | 0.1225 | 0.0005 | 0.0012 | 0.0005 | 0.0055 |
| Q2-4 | 0.4610 | 0.3830 | 0.4430 | 0.4763 | 0.4430 | 0.4528 | 0.0180 | 0.0153 | 0.0180 | 0.0082 |
| Q2-5 | 0.1540 | 0.2200 | 0.1477 | 0.1501 | 0.1477 | 0.1494 | 0.0063 | 0.0039 | 0.0063 | 0.0046 |
| Q2-6 | 0.1240 | N/A | 0.1016 | 0.0956 | 0.1016 | 0.1007 | 0.0224 | 0.0284 | 0.0224 | 0.0233 |
| Q2-3 | -0.1230 | N/A | -0.1264 | -0.1322 | -0.1264 | -0.1293 | 0.0034 | 0.0092 | 0.0034 | 0.0063 |
| Q3-2 | 0.0570 | N/A | 0.0657 | 0.0772 | 0.0657 | 0.0671 | 0.0087 | 0.0202 | 0.0087 | 0.0101 |
| Q3-5 | 0.2320 | N/A | 0.2396 | 0.2458 | 0.2396 | 0.2372 | 0.0076 | 0.0138 | 0.0076 | 0.0052 |
| Q3-6 | 0.6070 | 0.5830 | 0.5825 | 0.5830 | 0.5825 | 0.5797 | 0.0245 | 0.0240 | 0.0245 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.1947 | -0.2064 | -0.1947 | -0.2048 | 0.0043 | 0.0074 | 0.0043 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4411 | -0.4612 | -0.4411 | -0.4429 | 0.0099 | 0.0102 | 0.0099 | 0.0081 |
| Q4-5 | -0.0490 | N/A | -0.0481 | -0.0515 | -0.0481 | -0.0495 | 0.0009 | 0.0025 | 0.0009 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0242 | -0.0132 | -0.0242 | -0.0245 | 0.0038 | 0.0148 | 0.0038 | 0.0035 |
| Q5-1 | -0.1350 | N/A | -0.1341 | -0.1251 | -0.1341 | -0.1348 | 0.0009 | 0.0099 | 0.0009 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1721 | -0.1685 | -0.1721 | -0.1740 | 0.0079 | 0.0115 | 0.0079 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2679 | -0.2641 | -0.2679 | -0.2640 | 0.0069 | 0.0031 | 0.0069 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1043 | -0.1060 | -0.1043 | -0.1061 | 0.0073 | 0.0090 | 0.0073 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0502 | 0.0577 | 0.0502 | 0.0510 | 0.0112 | 0.0187 | 0.0112 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1295 | -0.1210 | -0.1295 | -0.1343 | 0.0305 | 0.0390 | 0.0305 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5539 | -0.5481 | -0.5539 | -0.5522 | 0.0251 | 0.0309 | 0.0251 | 0.0268 |
| **SUM** | | | | | | | **2.7291** | **2.1227** | **2.7291** | **1.3841** |

For the case, LAV and LTAV generate better results both overall and bad data estimation. LTS generates the same results with WLS since WLS does not detect any bad data.

**Comparison**

The comparison between full and median redundancy with the same condition of error are presented in Table 5.79. WLS and LTS generate worse results in both case, all estimators have better estimation in full redundancy case.

Table 5.79: Estimation errors of case 0, small noise, 6-bus AC system, meidan redundancy

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **3.0945** | **1.9207** | **1.2904** | **1.9207** | **1.1096** |
| **SUM** | **N/A** | **2.7291** | **2.1227** | **2.7291** | **1.3841** |

### Large bad data

Change the bad data in section 5.1.2.2 from reverse to large bad data by letting $P_{2-4} = 0.9280$, others remain the same both in full and median redundancy case.

#### Full redundancy

State variables are presented in Table 5.80, Table 5.81 shows the estimated value and errors between each estimator and actual value.

Table 5.80: State Variable of case 1, large, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0620 | -0.0674 | -0.0666 | -0.0666 | **V1** | 1.0447 | 1.0440 | 1.0462 | 1.0462 |
| $\theta 3$ | -0.0692 | -0.0781 | -0.0774 | -0.0774 | **V2** | 1.0432 | 1.0396 | 1.0423 | 1.0423 |
| $\theta 4$ | -0.0811 | -0.0770 | -0.0757 | -0.0757 | **V3** | 1.0644 | 1.0591 | 1.0626 | 1.0626 |
| $\theta 5$ | -0.0919 | -0.0973 | -0.0962 | -0.0962 | **V4** | 0.9780 | 0.9786 | 0.9827 | 0.9827 |
| $\theta 6$ | -0.0988 | -0.1080 | -0.1071 | -0.1071 | **V5** | 0.9798 | 0.9763 | 0.9794 | 0.9794 |
| | | | | | **V6** | 1.0010 | 0.9952 | 0.9991 | 0.9991 |

Table 5.81: Estimation Results and Errors of case 1, large, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0447 | 1.0440 | 1.0462 | 1.0462 | 0.0135 | 0.0053 | 0.0060 | 0.0038 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0432 | 1.0396 | 1.0423 | 1.0423 | 0.0161 | 0.0068 | 0.0104 | 0.0077 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0644 | 1.0591 | 1.0626 | 1.0626 | 0.0200 | 0.0056 | 0.0109 | 0.0074 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9780 | 0.9786 | 0.9827 | 0.9827 | 0.0083 | 0.0116 | 0.0110 | 0.0069 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9798 | 0.9763 | 0.9794 | 0.9794 | 0.0065 | 0.0059 | 0.0093 | 0.0062 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 1.0010 | 0.9952 | 0.9991 | 0.9991 | 0.0091 | 0.0033 | 0.0091 | 0.0053 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1050 | 1.1310 | 1.1190 | 1.1190 | 0.0520 | 0.0260 | 0.0520 | 0.0400 | 0.0400 |
| P2 | 0.5000 | 0.4840 | 0.5736 | 0.4840 | 0.4741 | 0.4741 | 0.0160 | 0.0736 | 0.0160 | 0.0259 | 0.0259 |
| P3 | 0.6000 | 0.5510 | 0.6268 | 0.5945 | 0.5967 | 0.5967 | 0.0490 | 0.0268 | 0.0055 | 0.0033 | 0.0033 |
| P4 | -0.7000 | -0.7180 | -0.8545 | -0.7180 | -0.7007 | -0.7007 | 0.0180 | 0.1545 | 0.0180 | 0.0007 | 0.0007 |
| P5 | -0.7000 | -0.7200 | -0.7097 | -0.7200 | -0.7197 | -0.7197 | 0.0200 | 0.0097 | 0.0200 | 0.0197 | 0.0197 |
| P6 | -0.7000 | -0.7230 | -0.6588 | -0.6902 | -0.6899 | -0.6899 | 0.0230 | 0.0412 | 0.0098 | 0.0101 | 0.0101 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q1 | 0.1600 | 0.2020 | 0.1965 | 0.2020 | 0.1893 | 0.1893 | 0.0420 | 0.0365 | 0.0420 | 0.0293 | 0.0293 |
| Q2 | 0.7440 | 0.7190 | 0.7087 | 0.7190 | 0.7017 | 0.7017 | 0.0250 | 0.0353 | 0.0250 | 0.0423 | 0.0423 |
| Q3 | 0.8960 | 0.9060 | 0.8632 | 0.8592 | 0.8647 | 0.8647 | 0.0100 | 0.0328 | 0.0368 | 0.0313 | 0.0313 |
| Q4 | -0.7000 | -0.7190 | -0.7055 | -0.7148 | -0.7000 | -0.7000 | 0.0190 | 0.0055 | 0.0148 | 0.0000 | 0.0000 |
| Q5 | -0.7000 | -0.6770 | -0.6831 | -0.6770 | -0.6828 | -0.6828 | 0.0230 | 0.0169 | 0.0230 | 0.0172 | 0.0172 |
| Q6 | -0.7000 | -0.6090 | -0.6643 | -0.6711 | -0.6641 | -0.6641 | 0.0910 | 0.0357 | 0.0289 | 0.0359 | 0.0359 |
| P1-2 | 0.2870 | 0.3150 | 0.2773 | 0.3064 | 0.3034 | 0.3034 | 0.0280 | 0.0097 | 0.0194 | 0.0164 | 0.0164 |
| P1-4 | 0.4360 | 0.3890 | 0.4756 | 0.4539 | 0.4473 | 0.4473 | 0.0470 | 0.0396 | 0.0179 | 0.0113 | 0.0113 |
| P1-5 | 0.3560 | 0.3570 | 0.3521 | 0.3708 | 0.3683 | 0.3683 | 0.0010 | 0.0039 | 0.0148 | 0.0123 | 0.0123 |
| P2-1 | -0.2780 | -0.3490 | -0.2689 | -0.2965 | -0.2937 | -0.2937 | 0.0710 | 0.0091 | 0.0185 | 0.0157 | 0.0157 |
| **P2-4** | **0.3310** | **0.9280** | **0.4294** | **0.3327** | **0.3226** | **0.3226** | **0.5970** | **0.0984** | **0.0017** | **0.0084** | **0.0084** |
| P2-5 | 0.1550 | 0.1740 | 0.1583 | 0.1573 | 0.1566 | 0.1566 | 0.0190 | 0.0033 | 0.0023 | 0.0016 | 0.0016 |
| P2-6 | 0.2620 | 0.2230 | 0.2409 | 0.2603 | 0.2591 | 0.2591 | 0.0390 | 0.0211 | 0.0017 | 0.0029 | 0.0029 |
| P2-3 | 0.0290 | 0.0860 | 0.0140 | 0.0301 | 0.0295 | 0.0295 | 0.0570 | 0.0150 | 0.0011 | 0.0005 | 0.0005 |
| P3-2 | -0.0290 | -0.0210 | -0.0136 | -0.0298 | -0.0291 | -0.0291 | 0.0080 | 0.0154 | 0.0008 | 0.0001 | 0.0001 |
| P3-5 | 0.1910 | 0.1770 | 0.2070 | 0.1912 | 0.1919 | 0.1919 | 0.0140 | 0.0160 | 0.0002 | 0.0009 | 0.0009 |
| P3-6 | 0.4380 | 0.4330 | 0.4333 | 0.4330 | 0.4338 | 0.4338 | 0.0050 | 0.0047 | 0.0050 | 0.0042 | 0.0042 |
| P4-1 | -0.4250 | -0.4010 | -0.4625 | -0.4417 | -0.4357 | -0.4357 | 0.0240 | 0.0375 | 0.0167 | 0.0107 | 0.0107 |
| P4-2 | -0.3160 | -0.2980 | -0.4109 | -0.3174 | -0.3081 | -0.3081 | 0.0180 | 0.0949 | 0.0014 | 0.0079 | 0.0079 |
| P4-5 | 0.0410 | 0.0070 | 0.0189 | 0.0411 | 0.0430 | 0.0430 | 0.0340 | 0.0221 | 0.0001 | 0.0020 | 0.0020 |
| P5-4 | -0.0400 | -0.0210 | -0.0187 | -0.0407 | -0.0426 | -0.0426 | 0.0190 | 0.0213 | 0.0007 | 0.0026 | 0.0026 |
| P5-1 | -0.3450 | -0.3660 | -0.3414 | -0.3590 | -0.3567 | -0.3567 | 0.0210 | 0.0036 | 0.0140 | 0.0117 | 0.0117 |
| P5-2 | -0.1500 | -0.1170 | -0.1533 | -0.1524 | -0.1518 | -0.1518 | 0.0330 | 0.0033 | 0.0024 | 0.0018 | 0.0018 |
| P5-3 | -0.1800 | -0.2510 | -0.1958 | -0.1807 | -0.1812 | -0.1812 | 0.0710 | 0.0158 | 0.0007 | 0.0012 | 0.0012 |
| P5-6 | 0.0160 | -0.0210 | -0.0005 | 0.0127 | 0.0126 | 0.0126 | 0.0370 | 0.0165 | 0.0033 | 0.0034 | 0.0034 |
| P6-5 | -0.0160 | 0.0100 | 0.0010 | -0.0123 | -0.0121 | -0.0121 | 0.0260 | 0.0170 | 0.0037 | 0.0039 | 0.0039 |
| P6-2 | 0.2570 | -0.1960 | -0.2359 | -0.2546 | -0.2535 | -0.2535 | 0.4530 | 0.4929 | 0.5116 | 0.5105 | 0.5105 |
| P6-3 | -0.4280 | -0.4680 | -0.4238 | -0.4234 | -0.4243 | -0.4243 | 0.0400 | 0.0042 | 0.0046 | 0.0037 | 0.0037 |
| Q1-2 | -0.1540 | -0.1320 | -0.1422 | -0.1397 | -0.1413 | -0.1413 | 0.0220 | 0.0118 | 0.0143 | 0.0127 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2248 | 0.2216 | 0.2130 | 0.2130 | 0.0110 | 0.0238 | 0.0206 | 0.0120 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1139 | 0.1201 | 0.1176 | 0.1176 | 0.0190 | 0.0009 | 0.0071 | 0.0046 | 0.0046 |
| Q2-1 | 0.1280 | 0.0970 | 0.1153 | 0.1160 | 0.1171 | 0.1171 | 0.0310 | 0.0127 | 0.0120 | 0.0109 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4571 | 0.4582 | 0.4495 | 0.4495 | 0.0780 | 0.0039 | 0.0028 | 0.0115 | 0.0115 |
| Q2-5 | 0.1540 | 0.2200 | 0.1477 | 0.1468 | 0.1461 | 0.1461 | 0.0660 | 0.0063 | 0.0072 | 0.0079 | 0.0079 |
| Q2-6 | 0.1240 | 0.1500 | 0.1122 | 0.1170 | 0.1119 | 0.1119 | 0.0260 | 0.0118 | 0.0070 | 0.0121 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1236 | -0.1190 | -0.1229 | -0.1229 | 0.0040 | 0.0006 | 0.0040 | 0.0001 | 0.0001 |
| Q3-2 | 0.0570 | 0.1020 | 0.0589 | 0.0549 | 0.0585 | 0.0585 | 0.0450 | 0.0019 | 0.0021 | 0.0015 | 0.0015 |
| Q3-5 | 0.2320 | 0.2390 | 0.2235 | 0.2214 | 0.2240 | 0.2240 | 0.0070 | 0.0085 | 0.0106 | 0.0080 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5809 | 0.5830 | 0.5822 | 0.5822 | 0.0240 | 0.0261 | 0.0240 | 0.0248 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2131 | -0.2139 | -0.2076 | -0.2076 | 0.0560 | 0.0141 | 0.0149 | 0.0086 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4404 | -0.4480 | -0.4410 | -0.4410 | 0.0080 | 0.0106 | 0.0030 | 0.0100 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0520 | -0.0529 | -0.0515 | -0.0515 | 0.1250 | 0.0030 | 0.0039 | 0.0025 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0245 | -0.0227 | -0.0246 | -0.0246 | 0.0130 | 0.0035 | 0.0053 | 0.0034 | 0.0034 |
| Q5-1 | -0.1350 | -0.1750 | -0.1355 | -0.1371 | -0.1358 | -0.1358 | 0.0400 | 0.0005 | 0.0021 | 0.0008 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1738 | -0.1727 | -0.1725 | -0.1725 | 0.0420 | 0.0062 | 0.0073 | 0.0075 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2514 | -0.2504 | -0.2531 | -0.2531 | 0.0620 | 0.0096 | 0.0106 | 0.0079 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0980 | -0.0941 | -0.0969 | -0.0969 | 0.0890 | 0.0010 | 0.0029 | 0.0001 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0406 | 0.0372 | 0.0397 | 0.0397 | 0.0100 | 0.0016 | 0.0018 | 0.0007 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1502 | -0.1524 | -0.1481 | -0.1481 | 0.0630 | 0.0098 | 0.0076 | 0.0119 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5547 | -0.5559 | -0.5556 | -0.5556 | 0.0680 | 0.0243 | 0.0231 | 0.0234 | 0.0234 |
| **SUM** | | | | | | | **3.0325** | **1.6906** | **1.1852** | **1.1096** | **1.1096** |

In this case, LTS and LTAV generates good results by excellent estimation of bad data, LAV is a little worse, WLS is the best though much better than measurements both for bad data and overall comparison.

### Median redundancy

The same bad data are applied as full redundancy case for consistency. State variables are presented in Table 5.82, Table 5.83 shows the estimated value and errors between each estimator and actual value.

Table 5.82: State Variable of case 1, large, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0627 | -0.0758 | -0.0701 | -0.0701 | **V1** | 1.0327 | 0.9837 | 1.0342 | 1.0342 |
| $\theta 3$ | -0.0696 | -0.0961 | -0.0854 | -0.0854 | **V2** | 1.0322 | 0.9814 | 1.0303 | 1.0303 |
| $\theta 4$ | -0.0862 | -0.0869 | -0.0791 | -0.0791 | **V3** | 1.0560 | 1.0050 | 1.0518 | 1.0518 |
| $\theta 5$ | -0.0924 | -0.1128 | -0.1006 | -0.1006 | **V4** | 0.9636 | 0.9143 | 0.9698 | 0.9698 |
| $\theta 6$ | -0.0994 | -0.1294 | -0.1155 | -0.1155 | **V5** | 0.9673 | 0.9131 | 0.9657 | 0.9657 |
| | | | | | **V6** | 0.9926 | 0.9379 | 0.9879 | 0.9879 |

Table 5.83: Estimation Results and Errors of case 1, large, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0327 | 0.9837 | 1.0342 | 1.0342 | 0.0173 | 0.0663 | 0.0158 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0322 | 0.9814 | 1.0303 | 1.0303 | 0.0178 | 0.0686 | 0.0197 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0560 | 1.0050 | 1.0518 | 1.0518 | 0.0140 | 0.0650 | 0.0182 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9636 | 0.9143 | 0.9698 | 0.9698 | 0.0260 | 0.0753 | 0.0198 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9673 | 0.9131 | 0.9657 | 0.9657 | 0.0183 | 0.0725 | 0.0199 | 0.0199 |
| V6 | 1.0043 | N/A | 0.9926 | 0.9379 | 0.9879 | 0.9879 | 0.0117 | 0.0664 | 0.0165 | 0.0165 |
| P1 | 1.0790 | 1.1310 | 1.1103 | 1.1310 | 1.1418 | 1.1418 | 0.0313 | 0.0520 | 0.0628 | 0.0628 |
| P2 | 0.5000 | 0.4840 | 0.6064 | 0.5668 | 0.4998 | 0.4998 | 0.1064 | 0.0668 | 0.0002 | 0.0002 |
| P3 | 0.6000 | 0.5510 | 0.6317 | 0.5585 | 0.5663 | 0.5663 | 0.0317 | 0.0415 | 0.0337 | 0.0337 |
| P4 | -0.7000 | -0.7180 | -0.9194 | -0.7180 | -0.7046 | -0.7046 | 0.2194 | 0.0180 | 0.0046 | 0.0046 |
| P5 | -0.7000 | -0.7200 | -0.6985 | -0.7200 | -0.7082 | -0.7082 | 0.0015 | 0.0200 | 0.0082 | 0.0082 |
| P6 | -0.7000 | -0.7230 | -0.6435 | -0.7230 | -0.7113 | -0.7113 | 0.0565 | 0.0230 | 0.0113 | 0.0113 |
| Q1 | 0.1600 | 0.2020 | 0.2016 | 0.2020 | 0.1902 | 0.1902 | 0.0416 | 0.0420 | 0.0302 | 0.0302 |
| Q2 | 0.7440 | 0.7190 | 0.7012 | 0.7190 | 0.6961 | 0.6961 | 0.0428 | 0.0250 | 0.0479 | 0.0479 |
| Q3 | 0.8960 | 0.9060 | 0.8821 | 0.9060 | 0.8840 | 0.8840 | 0.0139 | 0.0100 | 0.0120 | 0.0120 |
| Q4 | -0.7000 | -0.7190 | -0.7061 | -0.7190 | -0.6972 | -0.6972 | 0.0061 | 0.0190 | 0.0028 | 0.0028 |
| Q5 | -0.7000 | -0.6770 | -0.7037 | -0.6770 | -0.7034 | -0.7034 | 0.0037 | 0.0230 | 0.0034 | 0.0034 |
| Q6 | -0.7000 | -0.6090 | -0.6359 | -0.6111 | -0.6355 | -0.6355 | 0.0641 | 0.0889 | 0.0645 | 0.0645 |
| P1-2 | 0.2870 | N/A | 0.2724 | 0.3024 | 0.3119 | 0.3119 | 0.0146 | 0.0154 | 0.0249 | 0.0249 |
| P1-4 | 0.4360 | N/A | 0.4914 | 0.4515 | 0.4549 | 0.4549 | 0.0554 | 0.0155 | 0.0189 | 0.0189 |
| P1-5 | 0.3560 | 0.3570 | 0.3465 | 0.3770 | 0.3749 | 0.3749 | 0.0095 | 0.0210 | 0.0189 | 0.0189 |
| P2-1 | -0.2780 | N/A | -0.2640 | -0.2913 | -0.3014 | -0.3014 | 0.0140 | 0.0133 | 0.0234 | 0.0234 |
| **P2-4** | **0.3310** | **0.9280** | **0.4713** | **0.3433** | **0.3213** | **0.3213** | **0.1403** | **0.0123** | **0.0097** | **0.0097** |
| P2-5 | 0.1550 | 0.1740 | 0.1565 | 0.1672 | 0.1578 | 0.1578 | 0.0015 | 0.0122 | 0.0028 | 0.0028 |
| P2-6 | 0.2620 | N/A | 0.2325 | 0.2884 | 0.2754 | 0.2754 | 0.0295 | 0.0264 | 0.0134 | 0.0134 |
| P2-3 | 0.0290 | N/A | 0.0102 | 0.0593 | 0.0467 | 0.0467 | 0.0188 | 0.0303 | 0.0177 | 0.0177 |
| P3-2 | -0.0290 | N/A | -0.0097 | -0.0586 | -0.0461 | -0.0461 | 0.0193 | 0.0296 | 0.0171 | 0.0171 |
| P3-5 | 0.1910 | N/A | 0.2113 | 0.1841 | 0.1815 | 0.1815 | 0.0203 | 0.0069 | 0.0095 | 0.0095 |
| P3-6 | 0.4380 | 0.4330 | 0.4301 | 0.4330 | 0.4309 | 0.4309 | 0.0079 | 0.0050 | 0.0071 | 0.0071 |
| P4-1 | -0.4250 | N/A | -0.4771 | -0.4379 | -0.4427 | -0.4427 | 0.0521 | 0.0129 | 0.0177 | 0.0177 |
| P4-2 | -0.3160 | N/A | -0.4503 | -0.3248 | -0.3063 | -0.3063 | 0.1343 | 0.0088 | 0.0097 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0080 | 0.0447 | 0.0443 | 0.0443 | 0.0330 | 0.0037 | 0.0033 | 0.0033 |
| P5-4 | -0.0400 | N/A | -0.0080 | -0.0441 | -0.0439 | -0.0439 | 0.0320 | 0.0041 | 0.0039 | 0.0039 |
| P5-1 | -0.3450 | N/A | -0.3359 | -0.3634 | -0.3627 | -0.3627 | 0.0091 | 0.0184 | 0.0177 | 0.0177 |
| P5-2 | -0.1500 | N/A | -0.1514 | -0.1613 | -0.1527 | -0.1527 | 0.0014 | 0.0113 | 0.0027 | 0.0027 |
| P5-3 | -0.1800 | N/A | -0.1990 | -0.1713 | -0.1703 | -0.1703 | 0.0190 | 0.0087 | 0.0097 | 0.0097 |
| P5-6 | 0.0160 | N/A | -0.0042 | 0.0202 | 0.0214 | 0.0214 | 0.0202 | 0.0042 | 0.0054 | 0.0054 |
| P6-5 | -0.0160 | N/A | 0.0049 | -0.0193 | -0.0207 | -0.0207 | 0.0209 | 0.0033 | 0.0047 | 0.0047 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P6-2 | 0.2570 | N/A | -0.2279 | -0.2814 | -0.2693 | -0.2693 | 0.4849 | 0.5384 | 0.5263 | 0.5263 |
| P6-3 | -0.4280 | N/A | -0.4206 | -0.4223 | -0.4212 | -0.4212 | 0.0074 | 0.0057 | 0.0068 | 0.0068 |
| Q1-2 | -0.1540 | N/A | -0.1443 | -0.1452 | -0.1442 | -0.1442 | 0.0097 | 0.0088 | 0.0098 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.2311 | 0.2262 | 0.2136 | 0.2136 | 0.0301 | 0.0252 | 0.0126 | 0.0126 |
| Q1-5 | 0.1130 | 0.0940 | 0.1148 | 0.1210 | 0.1207 | 0.1207 | 0.0018 | 0.0080 | 0.0077 | 0.0077 |
| Q2-1 | 0.1280 | N/A | 0.1184 | 0.1288 | 0.1225 | 0.1225 | 0.0096 | 0.0008 | 0.0055 | 0.0055 |
| Q2-4 | 0.4610 | 0.3830 | 0.4643 | 0.4778 | 0.4528 | 0.4528 | 0.0033 | 0.0168 | 0.0082 | 0.0082 |
| Q2-5 | 0.1540 | 0.2200 | 0.1511 | 0.1504 | 0.1494 | 0.1494 | 0.0029 | 0.0036 | 0.0046 | 0.0046 |
| Q2-6 | 0.1240 | N/A | 0.0997 | 0.0948 | 0.1007 | 0.1007 | 0.0243 | 0.0292 | 0.0233 | 0.0233 |
| Q2-3 | -0.1230 | N/A | -0.1323 | -0.1328 | -0.1293 | -0.1293 | 0.0093 | 0.0098 | 0.0063 | 0.0063 |
| Q3-2 | 0.0570 | N/A | 0.0692 | 0.0773 | 0.0671 | 0.0671 | 0.0122 | 0.0203 | 0.0101 | 0.0101 |
| Q3-5 | 0.2320 | N/A | 0.2357 | 0.2457 | 0.2372 | 0.2372 | 0.0037 | 0.0137 | 0.0052 | 0.0052 |
| Q3-6 | 0.6070 | 0.5830 | 0.5771 | 0.5830 | 0.5797 | 0.5797 | 0.0299 | 0.0240 | 0.0273 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.2138 | -0.2077 | -0.2048 | -0.2048 | 0.0148 | 0.0087 | 0.0058 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4422 | -0.4589 | -0.4429 | -0.4429 | 0.0088 | 0.0079 | 0.0081 | 0.0081 |
| Q4-5 | -0.0490 | N/A | -0.0501 | -0.0524 | -0.0495 | -0.0495 | 0.0011 | 0.0034 | 0.0005 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0243 | -0.0133 | -0.0245 | -0.0245 | 0.0037 | 0.0147 | 0.0035 | 0.0035 |
| Q5-1 | -0.1350 | N/A | -0.1351 | -0.1240 | -0.1348 | -0.1348 | 0.0001 | 0.0110 | 0.0002 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1758 | -0.1687 | -0.1740 | -0.1740 | 0.0042 | 0.0113 | 0.0060 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2604 | -0.2641 | -0.2640 | -0.2640 | 0.0006 | 0.0031 | 0.0030 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1080 | -0.1069 | -0.1061 | -0.1061 | 0.0110 | 0.0099 | 0.0091 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0525 | 0.0581 | 0.0510 | 0.0510 | 0.0135 | 0.0191 | 0.0120 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1378 | -0.1206 | -0.1343 | -0.1343 | 0.0222 | 0.0394 | 0.0257 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5505 | -0.5485 | -0.5522 | -0.5522 | 0.0285 | 0.0305 | 0.0268 | 0.0268 |
| **SUM** | | | | | | | **2.1146** | **1.9699** | **1.3841** | **1.3841** |

The same situation as in full redundancy case, LTS and LTAV generate the same best results since they eliminate the same data in this case.

**Comparison**

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.23. Full redundancy case have better overall estimation than median case for all estimators.

Table 5.84: Estimation errors of case 1, single, 6-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **3.0325** | **1.6906** | **1.1852** | **1.1096** | **1.1096** |
| **SUM** | **N/A** | **2.1146** | **1.9699** | **1.3841** | **1.3841** |

#### 5.1.2.3 Case 2: Multiple non-interacting bad data

For multiple non-interacting bad data case, the same bad data are used for evaluation in all the four major categories. Two reverse bad data, two large bad

data and two non-conforming bad data, i.e, one reverse and one large bad data are conducted. Two non-interacting bad data $P_{2-4}$ and $P_{3-6}$ are used to replace the original one.

**Two reverse bad data**

The two reverse bad data are $P_{2-4} = -0.3280$ and $P_{3-6} = -0.4330$. Compared with conventional reverse bad data, the case is quiet different because the error with actual one is quiet large, even compared with large bad data case. The locations are still used is for consistency consideration.

**Full Redundancy**

Table 5.85 presents the state variables generated by the estimation of each estimator; The estimated system results and the error between them and the actual value are illustrated in Table 5.86.

Table 5.85: State Variable of case 2, multi reverse, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0715 | -0.0678 | -0.0715 | -0.0666 | **V1** | 1.0477 | 1.0447 | 1.0477 | 1.0462 |
| $\theta 3$ | -0.0931 | -0.0818 | -0.0931 | -0.0774 | **V2** | 1.0415 | 1.0400 | 1.0415 | 1.0423 |
| $\theta 4$ | -0.0699 | -0.0755 | -0.0699 | -0.0757 | **V3** | 1.0595 | 1.0588 | 1.0595 | 1.0626 |
| $\theta 5$ | -0.1015 | -0.0984 | -0.1015 | -0.0962 | **V4** | 0.9878 | 0.9798 | 0.9878 | 0.9827 |
| $\theta 6$ | -0.1094 | -0.1098 | -0.1094 | -0.1071 | **V5** | 0.9788 | 0.9767 | 0.9788 | 0.9794 |
| | | | | | **V6** | 0.9986 | 0.9952 | 0.9986 | 0.9990 |

Table 5.86: Estimation Results and Errors of case 2, multi reverse, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0477 | 1.0447 | 1.0477 | 1.0462 | 0.0135 | 0.0023 | 0.0053 | 0.0023 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0415 | 1.0400 | 1.0415 | 1.0423 | 0.0161 | 0.0085 | 0.0100 | 0.0085 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0595 | 1.0588 | 1.0595 | 1.0626 | 0.0200 | 0.0105 | 0.0112 | 0.0105 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9878 | 0.9798 | 0.9878 | 0.9827 | 0.0083 | 0.0017 | 0.0098 | 0.0017 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9788 | 0.9767 | 0.9788 | 0.9794 | 0.0065 | 0.0068 | 0.0090 | 0.0068 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 0.9986 | 0.9952 | 0.9986 | 0.9990 | 0.0091 | 0.0058 | 0.0091 | 0.0058 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1349 | 1.1310 | 1.1349 | 1.1190 | 0.0520 | 0.0559 | 0.0520 | 0.0559 | 0.0400 |
| P2 | 0.5000 | 0.4840 | 0.3726 | 0.4840 | 0.3726 | 0.4741 | 0.0160 | 0.1274 | 0.0160 | 0.1274 | 0.0259 |
| P3 | 0.6000 | 0.5510 | 0.3664 | 0.5510 | 0.3664 | 0.5969 | 0.0490 | 0.2336 | 0.0490 | 0.2336 | 0.0031 |
| P4 | -0.7000 | -0.7180 | -0.5355 | -0.6855 | -0.5355 | -0.7007 | 0.0180 | 0.1645 | 0.0145 | 0.1645 | 0.0007 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P5 | -0.7000 | -0.7200 | -0.7360 | -0.7200 | -0.7360 | -0.7197 | 0.0200 | 0.0360 | 0.0200 | 0.0360 | 0.0197 |
| P6 | -0.7000 | -0.7230 | -0.5275 | -0.6799 | -0.5275 | -0.6901 | 0.0230 | 0.1725 | 0.0201 | 0.1725 | 0.0099 |
| Q1 | 0.1600 | 0.2020 | 0.1818 | 0.2020 | 0.1818 | 0.1893 | 0.0420 | 0.0218 | 0.0420 | 0.0218 | 0.0293 |
| Q2 | 0.7440 | 0.7190 | 0.6987 | 0.7190 | 0.6987 | 0.7017 | 0.0250 | 0.0453 | 0.0250 | 0.0453 | 0.0423 |
| Q3 | 0.8960 | 0.9060 | 0.8661 | 0.8610 | 0.8661 | 0.8648 | 0.0100 | 0.0299 | 0.0350 | 0.0299 | 0.0312 |
| Q4 | -0.7000 | -0.7190 | -0.6920 | -0.7182 | -0.6920 | -0.7001 | 0.0190 | 0.0080 | 0.0182 | 0.0080 | 0.0001 |
| Q5 | -0.7000 | -0.6770 | -0.6877 | -0.6770 | -0.6877 | -0.6828 | 0.0230 | 0.0123 | 0.0230 | 0.0123 | 0.0172 |
| Q6 | -0.7000 | -0.6090 | -0.6737 | -0.6720 | -0.6737 | -0.6640 | 0.0910 | 0.0263 | 0.0280 | 0.0263 | 0.0360 |
| P1-2 | 0.2870 | 0.3150 | 0.3304 | 0.3096 | 0.3304 | 0.3034 | 0.0280 | 0.0434 | 0.0226 | 0.0434 | 0.0164 |
| P1-4 | 0.4360 | 0.3890 | 0.4170 | 0.4465 | 0.4170 | 0.4473 | 0.0470 | 0.0190 | 0.0105 | 0.0190 | 0.0113 |
| P1-5 | 0.3560 | 0.3570 | 0.3876 | 0.3750 | 0.3876 | 0.3683 | 0.0010 | 0.0316 | 0.0190 | 0.0316 | 0.0123 |
| P2-1 | -0.2780 | -0.3490 | -0.3192 | -0.2995 | -0.3192 | -0.2937 | 0.0710 | 0.0412 | 0.0215 | 0.0412 | 0.0157 |
| **P2-4** | **0.3310** | **-0.3280** | **0.2106** | **0.3127** | **0.2106** | **0.3226** | **0.6590** | **0.1204** | **0.0183** | **0.1204** | **0.0084** |
| P2-5 | 0.1550 | 0.1740 | 0.1574 | 0.1593 | 0.1574 | 0.1566 | 0.0190 | 0.0024 | 0.0043 | 0.0024 | 0.0016 |
| P2-6 | 0.2620 | 0.2230 | 0.2464 | 0.2672 | 0.2464 | 0.2591 | 0.0390 | 0.0156 | 0.0052 | 0.0156 | 0.0029 |
| P2-3 | 0.0290 | 0.0860 | 0.0774 | 0.0443 | 0.0774 | 0.0294 | 0.0570 | 0.0484 | 0.0153 | 0.0484 | 0.0004 |
| P3-2 | -0.0290 | -0.0210 | -0.0767 | -0.0438 | -0.0767 | -0.0290 | 0.0080 | 0.0477 | 0.0148 | 0.0477 | 0.0000 |
| P3-5 | 0.1910 | 0.1770 | 0.1527 | 0.1816 | 0.1527 | 0.1919 | 0.0140 | 0.0383 | 0.0094 | 0.0383 | 0.0009 |
| **P3-6** | **0.4380** | **-0.4330** | **0.2904** | **0.4132** | **0.2904** | **0.4340** | **0.8710** | **0.1476** | **0.0248** | **0.1476** | **0.0040** |
| P4-1 | -0.4250 | -0.4010 | -0.4068 | -0.4346 | -0.4068 | -0.4357 | 0.0240 | 0.0182 | 0.0096 | 0.0182 | 0.0107 |
| P4-2 | -0.3160 | -0.2980 | -0.1990 | -0.2980 | -0.1990 | -0.3081 | 0.0180 | 0.1170 | 0.0180 | 0.1170 | 0.0079 |
| P4-5 | 0.0410 | 0.0070 | 0.0704 | 0.0471 | 0.0704 | 0.0430 | 0.0340 | 0.0294 | 0.0061 | 0.0294 | 0.0020 |
| P5-4 | -0.0400 | -0.0210 | -0.0694 | -0.0466 | -0.0694 | -0.0426 | 0.0190 | 0.0294 | 0.0066 | 0.0294 | 0.0026 |
| P5-1 | -0.3450 | -0.3660 | -0.3749 | -0.3629 | -0.3749 | -0.3567 | 0.0210 | 0.0299 | 0.0179 | 0.0299 | 0.0117 |
| P5-2 | -0.1500 | -0.1170 | -0.1526 | -0.1543 | -0.1526 | -0.1518 | 0.0330 | 0.0026 | 0.0043 | 0.0026 | 0.0018 |
| P5-3 | -0.1800 | -0.2510 | -0.1431 | -0.1713 | -0.1431 | -0.1813 | 0.0710 | 0.0369 | 0.0087 | 0.0369 | 0.0013 |
| P5-6 | 0.0160 | -0.0210 | 0.0039 | 0.0152 | 0.0039 | 0.0126 | 0.0370 | 0.0121 | 0.0008 | 0.0121 | 0.0034 |
| P6-5 | -0.0160 | 0.0100 | -0.0035 | -0.0147 | -0.0035 | -0.0121 | 0.0260 | 0.0125 | 0.0013 | 0.0125 | 0.0039 |
| P6-2 | 0.2570 | -0.1960 | -0.2412 | -0.2613 | -0.2412 | -0.2535 | 0.4530 | 0.4982 | 0.5183 | 0.4982 | 0.5105 |
| P6-3 | -0.4280 | -0.4680 | -0.2828 | -0.4039 | -0.2828 | -0.4244 | 0.0400 | 0.1452 | 0.0241 | 0.1452 | 0.0036 |
| Q1-2 | -0.1540 | -0.1320 | -0.1405 | -0.1392 | -0.1405 | -0.1413 | 0.0220 | 0.0135 | 0.0148 | 0.0135 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2003 | 0.2205 | 0.2003 | 0.2130 | 0.0110 | 0.0007 | 0.0195 | 0.0007 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1220 | 0.1207 | 0.1220 | 0.1176 | 0.0190 | 0.0090 | 0.0077 | 0.0090 | 0.0046 |
| Q2-1 | 0.1280 | 0.0970 | 0.1192 | 0.1158 | 0.1192 | 0.1171 | 0.0310 | 0.0088 | 0.0122 | 0.0088 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4426 | 0.4592 | 0.4426 | 0.4495 | 0.0780 | 0.0184 | 0.0018 | 0.0184 | 0.0115 |
| Q2-5 | 0.1540 | 0.2200 | 0.1449 | 0.1462 | 0.1449 | 0.1461 | 0.0660 | 0.0091 | 0.0078 | 0.0091 | 0.0079 |
| Q2-6 | 0.1240 | 0.1500 | 0.1138 | 0.1167 | 0.1138 | 0.1119 | 0.0260 | 0.0102 | 0.0073 | 0.0102 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1219 | -0.1190 | -0.1219 | -0.1229 | 0.0040 | 0.0011 | 0.0040 | 0.0011 | 0.0001 |
| Q3-2 | 0.0570 | 0.1020 | 0.0589 | 0.0551 | 0.0589 | 0.0585 | 0.0450 | 0.0019 | 0.0019 | 0.0019 | 0.0015 |
| Q3-5 | 0.2320 | 0.2390 | 0.2302 | 0.2229 | 0.2302 | 0.2240 | 0.0070 | 0.0018 | 0.0091 | 0.0018 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5770 | 0.5830 | 0.5770 | 0.5822 | 0.0240 | 0.0300 | 0.0240 | 0.0300 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2010 | -0.2143 | -0.2010 | -0.2076 | 0.0560 | 0.0020 | 0.0153 | 0.0020 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4402 | -0.4502 | -0.4402 | -0.4410 | 0.0080 | 0.0108 | 0.0008 | 0.0108 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0508 | -0.0538 | -0.0508 | -0.0515 | 0.1250 | 0.0018 | 0.0048 | 0.0018 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0245 | -0.0217 | -0.0245 | -0.0246 | 0.0130 | 0.0035 | 0.0063 | 0.0035 | 0.0034 |
| Q5-1 | -0.1350 | -0.1750 | -0.1361 | -0.1369 | -0.1361 | -0.1358 | 0.0400 | 0.0011 | 0.0019 | 0.0011 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1713 | -0.1721 | -0.1713 | -0.1725 | 0.0420 | 0.0087 | 0.0079 | 0.0087 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2614 | -0.2525 | -0.2614 | -0.2531 | 0.0620 | 0.0004 | 0.0085 | 0.0004 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0945 | -0.0938 | -0.0945 | -0.0969 | 0.0890 | 0.0025 | 0.0032 | 0.0025 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0372 | 0.0368 | 0.0372 | 0.0397 | 0.0100 | 0.0018 | 0.0022 | 0.0018 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1509 | -0.1515 | -0.1509 | -0.1481 | 0.0630 | 0.0091 | 0.0085 | 0.0091 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5599 | -0.5574 | -0.5599 | -0.5556 | 0.0680 | 0.0191 | 0.0216 | 0.0191 | 0.0234 |
| **SUM** | | | | | | | **3.9605** | **2.6214** | **1.3695** | **2.6214** | **1.1088** |

WLS and LTS generate bad overall estimation with no bad data detected in this case, other estimators have good results. The reason is that LTS relay on WLS and does not trim all the two bad data, so the estimation is not acceptable.

### Median Redundancy

For median redundancy case, the same bad data as full redundancy case act as reverse errors. The state variables, estimation results and error with actual value are presented in the following two tables.

Table 5.87: State Variable of case 2, multi reverse, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0779 | -0.0791 | -0.0903 | -0.0701 | **V1** | 1.0356 | 0.9809 | 1.0405 | 1.0342 |
| $\theta 3$ | -0.1113 | -0.0995 | -0.1782 | -0.0854 | **V2** | 1.0284 | 0.9770 | 1.0269 | 1.0303 |
| $\theta 4$ | -0.0717 | -0.0834 | -0.0742 | -0.0791 | **V3** | 1.0459 | 1.0007 | 1.0305 | 1.0518 |
| $\theta 5$ | -0.1098 | -0.1148 | -0.1242 | -0.1006 | **V4** | 0.9763 | 0.9129 | 0.9810 | 0.9698 |
| $\theta 6$ | -0.1241 | -0.1329 | -0.1624 | -0.1155 | **V5** | 0.9637 | 0.9090 | 0.9652 | 0.9657 |
| | | | | | **V6** | 0.9852 | 0.9333 | 0.9759 | 0.9879 |

Table 5.88: Estimation Results and Errors of case 2, multi reverse, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0356 | 0.9809 | 1.0405 | 1.0342 | 0.0144 | 0.0691 | 0.0095 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0284 | 0.9770 | 1.0269 | 1.0303 | 0.0216 | 0.0730 | 0.0231 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0459 | 1.0007 | 1.0305 | 1.0518 | 0.0241 | 0.0693 | 0.0395 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9763 | 0.9129 | 0.9810 | 0.9698 | 0.0133 | 0.0767 | 0.0086 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9637 | 0.9090 | 0.9652 | 0.9657 | 0.0220 | 0.0766 | 0.0204 | 0.0199 |
| V6 | 1.0043 | N/A | 0.9852 | 0.9333 | 0.9759 | 0.9879 | 0.0191 | 0.0710 | 0.0284 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1763 | 1.1310 | 1.3134 | 1.1418 | 0.0973 | 0.0520 | 0.2344 | 0.0628 |
| P2 | 0.5000 | 0.4840 | 0.3918 | 0.4840 | 0.5839 | 0.4998 | 0.1082 | 0.0160 | 0.0839 | 0.0002 |
| P3 | 0.6000 | 0.5510 | 0.2467 | 0.5510 | -0.4644 | 0.5655 | 0.3533 | 0.0490 | 1.0644 | 0.0345 |
| P4 | -0.7000 | -0.7180 | -0.4755 | -0.6291 | -0.3605 | -0.7046 | 0.2245 | 0.0709 | 0.3395 | 0.0046 |
| P5 | -0.7000 | -0.7200 | -0.7245 | -0.7200 | -0.5308 | -0.7082 | 0.0245 | 0.0200 | 0.1692 | 0.0082 |
| P6 | -0.7000 | -0.7230 | -0.5340 | -0.7230 | -0.4419 | -0.7105 | 0.1660 | 0.0230 | 0.2581 | 0.0105 |
| Q1 | 0.1600 | 0.2020 | 0.1778 | 0.2020 | 0.1854 | 0.1902 | 0.0178 | 0.0420 | 0.0254 | 0.0302 |
| Q2 | 0.7440 | 0.7190 | 0.6963 | 0.7190 | 0.7066 | 0.6962 | 0.0477 | 0.0250 | 0.0374 | 0.0478 |
| Q3 | 0.8960 | 0.9060 | 0.8903 | 0.9060 | 0.9011 | 0.8840 | 0.0057 | 0.0100 | 0.0051 | 0.0120 |
| Q4 | -0.7000 | -0.7190 | -0.6835 | -0.7190 | -0.6774 | -0.6972 | 0.0165 | 0.0190 | 0.0226 | 0.0028 |
| Q5 | -0.7000 | -0.6770 | -0.7109 | -0.6770 | -0.6965 | -0.7035 | 0.0109 | 0.0230 | 0.0035 | 0.0035 |
| Q6 | -0.7000 | -0.6090 | -0.6461 | -0.6113 | -0.6313 | -0.6356 | 0.0539 | 0.0887 | 0.0687 | 0.0644 |
| P1-2 | 0.2870 | N/A | 0.3527 | 0.3167 | 0.4225 | 0.3119 | 0.0657 | 0.0297 | 0.1355 | 0.0249 |
| P1-4 | 0.4360 | N/A | 0.4163 | 0.4332 | 0.4324 | 0.4549 | 0.0197 | 0.0028 | 0.0036 | 0.0189 |
| P1-5 | 0.3560 | 0.3570 | 0.4073 | 0.3811 | 0.4585 | 0.3750 | 0.0513 | 0.0251 | 0.1025 | 0.0190 |
| P2-1 | -0.2780 | N/A | -0.3397 | -0.3046 | -0.4047 | -0.3014 | 0.0617 | 0.0266 | 0.1267 | 0.0234 |
| **P2-4** | **0.3310** | **-0.3280** | **0.1652** | **0.2811** | **0.0595** | **0.3213** | **0.1658** | **0.0499** | **0.2715** | **0.0097** |
| P2-5 | 0.1550 | 0.1740 | 0.1621 | 0.1619 | 0.1646 | 0.1578 | 0.0071 | 0.0069 | 0.0096 | 0.0028 |
| P2-6 | 0.2620 | N/A | 0.2794 | 0.2868 | 0.4072 | 0.2752 | 0.0174 | 0.0248 | 0.1452 | 0.0132 |
| P2-3 | 0.0290 | N/A | 0.1248 | 0.0589 | 0.3573 | 0.0468 | 0.0958 | 0.0299 | 0.3283 | 0.0178 |
| P3-2 | -0.0290 | N/A | -0.1237 | -0.0581 | -0.3510 | -0.0463 | 0.0947 | 0.0291 | 0.3220 | 0.0173 |
| P3-5 | 0.1910 | N/A | 0.1213 | 0.1785 | -0.0696 | 0.1814 | 0.0697 | 0.0125 | 0.2606 | 0.0096 |
| **P3-6** | **0.4380** | **-0.4330** | **0.2491** | **0.4307** | **-0.0438** | **0.4303** | **0.1889** | **0.0073** | **0.4818** | **0.0077** |
| P4-1 | -0.4250 | N/A | -0.4060 | -0.4205 | -0.4216 | -0.4427 | 0.0190 | 0.0045 | 0.0034 | 0.0177 |
| P4-2 | -0.3160 | N/A | -0.1541 | -0.2646 | -0.0500 | -0.3063 | 0.1619 | 0.0514 | 0.2660 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0847 | 0.0560 | 0.1112 | 0.0443 | 0.0437 | 0.0150 | 0.0702 | 0.0033 |
| P5-4 | -0.0400 | N/A | -0.0832 | -0.0551 | -0.1086 | -0.0439 | 0.0432 | 0.0151 | 0.0686 | 0.0039 |
| P5-1 | -0.3450 | N/A | -0.3930 | -0.3671 | -0.4410 | -0.3627 | 0.0480 | 0.0221 | 0.0960 | 0.0177 |
| P5-2 | -0.1500 | N/A | -0.1569 | -0.1561 | -0.1596 | -0.1527 | 0.0069 | 0.0061 | 0.0096 | 0.0027 |
| P5-3 | -0.1800 | N/A | -0.1114 | -0.1659 | 0.0800 | -0.1702 | 0.0686 | 0.0141 | 0.2600 | 0.0098 |
| P5-6 | 0.0160 | N/A | 0.0199 | 0.0242 | 0.0984 | 0.0214 | 0.0039 | 0.0082 | 0.0824 | 0.0054 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P6-5 | -0.0160 | N/A | -0.0192 | -0.0233 | -0.0969 | -0.0207 | 0.0032 | 0.0073 | 0.0809 | 0.0047 |
| P6-2 | 0.2570 | N/A | -0.2731 | -0.2797 | -0.3950 | -0.2692 | 0.5301 | 0.5367 | 0.6520 | 0.5262 |
| P6-3 | -0.4280 | N/A | -0.2416 | -0.4199 | 0.0500 | -0.4207 | 0.1864 | 0.0081 | 0.4780 | 0.0073 |
| Q1-2 | -0.1540 | N/A | -0.1445 | -0.1435 | -0.1404 | -0.1442 | 0.0095 | 0.0105 | 0.0136 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.1947 | 0.2215 | 0.1937 | 0.2136 | 0.0063 | 0.0205 | 0.0073 | 0.0126 |
| Q1-5 | 0.1130 | 0.0940 | 0.1276 | 0.1240 | 0.1320 | 0.1207 | 0.0146 | 0.0110 | 0.0190 | 0.0077 |
| Q2-1 | 0.1280 | N/A | 0.1279 | 0.1292 | 0.1332 | 0.1225 | 0.0001 | 0.0012 | 0.0052 | 0.0055 |
| Q2-4 | 0.4610 | 0.3830 | 0.4432 | 0.4763 | 0.4322 | 0.4528 | 0.0178 | 0.0153 | 0.0288 | 0.0082 |
| Q2-5 | 0.1540 | 0.2200 | 0.1484 | 0.1501 | 0.1370 | 0.1494 | 0.0056 | 0.0039 | 0.0170 | 0.0046 |
| Q2-6 | 0.1240 | N/A | 0.1032 | 0.0956 | 0.1058 | 0.1007 | 0.0208 | 0.0284 | 0.0182 | 0.0233 |
| Q2-3 | -0.1230 | N/A | -0.1264 | -0.1322 | -0.1016 | -0.1294 | 0.0034 | 0.0092 | 0.0214 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0677 | 0.0772 | 0.0695 | 0.0671 | 0.0107 | 0.0202 | 0.0125 | 0.0101 |
| Q3-5 | 0.2320 | N/A | 0.2476 | 0.2458 | 0.2698 | 0.2373 | 0.0156 | 0.0138 | 0.0378 | 0.0053 |
| Q3-6 | 0.6070 | 0.5830 | 0.5750 | 0.5830 | 0.5618 | 0.5797 | 0.0320 | 0.0240 | 0.0452 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.1942 | -0.2064 | -0.1915 | -0.2048 | 0.0048 | 0.0074 | 0.0075 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4413 | -0.4612 | -0.4334 | -0.4429 | 0.0097 | 0.0102 | 0.0176 | 0.0081 |
| Q4-5 | -0.0490 | N/A | -0.0480 | -0.0515 | -0.0525 | -0.0495 | 0.0010 | 0.0025 | 0.0035 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0242 | -0.0132 | -0.0181 | -0.0245 | 0.0038 | 0.0148 | 0.0099 | 0.0035 |
| Q5-1 | -0.1350 | N/A | -0.1341 | -0.1251 | -0.1267 | -0.1348 | 0.0009 | 0.0099 | 0.0083 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1725 | -0.1685 | -0.1619 | -0.1740 | 0.0075 | 0.0115 | 0.0181 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2767 | -0.2641 | -0.2969 | -0.2640 | 0.0157 | 0.0031 | 0.0359 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1034 | -0.1060 | -0.0928 | -0.1061 | 0.0064 | 0.0090 | 0.0042 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0484 | 0.0577 | 0.0408 | 0.0510 | 0.0094 | 0.0187 | 0.0018 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1359 | -0.1210 | -0.1212 | -0.1343 | 0.0241 | 0.0390 | 0.0388 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5586 | -0.5481 | -0.5509 | -0.5522 | 0.0204 | 0.0309 | 0.0281 | 0.0268 |
| **SUM** | | | | | | | **3.4341** | **2.1227** | **7.0961** | **1.3852** |

In this case, WLS has bad estimation, which induce wrong bad data detecting and elimination of LTS, so LTS generates even worse results. LAV and LTAV generate good results both for the bad data and overall summation, LTAV has best estimation on the basis of successful bad data elimination of LAV.

### Comparison

The comparison between full and median redundancy are presented in Table 5.89, only summation of errors are shown here. Both case have consistent results, LTAV generates best estimation based on the success of LAV. In median redundancy case, with few correct data, LTS detect the wrong bad data, making it produces much worse estimation.

Table 5.89: Estimation errors of case 2, multi reverse, 6-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| **SUM** | **3.9605** | **2.6214** | **1.3695** | **2.6214** | **1.1088** |
| **SUM** | **N/A** | **3.4341** | **2.1227** | **7.0961** | **1.3852** |

### Two non-conforming bad data

$P_{2-4} = -0.3280$ and $P_{3-6} = 0.9330$ instead of the original correct value are used to simulate the non-conforming bad data for full & median redundancy case.

### Full redundancy

The five system state variables are presented in Table 5.90, Table 5.91 illustrates the results and error between estimation and actual value.

Table 5.90: State Variable of case 2, non-conforning, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0718 | -0.0679 | -0.0717 | -0.0666 | **V1** | 1.0475 | 1.0449 | 1.0476 | 1.0462 |
| $\theta 3$ | -0.0822 | -0.0803 | -0.0861 | -0.0774 | **V2** | 1.0411 | 1.0402 | 1.0413 | 1.0423 |
| $\theta 4$ | -0.0701 | -0.0754 | -0.0700 | -0.0757 | **V3** | 1.0615 | 1.0592 | 1.0608 | 1.0626 |
| $\theta 5$ | -0.1006 | -0.0983 | -0.1009 | -0.0962 | **V4** | 0.9875 | 0.9799 | 0.9876 | 0.9827 |
| $\theta 6$ | -0.1198 | -0.1112 | -0.1160 | -0.1071 | **V5** | 0.9790 | 0.9769 | 0.9790 | 0.9794 |
| | | | | | **V6** | 0.9960 | 0.9952 | 0.9969 | 0.9990 |

Table 5.91: Estimation Results and Errors of case 2, non-conforning, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0475 | 1.0449 | 1.0476 | 1.0462 | 0.0135 | 0.0025 | 0.0051 | 0.0024 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0411 | 1.0402 | 1.0413 | 1.0423 | 0.0161 | 0.0089 | 0.0098 | 0.0087 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0615 | 1.0592 | 1.0608 | 1.0626 | 0.0200 | 0.0085 | 0.0108 | 0.0092 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9875 | 0.9799 | 0.9876 | 0.9827 | 0.0083 | 0.0021 | 0.0096 | 0.0020 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9790 | 0.9769 | 0.9790 | 0.9794 | 0.0065 | 0.0066 | 0.0088 | 0.0067 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 0.9960 | 0.9952 | 0.9969 | 0.9990 | 0.0091 | 0.0084 | 0.0091 | 0.0074 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1343 | 1.1310 | 1.1345 | 1.1190 | 0.0520 | 0.0553 | 0.0520 | 0.0555 | 0.0400 |
| P2 | 0.5000 | 0.4840 | 0.3664 | 0.4840 | 0.3686 | 0.4741 | 0.0160 | 0.1336 | 0.0160 | 0.1314 | 0.0259 |
| P3 | 0.6000 | 0.5510 | 0.6798 | 0.5952 | 0.5652 | 0.5969 | 0.0490 | 0.0798 | 0.0048 | 0.0348 | 0.0031 |
| P4 | -0.7000 | -0.7180 | -0.5379 | -0.6858 | -0.5370 | -0.7007 | 0.0180 | 0.1621 | 0.0142 | 0.1630 | 0.0007 |
| P5 | -0.7000 | -0.7200 | -0.7260 | -0.7200 | -0.7298 | -0.7197 | 0.0200 | 0.0260 | 0.0200 | 0.0298 | 0.0197 |
| P6 | -0.7000 | -0.7230 | -0.8358 | -0.7230 | -0.7232 | -0.6901 | 0.0230 | 0.1358 | 0.0230 | 0.0232 | 0.0099 |
| Q1 | 0.1600 | 0.2020 | 0.1815 | 0.2020 | 0.1816 | 0.1893 | 0.0420 | 0.0215 | 0.0420 | 0.0216 | 0.0293 |
| Q2 | 0.7440 | 0.7190 | 0.6941 | 0.7190 | 0.6958 | 0.7017 | 0.0250 | 0.0499 | 0.0250 | 0.0482 | 0.0423 |
| Q3 | 0.8960 | 0.9060 | 0.8685 | 0.8597 | 0.8669 | 0.8648 | 0.0100 | 0.0275 | 0.0363 | 0.0291 | 0.0312 |
| Q4 | -0.7000 | -0.7190 | -0.6929 | -0.7190 | -0.6926 | -0.7001 | 0.0190 | 0.0071 | 0.0190 | 0.0074 | 0.0001 |
| Q5 | -0.7000 | -0.6770 | -0.6794 | -0.6770 | -0.6824 | -0.6828 | 0.0230 | 0.0206 | 0.0230 | 0.0176 | 0.0172 |
| Q6 | -0.7000 | -0.6090 | -0.6558 | -0.6669 | -0.6632 | -0.6640 | 0.0910 | 0.0442 | 0.0331 | 0.0368 | 0.0360 |
| P1-2 | 0.2870 | 0.3150 | 0.3320 | 0.3097 | 0.3314 | 0.3034 | 0.0280 | 0.0450 | 0.0227 | 0.0444 | 0.0164 |
| P1-4 | 0.4360 | 0.3890 | 0.4179 | 0.4465 | 0.4176 | 0.4473 | 0.0470 | 0.0181 | 0.0105 | 0.0184 | 0.0113 |
| P1-5 | 0.3560 | 0.3570 | 0.3844 | 0.3748 | 0.3855 | 0.3683 | 0.0010 | 0.0284 | 0.0188 | 0.0295 | 0.0123 |
| P2-1 | -0.2780 | -0.3490 | -0.3207 | -0.2996 | -0.3201 | -0.2937 | 0.0710 | 0.0427 | 0.0216 | 0.0421 | 0.0157 |
| **P2-4** | **0.3310** | **-0.3280** | **0.2094** | **0.3128** | **0.2099** | **0.3226** | **0.6590** | **0.1216** | **0.0182** | **0.1211** | **0.0084** |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P2-5 | 0.1550 | 0.1740 | 0.1531 | 0.1591 | 0.1547 | 0.1566 | 0.0190 | 0.0019 | 0.0041 | 0.0003 | 0.0016 |
| P2-6 | 0.2620 | 0.2230 | 0.2968 | 0.2744 | 0.2783 | 0.2591 | 0.0390 | 0.0348 | 0.0124 | 0.0163 | 0.0029 |
| P2-3 | 0.0290 | 0.0860 | 0.0277 | 0.0374 | 0.0458 | 0.0294 | 0.0570 | 0.0013 | 0.0084 | 0.0168 | 0.0004 |
| P3-2 | -0.0290 | -0.0210 | -0.0273 | -0.0370 | -0.0453 | -0.0290 | 0.0080 | 0.0017 | 0.0080 | 0.0163 | 0.0000 |
| P3-5 | 0.1910 | 0.1770 | 0.1892 | 0.1870 | 0.1759 | 0.1919 | 0.0140 | 0.0018 | 0.0040 | 0.0151 | 0.0009 |
| **P3-6** | **0.4380** | **0.9330** | **0.5180** | **0.4452** | **0.4347** | **0.4340** | **0.4950** | **0.0800** | **0.0072** | **0.0033** | **0.0040** |
| P4-1 | -0.4250 | -0.4010 | -0.4077 | -0.4347 | -0.4074 | -0.4357 | 0.0240 | 0.0173 | 0.0097 | 0.0176 | 0.0107 |
| P4-2 | -0.3160 | -0.2980 | -0.1979 | -0.2980 | -0.1983 | -0.3081 | 0.0180 | 0.1181 | 0.0180 | 0.1177 | 0.0079 |
| P4-5 | 0.0410 | 0.0070 | 0.0678 | 0.0469 | 0.0687 | 0.0430 | 0.0340 | 0.0268 | 0.0059 | 0.0277 | 0.0020 |
| P5-4 | -0.0400 | -0.0210 | -0.0668 | -0.0464 | -0.0677 | -0.0426 | 0.0190 | 0.0268 | 0.0064 | 0.0277 | 0.0026 |
| P5-1 | -0.3450 | -0.3660 | -0.3719 | -0.3628 | -0.3730 | -0.3567 | 0.0210 | 0.0269 | 0.0178 | 0.0280 | 0.0117 |
| P5-2 | -0.1500 | -0.1170 | -0.1484 | -0.1541 | -0.1499 | -0.1518 | 0.0330 | 0.0016 | 0.0041 | 0.0001 | 0.0018 |
| P5-3 | -0.1800 | -0.2510 | -0.1787 | -0.1766 | -0.1657 | -0.1813 | 0.0710 | 0.0013 | 0.0034 | 0.0143 | 0.0013 |
| P5-6 | 0.0160 | -0.0210 | 0.0398 | 0.0199 | 0.0266 | 0.0126 | 0.0370 | 0.0238 | 0.0039 | 0.0106 | 0.0034 |
| P6-5 | -0.0160 | 0.0100 | -0.0391 | -0.0194 | -0.0261 | -0.0121 | 0.0260 | 0.0231 | 0.0034 | 0.0101 | 0.0039 |
| P6-2 | 0.2570 | -0.1960 | -0.2899 | -0.2682 | -0.2721 | -0.2535 | 0.4530 | 0.5469 | 0.5252 | 0.5291 | 0.5105 |
| P6-3 | -0.4280 | -0.4680 | -0.5068 | -0.4354 | -0.4250 | -0.4244 | 0.0400 | 0.0788 | 0.0074 | 0.0030 | 0.0036 |
| Q1-2 | -0.1540 | -0.1320 | -0.1404 | -0.1394 | -0.1404 | -0.1413 | 0.0220 | 0.0136 | 0.0146 | 0.0136 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2008 | 0.2207 | 0.2006 | 0.2130 | 0.0110 | 0.0002 | 0.0197 | 0.0004 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1211 | 0.1207 | 0.1214 | 0.1176 | 0.0190 | 0.0081 | 0.0077 | 0.0084 | 0.0046 |
| Q2-1 | 0.1280 | 0.0970 | 0.1194 | 0.1160 | 0.1193 | 0.1171 | 0.0310 | 0.0086 | 0.0120 | 0.0087 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4431 | 0.4599 | 0.4429 | 0.4495 | 0.0780 | 0.0179 | 0.0011 | 0.0181 | 0.0115 |
| Q2-5 | 0.1540 | 0.2200 | 0.1442 | 0.1464 | 0.1445 | 0.1461 | 0.0660 | 0.0098 | 0.0076 | 0.0095 | 0.0079 |
| Q2-6 | 0.1240 | 0.1500 | 0.1101 | 0.1156 | 0.1114 | 0.1119 | 0.0260 | 0.0139 | 0.0084 | 0.0126 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1227 | -0.1189 | -0.1224 | -0.1229 | 0.0040 | 0.0003 | 0.0041 | 0.0006 | 0.0001 |
| Q3-2 | 0.0570 | 0.1020 | 0.0584 | 0.0549 | 0.0585 | 0.0585 | 0.0450 | 0.0014 | 0.0021 | 0.0015 | 0.0015 |
| Q3-5 | 0.2320 | 0.2390 | 0.2219 | 0.2219 | 0.2249 | 0.2240 | 0.0070 | 0.0101 | 0.0101 | 0.0071 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5882 | 0.5830 | 0.5836 | 0.5822 | 0.0240 | 0.0188 | 0.0240 | 0.0234 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2014 | -0.2144 | -0.2013 | -0.2076 | 0.0560 | 0.0024 | 0.0154 | 0.0023 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4406 | -0.4508 | -0.4405 | -0.4410 | 0.0080 | 0.0104 | 0.0002 | 0.0105 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0509 | -0.0538 | -0.0509 | -0.0515 | 0.1250 | 0.0019 | 0.0048 | 0.0019 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0245 | -0.0218 | -0.0245 | -0.0246 | 0.0130 | 0.0035 | 0.0062 | 0.0035 | 0.0034 |
| Q5-1 | -0.1350 | -0.1750 | -0.1359 | -0.1370 | -0.1359 | -0.1358 | 0.0400 | 0.0009 | 0.0020 | 0.0009 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1710 | -0.1723 | -0.1711 | -0.1725 | 0.0420 | 0.0090 | 0.0077 | 0.0089 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2514 | -0.2512 | -0.2550 | -0.2531 | 0.0620 | 0.0096 | 0.0098 | 0.0060 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0967 | -0.0947 | -0.0959 | -0.0969 | 0.0890 | 0.0003 | 0.0023 | 0.0011 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0401 | 0.0379 | 0.0390 | 0.0397 | 0.0100 | 0.0011 | 0.0011 | 0.0000 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1422 | -0.1498 | -0.1456 | -0.1481 | 0.0630 | 0.0178 | 0.0102 | 0.0144 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5537 | -0.5550 | -0.5565 | -0.5556 | 0.0680 | 0.0253 | 0.0240 | 0.0225 | 0.0234 |
| **SUM** | | | | | | | **3.5845** | **2.2541** | **1.2982** | **1.9201** | **1.1088** |

LTAV generate best estimation by correctly detect and eliminate the two bad data, LAV's results are also very good. WLS can detect part of all bad data, it's estimation is much better than measurements though far away from LAV and LTAV, LTS generates relative better estimation after eliminating bad data detected by WLS.

**Median redundancy**

The same bad data as in full redundancy case, the following two tables shows the results and error with actual value of the system.

Table 5.92: State Variable of case 2, non-conforming, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0785 | -0.0791 | -0.0782 | -0.0701 | **V1** | 1.0343 | 0.9809 | 1.0349 | 1.0342 |
| $\theta 3$ | -0.0975 | -0.0995 | -0.1025 | -0.0854 | **V2** | 1.0266 | 0.9770 | 1.0275 | 1.0303 |
| $\theta 4$ | -0.0722 | -0.0834 | -0.0720 | -0.0791 | **V3** | 1.0468 | 1.0007 | 1.0467 | 1.0518 |
| $\theta 5$ | -0.1091 | -0.1148 | -0.1094 | -0.1006 | **V4** | 0.9745 | 0.9129 | 0.9753 | 0.9698 |
| $\theta 6$ | -0.1382 | -0.1329 | -0.1329 | -0.1155 | **V5** | 0.9626 | 0.9090 | 0.9632 | 0.9657 |
| | | | | | **V6** | 0.9801 | 0.9333 | 0.9822 | 0.9879 |

Table 5.93: Estimation Results and Errors of case 2, non-conforming, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0343 | 0.9809 | 1.0349 | 1.0342 | 0.0157 | 0.0691 | 0.0151 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0266 | 0.9770 | 1.0275 | 1.0303 | 0.0234 | 0.0730 | 0.0225 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0468 | 1.0007 | 1.0467 | 1.0518 | 0.0232 | 0.0693 | 0.0233 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9745 | 0.9129 | 0.9753 | 0.9698 | 0.0150 | 0.0767 | 0.0142 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9626 | 0.9090 | 0.9632 | 0.9657 | 0.0231 | 0.0766 | 0.0225 | 0.0199 |
| V6 | 1.0043 | N/A | 0.9801 | 0.9333 | 0.9822 | 0.9879 | 0.0242 | 0.0710 | 0.0222 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1772 | 1.1310 | 1.1767 | 1.1418 | 0.0982 | 0.0520 | 0.0977 | 0.0628 |
| P2 | 0.5000 | 0.4840 | 0.3853 | 0.4840 | 0.3875 | 0.4998 | 0.1147 | 0.0160 | 0.1125 | 0.0002 |
| P3 | 0.6000 | 0.5510 | 0.6423 | 0.5510 | 0.4969 | 0.5655 | 0.0423 | 0.0490 | 0.1031 | 0.0345 |
| P4 | -0.7000 | -0.7180 | -0.4781 | -0.6291 | -0.4773 | -0.7046 | 0.2219 | 0.0709 | 0.2227 | 0.0046 |
| P5 | -0.7000 | -0.7200 | -0.7121 | -0.7200 | -0.7169 | -0.7082 | 0.0121 | 0.0200 | 0.0169 | 0.0082 |
| P6 | -0.7000 | -0.7230 | -0.9264 | -0.7230 | -0.7821 | -0.7105 | 0.2264 | 0.0230 | 0.0821 | 0.0105 |
| Q1 | 0.1600 | 0.2020 | 0.1799 | 0.2020 | 0.1790 | 0.1902 | 0.0199 | 0.0420 | 0.0190 | 0.0302 |
| Q2 | 0.7440 | 0.7190 | 0.6919 | 0.7190 | 0.6935 | 0.6962 | 0.0521 | 0.0250 | 0.0505 | 0.0478 |
| Q3 | 0.8960 | 0.9060 | 0.8878 | 0.9060 | 0.8878 | 0.8840 | 0.0082 | 0.0100 | 0.0082 | 0.0120 |
| Q4 | -0.7000 | -0.7190 | -0.6840 | -0.7190 | -0.6839 | -0.6972 | 0.0160 | 0.0190 | 0.0161 | 0.0028 |
| Q5 | -0.7000 | -0.6770 | -0.6977 | -0.6770 | -0.7026 | -0.7035 | 0.0023 | 0.0230 | 0.0026 | 0.0035 |
| Q6 | -0.7000 | -0.6090 | -0.6234 | -0.6113 | -0.6333 | -0.6356 | 0.0766 | 0.0887 | 0.0667 | 0.0644 |
| P1-2 | 0.2870 | N/A | 0.3554 | 0.3167 | 0.3544 | 0.3119 | 0.0684 | 0.0297 | 0.0674 | 0.0249 |
| P1-4 | 0.4360 | N/A | 0.4178 | 0.4332 | 0.4172 | 0.4549 | 0.0182 | 0.0028 | 0.0188 | 0.0189 |
| P1-5 | 0.3560 | 0.3570 | 0.4039 | 0.3811 | 0.4051 | 0.3750 | 0.0479 | 0.0251 | 0.0491 | 0.0190 |
| P2-1 | -0.2780 | N/A | -0.3422 | -0.3046 | -0.3412 | -0.3014 | 0.0642 | 0.0266 | 0.0632 | 0.0234 |
| **P2-4** | **0.3310** | **-0.3280** | **0.1633** | **0.2811** | **0.1641** | **0.3213** | **0.1677** | **0.0499** | **0.1669** | **0.0097** |
| P2-5 | 0.1550 | 0.1740 | 0.1570 | 0.1619 | 0.1589 | 0.1578 | 0.0020 | 0.0069 | 0.0039 | 0.0028 |
| P2-6 | 0.2620 | N/A | 0.3447 | 0.2868 | 0.3205 | 0.2752 | 0.0827 | 0.0248 | 0.0585 | 0.0132 |
| P2-3 | 0.0290 | N/A | 0.0626 | 0.0589 | 0.0853 | 0.0468 | 0.0336 | 0.0299 | 0.0563 | 0.0178 |
| P3-2 | -0.0290 | N/A | -0.0619 | -0.0581 | -0.0845 | -0.0463 | 0.0329 | 0.0291 | 0.0555 | 0.0173 |
| P3-5 | 0.1910 | N/A | 0.1664 | 0.1785 | 0.1499 | 0.1814 | 0.0246 | 0.0125 | 0.0411 | 0.0096 |
| **P3-6** | **0.4380** | **0.9330** | **0.5378** | **0.4307** | **0.4315** | **0.4303** | **0.0998** | **0.0073** | **0.0065** | **0.0077** |
| P4-1 | -0.4250 | N/A | -0.4074 | -0.4205 | -0.4069 | -0.4427 | 0.0176 | 0.0045 | 0.0181 | 0.0177 |
| P4-2 | -0.3160 | N/A | -0.1523 | -0.2646 | -0.1531 | -0.3063 | 0.1637 | 0.0514 | 0.1629 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0816 | 0.0560 | 0.0827 | 0.0443 | 0.0406 | 0.0150 | 0.0417 | 0.0033 |
| P5-4 | -0.0400 | N/A | -0.0802 | -0.0551 | -0.0813 | -0.0439 | 0.0402 | 0.0151 | 0.0413 | 0.0039 |
| P5-1 | -0.3450 | N/A | -0.3898 | -0.3671 | -0.3909 | -0.3627 | 0.0448 | 0.0221 | 0.0459 | 0.0177 |
| P5-2 | -0.1500 | N/A | -0.1520 | -0.1561 | -0.1538 | -0.1527 | 0.0020 | 0.0061 | 0.0038 | 0.0027 |
| P5-3 | -0.1800 | N/A | -0.1559 | -0.1659 | -0.1397 | -0.1702 | 0.0241 | 0.0141 | 0.0403 | 0.0098 |
| P5-6 | 0.0160 | N/A | 0.0658 | 0.0242 | 0.0488 | 0.0214 | 0.0498 | 0.0082 | 0.0328 | 0.0054 |
| P6-5 | -0.0160 | N/A | -0.0647 | -0.0233 | -0.0479 | -0.0207 | 0.0487 | 0.0073 | 0.0319 | 0.0047 |
| P6-2 | 0.2570 | N/A | -0.3357 | -0.2797 | -0.3126 | -0.2692 | 0.5927 | 0.5367 | 0.5696 | 0.5262 |
| P6-3 | -0.4280 | N/A | -0.5260 | -0.4199 | -0.4216 | -0.4207 | 0.0980 | 0.0081 | 0.0064 | 0.0073 |
| Q1-2 | -0.1540 | N/A | -0.1433 | -0.1435 | -0.1438 | -0.1442 | 0.0107 | 0.0105 | 0.0102 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.1961 | 0.2215 | 0.1956 | 0.2136 | 0.0049 | 0.0205 | 0.0054 | 0.0126 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q1-5 | 0.1130 | 0.0940 | 0.1271 | 0.1240 | 0.1272 | 0.1207 | 0.0141 | 0.0110 | 0.0142 | 0.0077 |
| Q2-1 | 0.1280 | N/A | 0.1272 | 0.1292 | 0.1275 | 0.1225 | 0.0008 | 0.0012 | 0.0005 | 0.0055 |
| Q2-4 | 0.4610 | 0.3830 | 0.4429 | 0.4763 | 0.4430 | 0.4528 | 0.0181 | 0.0153 | 0.0180 | 0.0082 |
| Q2-5 | 0.1540 | 0.2200 | 0.1473 | 0.1501 | 0.1477 | 0.1494 | 0.0067 | 0.0039 | 0.0063 | 0.0046 |
| Q2-6 | 0.1240 | N/A | 0.1007 | 0.0956 | 0.1016 | 0.1007 | 0.0233 | 0.0284 | 0.0224 | 0.0233 |
| Q2-3 | -0.1230 | N/A | -0.1262 | -0.1322 | -0.1264 | -0.1294 | 0.0032 | 0.0092 | 0.0034 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0647 | 0.0772 | 0.0657 | 0.0671 | 0.0077 | 0.0202 | 0.0087 | 0.0101 |
| Q3-5 | 0.2320 | N/A | 0.2352 | 0.2458 | 0.2396 | 0.2373 | 0.0032 | 0.0138 | 0.0076 | 0.0053 |
| Q3-6 | 0.6070 | 0.5830 | 0.5879 | 0.5830 | 0.5824 | 0.5797 | 0.0191 | 0.0240 | 0.0246 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.1950 | -0.2064 | -0.1947 | -0.2048 | 0.0040 | 0.0074 | 0.0043 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4409 | -0.4612 | -0.4411 | -0.4429 | 0.0101 | 0.0102 | 0.0099 | 0.0081 |
| Q4-5 | -0.0490 | N/A | -0.0481 | -0.0515 | -0.0481 | -0.0495 | 0.0009 | 0.0025 | 0.0009 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0241 | -0.0132 | -0.0242 | -0.0245 | 0.0039 | 0.0148 | 0.0038 | 0.0035 |
| Q5-1 | -0.1350 | N/A | -0.1341 | -0.1251 | -0.1341 | -0.1348 | 0.0009 | 0.0099 | 0.0009 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1719 | -0.1685 | -0.1721 | -0.1740 | 0.0081 | 0.0115 | 0.0079 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2628 | -0.2641 | -0.2680 | -0.2640 | 0.0018 | 0.0031 | 0.0070 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1047 | -0.1060 | -0.1043 | -0.1061 | 0.0077 | 0.0090 | 0.0073 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0514 | 0.0577 | 0.0502 | 0.0510 | 0.0124 | 0.0187 | 0.0112 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1254 | -0.1210 | -0.1295 | -0.1343 | 0.0346 | 0.0390 | 0.0305 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5493 | -0.5481 | -0.5539 | -0.5522 | 0.0297 | 0.0309 | 0.0251 | 0.0268 |
| **SUM** | | | | | | | **3.0055** | **2.1227** | **2.7298** | **1.3852** |

LTAV has best results with both bad data and overall estimation, other three estimators' estimation are on the same level.

### Comparison

Table 5.94 presents the comparison between full and median redundancy case. Full redundancy case have great advantage for all estimators.

Table 5.94: Estimation errors of case 2, non-conforming, 6-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **3.5845** | **2.2541** | **1.2982** | **1.9201** | **1.1088** |
| **SUM** | **N/A** | **3.0055** | **2.1227** | **2.7298** | **1.3852** |

## Two large bad data

Compared with the previous two cases, $P_{2-4} = 0.9280$ and $P_{3-6} = 0.9330$ are introduced to simulate the two large bad data in both full and median redundancy case at the same location. The results are presented below.

### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.95 gives the state variables, and another table shows the estimation results and error with actual values.

Table 5.95: State variable of case 2, multi large, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0621 | -0.0674 | -0.0666 | -0.0666 | **V1** | 1.0446 | 1.0441 | 1.0462 | 1.0462 |
| $\theta 3$ | -0.0653 | -0.0770 | -0.0774 | -0.0774 | **V2** | 1.0431 | 1.0398 | 1.0423 | 1.0423 |
| $\theta 4$ | -0.0812 | -0.0770 | -0.0757 | -0.0757 | **V3** | 1.0651 | 1.0594 | 1.0626 | 1.0626 |
| $\theta 5$ | -0.0915 | -0.0972 | -0.0962 | -0.0962 | **V4** | 0.9778 | 0.9786 | 0.9827 | 0.9827 |
| $\theta 6$ | -0.1026 | -0.1091 | -0.1071 | -0.1071 | **V5** | 0.9798 | 0.9765 | 0.9794 | 0.9794 |
| | | | | | **V6** | 1.0001 | 0.9952 | 0.9990 | 0.9990 |

Table 5.96: Estimation Results and Errors of case 2, multi large, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0446 | 1.0441 | 1.0462 | 1.0462 | 0.0135 | 0.0054 | 0.0059 | 0.0038 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0431 | 1.0398 | 1.0423 | 1.0423 | 0.0161 | 0.0069 | 0.0102 | 0.0077 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0651 | 1.0594 | 1.0626 | 1.0626 | 0.0200 | 0.0049 | 0.0106 | 0.0074 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9778 | 0.9786 | 0.9827 | 0.9827 | 0.0083 | 0.0118 | 0.0109 | 0.0069 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9798 | 0.9765 | 0.9794 | 0.9794 | 0.0065 | 0.0058 | 0.0092 | 0.0062 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 1.0001 | 0.9952 | 0.9990 | 0.9990 | 0.0091 | 0.0043 | 0.0091 | 0.0053 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1048 | 1.1310 | 1.1190 | 1.1190 | 0.0520 | 0.0258 | 0.0520 | 0.0400 | 0.0400 |
| P2 | 0.5000 | 0.4840 | 0.5714 | 0.4840 | 0.4741 | 0.4741 | 0.0160 | 0.0714 | 0.0160 | 0.0259 | 0.0259 |
| P3 | 0.6000 | 0.5510 | 0.7417 | 0.6280 | 0.5969 | 0.5969 | 0.0490 | 0.1417 | 0.0280 | 0.0031 | 0.0031 |
| P4 | -0.7000 | -0.7180 | -0.8553 | -0.7180 | -0.7007 | -0.7007 | 0.0180 | 0.1553 | 0.0180 | 0.0007 | 0.0007 |
| P5 | -0.7000 | -0.7200 | -0.7060 | -0.7200 | -0.7197 | -0.7197 | 0.0200 | 0.0060 | 0.0200 | 0.0197 | 0.0197 |
| P6 | -0.7000 | -0.7230 | -0.7717 | -0.7230 | -0.6901 | -0.6901 | 0.0230 | 0.0717 | 0.0230 | 0.0099 | 0.0099 |
| Q1 | 0.1600 | 0.2020 | 0.1964 | 0.2020 | 0.1893 | 0.1893 | 0.0420 | 0.0364 | 0.0420 | 0.0293 | 0.0293 |
| Q2 | 0.7440 | 0.7190 | 0.7070 | 0.7190 | 0.7017 | 0.7017 | 0.0250 | 0.0370 | 0.0250 | 0.0423 | 0.0423 |
| Q3 | 0.8960 | 0.9060 | 0.8648 | 0.8584 | 0.8648 | 0.8648 | 0.0100 | 0.0312 | 0.0376 | 0.0312 | 0.0312 |
| Q4 | -0.7000 | -0.7190 | -0.7059 | -0.7156 | -0.7001 | -0.7001 | 0.0190 | 0.0059 | 0.0156 | 0.0001 | 0.0001 |
| Q5 | -0.7000 | -0.6770 | -0.6802 | -0.6770 | -0.6828 | -0.6828 | 0.0230 | 0.0198 | 0.0230 | 0.0172 | 0.0172 |
| Q6 | -0.7000 | -0.6090 | -0.6569 | -0.6670 | -0.6640 | -0.6640 | 0.0910 | 0.0431 | 0.0330 | 0.0360 | 0.0360 |
| P1-2 | 0.2870 | 0.3150 | 0.2779 | 0.3065 | 0.3034 | 0.3034 | 0.0280 | 0.0091 | 0.0195 | 0.0164 | 0.0164 |
| P1-4 | 0.4360 | 0.3890 | 0.4760 | 0.4539 | 0.4473 | 0.4473 | 0.0470 | 0.0400 | 0.0179 | 0.0113 | 0.0113 |
| P1-5 | 0.3560 | 0.3570 | 0.3509 | 0.3706 | 0.3683 | 0.3683 | 0.0010 | 0.0051 | 0.0146 | 0.0123 | 0.0123 |
| P2-1 | -0.2780 | -0.3490 | -0.2695 | -0.2966 | -0.2937 | -0.2937 | 0.0710 | 0.0085 | 0.0186 | 0.0157 | 0.0157 |
| **P2-4** | **0.3310** | **0.9280** | **0.4290** | **0.3326** | **0.3226** | **0.3226** | **0.5970** | **0.0980** | **0.0016** | **0.0084** | **0.0084** |
| P2-5 | 0.1550 | 0.1740 | 0.1567 | 0.1572 | 0.1566 | 0.1566 | 0.0190 | 0.0017 | 0.0022 | 0.0016 | 0.0016 |
| P2-6 | 0.2620 | 0.2230 | 0.2593 | 0.2658 | 0.2591 | 0.2591 | 0.0390 | 0.0027 | 0.0038 | 0.0029 | 0.0029 |
| P2-3 | 0.0290 | 0.0860 | -0.0041 | 0.0250 | 0.0294 | 0.0294 | 0.0570 | 0.0331 | 0.0040 | 0.0004 | 0.0004 |
| P3-2 | -0.0290 | -0.0210 | 0.0045 | -0.0246 | -0.0290 | -0.0290 | 0.0080 | 0.0335 | 0.0044 | 0.0000 | 0.0000 |
| P3-5 | 0.1910 | 0.1770 | 0.2204 | 0.1953 | 0.1919 | 0.1919 | 0.0140 | 0.0294 | 0.0043 | 0.0009 | 0.0009 |
| **P3-6** | **0.4380** | **0.9330** | **0.5168** | **0.4573** | **0.4340** | **0.4340** | **0.4950** | **0.0788** | **0.0193** | **0.0040** | **0.0040** |
| P4-1 | -0.4250 | -0.4010 | -0.4628 | -0.4417 | -0.4357 | -0.4357 | 0.0240 | 0.0378 | 0.0167 | 0.0107 | 0.0107 |
| P4-2 | -0.3160 | -0.2980 | -0.4104 | -0.3173 | -0.3081 | -0.3081 | 0.0180 | 0.0944 | 0.0013 | 0.0079 | 0.0079 |
| P4-5 | 0.0410 | 0.0070 | 0.0179 | 0.0410 | 0.0430 | 0.0430 | 0.0340 | 0.0231 | 0.0000 | 0.0020 | 0.0020 |
| P5-4 | -0.0400 | -0.0210 | -0.0178 | -0.0406 | -0.0426 | -0.0426 | 0.0190 | 0.0222 | 0.0006 | 0.0026 | 0.0026 |
| P5-1 | -0.3450 | -0.3660 | -0.3403 | -0.3589 | -0.3567 | -0.3567 | 0.0210 | 0.0047 | 0.0139 | 0.0117 | 0.0117 |
| P5-2 | -0.1500 | -0.1170 | -0.1518 | -0.1522 | -0.1518 | -0.1518 | 0.0330 | 0.0018 | 0.0022 | 0.0018 | 0.0018 |
| P5-3 | -0.1800 | -0.2510 | -0.2087 | -0.1846 | -0.1813 | -0.1813 | 0.0710 | 0.0287 | 0.0046 | 0.0013 | 0.0013 |
| P5-6 | 0.0160 | -0.0210 | 0.0127 | 0.0163 | 0.0126 | 0.0126 | 0.0370 | 0.0033 | 0.0003 | 0.0034 | 0.0034 |
| P6-5 | -0.0160 | 0.0100 | -0.0121 | -0.0159 | -0.0121 | -0.0121 | 0.0260 | 0.0039 | 0.0001 | 0.0039 | 0.0039 |
| P6-2 | 0.2570 | -0.1960 | -0.2538 | -0.2599 | -0.2535 | -0.2535 | 0.4530 | 0.5108 | 0.5169 | 0.5105 | 0.5105 |
| P6-3 | -0.4280 | -0.4680 | -0.5058 | -0.4472 | -0.4244 | -0.4244 | 0.0400 | 0.0778 | 0.0192 | 0.0036 | 0.0036 |
| Q1-2 | -0.1540 | -0.1320 | -0.1422 | -0.1399 | -0.1413 | -0.1413 | 0.0220 | 0.0118 | 0.0141 | 0.0127 | 0.0127 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q1-4 | 0.2010 | 0.2120 | 0.2250 | 0.2218 | 0.2130 | 0.2130 | 0.0110 | 0.0240 | 0.0208 | 0.0120 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1136 | 0.1201 | 0.1176 | 0.1176 | 0.0190 | 0.0006 | 0.0071 | 0.0046 | 0.0046 |
| Q2-1 | 0.1280 | 0.0970 | 0.1154 | 0.1162 | 0.1171 | 0.1171 | 0.0310 | 0.0126 | 0.0118 | 0.0109 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4573 | 0.4588 | 0.4495 | 0.4495 | 0.0780 | 0.0037 | 0.0022 | 0.0115 | 0.0115 |
| Q2-5 | 0.1540 | 0.2200 | 0.1474 | 0.1469 | 0.1461 | 0.1461 | 0.0660 | 0.0066 | 0.0071 | 0.0079 | 0.0079 |
| Q2-6 | 0.1240 | 0.1500 | 0.1106 | 0.1160 | 0.1119 | 0.1119 | 0.0260 | 0.0134 | 0.0080 | 0.0121 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1237 | -0.1190 | -0.1229 | -0.1229 | 0.0040 | 0.0007 | 0.0040 | 0.0001 | 0.0001 |
| Q3-2 | 0.0570 | 0.1020 | 0.0589 | 0.0548 | 0.0585 | 0.0585 | 0.0450 | 0.0019 | 0.0022 | 0.0015 | 0.0015 |
| Q3-5 | 0.2320 | 0.2390 | 0.2206 | 0.2207 | 0.2240 | 0.2240 | 0.0070 | 0.0114 | 0.0113 | 0.0080 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5852 | 0.5830 | 0.5822 | 0.5822 | 0.0240 | 0.0218 | 0.0240 | 0.0248 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2132 | -0.2141 | -0.2076 | -0.2076 | 0.0560 | 0.0142 | 0.0151 | 0.0086 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4406 | -0.4486 | -0.4410 | -0.4410 | 0.0080 | 0.0104 | 0.0024 | 0.0100 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0520 | -0.0530 | -0.0515 | -0.0515 | 0.1250 | 0.0030 | 0.0040 | 0.0025 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0244 | -0.0227 | -0.0246 | -0.0246 | 0.0130 | 0.0036 | 0.0053 | 0.0034 | 0.0034 |
| Q5-1 | -0.1350 | -0.1750 | -0.1354 | -0.1372 | -0.1358 | -0.1358 | 0.0400 | 0.0004 | 0.0022 | 0.0008 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1737 | -0.1729 | -0.1725 | -0.1725 | 0.0420 | 0.0063 | 0.0071 | 0.0075 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2476 | -0.2494 | -0.2531 | -0.2531 | 0.0620 | 0.0134 | 0.0116 | 0.0079 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0990 | -0.0949 | -0.0969 | -0.0969 | 0.0890 | 0.0020 | 0.0021 | 0.0001 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0417 | 0.0380 | 0.0397 | 0.0397 | 0.0100 | 0.0027 | 0.0010 | 0.0007 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1469 | -0.1510 | -0.1481 | -0.1481 | 0.0630 | 0.0131 | 0.0090 | 0.0119 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5517 | -0.5540 | -0.5556 | -0.5556 | 0.0680 | 0.0273 | 0.0250 | 0.0234 | 0.0234 |
| **SUM** | | | | | | | **3.5225** | **2.0376** | **1.2724** | **1.1088** | **1.1088** |

In this case, LAV, LTS and LTAV's estimation are on the same level, WLS has a little worse results. LTS and LTAV share the best estimation since both estimator successfully detect the bad data.

### Median redundancy

For median redundancy case, the same bad data are applied. The following two tables show the results and errors in detail.

Table 5.97: State Variable of case 2, multi large, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0629 | -0.0774 | -0.0701 | -0.0701 | **V1** | 1.0324 | 0.9812 | 1.0342 | 1.0342 |
| $\theta 3$ | -0.0646 | -0.0891 | -0.0854 | -0.0854 | **V2** | 1.0317 | 0.9785 | 1.0303 | 1.0303 |
| $\theta 4$ | -0.0864 | -0.0877 | -0.0791 | -0.0791 | **V3** | 1.0565 | 1.0044 | 1.0518 | 1.0518 |
| $\theta 5$ | -0.0921 | -0.1110 | -0.1006 | -0.1006 | **V4** | 0.9632 | 0.9114 | 0.9698 | 0.9698 |
| $\theta 6$ | -0.1045 | -0.1258 | -0.1155 | -0.1155 | **V5** | 0.9672 | 0.9110 | 0.9657 | 0.9657 |
| | | | | | **V6** | 0.9911 | 0.9367 | 0.9879 | 0.9879 |

Table 5.98: Estimation Results and Errors of case 2, multi large, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0324 | 0.9812 | 1.0342 | 1.0342 | 0.0176 | 0.0688 | 0.0158 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0317 | 0.9785 | 1.0303 | 1.0303 | 0.0183 | 0.0715 | 0.0197 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0565 | 1.0044 | 1.0518 | 1.0518 | 0.0135 | 0.0656 | 0.0182 | 0.0182 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V4 | 0.9896 | N/A | 0.9632 | 0.9114 | 0.9698 | 0.9698 | 0.0264 | 0.0781 | 0.0198 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9672 | 0.9110 | 0.9657 | 0.9657 | 0.0185 | 0.0747 | 0.0199 | 0.0199 |
| V6 | 1.0043 | N/A | 0.9911 | 0.9367 | 0.9879 | 0.9879 | 0.0133 | 0.0676 | 0.0164 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1107 | 1.1310 | 1.1418 | 1.1418 | 0.0317 | 0.0520 | 0.0628 | 0.0628 |
| P2 | 0.5000 | 0.4840 | 0.6040 | 0.4840 | 0.4998 | 0.4998 | 0.1040 | 0.0160 | 0.0002 | 0.0002 |
| P3 | 0.6000 | 0.5510 | 0.7765 | 0.6411 | 0.5655 | 0.5655 | 0.1765 | 0.0411 | 0.0345 | 0.0345 |
| P4 | -0.7000 | -0.7180 | -0.9204 | -0.7180 | -0.7046 | -0.7046 | 0.2204 | 0.0180 | 0.0046 | 0.0046 |
| P5 | -0.7000 | -0.7200 | -0.6938 | -0.7200 | -0.7082 | -0.7082 | 0.0062 | 0.0200 | 0.0082 | 0.0082 |
| P6 | -0.7000 | -0.7230 | -0.7869 | -0.7230 | -0.7105 | -0.7105 | 0.0869 | 0.0230 | 0.0105 | 0.0105 |
| Q1 | 0.1600 | 0.2020 | 0.2021 | 0.2020 | 0.1902 | 0.1902 | 0.0421 | 0.0420 | 0.0302 | 0.0302 |
| Q2 | 0.7440 | 0.7190 | 0.6996 | 0.7190 | 0.6962 | 0.6962 | 0.0444 | 0.0250 | 0.0478 | 0.0478 |
| Q3 | 0.8960 | 0.9060 | 0.8817 | 0.9055 | 0.8840 | 0.8840 | 0.0143 | 0.0095 | 0.0120 | 0.0120 |
| Q4 | -0.7000 | -0.7190 | -0.7064 | -0.7190 | -0.6972 | -0.6972 | 0.0064 | 0.0190 | 0.0028 | 0.0028 |
| Q5 | -0.7000 | -0.6770 | -0.6990 | -0.6770 | -0.7035 | -0.7035 | 0.0010 | 0.0230 | 0.0035 | 0.0035 |
| Q6 | -0.7000 | -0.6090 | -0.6256 | -0.6090 | -0.6356 | -0.6356 | 0.0744 | 0.0910 | 0.0644 | 0.0644 |
| P1-2 | 0.2870 | N/A | 0.2734 | 0.3079 | 0.3119 | 0.3119 | 0.0136 | 0.0209 | 0.0249 | 0.0249 |
| P1-4 | 0.4360 | N/A | 0.4920 | 0.4532 | 0.4549 | 0.4549 | 0.0560 | 0.0172 | 0.0189 | 0.0189 |
| P1-5 | 0.3560 | 0.3570 | 0.3453 | 0.3699 | 0.3750 | 0.3750 | 0.0107 | 0.0139 | 0.0190 | 0.0190 |
| P2-1 | -0.2780 | N/A | -0.2649 | -0.2964 | -0.3014 | -0.3014 | 0.0131 | 0.0184 | 0.0234 | 0.0234 |
| **P2-4** | **0.3310** | **0.9280** | **0.4706** | **0.3364** | **0.3213** | **0.3213** | **0.1396** | **0.0054** | **0.0097** | **0.0097** |
| P2-5 | 0.1550 | 0.1740 | 0.1546 | 0.1565 | 0.1578 | 0.1578 | 0.0004 | 0.0015 | 0.0028 | 0.0028 |
| P2-6 | 0.2620 | N/A | 0.2562 | 0.2629 | 0.2752 | 0.2752 | 0.0058 | 0.0009 | 0.0132 | 0.0132 |
| P2-3 | 0.0290 | N/A | -0.0124 | 0.0247 | 0.0468 | 0.0468 | 0.0414 | 0.0043 | 0.0178 | 0.0178 |
| P3-2 | -0.0290 | N/A | 0.0129 | -0.0241 | -0.0463 | -0.0463 | 0.0419 | 0.0049 | 0.0173 | 0.0173 |
| P3-5 | 0.1910 | N/A | 0.2279 | 0.2013 | 0.1814 | 0.1814 | 0.0369 | 0.0103 | 0.0096 | 0.0096 |
| **P3-6** | **0.4380** | **0.9330** | **0.5358** | **0.4639** | **0.4303** | **0.4303** | **0.0978** | **0.0259** | **0.0077** | **0.0077** |
| P4-1 | -0.4250 | N/A | -0.4777 | -0.4394 | -0.4427 | -0.4427 | 0.0527 | 0.0144 | 0.0177 | 0.0177 |
| P4-2 | -0.3160 | N/A | -0.4496 | -0.3180 | -0.3063 | -0.3063 | 0.1336 | 0.0020 | 0.0097 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0069 | 0.0394 | 0.0443 | 0.0443 | 0.0341 | 0.0016 | 0.0033 | 0.0033 |
| P5-4 | -0.0400 | N/A | -0.0069 | -0.0389 | -0.0439 | -0.0439 | 0.0331 | 0.0011 | 0.0039 | 0.0039 |
| P5-1 | -0.3450 | N/A | -0.3347 | -0.3567 | -0.3627 | -0.3627 | 0.0103 | 0.0117 | 0.0177 | 0.0177 |
| P5-2 | -0.1500 | N/A | -0.1496 | -0.1509 | -0.1527 | -0.1527 | 0.0004 | 0.0009 | 0.0027 | 0.0027 |
| P5-3 | -0.1800 | N/A | -0.2150 | -0.1879 | -0.1702 | -0.1702 | 0.0350 | 0.0079 | 0.0098 | 0.0098 |
| P5-6 | 0.0160 | N/A | 0.0125 | 0.0144 | 0.0214 | 0.0214 | 0.0035 | 0.0016 | 0.0054 | 0.0054 |
| P6-5 | -0.0160 | N/A | -0.0117 | -0.0135 | -0.0207 | -0.0207 | 0.0043 | 0.0025 | 0.0047 | 0.0047 |
| P6-2 | 0.2570 | N/A | -0.2508 | -0.2568 | -0.2692 | -0.2692 | 0.5078 | 0.5138 | 0.5262 | 0.5262 |
| P6-3 | -0.4280 | N/A | -0.5244 | -0.4527 | -0.4207 | -0.4207 | 0.0964 | 0.0247 | 0.0073 | 0.0073 |
| Q1-2 | -0.1540 | N/A | -0.1441 | -0.1456 | -0.1442 | -0.1442 | 0.0099 | 0.0084 | 0.0098 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.2316 | 0.2270 | 0.2136 | 0.2136 | 0.0306 | 0.0260 | 0.0126 | 0.0126 |
| Q1-5 | 0.1130 | 0.0940 | 0.1146 | 0.1206 | 0.1207 | 0.1207 | 0.0016 | 0.0076 | 0.0077 | 0.0077 |
| Q2-1 | 0.1280 | N/A | 0.1183 | 0.1302 | 0.1225 | 0.1225 | 0.0097 | 0.0022 | 0.0055 | 0.0055 |
| Q2-4 | 0.4610 | 0.3830 | 0.4643 | 0.4790 | 0.4528 | 0.4528 | 0.0033 | 0.0180 | 0.0082 | 0.0082 |
| Q2-5 | 0.1540 | 0.2200 | 0.1507 | 0.1507 | 0.1494 | 0.1494 | 0.0033 | 0.0033 | 0.0046 | 0.0046 |
| Q2-6 | 0.1240 | N/A | 0.0980 | 0.0938 | 0.1007 | 0.1007 | 0.0260 | 0.0302 | 0.0233 | 0.0233 |
| Q2-3 | -0.1230 | N/A | -0.1317 | -0.1347 | -0.1294 | -0.1294 | 0.0087 | 0.0117 | 0.0064 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0687 | 0.0788 | 0.0671 | 0.0671 | 0.0117 | 0.0218 | 0.0101 | 0.0101 |
| Q3-5 | 0.2320 | N/A | 0.2315 | 0.2437 | 0.2373 | 0.2373 | 0.0005 | 0.0117 | 0.0053 | 0.0053 |
| Q3-6 | 0.6070 | 0.5830 | 0.5815 | 0.5830 | 0.5797 | 0.5797 | 0.0255 | 0.0240 | 0.0273 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.2140 | -0.2076 | -0.2048 | -0.2048 | 0.0150 | 0.0086 | 0.0058 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4422 | -0.4602 | -0.4429 | -0.4429 | 0.0088 | 0.0092 | 0.0081 | 0.0081 |
| Q4-5 | -0.0490 | N/A | -0.0501 | -0.0512 | -0.0495 | -0.0495 | 0.0011 | 0.0022 | 0.0005 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0243 | -0.0143 | -0.0245 | -0.0245 | 0.0037 | 0.0137 | 0.0035 | 0.0035 |
| Q5-1 | -0.1350 | N/A | -0.1350 | -0.1248 | -0.1348 | -0.1348 | 0.0000 | 0.0102 | 0.0002 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1756 | -0.1697 | -0.1740 | -0.1740 | 0.0044 | 0.0103 | 0.0060 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2551 | -0.2605 | -0.2640 | -0.2640 | 0.0059 | 0.0005 | 0.0030 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1090 | -0.1077 | -0.1061 | -0.1061 | 0.0120 | 0.0107 | 0.0091 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0537 | 0.0590 | 0.0510 | 0.0510 | 0.0147 | 0.0200 | 0.0120 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1339 | -0.1223 | -0.1343 | -0.1343 | 0.0261 | 0.0377 | 0.0257 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5453 | -0.5457 | -0.5522 | -0.5522 | 0.0337 | 0.0333 | 0.0268 | 0.0268 |
| **SUM** | | | | | | | **2.5405** | **1.8329** | **1.3852** | **1.3852** |

The same condition with full redundancy case, LTS and LTAV share best results and they correct the bad data.

**Comparison**

Full redundancy and median redundancy estimation with the same condition of error are presented in Table 5.99. From the results, it is easy to find that full redundancy case has better estimation results for all estimator.

Table 5.99: Estimation errors of case 2, multi large, 6-bus DC system, comparison

| | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| SUM | 3.5225 | 2.0376 | 1.2724 | 1.1088 | 1.1088 |
| SUM | N/A | 2.5405 | 1.8329 | 1.3852 | 1.3852 |

#### 5.1.2.4 Case 3: Multiple interacting bad data

In this case, two bad data interacting with each other are used for performance evaluation. The locations of these two bad data are the same as in DC part to guarantee consistency. Three cases are conducted as usual.

**Two reverse bad data**

The two reverse bad data are $P_{2-4} = -0.3280$ and $P_{2-5} = -0.1740$ by add a negative sign to original measurements. Full and median redundancy are used to test the performance.

**Full redundancy**

Full redundancy take all measurements as available value, the system state variables are presented in Table 5.100, Table 5.101 gives the estimation results and error with actual value.

Table 5.100: State Variable of case 3, multi reverse, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0725 | -0.0679 | -0.0725 | -0.0667 | **V1** | 1.0474 | 1.0448 | 1.0474 | 1.0462 |
| $\theta 3$ | -0.0854 | -0.0809 | -0.0854 | -0.0773 | **V2** | 1.0408 | 1.0401 | 1.0408 | 1.0423 |
| $\theta 4$ | -0.0706 | -0.0755 | -0.0706 | -0.0757 | **V3** | 1.0609 | 1.0591 | 1.0609 | 1.0626 |
| $\theta 5$ | -0.0980 | -0.0983 | -0.0980 | -0.0961 | **V4** | 0.9871 | 0.9798 | 0.9871 | 0.9827 |
| $\theta 6$ | -0.1153 | -0.1107 | -0.1153 | -0.1070 | **V5** | 0.9799 | 0.9768 | 0.9799 | 0.9795 |
| | | | | | **V6** | 0.9971 | 0.9952 | 0.9971 | 0.9991 |

Table 5.101: Estimation Results and Errors of case 3, multi reverse, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0474 | 1.0448 | 1.0474 | 1.0462 | 0.0135 | 0.0026 | 0.0052 | 0.0026 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0408 | 1.0401 | 1.0408 | 1.0423 | 0.0161 | 0.0092 | 0.0099 | 0.0092 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0609 | 1.0591 | 1.0609 | 1.0626 | 0.0200 | 0.0091 | 0.0109 | 0.0091 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9871 | 0.9798 | 0.9871 | 0.9827 | 0.0083 | 0.0025 | 0.0097 | 0.0025 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9799 | 0.9768 | 0.9799 | 0.9795 | 0.0065 | 0.0057 | 0.0089 | 0.0057 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 0.9971 | 0.9952 | 0.9971 | 0.9991 | 0.0091 | 0.0073 | 0.0091 | 0.0073 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1311 | 1.1310 | 1.1311 | 1.1188 | 0.0520 | 0.0521 | 0.0520 | 0.0521 | 0.0398 |
| P2 | 0.5000 | 0.4840 | 0.3366 | 0.4840 | 0.3366 | 0.4724 | 0.0160 | 0.1634 | 0.0160 | 0.1634 | 0.0276 |
| P3 | 0.6000 | 0.5510 | 0.5644 | 0.5783 | 0.5644 | 0.5967 | 0.0490 | 0.0356 | 0.0217 | 0.0356 | 0.0033 |
| P4 | -0.7000 | -0.7180 | -0.5473 | -0.6857 | -0.5473 | -0.7013 | 0.0180 | 0.1527 | 0.0143 | 0.1527 | 0.0013 |
| P5 | -0.7000 | -0.7200 | -0.6837 | -0.7200 | -0.6837 | -0.7173 | 0.0200 | 0.0163 | 0.0200 | 0.0163 | 0.0173 |
| P6 | -0.7000 | -0.7230 | -0.7240 | -0.7066 | -0.7240 | -0.6899 | 0.0230 | 0.0240 | 0.0066 | 0.0240 | 0.0101 |
| Q1 | 0.1600 | 0.2020 | 0.1795 | 0.2020 | 0.1795 | 0.1892 | 0.0420 | 0.0195 | 0.0420 | 0.0195 | 0.0292 |
| Q2 | 0.7440 | 0.7190 | 0.6949 | 0.7190 | 0.6949 | 0.7016 | 0.0250 | 0.0491 | 0.0250 | 0.0491 | 0.0424 |
| Q3 | 0.8960 | 0.9060 | 0.8685 | 0.8603 | 0.8685 | 0.8648 | 0.0100 | 0.0275 | 0.0357 | 0.0275 | 0.0312 |
| Q4 | -0.7000 | -0.7190 | -0.6935 | -0.7188 | -0.6935 | -0.7001 | 0.0190 | 0.0065 | 0.0188 | 0.0065 | 0.0001 |
| Q5 | -0.7000 | -0.6770 | -0.6842 | -0.6770 | -0.6842 | -0.6829 | 0.0230 | 0.0158 | 0.0230 | 0.0158 | 0.0171 |
| Q6 | -0.7000 | -0.6090 | -0.6632 | -0.6688 | -0.6632 | -0.6641 | 0.0910 | 0.0368 | 0.0312 | 0.0368 | 0.0359 |
| P1-2 | 0.2870 | 0.3150 | 0.3351 | 0.3096 | 0.3351 | 0.3036 | 0.0280 | 0.0481 | 0.0226 | 0.0481 | 0.0166 |
| P1-4 | 0.4360 | 0.3890 | 0.4208 | 0.4465 | 0.4208 | 0.4475 | 0.0470 | 0.0152 | 0.0105 | 0.0152 | 0.0115 |
| P1-5 | 0.3560 | 0.3570 | 0.3752 | 0.3749 | 0.3752 | 0.3678 | 0.0010 | 0.0192 | 0.0189 | 0.0192 | 0.0118 |
| P2-1 | -0.2780 | -0.3490 | -0.3236 | -0.2996 | -0.3236 | -0.2939 | 0.0710 | 0.0456 | 0.0216 | 0.0456 | 0.0159 |
| **P2-4** | **0.3310** | **-0.3280** | **0.2088** | **0.3128** | **0.2088** | **0.3226** | **0.6590** | **0.1222** | **0.0182** | **0.1222** | **0.0084** |
| **P2-5** | **0.1550** | **-0.1740** | **0.1419** | **0.1592** | **0.1419** | **0.1560** | **0.3290** | **0.0131** | **0.0042** | **0.0131** | **0.0010** |
| P2-6 | 0.2620 | 0.2230 | 0.2706 | 0.2717 | 0.2706 | 0.2587 | 0.0390 | 0.0086 | 0.0097 | 0.0086 | 0.0033 |
| P2-3 | 0.0290 | 0.0860 | 0.0389 | 0.0400 | 0.0389 | 0.0291 | 0.0570 | 0.0099 | 0.0110 | 0.0099 | 0.0001 |
| P3-2 | -0.0290 | -0.0210 | -0.0385 | -0.0396 | -0.0385 | -0.0287 | 0.0080 | 0.0095 | 0.0106 | 0.0095 | 0.0003 |
| P3-5 | 0.1910 | 0.1770 | 0.1674 | 0.1850 | 0.1674 | 0.1915 | 0.0140 | 0.0236 | 0.0060 | 0.0236 | 0.0005 |
| P3-6 | 0.4380 | 0.4330 | 0.4355 | 0.4330 | 0.4355 | 0.4339 | 0.0050 | 0.0025 | 0.0050 | 0.0025 | 0.0041 |
| P4-1 | -0.4250 | -0.4010 | -0.4104 | -0.4347 | -0.4104 | -0.4358 | 0.0240 | 0.0146 | 0.0097 | 0.0146 | 0.0108 |
| P4-2 | -0.3160 | -0.2980 | -0.1973 | -0.2980 | -0.1973 | -0.3080 | 0.0180 | 0.1187 | 0.0180 | 0.1187 | 0.0080 |
| P4-5 | 0.0410 | 0.0070 | 0.0603 | 0.0470 | 0.0603 | 0.0426 | 0.0340 | 0.0193 | 0.0060 | 0.0193 | 0.0016 |
| P5-4 | -0.0400 | -0.0210 | -0.0596 | -0.0465 | -0.0596 | -0.0422 | 0.0190 | 0.0196 | 0.0065 | 0.0196 | 0.0022 |
| P5-1 | -0.3450 | -0.3660 | -0.3633 | -0.3628 | -0.3633 | -0.3562 | 0.0210 | 0.0183 | 0.0178 | 0.0183 | 0.0112 |
| P5-2 | -0.1500 | -0.1170 | -0.1375 | -0.1542 | -0.1375 | -0.1511 | 0.0330 | 0.0125 | 0.0042 | 0.0125 | 0.0011 |
| P5-3 | -0.1800 | -0.2510 | -0.1575 | -0.1746 | -0.1575 | -0.1808 | 0.0710 | 0.0225 | 0.0054 | 0.0225 | 0.0008 |
| P5-6 | 0.0160 | -0.0210 | 0.0341 | 0.0181 | 0.0341 | 0.0130 | 0.0370 | 0.0181 | 0.0021 | 0.0181 | 0.0030 |
| P6-5 | -0.0160 | 0.0100 | -0.0335 | -0.0176 | -0.0335 | -0.0125 | 0.0260 | 0.0175 | 0.0016 | 0.0175 | 0.0035 |
| P6-2 | 0.2570 | -0.1960 | -0.2646 | -0.2656 | -0.2646 | -0.2531 | 0.4530 | 0.5216 | 0.5226 | 0.5216 | 0.5101 |
| P6-3 | -0.4280 | -0.4680 | -0.4258 | -0.4234 | -0.4258 | -0.4243 | 0.0400 | 0.0022 | 0.0046 | 0.0022 | 0.0037 |
| Q1-2 | -0.1540 | -0.1320 | -0.1409 | -0.1393 | -0.1409 | -0.1413 | 0.0220 | 0.0131 | 0.0147 | 0.0131 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2014 | 0.2207 | 0.2014 | 0.2130 | 0.0110 | 0.0004 | 0.0197 | 0.0004 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1189 | 0.1207 | 0.1189 | 0.1174 | 0.0190 | 0.0059 | 0.0077 | 0.0059 | 0.0044 |
| Q2-1 | 0.1280 | 0.0970 | 0.1203 | 0.1160 | 0.1203 | 0.1171 | 0.0310 | 0.0077 | 0.0120 | 0.0077 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4440 | 0.4597 | 0.4440 | 0.4496 | 0.0780 | 0.0170 | 0.0013 | 0.0170 | 0.0114 |
| Q2-5 | 0.1540 | 0.2200 | 0.1434 | 0.1463 | 0.1434 | 0.1460 | 0.0660 | 0.0106 | 0.0077 | 0.0106 | 0.0080 |
| Q2-6 | 0.1240 | 0.1500 | 0.1106 | 0.1160 | 0.1106 | 0.1119 | 0.0260 | 0.0134 | 0.0080 | 0.0134 | 0.0121 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q2-3 | -0.1230 | -0.1190 | -0.1235 | -0.1190 | -0.1235 | -0.1230 | 0.0040 | 0.0005 | 0.0040 | 0.0005 | 0.0000 |
| Q3-2 | 0.0570 | 0.1020 | 0.0595 | 0.0550 | 0.0595 | 0.0586 | 0.0450 | 0.0025 | 0.0020 | 0.0025 | 0.0016 |
| Q3-5 | 0.2320 | 0.2390 | 0.2254 | 0.2223 | 0.2254 | 0.2240 | 0.0070 | 0.0066 | 0.0097 | 0.0066 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5836 | 0.5830 | 0.5836 | 0.5822 | 0.0240 | 0.0234 | 0.0240 | 0.0234 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2015 | -0.2144 | -0.2015 | -0.2076 | 0.0560 | 0.0025 | 0.0154 | 0.0025 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4415 | -0.4506 | -0.4415 | -0.4410 | 0.0080 | 0.0095 | 0.0004 | 0.0095 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0506 | -0.0538 | -0.0506 | -0.0515 | 0.1250 | 0.0016 | 0.0048 | 0.0016 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0253 | -0.0217 | -0.0253 | -0.0247 | 0.0130 | 0.0027 | 0.0063 | 0.0027 | 0.0033 |
| Q5-1 | -0.1350 | -0.1750 | -0.1358 | -0.1370 | -0.1358 | -0.1358 | 0.0400 | 0.0008 | 0.0020 | 0.0008 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1712 | -0.1722 | -0.1712 | -0.1725 | 0.0420 | 0.0088 | 0.0078 | 0.0088 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2562 | -0.2517 | -0.2562 | -0.2531 | 0.0620 | 0.0048 | 0.0093 | 0.0048 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0957 | -0.0944 | -0.0957 | -0.0969 | 0.0890 | 0.0013 | 0.0026 | 0.0013 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0388 | 0.0375 | 0.0388 | 0.0397 | 0.0100 | 0.0002 | 0.0015 | 0.0002 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1456 | -0.1504 | -0.1456 | -0.1481 | 0.0630 | 0.0144 | 0.0096 | 0.0144 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5565 | -0.5559 | -0.5565 | -0.5556 | 0.0680 | 0.0225 | 0.0231 | 0.0225 | 0.0234 |
| **SUM** | | | | | | | **3.4045** | **1.9087** | **1.2904** | **1.9087** | **1.1044** |

LTAV performs best both in bad data correction and overall summation, LAV follows up. WLS and LTS generate the same unsatisfied results though much better than measurements.

**Median redundancy**

System state variables, estimation results and error are presented in the following two tables:

Table 5.102: State Variable of case 3, multi reverse, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0793 | -0.0791 | -0.0793 | -0.0702 | **V1** | 1.0350 | 0.9809 | 1.0350 | 1.0342 |
| $\theta 3$ | -0.1013 | -0.0995 | -0.1013 | -0.0853 | **V2** | 1.0273 | 0.9770 | 1.0273 | 1.0303 |
| $\theta 4$ | -0.0730 | -0.0834 | -0.0730 | -0.0791 | **V3** | 1.0474 | 1.0007 | 1.0474 | 1.0518 |
| $\theta 5$ | -0.1056 | -0.1148 | -0.1056 | -0.1004 | **V4** | 0.9749 | 0.9129 | 0.9749 | 0.9697 |
| $\theta 6$ | -0.1319 | -0.1329 | -0.1319 | -0.1154 | **V5** | 0.9649 | 0.9090 | 0.9649 | 0.9658 |
| | | | | | **V6** | 0.9829 | 0.9333 | 0.9829 | 0.9879 |

Table 5.103: Estimation Results and Errors of case 3, multi reverse, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0350 | 0.9809 | 1.0350 | 1.0342 | 0.0150 | 0.0691 | 0.0150 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0273 | 0.9770 | 1.0273 | 1.0303 | 0.0227 | 0.0730 | 0.0227 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0474 | 1.0007 | 1.0474 | 1.0518 | 0.0226 | 0.0693 | 0.0226 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9749 | 0.9129 | 0.9749 | 0.9697 | 0.0146 | 0.0767 | 0.0146 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9649 | 0.9090 | 0.9649 | 0.9658 | 0.0208 | 0.0766 | 0.0208 | 0.0198 |
| V6 | 1.0043 | N/A | 0.9829 | 0.9333 | 0.9829 | 0.9879 | 0.0214 | 0.0710 | 0.0214 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1741 | 1.1310 | 1.1741 | 1.1416 | 0.0951 | 0.0520 | 0.0951 | 0.0626 |
| P2 | 0.5000 | 0.4840 | 0.3468 | 0.4840 | 0.3468 | 0.4978 | 0.1532 | 0.0160 | 0.1532 | 0.0022 |
| P3 | 0.6000 | 0.5510 | 0.4981 | 0.5510 | 0.4981 | 0.5663 | 0.1019 | 0.0490 | 0.1019 | 0.0337 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P4 | -0.7000 | -0.7180 | -0.4937 | -0.6291 | -0.4937 | -0.7055 | 0.2063 | 0.0709 | 0.2063 | 0.0055 |
| P5 | -0.7000 | -0.7200 | -0.6598 | -0.7200 | -0.6598 | -0.7054 | 0.0402 | 0.0200 | 0.0402 | 0.0054 |
| P6 | -0.7000 | -0.7230 | -0.7825 | -0.7230 | -0.7825 | -0.7112 | 0.0825 | 0.0230 | 0.0825 | 0.0112 |
| Q1 | 0.1600 | 0.2020 | 0.1760 | 0.2020 | 0.1760 | 0.1900 | 0.0160 | 0.0420 | 0.0160 | 0.0300 |
| Q2 | 0.7440 | 0.7190 | 0.6920 | 0.7190 | 0.6920 | 0.6961 | 0.0520 | 0.0250 | 0.0520 | 0.0479 |
| Q3 | 0.8960 | 0.9060 | 0.8899 | 0.9060 | 0.8899 | 0.8841 | 0.0061 | 0.0100 | 0.0061 | 0.0119 |
| Q4 | -0.7000 | -0.7190 | -0.6861 | -0.7190 | -0.6861 | -0.6973 | 0.0139 | 0.0190 | 0.0139 | 0.0027 |
| Q5 | -0.7000 | -0.6770 | -0.7045 | -0.6770 | -0.7045 | -0.7035 | 0.0045 | 0.0230 | 0.0045 | 0.0035 |
| Q6 | -0.7000 | -0.6090 | -0.6329 | -0.6113 | -0.6329 | -0.6355 | 0.0671 | 0.0887 | 0.0671 | 0.0645 |
| P1-2 | 0.2870 | N/A | 0.3594 | 0.3167 | 0.3594 | 0.3121 | 0.0724 | 0.0297 | 0.0724 | 0.0251 |
| P1-4 | 0.4360 | N/A | 0.4224 | 0.4332 | 0.4224 | 0.4552 | 0.0136 | 0.0028 | 0.0136 | 0.0192 |
| P1-5 | 0.3560 | 0.3570 | 0.3923 | 0.3811 | 0.3923 | 0.3743 | 0.0363 | 0.0251 | 0.0363 | 0.0183 |
| P2-1 | -0.2780 | N/A | -0.3459 | -0.3046 | -0.3459 | -0.3016 | 0.0679 | 0.0266 | 0.0679 | 0.0236 |
| **P2-4** | **0.3310** | **-0.3280** | **0.1646** | **0.2811** | **0.1646** | **0.3213** | **0.1664** | **0.0499** | **0.1664** | **0.0097** |
| **P2-5** | **0.1550** | **-0.1740** | **0.1427** | **0.1619** | **0.1427** | **0.1570** | **0.0123** | **0.0069** | **0.0123** | **0.0020** |
| P2-6 | 0.2620 | N/A | 0.3098 | 0.2868 | 0.3098 | 0.2748 | 0.0478 | 0.0248 | 0.0478 | 0.0128 |
| P2-3 | 0.0290 | N/A | 0.0757 | 0.0589 | 0.0757 | 0.0462 | 0.0467 | 0.0299 | 0.0467 | 0.0172 |
| P3-2 | -0.0290 | N/A | -0.0750 | -0.0581 | -0.0750 | -0.0457 | 0.0460 | 0.0291 | 0.0460 | 0.0167 |
| P3-5 | 0.1910 | N/A | 0.1400 | 0.1785 | 0.1400 | 0.1810 | 0.0510 | 0.0125 | 0.0510 | 0.0100 |
| P3-6 | 0.4380 | 0.4330 | 0.4331 | 0.4307 | 0.4331 | 0.4310 | 0.0049 | 0.0073 | 0.0049 | 0.0070 |
| P4-1 | -0.4250 | N/A | -0.4118 | -0.4205 | -0.4118 | -0.4429 | 0.0132 | 0.0045 | 0.0132 | 0.0179 |
| P4-2 | -0.3160 | N/A | -0.1535 | -0.2646 | -0.1535 | -0.3063 | 0.1625 | 0.0514 | 0.1625 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0717 | 0.0560 | 0.0717 | 0.0438 | 0.0307 | 0.0150 | 0.0307 | 0.0028 |
| P5-4 | -0.0400 | N/A | -0.0706 | -0.0551 | -0.0706 | -0.0434 | 0.0306 | 0.0151 | 0.0306 | 0.0034 |
| P5-1 | -0.3450 | N/A | -0.3790 | -0.3671 | -0.3790 | -0.3621 | 0.0340 | 0.0221 | 0.0340 | 0.0171 |
| P5-2 | -0.1500 | N/A | -0.1381 | -0.1561 | -0.1381 | -0.1520 | 0.0119 | 0.0061 | 0.0119 | 0.0020 |
| P5-3 | -0.1800 | N/A | -0.1300 | -0.1659 | -0.1300 | -0.1699 | 0.0500 | 0.0141 | 0.0500 | 0.0101 |
| P5-6 | 0.0160 | N/A | 0.0578 | 0.0242 | 0.0578 | 0.0219 | 0.0418 | 0.0082 | 0.0418 | 0.0059 |
| P6-5 | -0.0160 | N/A | -0.0568 | -0.0233 | -0.0568 | -0.0212 | 0.0408 | 0.0073 | 0.0408 | 0.0052 |
| P6-2 | 0.2570 | N/A | -0.3024 | -0.2797 | -0.3024 | -0.2688 | 0.5594 | 0.5367 | 0.5594 | 0.5258 |
| P6-3 | -0.4280 | N/A | -0.4232 | -0.4199 | -0.4232 | -0.4213 | 0.0048 | 0.0081 | 0.0048 | 0.0067 |
| Q1-2 | -0.1540 | N/A | -0.1446 | -0.1435 | -0.1446 | -0.1442 | 0.0094 | 0.0105 | 0.0094 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.1970 | 0.2215 | 0.1970 | 0.2137 | 0.0040 | 0.0205 | 0.0040 | 0.0127 |
| Q1-5 | 0.1130 | 0.0940 | 0.1236 | 0.1240 | 0.1236 | 0.1205 | 0.0106 | 0.0110 | 0.0106 | 0.0075 |
| Q2-1 | 0.1280 | N/A | 0.1290 | 0.1292 | 0.1290 | 0.1226 | 0.0010 | 0.0012 | 0.0010 | 0.0054 |
| Q2-4 | 0.4610 | 0.3830 | 0.4450 | 0.4763 | 0.4450 | 0.4529 | 0.0160 | 0.0153 | 0.0160 | 0.0081 |
| Q2-5 | 0.1540 | 0.2200 | 0.1462 | 0.1501 | 0.1462 | 0.1493 | 0.0078 | 0.0039 | 0.0078 | 0.0047 |
| Q2-6 | 0.1240 | N/A | 0.1001 | 0.0956 | 0.1001 | 0.1006 | 0.0239 | 0.0284 | 0.0239 | 0.0234 |
| Q2-3 | -0.1230 | N/A | -0.1283 | -0.1322 | -0.1283 | -0.1294 | 0.0053 | 0.0092 | 0.0053 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0673 | 0.0772 | 0.0673 | 0.0672 | 0.0103 | 0.0202 | 0.0103 | 0.0102 |
| Q3-5 | 0.2320 | N/A | 0.2403 | 0.2458 | 0.2403 | 0.2373 | 0.0083 | 0.0138 | 0.0083 | 0.0053 |
| Q3-6 | 0.6070 | 0.5830 | 0.5823 | 0.5830 | 0.5823 | 0.5797 | 0.0247 | 0.0240 | 0.0247 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.1952 | -0.2064 | -0.1952 | -0.2048 | 0.0038 | 0.0074 | 0.0038 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4428 | -0.4612 | -0.4428 | -0.4430 | 0.0082 | 0.0102 | 0.0082 | 0.0080 |
| Q4-5 | -0.0490 | N/A | -0.0480 | -0.0515 | -0.0480 | -0.0495 | 0.0010 | 0.0025 | 0.0010 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0250 | -0.0132 | -0.0250 | -0.0246 | 0.0030 | 0.0148 | 0.0030 | 0.0034 |
| Q5-1 | -0.1350 | N/A | -0.1338 | -0.1251 | -0.1338 | -0.1348 | 0.0012 | 0.0099 | 0.0012 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1722 | -0.1685 | -0.1722 | -0.1740 | 0.0078 | 0.0115 | 0.0078 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2694 | -0.2641 | -0.2694 | -0.2640 | 0.0084 | 0.0031 | 0.0084 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1041 | -0.1060 | -0.1041 | -0.1061 | 0.0071 | 0.0090 | 0.0071 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0502 | 0.0577 | 0.0502 | 0.0510 | 0.0112 | 0.0187 | 0.0112 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1294 | -0.1210 | -0.1294 | -0.1343 | 0.0306 | 0.0390 | 0.0306 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5537 | -0.5481 | -0.5537 | -0.5522 | 0.0253 | 0.0309 | 0.0253 | 0.0268 |
| **SUM** | | | | | | | **2.7297** | **2.1227** | **2.7297** | **1.3805** |

The results have the same pattern as full redundancy case, LTAV generates the best estimation.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.104. Full redundancy case defects median ones for all estimators.

Table 5.104: Estimation errors of case 3, multi reverse, 6-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **3.4045** | **1.9087** | **1.2904** | **1.9087** | **1.1044** |
| **SUM** | **N/A** | **2.7297** | **2.1227** | **2.7297** | **1.3805** |

## Two non-conforming bad data

One large, $P_{2-5} = 0.9740$, and one reverse bad data, $P_{2-4} = -0.3280$, consist the non-conforming case, full and median redundancy case are considered.

### Full redundancy

The following two tables present state variables, estimation results and error respectively in detail.

Table 5.105: State Variable of case 3, non-conforming, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0699 | -0.0679 | -0.0717 | -0.0667 | **V1** | 1.0482 | 1.0448 | 1.0476 | 1.0462 |
| $\theta 3$ | -0.0878 | -0.0809 | -0.0861 | -0.0773 | **V2** | 1.0423 | 1.0401 | 1.0412 | 1.0423 |
| $\theta 4$ | -0.0686 | -0.0755 | -0.0701 | -0.0757 | **V3** | 1.0604 | 1.0591 | 1.0608 | 1.0626 |
| $\theta 5$ | -0.1077 | -0.0983 | -0.1007 | -0.0961 | **V4** | 0.9888 | 0.9798 | 0.9876 | 0.9827 |
| $\theta 6$ | -0.1174 | -0.1107 | -0.1159 | -0.1070 | **V5** | 0.9767 | 0.9768 | 0.9790 | 0.9795 |
| | | | | | **V6** | 0.9966 | 0.9952 | 0.9969 | 0.9991 |

Table 5.106: Estimation Results and Errors of case 3, non-conforming, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0482 | 1.0448 | 1.0476 | 1.0462 | 0.0135 | 0.0018 | 0.0052 | 0.0024 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0423 | 1.0401 | 1.0412 | 1.0423 | 0.0161 | 0.0077 | 0.0099 | 0.0088 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0604 | 1.0591 | 1.0608 | 1.0626 | 0.0200 | 0.0096 | 0.0109 | 0.0092 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9888 | 0.9798 | 0.9876 | 0.9827 | 0.0083 | 0.0008 | 0.0097 | 0.0020 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9767 | 0.9768 | 0.9790 | 0.9795 | 0.0065 | 0.0090 | 0.0089 | 0.0066 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 0.9966 | 0.9952 | 0.9969 | 0.9991 | 0.0091 | 0.0078 | 0.0091 | 0.0074 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1425 | 1.1310 | 1.1343 | 1.1188 | 0.0520 | 0.0635 | 0.0520 | 0.0553 | 0.0398 |
| P2 | 0.5000 | 0.4840 | 0.4424 | 0.4840 | 0.3667 | 0.4724 | 0.0160 | 0.0576 | 0.0160 | 0.1333 | 0.0276 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P3 | 0.6000 | 0.5510 | 0.5661 | 0.5783 | 0.5648 | 0.5967 | 0.0490 | 0.0339 | 0.0217 | 0.0352 | 0.0033 |
| P4 | -0.7000 | -0.7180 | -0.5132 | -0.6857 | -0.5376 | -0.7013 | 0.0180 | 0.1868 | 0.0143 | 0.1624 | 0.0013 |
| P5 | -0.7000 | -0.7200 | -0.8360 | -0.7200 | -0.7271 | -0.7173 | 0.0200 | 0.1360 | 0.0200 | 0.0271 | 0.0173 |
| P6 | -0.7000 | -0.7230 | -0.7199 | -0.7066 | -0.7229 | -0.6899 | 0.0230 | 0.0199 | 0.0066 | 0.0229 | 0.0101 |
| Q1 | 0.1600 | 0.2020 | 0.1866 | 0.2020 | 0.1815 | 0.1892 | 0.0420 | 0.0266 | 0.0420 | 0.0215 | 0.0292 |
| Q2 | 0.7440 | 0.7190 | 0.6985 | 0.7190 | 0.6957 | 0.7016 | 0.0250 | 0.0455 | 0.0250 | 0.0483 | 0.0424 |
| Q3 | 0.8960 | 0.9060 | 0.8633 | 0.8603 | 0.8670 | 0.8648 | 0.0100 | 0.0327 | 0.0357 | 0.0290 | 0.0312 |
| Q4 | -0.7000 | -0.7190 | -0.6903 | -0.7188 | -0.6926 | -0.7001 | 0.0190 | 0.0097 | 0.0188 | 0.0074 | 0.0001 |
| Q5 | -0.7000 | -0.6770 | -0.6778 | -0.6770 | -0.6825 | -0.6829 | 0.0230 | 0.0222 | 0.0230 | 0.0175 | 0.0171 |
| Q6 | -0.7000 | -0.6090 | -0.6631 | -0.6688 | -0.6632 | -0.6641 | 0.0910 | 0.0369 | 0.0312 | 0.0368 | 0.0359 |
| P1-2 | 0.2870 | 0.3150 | 0.3228 | 0.3096 | 0.3316 | 0.3036 | 0.0280 | 0.0358 | 0.0226 | 0.0446 | 0.0166 |
| P1-4 | 0.4360 | 0.3890 | 0.4103 | 0.4465 | 0.4178 | 0.4475 | 0.0470 | 0.0257 | 0.0105 | 0.0182 | 0.0115 |
| P1-5 | 0.3560 | 0.3570 | 0.4094 | 0.3749 | 0.3849 | 0.3678 | 0.0010 | 0.0534 | 0.0189 | 0.0289 | 0.0118 |
| P2-1 | -0.2780 | -0.3490 | -0.3120 | -0.2996 | -0.3203 | -0.2939 | 0.0710 | 0.0340 | 0.0216 | 0.0423 | 0.0159 |
| **P2-4** | **0.3310** | **-0.3280** | **0.2123** | **0.3128** | **0.2098** | **0.3226** | **0.6590** | **0.1187** | **0.0182** | **0.1212** | **0.0084** |
| **P2-5** | **0.1550** | **0.9740** | **0.1844** | **0.1592** | **0.1540** | **0.1560** | **0.8190** | **0.0294** | **0.0042** | **0.0010** | **0.0010** |
| P2-6 | 0.2620 | 0.2230 | 0.2959 | 0.2717 | 0.2778 | 0.2587 | 0.0390 | 0.0339 | 0.0097 | 0.0158 | 0.0033 |
| P2-3 | 0.0290 | 0.0860 | 0.0617 | 0.0400 | 0.0455 | 0.0291 | 0.0570 | 0.0327 | 0.0110 | 0.0165 | 0.0001 |
| P3-2 | -0.0290 | -0.0210 | -0.0612 | -0.0396 | -0.0450 | -0.0287 | 0.0080 | 0.0322 | 0.0106 | 0.0160 | 0.0003 |
| P3-5 | 0.1910 | 0.1770 | 0.1954 | 0.1850 | 0.1753 | 0.1915 | 0.0140 | 0.0044 | 0.0060 | 0.0157 | 0.0005 |
| P3-6 | 0.4380 | 0.4330 | 0.4319 | 0.4330 | 0.4345 | 0.4339 | 0.0050 | 0.0061 | 0.0050 | 0.0035 | 0.0041 |
| P4-1 | -0.4250 | -0.4010 | -0.4004 | -0.4347 | -0.4076 | -0.4358 | 0.0240 | 0.0246 | 0.0097 | 0.0174 | 0.0108 |
| P4-2 | -0.3160 | -0.2980 | -0.2009 | -0.2980 | -0.1983 | -0.3080 | 0.0180 | 0.1151 | 0.0180 | 0.1177 | 0.0080 |
| P4-5 | 0.0410 | 0.0070 | 0.0882 | 0.0470 | 0.0682 | 0.0426 | 0.0340 | 0.0472 | 0.0060 | 0.0272 | 0.0016 |
| P5-4 | -0.0400 | -0.0210 | -0.0865 | -0.0465 | -0.0673 | -0.0422 | 0.0190 | 0.0465 | 0.0065 | 0.0273 | 0.0022 |
| P5-1 | -0.3450 | -0.3660 | -0.3953 | -0.3628 | -0.3724 | -0.3562 | 0.0210 | 0.0503 | 0.0178 | 0.0274 | 0.0112 |
| P5-2 | -0.1500 | -0.1170 | -0.1787 | -0.1542 | -0.1492 | -0.1511 | 0.0330 | 0.0287 | 0.0042 | 0.0008 | 0.0011 |
| P5-3 | -0.1800 | -0.2510 | -0.1846 | -0.1746 | -0.1652 | -0.1808 | 0.0710 | 0.0046 | 0.0054 | 0.0148 | 0.0008 |
| P5-6 | 0.0160 | -0.0210 | 0.0091 | 0.0181 | 0.0270 | 0.0130 | 0.0370 | 0.0069 | 0.0021 | 0.0110 | 0.0030 |
| P6-5 | -0.0160 | 0.0100 | -0.0086 | -0.0176 | -0.0265 | -0.0125 | 0.0260 | 0.0074 | 0.0016 | 0.0105 | 0.0035 |
| P6-2 | 0.2570 | -0.1960 | -0.2890 | -0.2656 | -0.2716 | -0.2531 | 0.4530 | 0.5460 | 0.5226 | 0.5286 | 0.5101 |
| P6-3 | -0.4280 | -0.4680 | -0.4223 | -0.4234 | -0.4248 | -0.4243 | 0.0400 | 0.0057 | 0.0046 | 0.0032 | 0.0037 |
| Q1-2 | -0.1540 | -0.1320 | -0.1393 | -0.1393 | -0.1404 | -0.1413 | 0.0220 | 0.0147 | 0.0147 | 0.0136 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.1987 | 0.2207 | 0.2007 | 0.2130 | 0.0110 | 0.0023 | 0.0197 | 0.0003 | 0.0120 |
| Q1-5 | 0.1130 | 0.0940 | 0.1273 | 0.1207 | 0.1213 | 0.1174 | 0.0190 | 0.0143 | 0.0077 | 0.0083 | 0.0044 |
| Q2-1 | 0.1280 | 0.0970 | 0.1171 | 0.1160 | 0.1194 | 0.1171 | 0.0310 | 0.0109 | 0.0120 | 0.0086 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4405 | 0.4597 | 0.4430 | 0.4496 | 0.0780 | 0.0205 | 0.0013 | 0.0180 | 0.0114 |
| Q2-5 | 0.1540 | 0.2200 | 0.1471 | 0.1463 | 0.1444 | 0.1460 | 0.0660 | 0.0069 | 0.0077 | 0.0096 | 0.0080 |
| Q2-6 | 0.1240 | 0.1500 | 0.1134 | 0.1160 | 0.1114 | 0.1119 | 0.0260 | 0.0106 | 0.0080 | 0.0126 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1196 | -0.1190 | -0.1225 | -0.1230 | 0.0040 | 0.0034 | 0.0040 | 0.0005 | 0.0000 |
| Q3-2 | 0.0570 | 0.1020 | 0.0559 | 0.0550 | 0.0585 | 0.0586 | 0.0450 | 0.0011 | 0.0020 | 0.0015 | 0.0016 |
| Q3-5 | 0.2320 | 0.2390 | 0.2238 | 0.2223 | 0.2249 | 0.2240 | 0.0070 | 0.0082 | 0.0097 | 0.0071 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5836 | 0.5830 | 0.5836 | 0.5822 | 0.0240 | 0.0234 | 0.0240 | 0.0234 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2007 | -0.2144 | -0.2013 | -0.2076 | 0.0560 | 0.0017 | 0.0154 | 0.0023 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4382 | -0.4506 | -0.4405 | -0.4410 | 0.0080 | 0.0128 | 0.0004 | 0.0105 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0514 | -0.0538 | -0.0508 | -0.0515 | 0.1250 | 0.0024 | 0.0048 | 0.0018 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0226 | -0.0217 | -0.0245 | -0.0247 | 0.0130 | 0.0054 | 0.0063 | 0.0035 | 0.0033 |
| Q5-1 | -0.1350 | -0.1750 | -0.1361 | -0.1370 | -0.1359 | -0.1358 | 0.0400 | 0.0011 | 0.0020 | 0.0009 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1707 | -0.1722 | -0.1711 | -0.1725 | 0.0420 | 0.0093 | 0.0078 | 0.0089 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2522 | -0.2517 | -0.2551 | -0.2531 | 0.0620 | 0.0088 | 0.0093 | 0.0059 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0962 | -0.0944 | -0.0959 | -0.0969 | 0.0890 | 0.0008 | 0.0026 | 0.0011 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0392 | 0.0375 | 0.0389 | 0.0397 | 0.0100 | 0.0002 | 0.0015 | 0.0001 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1457 | -0.1504 | -0.1456 | -0.1481 | 0.0630 | 0.0143 | 0.0096 | 0.0144 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5567 | -0.5559 | -0.5566 | -0.5556 | 0.0680 | 0.0223 | 0.0231 | 0.0224 | 0.0234 |
| **SUM** | | | | | | | **3.8945** | **2.1926** | **1.2904** | **1.9180** | **1.1044** |

The results of LTAV are the best as usual.

### Median redundancy

Same locations and bad data value as full case, all results and comparison are shown in the following two tables:

Table 5.107: State Variable of case 3, non-conforming, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0759 | -0.0791 | -0.0783 | -0.0702 | **V1** | 1.0346 | 0.9809 | 1.0349 | 1.0342 |
| $\theta 3$ | -0.1050 | -0.0995 | -0.1024 | -0.0853 | **V2** | 1.0277 | 0.9770 | 1.0274 | 1.0303 |
| $\theta 4$ | -0.0698 | -0.0834 | -0.0720 | -0.0791 | **V3** | 1.0449 | 1.0007 | 1.0467 | 1.0518 |
| $\theta 5$ | -0.1182 | -0.1148 | -0.1092 | -0.1004 | **V4** | 0.9761 | 0.9129 | 0.9753 | 0.9697 |
| $\theta 6$ | -0.1353 | -0.1329 | -0.1329 | -0.1154 | **V5** | 0.9590 | 0.9090 | 0.9632 | 0.9658 |
| | | | | | **V6** | 0.9803 | 0.9333 | 0.9822 | 0.9879 |

Table 5.108: Estimation Results and Errors of case 3, non-conforming, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0346 | 0.9809 | 1.0349 | 1.0342 | 0.0154 | 0.0691 | 0.0151 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0277 | 0.9770 | 1.0274 | 1.0303 | 0.0223 | 0.0730 | 0.0226 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0449 | 1.0007 | 1.0467 | 1.0518 | 0.0251 | 0.0693 | 0.0233 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9761 | 0.9129 | 0.9753 | 0.9697 | 0.0135 | 0.0767 | 0.0142 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9590 | 0.9090 | 0.9632 | 0.9658 | 0.0267 | 0.0766 | 0.0224 | 0.0198 |
| V6 | 1.0043 | N/A | 0.9803 | 0.9333 | 0.9822 | 0.9879 | 0.0241 | 0.0710 | 0.0221 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1828 | 1.1310 | 1.1766 | 1.1416 | 0.1038 | 0.0520 | 0.0976 | 0.0626 |
| P2 | 0.5000 | 0.4840 | 0.4813 | 0.4840 | 0.3856 | 0.4978 | 0.0187 | 0.0160 | 0.1144 | 0.0022 |
| P3 | 0.6000 | 0.5510 | 0.4959 | 0.5510 | 0.4974 | 0.5663 | 0.1041 | 0.0490 | 0.1026 | 0.0337 |
| P4 | -0.7000 | -0.7180 | -0.4394 | -0.6291 | -0.4780 | -0.7055 | 0.2606 | 0.0709 | 0.2220 | 0.0055 |
| P5 | -0.7000 | -0.7200 | -0.8487 | -0.7200 | -0.7143 | -0.7054 | 0.1487 | 0.0200 | 0.0143 | 0.0054 |
| P6 | -0.7000 | -0.7230 | -0.7823 | -0.7230 | -0.7825 | -0.7112 | 0.0823 | 0.0230 | 0.0825 | 0.0112 |
| Q1 | 0.1600 | 0.2020 | 0.1860 | 0.2020 | 0.1789 | 0.1900 | 0.0260 | 0.0420 | 0.0189 | 0.0300 |
| Q2 | 0.7440 | 0.7190 | 0.6978 | 0.7190 | 0.6934 | 0.6961 | 0.0462 | 0.0250 | 0.0506 | 0.0479 |
| Q3 | 0.8960 | 0.9060 | 0.8830 | 0.9060 | 0.8879 | 0.8841 | 0.0130 | 0.0100 | 0.0081 | 0.0119 |
| Q4 | -0.7000 | -0.7190 | -0.6783 | -0.7190 | -0.6840 | -0.6973 | 0.0217 | 0.0190 | 0.0160 | 0.0027 |
| Q5 | -0.7000 | -0.6770 | -0.6972 | -0.6770 | -0.7027 | -0.7035 | 0.0028 | 0.0230 | 0.0027 | 0.0035 |
| Q6 | -0.7000 | -0.6090 | -0.6340 | -0.6113 | -0.6333 | -0.6355 | 0.0660 | 0.0887 | 0.0667 | 0.0645 |
| P1-2 | 0.2870 | N/A | 0.3428 | 0.3167 | 0.3546 | 0.3121 | 0.0558 | 0.0297 | 0.0676 | 0.0251 |
| P1-4 | 0.4360 | N/A | 0.4053 | 0.4332 | 0.4175 | 0.4552 | 0.0307 | 0.0028 | 0.0185 | 0.0192 |
| P1-5 | 0.3560 | 0.3570 | 0.4347 | 0.3811 | 0.4045 | 0.3743 | 0.0787 | 0.0251 | 0.0485 | 0.0183 |
| P2-1 | -0.2780 | N/A | -0.3305 | -0.3046 | -0.3415 | -0.3016 | 0.0525 | 0.0266 | 0.0635 | 0.0236 |
| **P2-4** | **0.3310** | **-0.3280** | **0.1631** | **0.2811** | **0.1641** | **0.3213** | **0.1679** | **0.0499** | **0.1669** | **0.0097** |
| **P2-5** | **0.1550** | **0.9740** | **0.1965** | **0.1619** | **0.1581** | **0.1570** | **0.0415** | **0.0069** | **0.0031** | **0.0020** |
| P2-6 | 0.2620 | N/A | 0.3452 | 0.2868 | 0.3200 | 0.2748 | 0.0832 | 0.0248 | 0.0580 | 0.0128 |
| P2-3 | 0.0290 | N/A | 0.1071 | 0.0589 | 0.0848 | 0.0462 | 0.0781 | 0.0299 | 0.0558 | 0.0172 |
| P3-2 | -0.0290 | N/A | -0.1061 | -0.0581 | -0.0840 | -0.0457 | 0.0771 | 0.0291 | 0.0550 | 0.0167 |
| P3-5 | 0.1910 | N/A | 0.1733 | 0.1785 | 0.1495 | 0.1810 | 0.0177 | 0.0125 | 0.0415 | 0.0100 |
| P3-6 | 0.4380 | 0.4330 | 0.4288 | 0.4307 | 0.4318 | 0.4310 | 0.0092 | 0.0073 | 0.0062 | 0.0070 |
| P4-1 | -0.4250 | N/A | -0.3955 | -0.4205 | -0.4071 | -0.4429 | 0.0295 | 0.0045 | 0.0179 | 0.0179 |
| P4-2 | -0.3160 | N/A | -0.1523 | -0.2646 | -0.1531 | -0.3063 | 0.1637 | 0.0514 | 0.1629 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.1084 | 0.0560 | 0.0822 | 0.0438 | 0.0674 | 0.0150 | 0.0412 | 0.0028 |
| P5-4 | -0.0400 | N/A | -0.1059 | -0.0551 | -0.0808 | -0.0434 | 0.0659 | 0.0151 | 0.0408 | 0.0034 |
| P5-1 | -0.3450 | N/A | -0.4185 | -0.3671 | -0.3904 | -0.3621 | 0.0735 | 0.0221 | 0.0454 | 0.0171 |
| P5-2 | -0.1500 | N/A | -0.1900 | -0.1561 | -0.1531 | -0.1520 | 0.0400 | 0.0061 | 0.0031 | 0.0020 |
| P5-3 | -0.1800 | N/A | -0.1622 | -0.1659 | -0.1393 | -0.1699 | 0.0178 | 0.0141 | 0.0407 | 0.0101 |
| P5-6 | 0.0160 | N/A | 0.0280 | 0.0242 | 0.0492 | 0.0219 | 0.0120 | 0.0082 | 0.0332 | 0.0059 |
| P6-5 | -0.0160 | N/A | -0.0272 | -0.0233 | -0.0483 | -0.0212 | 0.0112 | 0.0073 | 0.0323 | 0.0052 |
| P6-2 | 0.2570 | N/A | -0.3361 | -0.2797 | -0.3122 | -0.2688 | 0.5931 | 0.5367 | 0.5692 | 0.5258 |
| P6-3 | -0.4280 | N/A | -0.4190 | -0.4199 | -0.4220 | -0.4213 | 0.0090 | 0.0081 | 0.0060 | 0.0067 |
| Q1-2 | -0.1540 | N/A | -0.1419 | -0.1435 | -0.1438 | -0.1442 | 0.0121 | 0.0105 | 0.0102 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.1922 | 0.2215 | 0.1956 | 0.2137 | 0.0088 | 0.0205 | 0.0054 | 0.0127 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q1-5 | 0.1130 | 0.0940 | 0.1357 | 0.1240 | 0.1270 | 0.1205 | 0.0227 | 0.0110 | 0.0140 | 0.0075 |
| Q2-1 | 0.1280 | N/A | 0.1240 | 0.1292 | 0.1276 | 0.1226 | 0.0040 | 0.0012 | 0.0004 | 0.0054 |
| Q2-4 | 0.4610 | 0.3830 | 0.4386 | 0.4763 | 0.4431 | 0.4529 | 0.0224 | 0.0153 | 0.0179 | 0.0081 |
| Q2-5 | 0.1540 | 0.2200 | 0.1517 | 0.1501 | 0.1477 | 0.1493 | 0.0023 | 0.0039 | 0.0063 | 0.0047 |
| Q2-6 | 0.1240 | N/A | 0.1054 | 0.0956 | 0.1015 | 0.1006 | 0.0186 | 0.0284 | 0.0225 | 0.0234 |
| Q2-3 | -0.1230 | N/A | -0.1218 | -0.1322 | -0.1265 | -0.1294 | 0.0012 | 0.0092 | 0.0035 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0620 | 0.0772 | 0.0658 | 0.0672 | 0.0050 | 0.0202 | 0.0088 | 0.0102 |
| Q3-5 | 0.2320 | N/A | 0.2381 | 0.2458 | 0.2397 | 0.2373 | 0.0061 | 0.0138 | 0.0077 | 0.0053 |
| Q3-6 | 0.6070 | 0.5830 | 0.5828 | 0.5830 | 0.5824 | 0.5797 | 0.0242 | 0.0240 | 0.0246 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.1934 | -0.2064 | -0.1947 | -0.2048 | 0.0056 | 0.0074 | 0.0043 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4370 | -0.4612 | -0.4411 | -0.4430 | 0.0140 | 0.0102 | 0.0099 | 0.0080 |
| Q4-5 | -0.0490 | N/A | -0.0479 | -0.0515 | -0.0481 | -0.0495 | 0.0011 | 0.0025 | 0.0009 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0220 | -0.0132 | -0.0242 | -0.0246 | 0.0060 | 0.0148 | 0.0038 | 0.0034 |
| Q5-1 | -0.1350 | N/A | -0.1345 | -0.1251 | -0.1341 | -0.1348 | 0.0005 | 0.0099 | 0.0009 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1717 | -0.1685 | -0.1721 | -0.1740 | 0.0083 | 0.0115 | 0.0079 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2645 | -0.2641 | -0.2680 | -0.2640 | 0.0035 | 0.0031 | 0.0070 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1045 | -0.1060 | -0.1043 | -0.1061 | 0.0075 | 0.0090 | 0.0073 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0502 | 0.0577 | 0.0502 | 0.0510 | 0.0112 | 0.0187 | 0.0112 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1300 | -0.1210 | -0.1295 | -0.1343 | 0.0300 | 0.0390 | 0.0305 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5542 | -0.5481 | -0.5539 | -0.5522 | 0.0248 | 0.0309 | 0.0251 | 0.0268 |
| | | | | | | | **3.0364** | **2.1227** | **2.7232** | **1.3805** |

LTAV generate much better estimation when compared with other estimators.

### Comparison

The results are compared in Table 5.109, median redundancy case have better results with LMR, full redundancy have advantage with all other estimators.

Table 5.109: Estimation errors of case 3, non-conforming, 6-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **3.8945** | **2.1926** | **1.2904** | **1.9180** | **1.1044** |
| **SUM** | **N/A** | **3.0364** | **2.1227** | **2.7232** | **1.3805** |

The same situation as former case, LTAV and LAV performs good for median redundancy case.

### Two large bad data

The two large bad data are simulated just by changing the first decimal to 9, with $P_{2-4} = 0.9280$ and $P_{2-5} = 0.9740$. Two cases are considered.

### Full redundancy

Detail results of estimation are presented in the following two tables:

Table 5.110: State Variable of case 3, multi large, 6-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0602 | -0.0674 | -0.0667 | -0.0667 | **V1** | 1.0453 | 1.0440 | 1.0462 | 1.0462 |
| $\theta 3$ | -0.0708 | -0.0781 | -0.0773 | -0.0773 | **V2** | 1.0443 | 1.0396 | 1.0423 | 1.0423 |
| $\theta 4$ | -0.0796 | -0.0770 | -0.0757 | -0.0757 | **V3** | 1.0640 | 1.0591 | 1.0626 | 1.0626 |
| $\theta 5$ | -0.0986 | -0.0973 | -0.0961 | -0.0961 | **V4** | 0.9792 | 0.9786 | 0.9827 | 0.9827 |
| $\theta 6$ | -0.1002 | -0.1080 | -0.1070 | -0.1070 | **V5** | 0.9776 | 0.9763 | 0.9795 | 0.9795 |
| | | | | | **V6** | 1.0006 | 0.9952 | 0.9991 | 0.9991 |

Table 5.111: Estimation Results and Errors of case 3, multi large, 6-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0453 | 1.0440 | 1.0462 | 1.0462 | 0.0135 | 0.0047 | 0.0060 | 0.0038 | 0.0038 |
| V2 | 1.0500 | 1.0339 | 1.0443 | 1.0396 | 1.0423 | 1.0423 | 0.0161 | 0.0057 | 0.0104 | 0.0077 | 0.0077 |
| V3 | 1.0700 | 1.0900 | 1.0640 | 1.0591 | 1.0626 | 1.0626 | 0.0200 | 0.0060 | 0.0109 | 0.0074 | 0.0074 |
| V4 | 0.9896 | 0.9813 | 0.9792 | 0.9786 | 0.9827 | 0.9827 | 0.0083 | 0.0103 | 0.0110 | 0.0069 | 0.0069 |
| V5 | 0.9857 | 0.9791 | 0.9776 | 0.9763 | 0.9795 | 0.9795 | 0.0065 | 0.0081 | 0.0093 | 0.0062 | 0.0062 |
| V6 | 1.0043 | 0.9952 | 1.0006 | 0.9952 | 0.9991 | 0.9991 | 0.0091 | 0.0037 | 0.0091 | 0.0053 | 0.0053 |
| P1 | 1.0790 | 1.1310 | 1.1129 | 1.1310 | 1.1188 | 1.1188 | 0.0520 | 0.0339 | 0.0520 | 0.0398 | 0.0398 |
| P2 | 0.5000 | 0.4840 | 0.6471 | 0.4840 | 0.4724 | 0.4724 | 0.0160 | 0.1471 | 0.0160 | 0.0276 | 0.0276 |
| P3 | 0.6000 | 0.5510 | 0.6280 | 0.5945 | 0.5967 | 0.5967 | 0.0490 | 0.0280 | 0.0055 | 0.0033 | 0.0033 |
| P4 | -0.7000 | -0.7180 | -0.8304 | -0.7180 | -0.7013 | -0.7013 | 0.0180 | 0.1304 | 0.0180 | 0.0013 | 0.0013 |
| P5 | -0.7000 | -0.7200 | -0.8162 | -0.7200 | -0.7173 | -0.7173 | 0.0200 | 0.1162 | 0.0200 | 0.0173 | 0.0173 |
| P6 | -0.7000 | -0.7230 | -0.6559 | -0.6902 | -0.6899 | -0.6899 | 0.0230 | 0.0441 | 0.0098 | 0.0101 | 0.0101 |
| Q1 | 0.1600 | 0.2020 | 0.2015 | 0.2020 | 0.1892 | 0.1892 | 0.0420 | 0.0415 | 0.0420 | 0.0292 | 0.0292 |
| Q2 | 0.7440 | 0.7190 | 0.7109 | 0.7190 | 0.7016 | 0.7016 | 0.0250 | 0.0331 | 0.0250 | 0.0424 | 0.0424 |
| Q3 | 0.8960 | 0.9060 | 0.8593 | 0.8592 | 0.8648 | 0.8648 | 0.0100 | 0.0367 | 0.0368 | 0.0312 | 0.0312 |
| Q4 | -0.7000 | -0.7190 | -0.7033 | -0.7148 | -0.7001 | -0.7001 | 0.0190 | 0.0033 | 0.0148 | 0.0001 | 0.0001 |
| Q5 | -0.7000 | -0.6770 | -0.6785 | -0.6770 | -0.6829 | -0.6829 | 0.0230 | 0.0215 | 0.0230 | 0.0171 | 0.0171 |
| Q6 | -0.7000 | -0.6090 | -0.6646 | -0.6711 | -0.6641 | -0.6641 | 0.0910 | 0.0354 | 0.0289 | 0.0359 | 0.0359 |
| P1-2 | 0.2870 | 0.3150 | 0.2687 | 0.3064 | 0.3036 | 0.3036 | 0.0280 | 0.0183 | 0.0194 | 0.0166 | 0.0166 |
| P1-4 | 0.4360 | 0.3890 | 0.4683 | 0.4539 | 0.4475 | 0.4475 | 0.0470 | 0.0323 | 0.0179 | 0.0115 | 0.0115 |
| P1-5 | 0.3560 | 0.3570 | 0.3759 | 0.3708 | 0.3678 | 0.3678 | 0.0010 | 0.0199 | 0.0148 | 0.0118 | 0.0118 |
| P2-1 | -0.2780 | -0.3490 | -0.2608 | -0.2965 | -0.2939 | -0.2939 | 0.0710 | 0.0172 | 0.0185 | 0.0159 | 0.0159 |
| **P2-4** | **0.3310** | **0.9280** | **0.4315** | **0.3327** | **0.3226** | **0.3226** | **0.5970** | **0.1005** | **0.0017** | **0.0084** | **0.0084** |
| **P2-5** | **0.1550** | **0.9740** | **0.1881** | **0.1573** | **0.1560** | **0.1560** | **0.8190** | **0.0331** | **0.0023** | **0.0010** | **0.0010** |
| P2-6 | 0.2620 | 0.2230 | 0.2585 | 0.2603 | 0.2587 | 0.2587 | 0.0390 | 0.0035 | 0.0017 | 0.0033 | 0.0033 |
| P2-3 | 0.0290 | 0.0860 | 0.0298 | 0.0301 | 0.0291 | 0.0291 | 0.0570 | 0.0008 | 0.0011 | 0.0001 | 0.0001 |
| P3-2 | -0.0290 | -0.0210 | -0.0294 | -0.0298 | -0.0287 | -0.0287 | 0.0080 | 0.0004 | 0.0008 | 0.0003 | 0.0003 |
| P3-5 | 0.1910 | 0.1770 | 0.2266 | 0.1912 | 0.1915 | 0.1915 | 0.0140 | 0.0356 | 0.0002 | 0.0005 | 0.0005 |
| P3-6 | 0.4380 | 0.4330 | 0.4308 | 0.4330 | 0.4339 | 0.4339 | 0.0050 | 0.0072 | 0.0050 | 0.0041 | 0.0041 |
| P4-1 | -0.4250 | -0.4010 | -0.4555 | -0.4417 | -0.4358 | -0.4358 | 0.0240 | 0.0305 | 0.0167 | 0.0108 | 0.0108 |
| P4-2 | -0.3160 | -0.2980 | -0.4131 | -0.3174 | -0.3080 | -0.3080 | 0.0180 | 0.0971 | 0.0014 | 0.0080 | 0.0080 |
| P4-5 | 0.0410 | 0.0070 | 0.0381 | 0.0411 | 0.0426 | 0.0426 | 0.0340 | 0.0029 | 0.0001 | 0.0016 | 0.0016 |
| P5-4 | -0.0400 | -0.0210 | -0.0378 | -0.0407 | -0.0422 | -0.0422 | 0.0190 | 0.0022 | 0.0007 | 0.0022 | 0.0022 |
| P5-1 | -0.3450 | -0.3660 | -0.3639 | -0.3590 | -0.3562 | -0.3562 | 0.0210 | 0.0189 | 0.0140 | 0.0112 | 0.0112 |
| P5-2 | -0.1500 | -0.1170 | -0.1821 | -0.1524 | -0.1511 | -0.1511 | 0.0330 | 0.0321 | 0.0024 | 0.0011 | 0.0011 |
| P5-3 | -0.1800 | -0.2510 | -0.2145 | -0.1807 | -0.1808 | -0.1808 | 0.0710 | 0.0345 | 0.0007 | 0.0008 | 0.0008 |
| P5-6 | 0.0160 | -0.0210 | -0.0179 | 0.0127 | 0.0130 | 0.0130 | 0.0370 | 0.0339 | 0.0033 | 0.0030 | 0.0030 |
| P6-5 | -0.0160 | 0.0100 | 0.0184 | -0.0123 | -0.0125 | -0.0125 | 0.0260 | 0.0344 | 0.0037 | 0.0035 | 0.0035 |
| P6-2 | 0.2570 | -0.1960 | -0.2530 | -0.2546 | -0.2531 | -0.2531 | 0.4530 | 0.5100 | 0.5116 | 0.5101 | 0.5101 |
| P6-3 | -0.4280 | -0.4680 | -0.4213 | -0.4234 | -0.4243 | -0.4243 | 0.0400 | 0.0067 | 0.0046 | 0.0037 | 0.0037 |
| Q1-2 | -0.1540 | -0.1320 | -0.1408 | -0.1397 | -0.1413 | -0.1413 | 0.0220 | 0.0132 | 0.0143 | 0.0127 | 0.0127 |
| Q1-4 | 0.2010 | 0.2120 | 0.2227 | 0.2216 | 0.2130 | 0.2130 | 0.0110 | 0.0217 | 0.0206 | 0.0120 | 0.0120 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q1-5 | 0.1130 | 0.0940 | 0.1196 | 0.1201 | 0.1174 | 0.1174 | 0.0190 | 0.0066 | 0.0071 | 0.0044 | 0.0044 |
| Q2-1 | 0.1280 | 0.0970 | 0.1130 | 0.1160 | 0.1171 | 0.1171 | 0.0310 | 0.0150 | 0.0120 | 0.0109 | 0.0109 |
| Q2-4 | 0.4610 | 0.3830 | 0.4542 | 0.4582 | 0.4496 | 0.4496 | 0.0780 | 0.0068 | 0.0028 | 0.0114 | 0.0114 |
| Q2-5 | 0.1540 | 0.2200 | 0.1502 | 0.1468 | 0.1460 | 0.1460 | 0.0660 | 0.0038 | 0.0072 | 0.0080 | 0.0080 |
| Q2-6 | 0.1240 | 0.1500 | 0.1142 | 0.1170 | 0.1119 | 0.1119 | 0.0260 | 0.0098 | 0.0070 | 0.0121 | 0.0121 |
| Q2-3 | -0.1230 | -0.1190 | -0.1208 | -0.1190 | -0.1230 | -0.1230 | 0.0040 | 0.0022 | 0.0040 | 0.0000 | 0.0000 |
| Q3-2 | 0.0570 | 0.1020 | 0.0561 | 0.0549 | 0.0586 | 0.0586 | 0.0450 | 0.0009 | 0.0021 | 0.0016 | 0.0016 |
| Q3-5 | 0.2320 | 0.2390 | 0.2223 | 0.2214 | 0.2240 | 0.2240 | 0.0070 | 0.0097 | 0.0106 | 0.0080 | 0.0080 |
| Q3-6 | 0.6070 | 0.5830 | 0.5809 | 0.5830 | 0.5822 | 0.5822 | 0.0240 | 0.0261 | 0.0240 | 0.0248 | 0.0248 |
| Q4-1 | -0.1990 | -0.1430 | -0.2126 | -0.2139 | -0.2076 | -0.2076 | 0.0560 | 0.0136 | 0.0149 | 0.0086 | 0.0086 |
| Q4-2 | -0.4510 | -0.4430 | -0.4378 | -0.4480 | -0.4410 | -0.4410 | 0.0080 | 0.0132 | 0.0030 | 0.0100 | 0.0100 |
| Q4-5 | -0.0490 | -0.1740 | -0.0528 | -0.0529 | -0.0515 | -0.0515 | 0.1250 | 0.0038 | 0.0039 | 0.0025 | 0.0025 |
| Q5-4 | -0.0280 | -0.0150 | -0.0230 | -0.0227 | -0.0247 | -0.0247 | 0.0130 | 0.0050 | 0.0053 | 0.0033 | 0.0033 |
| Q5-1 | -0.1350 | -0.1750 | -0.1359 | -0.1371 | -0.1358 | -0.1358 | 0.0400 | 0.0009 | 0.0021 | 0.0008 | 0.0008 |
| Q5-2 | -0.1800 | -0.2220 | -0.1733 | -0.1727 | -0.1725 | -0.1725 | 0.0420 | 0.0067 | 0.0073 | 0.0075 | 0.0075 |
| Q5-3 | -0.2610 | -0.1990 | -0.2483 | -0.2504 | -0.2531 | -0.2531 | 0.0620 | 0.0127 | 0.0106 | 0.0079 | 0.0079 |
| Q5-6 | -0.0970 | -0.0080 | -0.0979 | -0.0941 | -0.0969 | -0.0969 | 0.0890 | 0.0009 | 0.0029 | 0.0001 | 0.0001 |
| Q6-5 | 0.0390 | 0.0290 | 0.0408 | 0.0372 | 0.0397 | 0.0397 | 0.0100 | 0.0018 | 0.0018 | 0.0007 | 0.0007 |
| Q6-2 | -0.1600 | -0.2230 | -0.1506 | -0.1524 | -0.1481 | -0.1481 | 0.0630 | 0.0094 | 0.0076 | 0.0119 | 0.0119 |
| Q6-3 | -0.5790 | -0.5110 | -0.5549 | -0.5559 | -0.5556 | -0.5556 | 0.0680 | 0.0241 | 0.0231 | 0.0234 | 0.0234 |
| **SUM** | | | | | | | **3.8325** | **1.9897** | **1.1852** | **1.1044** | **1.1044** |

LAV, LTS and LTAV generate almost the same best estimation, much better than WLS.

### Median redundancy

Table 5.112 gives system state variables, Table 5.113 presents the estimation results and the comparison results with actual value.

Table 5.112: State variable of case 3, multi large, 6-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0793 | -0.0791 | -0.0793 | -0.0702 | **V1** | 1.0350 | 0.9809 | 1.0350 | 1.0342 |
| $\theta 3$ | -0.1013 | -0.0995 | -0.1013 | -0.0853 | **V2** | 1.0273 | 0.9770 | 1.0273 | 1.0303 |
| $\theta 4$ | -0.0730 | -0.0834 | -0.0730 | -0.0791 | **V3** | 1.0474 | 1.0007 | 1.0474 | 1.0518 |
| $\theta 5$ | -0.1056 | -0.1148 | -0.1056 | -0.1004 | **V4** | 0.9749 | 0.9129 | 0.9749 | 0.9697 |
| $\theta 6$ | -0.1319 | -0.1329 | -0.1319 | -0.1154 | **V5** | 0.9649 | 0.9090 | 0.9649 | 0.9658 |
| | | | | | **V6** | 0.9829 | 0.9333 | 0.9829 | 0.9879 |

Table 5.113: Estimation Results and Errors of case 3, multi large, 6-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0500 | 1.0365 | 1.0350 | 0.9809 | 1.0350 | 1.0342 | 0.0150 | 0.0691 | 0.0150 | 0.0158 |
| V2 | 1.0500 | N/A | 1.0273 | 0.9770 | 1.0273 | 1.0303 | 0.0227 | 0.0730 | 0.0227 | 0.0197 |
| V3 | 1.0700 | N/A | 1.0474 | 1.0007 | 1.0474 | 1.0518 | 0.0226 | 0.0693 | 0.0226 | 0.0182 |
| V4 | 0.9896 | N/A | 0.9749 | 0.9129 | 0.9749 | 0.9697 | 0.0146 | 0.0767 | 0.0146 | 0.0198 |
| V5 | 0.9857 | N/A | 0.9649 | 0.9090 | 0.9649 | 0.9658 | 0.0208 | 0.0766 | 0.0208 | 0.0198 |
| V6 | 1.0043 | N/A | 0.9829 | 0.9333 | 0.9829 | 0.9879 | 0.0214 | 0.0710 | 0.0214 | 0.0164 |
| P1 | 1.0790 | 1.1310 | 1.1741 | 1.1310 | 1.1741 | 1.1416 | 0.0951 | 0.0520 | 0.0951 | 0.0626 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P2 | 0.5000 | 0.4840 | 0.3468 | 0.4840 | 0.3468 | 0.4978 | 0.1532 | 0.0160 | 0.1532 | 0.0022 |
| P3 | 0.6000 | 0.5510 | 0.4981 | 0.5510 | 0.4981 | 0.5663 | 0.1019 | 0.0490 | 0.1019 | 0.0337 |
| P4 | -0.7000 | -0.7180 | -0.4937 | -0.6291 | -0.4937 | -0.7055 | 0.2063 | 0.0709 | 0.2063 | 0.0055 |
| P5 | -0.7000 | -0.7200 | -0.6598 | -0.7200 | -0.6598 | -0.7054 | 0.0402 | 0.0200 | 0.0402 | 0.0054 |
| P6 | -0.7000 | -0.7230 | -0.7825 | -0.7230 | -0.7825 | -0.7112 | 0.0825 | 0.0230 | 0.0825 | 0.0112 |
| Q1 | 0.1600 | 0.2020 | 0.1760 | 0.2020 | 0.1760 | 0.1900 | 0.0160 | 0.0420 | 0.0160 | 0.0300 |
| Q2 | 0.7440 | 0.7190 | 0.6920 | 0.7190 | 0.6920 | 0.6961 | 0.0520 | 0.0250 | 0.0520 | 0.0479 |
| Q3 | 0.8960 | 0.9060 | 0.8899 | 0.9060 | 0.8899 | 0.8841 | 0.0061 | 0.0100 | 0.0061 | 0.0119 |
| Q4 | -0.7000 | -0.7190 | -0.6861 | -0.7190 | -0.6861 | -0.6973 | 0.0139 | 0.0190 | 0.0139 | 0.0027 |
| Q5 | -0.7000 | -0.6770 | -0.7045 | -0.6770 | -0.7045 | -0.7035 | 0.0045 | 0.0230 | 0.0045 | 0.0035 |
| Q6 | -0.7000 | -0.6090 | -0.6329 | -0.6113 | -0.6329 | -0.6355 | 0.0671 | 0.0887 | 0.0671 | 0.0645 |
| P1-2 | 0.2870 | N/A | 0.3594 | 0.3167 | 0.3594 | 0.3121 | 0.0724 | 0.0297 | 0.0724 | 0.0251 |
| P1-4 | 0.4360 | N/A | 0.4224 | 0.4332 | 0.4224 | 0.4552 | 0.0136 | 0.0028 | 0.0136 | 0.0192 |
| P1-5 | 0.3560 | 0.3570 | 0.3923 | 0.3811 | 0.3923 | 0.3743 | 0.0363 | 0.0251 | 0.0363 | 0.0183 |
| P2-1 | -0.2780 | N/A | -0.3459 | -0.3046 | -0.3459 | -0.3016 | 0.0679 | 0.0266 | 0.0679 | 0.0236 |
| **P2-4** | **0.3310** | **0.9280** | **0.1646** | **0.2811** | **0.1646** | **0.3213** | **0.1664** | **0.0499** | **0.1664** | **0.0097** |
| **P2-5** | **0.1550** | **0.9740** | **0.1427** | **0.1619** | **0.1427** | **0.1570** | **0.0123** | **0.0069** | **0.0123** | **0.0020** |
| P2-6 | 0.2620 | N/A | 0.3098 | 0.2868 | 0.3098 | 0.2748 | 0.0478 | 0.0248 | 0.0478 | 0.0128 |
| P2-3 | 0.0290 | N/A | 0.0757 | 0.0589 | 0.0757 | 0.0462 | 0.0467 | 0.0299 | 0.0467 | 0.0172 |
| P3-2 | -0.0290 | N/A | -0.0750 | -0.0581 | -0.0750 | -0.0457 | 0.0460 | 0.0291 | 0.0460 | 0.0167 |
| P3-5 | 0.1910 | N/A | 0.1400 | 0.1785 | 0.1400 | 0.1810 | 0.0510 | 0.0125 | 0.0510 | 0.0100 |
| P3-6 | 0.4380 | 0.4330 | 0.4331 | 0.4307 | 0.4331 | 0.4310 | 0.0049 | 0.0073 | 0.0049 | 0.0070 |
| P4-1 | -0.4250 | N/A | -0.4118 | -0.4205 | -0.4118 | -0.4429 | 0.0132 | 0.0045 | 0.0132 | 0.0179 |
| P4-2 | -0.3160 | N/A | -0.1535 | -0.2646 | -0.1535 | -0.3063 | 0.1625 | 0.0514 | 0.1625 | 0.0097 |
| P4-5 | 0.0410 | N/A | 0.0717 | 0.0560 | 0.0717 | 0.0438 | 0.0307 | 0.0150 | 0.0307 | 0.0028 |
| P5-4 | -0.0400 | N/A | -0.0706 | -0.0551 | -0.0706 | -0.0434 | 0.0306 | 0.0151 | 0.0306 | 0.0034 |
| P5-1 | -0.3450 | N/A | -0.3790 | -0.3671 | -0.3790 | -0.3621 | 0.0340 | 0.0221 | 0.0340 | 0.0171 |
| P5-2 | -0.1500 | N/A | -0.1381 | -0.1561 | -0.1381 | -0.1520 | 0.0119 | 0.0061 | 0.0119 | 0.0020 |
| P5-3 | -0.1800 | N/A | -0.1300 | -0.1659 | -0.1300 | -0.1699 | 0.0500 | 0.0141 | 0.0500 | 0.0101 |
| P5-6 | 0.0160 | N/A | 0.0578 | 0.0242 | 0.0578 | 0.0219 | 0.0418 | 0.0082 | 0.0418 | 0.0059 |
| P6-5 | -0.0160 | N/A | -0.0568 | -0.0233 | -0.0568 | -0.0212 | 0.0408 | 0.0073 | 0.0408 | 0.0052 |
| P6-2 | 0.2570 | N/A | -0.3024 | -0.2797 | -0.3024 | -0.2688 | 0.5594 | 0.5367 | 0.5594 | 0.5258 |
| P6-3 | -0.4280 | N/A | -0.4232 | -0.4199 | -0.4232 | -0.4213 | 0.0048 | 0.0081 | 0.0048 | 0.0067 |
| Q1-2 | -0.1540 | N/A | -0.1446 | -0.1435 | -0.1446 | -0.1442 | 0.0094 | 0.0105 | 0.0094 | 0.0098 |
| Q1-4 | 0.2010 | N/A | 0.1970 | 0.2215 | 0.1970 | 0.2137 | 0.0040 | 0.0205 | 0.0040 | 0.0127 |
| Q1-5 | 0.1130 | 0.0940 | 0.1236 | 0.1240 | 0.1236 | 0.1205 | 0.0106 | 0.0110 | 0.0106 | 0.0075 |
| Q2-1 | 0.1280 | N/A | 0.1290 | 0.1292 | 0.1290 | 0.1226 | 0.0010 | 0.0012 | 0.0010 | 0.0054 |
| Q2-4 | 0.4610 | 0.3830 | 0.4450 | 0.4763 | 0.4450 | 0.4529 | 0.0160 | 0.0153 | 0.0160 | 0.0081 |
| Q2-5 | 0.1540 | 0.2200 | 0.1462 | 0.1501 | 0.1462 | 0.1493 | 0.0078 | 0.0039 | 0.0078 | 0.0047 |
| Q2-6 | 0.1240 | N/A | 0.1001 | 0.0956 | 0.1001 | 0.1006 | 0.0239 | 0.0284 | 0.0239 | 0.0234 |
| Q2-3 | -0.1230 | N/A | -0.1283 | -0.1322 | -0.1283 | -0.1294 | 0.0053 | 0.0092 | 0.0053 | 0.0064 |
| Q3-2 | 0.0570 | N/A | 0.0673 | 0.0772 | 0.0673 | 0.0672 | 0.0103 | 0.0202 | 0.0103 | 0.0102 |
| Q3-5 | 0.2320 | N/A | 0.2403 | 0.2458 | 0.2403 | 0.2373 | 0.0083 | 0.0138 | 0.0083 | 0.0053 |
| Q3-6 | 0.6070 | 0.5830 | 0.5823 | 0.5830 | 0.5823 | 0.5797 | 0.0247 | 0.0240 | 0.0247 | 0.0273 |
| Q4-1 | -0.1990 | N/A | -0.1952 | -0.2064 | -0.1952 | -0.2048 | 0.0038 | 0.0074 | 0.0038 | 0.0058 |
| Q4-2 | -0.4510 | N/A | -0.4428 | -0.4612 | -0.4428 | -0.4430 | 0.0082 | 0.0102 | 0.0082 | 0.0080 |
| Q4-5 | -0.0490 | N/A | -0.0480 | -0.0515 | -0.0480 | -0.0495 | 0.0010 | 0.0025 | 0.0010 | 0.0005 |
| Q5-4 | -0.0280 | N/A | -0.0250 | -0.0132 | -0.0250 | -0.0246 | 0.0030 | 0.0148 | 0.0030 | 0.0034 |
| Q5-1 | -0.1350 | N/A | -0.1338 | -0.1251 | -0.1338 | -0.1348 | 0.0012 | 0.0099 | 0.0012 | 0.0002 |
| Q5-2 | -0.1800 | N/A | -0.1722 | -0.1685 | -0.1722 | -0.1740 | 0.0078 | 0.0115 | 0.0078 | 0.0060 |
| Q5-3 | -0.2610 | N/A | -0.2694 | -0.2641 | -0.2694 | -0.2640 | 0.0084 | 0.0031 | 0.0084 | 0.0030 |
| Q5-6 | -0.0970 | N/A | -0.1041 | -0.1060 | -0.1041 | -0.1061 | 0.0071 | 0.0090 | 0.0071 | 0.0091 |
| Q6-5 | 0.0390 | N/A | 0.0502 | 0.0577 | 0.0502 | 0.0510 | 0.0112 | 0.0187 | 0.0112 | 0.0120 |
| Q6-2 | -0.1600 | N/A | -0.1294 | -0.1210 | -0.1294 | -0.1343 | 0.0306 | 0.0390 | 0.0306 | 0.0257 |
| Q6-3 | -0.5790 | N/A | -0.5537 | -0.5481 | -0.5537 | -0.5522 | 0.0253 | 0.0309 | 0.0253 | 0.0268 |
| **SUM** | | | | | | | **2.7297** | **2.1227** | **2.7297** | **1.3805** |

## Comparison

The comparison are presented in Table 5.114. From the results, it is very easy to find that full redundancy case has better estimation results for all estimators.

Table 5.114: Estimation errors of case 3, multi large, 6-bus AC system, comparison

| | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| **SUM** | **3.8325** | **1.9897** | **1.1852** | **1.1044** | **1.1044** |
| **SUM** | **N/A** | **2.7297** | **2.1227** | **2.7297** | **1.3805** |

#### 5.1.2.5 Summary

Table 5.115 and Table 5.116 presents the comparison between each estimator for full and median redundancy cases respectively.

Table 5.115: Comparison of full redundancy 6-bus estimation between estimators

| N.o | Type | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|---|
| **Case 0** | **Random** | 2.4385 | **1.1079** | 1.1898 | **1.1079** | **1.1079** |
| **Case 1** | **Reverse** | 3.0945 | 1.9207 | 1.2904 | 1.9207 | **1.1096** |
| | **Large** | 3.0325 | 1.6906 | 1.1852 | **1.1096** | **1.1096** |
| **Case 2** | **Reverse** | 3.9605 | 2.6214 | 1.3695 | 2.6214 | **1.1088** |
| | **Non-conf** | 3.5845 | 2.2541 | 1.2982 | 1.9201 | **1.1088** |
| | **Large** | 3.5225 | 2.0376 | 1.2724 | **1.1088** | **1.1088** |
| **Case 3** | **Reverse** | 3.4045 | 1.9087 | 1.2904 | 1.9087 | **1.1044** |
| | **Non-conf** | 3.8945 | 2.1926 | 1.2904 | 1.9180 | **1.1044** |
| | **Large** | 3.8325 | 1.9897 | 1.1852 | **1.1044** | **1.1044** |

Table 5.116: Comparison of median redundancy 6-bus estimation between estimators

| N.o | Type | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| **Case 0** | **Random** | **1.3777** | 1.9299 | **1.3777** | **1.3777** |
| **Case 1** | **Reverse** | 2.7291 | 2.1227 | 2.7291 | **1.3841** |
| | **Large** | 2.1146 | 1.9699 | **1.3841** | **1.3841** |
| **Case 2** | **Reverse** | 3.4341 | 2.1227 | 7.0961 | **1.3852** |
| | **Non-conf** | 3.0055 | 2.1227 | 2.7298 | **1.3852** |
| | **Large** | 2.5405 | 1.8329 | **1.3852** | **1.3852** |
| **Case 3** | **Reverse** | 2.7297 | 2.1227 | 2.7297 | **1.3805** |
| | **Non-conf** | 3.0364 | 2.1227 | 2.7232 | **1.3805** |
| | **Large** | 2.7297 | 2.1227 | 2.7297 | **1.3805** |

It's clear that LTAV generate best estimation in all cases, for some cases, it shares the best with other estimators, no matter it's full or median redundancy. WLS always have the worst performance, LAV performs good in estimate bad data, which makes a solid background for LTAV estimator.

LTS and LTAV all performs good in multiple large bad data case, the reason is that when their are large bad data, it's easy to detect and trim them. LTAV generates the same results within the same case for both full and median redundancy case, means that within the case, the same bad data are detected no matter what kind of bad data they are.

Comparison between full and median redundancy shows that full redundancy cases always generate better estimation than median counterpart regardless of bad data type. Since full redundancy cases have almost three times of raw data than median ones, when their are bad data, it have more correct data for reference, making it performs good both in bad data detection and overall estimation.

## 5.2 14-Bus System

The estimation results of 14-bus system are presented in this section, DC estimation comes first, then AC state estimation are conducted with both full and median redundancy in various scenarios.

### 5.2.1 14-bus DC State Estimation

As stated in the theory part, all voltage magnitude are regarded as 1 and branch resistance are 0, branch reactance are also regarded as 1 for simplicity. All the system data are presented in the Appendix 6.2.

Four major categories that are conducted in this section are the same as in 6-bus system with a few modification. In multiple bad data case, three locations

are used to put bad data instead of two, and in multiple cases, one reverse two large bad data and one large two reverse bad data are conducted respectively. This modification applies to both full and median redundancy.

The parameters used in the estimation are:

1. Tolerance of iterative loop for all estimators: 0.0001;

2. Fixed trimmed number for LTS: $K = 3$;

3. $M$ for LMS: 3; $M$ for LMR: 500; $M$ for LTAV: 3;

#### 5.2.1.1 Case 0: basic case with only random noise

Raw data with no bad data are used in this section. Full and median redundancy evaluation are performed.

**Full redundancy**

Table 5.117 shows the actual values of real power injection and flow in the system from load flow. Random noise are generated using normal distribution with $\sigma = 0.02$ and added to the actual value to simulate the measurements. As for full redundancy case, all the data are used for estimation.

Table 5.117: Raw data for 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | P13 | -0.0900 | -0.0909 | P6-11 | 0.0368 | 0.0372 |
| P2 | 0.1500 | 0.1483 | P14 | -0.1250 | -0.1227 | P6-12 | 0.0688 | 0.0688 |
| P3 | 0.0750 | 0.0755 | P1-2 | 0.0669 | 0.0658 | P6-13 | 0.0676 | 0.0667 |
| P4 | 0.0600 | 0.0599 | P1-5 | 0.1831 | 0.1832 | P7-8 | -0.0950 | -0.0954 |
| P5 | -0.0750 | -0.0751 | P2-3 | 0.0086 | 0.0086 | P7-9 | 0.1239 | 0.1236 |
| P6 | -0.0750 | -0.0744 | P2-4 | 0.0922 | 0.0900 | P9-10 | 0.0982 | 0.0987 |
| P7 | -0.0150 | -0.0147 | P2-5 | 0.1162 | 0.1159 | P9-14 | 0.1486 | 0.1506 |
| P8 | 0.0950 | 0.0954 | P3-4 | 0.0836 | 0.0823 | P10-11 | 0.0432 | 0.0435 |
| P9 | -0.0450 | -0.0452 | P4-5 | 0.0240 | 0.0240 | P12-13 | -0.0012 | -0.0012 |
| P10 | -0.0550 | -0.0558 | P4-7 | 0.0439 | 0.0441 | P13-14 | -0.0236 | -0.0232 |
| P11 | -0.0800 | -0.0798 | P4-9 | 0.1678 | 0.1682 | | | |
| P12 | -0.0700 | -0.0700 | P5-6 | 0.2483 | 0.2520 | | | |

Table 5.118 shows the state variables, Table 5.119 shows the estimation results of each estimator and original actual value, measured value. Table 5.120 shows the error between each estimator and actual value.

Table 5.118: State Variable of case 0, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0644 | -0.0660 | -0.0660 | -0.0672 | -0.0667 | -0.0675 |
| $\theta 3$ | -0.0721 | -0.0740 | -0.0739 | -0.0754 | -0.0749 | -0.0756 |
| $\theta 4$ | -0.1555 | -0.1575 | -0.1566 | -0.1590 | -0.1581 | -0.1593 |
| $\theta 5$ | -0.1804 | -0.1818 | -0.1813 | -0.1833 | -0.1824 | -0.1832 |
| $\theta 6$ | -0.4293 | -0.4295 | -0.4279 | -0.4314 | -0.4299 | -0.4312 |
| $\theta 7$ | -0.1990 | -0.2009 | -0.2007 | -0.2030 | -0.2017 | -0.2032 |
| $\theta 8$ | -0.1035 | -0.1055 | -0.1059 | -0.1081 | -0.1063 | -0.1078 |
| $\theta 9$ | -0.3234 | -0.3248 | -0.3242 | -0.3267 | -0.3255 | -0.3268 |
| $\theta 10$ | -0.4225 | -0.4236 | -0.4222 | -0.4253 | -0.4241 | -0.4255 |
| $\theta 11$ | -0.4660 | -0.4665 | -0.4651 | -0.4685 | -0.4671 | -0.4683 |
| $\theta 12$ | -0.4982 | -0.4982 | -0.4963 | -0.5002 | -0.4984 | -0.4999 |
| $\theta 13$ | -0.4969 | -0.4969 | -0.4953 | -0.4986 | -0.4971 | -0.4987 |
| $\theta 14$ | -0.4724 | -0.4722 | -0.4714 | -0.4738 | -0.4729 | -0.4741 |

Table 5.119: Estimation Results of case 0, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2448 | 0.2478 | 0.2473 | 0.2505 | 0.2492 | 0.2507 |
| P2 | 0.1500 | 0.1483 | 0.1503 | 0.1495 | 0.1477 | 0.1488 | 0.1485 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0758 | 0.0755 | 0.0749 | 0.0755 | 0.0750 | 0.0755 |
| P4 | 0.0600 | 0.0599 | 0.0618 | 0.0599 | 0.0629 | 0.0604 | 0.0608 | 0.0599 |
| P5 | -0.0750 | -0.0751 | -0.0724 | -0.0744 | -0.0744 | -0.0756 | -0.0750 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0754 | -0.0744 | -0.0738 | -0.0749 | -0.0744 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0145 | -0.0148 | -0.0154 | -0.0153 | -0.0150 | -0.0156 |
| P8 | 0.0950 | 0.0954 | 0.0955 | 0.0954 | 0.0948 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0443 | -0.0452 | -0.0457 | -0.0457 | -0.0453 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0555 | -0.0558 | -0.0552 | -0.0555 | -0.0557 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0804 | -0.0799 | -0.0800 | -0.0803 | -0.0801 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0703 | -0.0700 | -0.0694 | -0.0705 | -0.0698 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0908 | -0.0909 | -0.0902 | -0.0904 | -0.0902 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1245 | -0.1227 | -0.1234 | -0.1222 | -0.1232 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0644 | 0.0660 | 0.0660 | 0.0672 | 0.0667 | 0.0675 |
| P1-5 | 0.1831 | 0.1832 | 0.1804 | 0.1818 | 0.1813 | 0.1833 | 0.1824 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | 0.0076 | 0.0080 | 0.0079 | 0.0081 | 0.0082 | 0.0082 |
| P2-4 | 0.0922 | 0.0900 | 0.0911 | 0.0915 | 0.0906 | 0.0918 | 0.0914 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1160 | 0.1159 | 0.1152 | 0.1161 | 0.1157 | 0.1158 |
| P3-4 | 0.0836 | 0.0823 | 0.0834 | 0.0835 | 0.0827 | 0.0837 | 0.0832 | 0.0836 |
| P4-5 | 0.0240 | 0.0240 | 0.0249 | 0.0243 | 0.0246 | 0.0243 | 0.0243 | 0.0240 |
| P4-7 | 0.0439 | 0.0441 | 0.0435 | 0.0434 | 0.0441 | 0.0440 | 0.0436 | 0.0439 |
| P4-9 | 0.1678 | 0.1682 | 0.1679 | 0.1673 | 0.1675 | 0.1677 | 0.1674 | 0.1675 |
| P5-6 | 0.2483 | 0.2520 | 0.2489 | 0.2476 | 0.2467 | 0.2481 | 0.2474 | 0.2479 |
| P6-11 | 0.0368 | 0.0372 | 0.0368 | 0.0370 | 0.0372 | 0.0371 | 0.0372 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.0690 | 0.0687 | 0.0684 | 0.0689 | 0.0686 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.0676 | 0.0675 | 0.0674 | 0.0672 | 0.0673 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.0955 | -0.0954 | -0.0948 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1245 | 0.1240 | 0.1235 | 0.1237 | 0.1239 | 0.1236 |
| P9-10 | 0.0982 | 0.0987 | 0.0991 | 0.0987 | 0.0981 | 0.0986 | 0.0986 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1490 | 0.1474 | 0.1472 | 0.1470 | 0.1474 | 0.1473 |
| P10-11 | 0.0432 | 0.0435 | 0.0436 | 0.0429 | 0.0429 | 0.0432 | 0.0429 | 0.0428 |
| P12-13 | -0.0012 | -0.0012 | -0.0013 | -0.0013 | -0.0010 | -0.0017 | -0.0013 | -0.0012 |
| P13-14 | -0.0236 | -0.0232 | -0.0245 | -0.0247 | -0.0239 | -0.0248 | -0.0242 | -0.0246 |

Table 5.120: Estimation errors of case 0, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0052 | 0.0022 | 0.0027 | 0.0005 | 0.0008 | 0.0007 |
| P2 | 0.0017 | 0.0003 | 0.0005 | 0.0023 | 0.0012 | 0.0015 | 0.0017 |
| P3 | 0.0005 | 0.0008 | 0.0005 | 0.0001 | 0.0005 | 0.0000 | 0.0005 |
| P4 | 0.0001 | 0.0018 | 0.0001 | 0.0028 | 0.0004 | 0.0008 | 0.0001 |
| P5 | 0.0001 | 0.0026 | 0.0006 | 0.0006 | 0.0006 | 0.0000 | 0.0001 |
| P6 | 0.0006 | 0.0004 | 0.0006 | 0.0012 | 0.0001 | 0.0006 | 0.0006 |
| P7 | 0.0002 | 0.0005 | 0.0002 | 0.0004 | 0.0003 | 0.0000 | 0.0006 |
| P8 | 0.0004 | 0.0005 | 0.0004 | 0.0002 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.0007 | 0.0002 | 0.0007 | 0.0007 | 0.0003 | 0.0002 |
| P10 | 0.0008 | 0.0005 | 0.0008 | 0.0002 | 0.0005 | 0.0007 | 0.0008 |
| P11 | 0.0002 | 0.0004 | 0.0001 | 0.0000 | 0.0003 | 0.0001 | 0.0000 |
| P12 | 0.0000 | 0.0003 | 0.0000 | 0.0006 | 0.0005 | 0.0002 | 0.0000 |
| P13 | 0.0009 | 0.0008 | 0.0009 | 0.0002 | 0.0004 | 0.0002 | 0.0009 |
| P14 | 0.0023 | 0.0005 | 0.0023 | 0.0016 | 0.0028 | 0.0018 | 0.0023 |
| P1-2 | 0.0011 | 0.0025 | 0.0010 | 0.0009 | 0.0003 | 0.0002 | 0.0005 |
| P1-5 | 0.0002 | 0.0027 | 0.0013 | 0.0018 | 0.0002 | 0.0007 | 0.0002 |
| P2-3 | 0.0001 | 0.0009 | 0.0006 | 0.0007 | 0.0004 | 0.0004 | 0.0004 |
| P2-4 | 0.0022 | 0.0011 | 0.0006 | 0.0015 | 0.0003 | 0.0008 | 0.0003 |
| P2-5 | 0.0003 | 0.0002 | 0.0003 | 0.0010 | 0.0001 | 0.0005 | 0.0004 |
| P3-4 | 0.0012 | 0.0001 | 0.0000 | 0.0008 | 0.0001 | 0.0004 | 0.0001 |
| P4-5 | 0.0001 | 0.0009 | 0.0003 | 0.0006 | 0.0002 | 0.0003 | 0.0001 |
| P4-7 | 0.0002 | 0.0004 | 0.0005 | 0.0002 | 0.0001 | 0.0004 | 0.0000 |
| P4-9 | 0.0004 | 0.0001 | 0.0005 | 0.0003 | 0.0001 | 0.0004 | 0.0003 |
| P5-6 | 0.0037 | 0.0006 | 0.0006 | 0.0016 | 0.0002 | 0.0008 | 0.0004 |
| P6-11 | 0.0003 | 0.0001 | 0.0002 | 0.0003 | 0.0003 | 0.0004 | 0.0003 |
| P6-12 | 0.0000 | 0.0002 | 0.0001 | 0.0004 | 0.0000 | 0.0003 | 0.0000 |
| P6-13 | 0.0009 | 0.0000 | 0.0002 | 0.0003 | 0.0004 | 0.0004 | 0.0001 |
| P7-8 | 0.0004 | 0.0005 | 0.0004 | 0.0002 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0005 | 0.0001 | 0.0005 | 0.0002 | 0.0000 | 0.0003 |
| P9-10 | 0.0006 | 0.0009 | 0.0006 | 0.0001 | 0.0004 | 0.0004 | 0.0005 |
| P9-14 | 0.0020 | 0.0004 | 0.0012 | 0.0013 | 0.0015 | 0.0011 | 0.0013 |
| P10-11 | 0.0003 | 0.0004 | 0.0003 | 0.0003 | 0.0000 | 0.0002 | 0.0003 |
| P12-13 | 0.0000 | 0.0001 | 0.0001 | 0.0002 | 0.0005 | 0.0001 | 0.0001 |
| P13-14 | 0.0004 | 0.0009 | 0.0011 | 0.0003 | 0.0013 | 0.0006 | 0.0010 |
| **SUM** | **0.0335** | **0.0288** | **0.0193** | **0.0272** | **0.0156** | **0.0160** | **0.0157** |

For the base case, all estimators generate better results than measurements, LMR and LTAV have best estimation, but all other results are acceptable.

### Median redundancy

The median redundancy rate are selected to be 2.0 for 14 bus DC state estimation, i.e, the number of measurement equal to $2.0 \times Ns$, where $Ns$ is the number of state variable, $Ns = 13$ in this section.

The next step is topology planning, it's the key point to guarantee the observability of the system. Following the detail steps in section 3.2, Chapter 3, the

location to put meters are determined.

To keep consistency, power injection at each bus are selected at first, the involvement of power injection ensure a good coverage of power flow connected with each bus. Then other 12 meters are placed according to topology algorithm step by step. The placement of meters are shown in Figure 5.5. The raw data are presented in Table. 5.121.

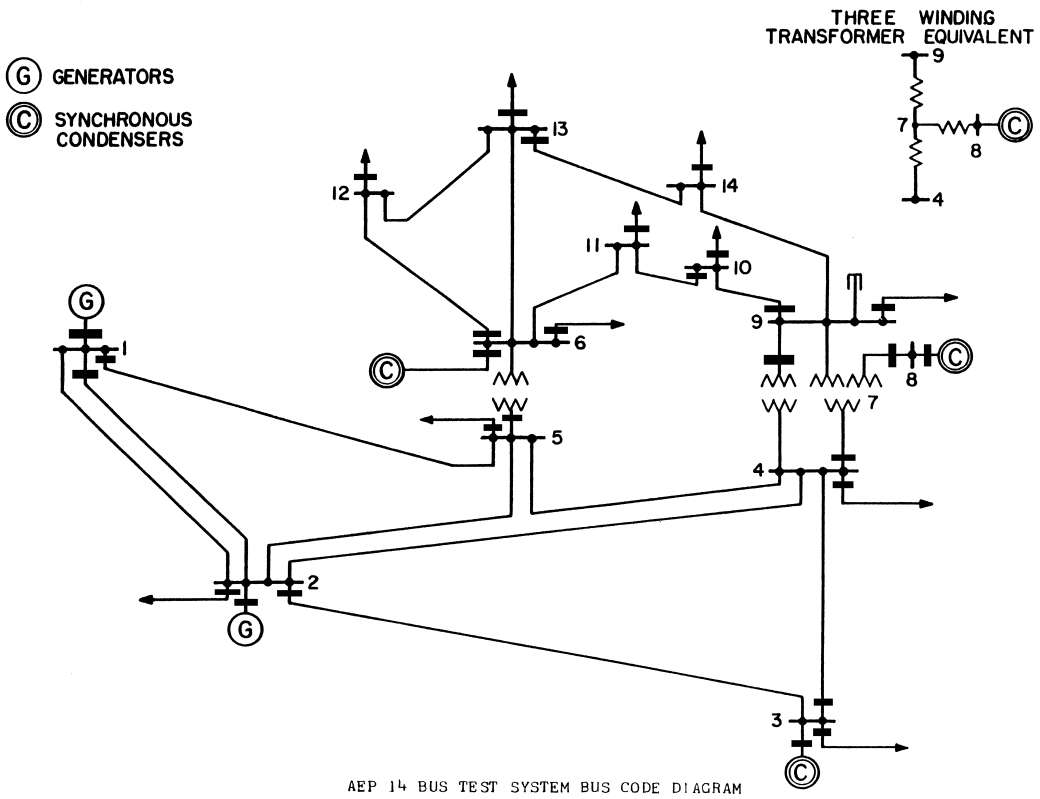


Figure 5.5: 14-bus system with meters for median redundancy.

Table 5.121: raw data for 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | P10 | -0.0550 | -0.0558 | P4-7 | 0.0439 | 0.0441 |
| P2 | 0.1500 | 0.1483 | P11 | -0.0800 | -0.0798 | P5-6 | 0.2483 | 0.2520 |
| P3 | 0.0750 | 0.0755 | P12 | -0.0700 | -0.0700 | P6-12 | 0.0688 | 0.0688 |
| P4 | 0.0600 | 0.0599 | P13 | -0.0900 | -0.0909 | P7-8 | -0.0950 | -0.0954 |
| P5 | -0.0750 | -0.0751 | P14 | -0.1250 | -0.1227 | P7-9 | 0.1239 | 0.1236 |
| P6 | -0.0750 | -0.0744 | P1-2 | 0.0669 | 0.0658 | P9-10 | 0.0982 | 0.0987 |
| P7 | -0.0150 | -0.0147 | P1-5 | 0.1831 | 0.1832 | P10-11 | 0.0432 | 0.0435 |
| P8 | 0.0950 | 0.0954 | P2-3 | 0.0086 | 0.0086 | P13-14 | -0.0236 | -0.0232 |
| P9 | -0.0450 | -0.0452 | P3-4 | 0.0836 | 0.0823 | | | |

Table 5.122 shows the system state variables generated by each estimator; Table 5.123 presents the estimation results and Table 5.124 gives the summation of the absolute value of errors between estimation results, the summation of raw measurements are also presented for comparison. $N/A$ means that the data are not used in the estimation.

Table 5.122: State Variable of case 0, small noise, 14-bus DC system, meidan redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0639 | -0.0658 | -0.0661 | -0.0678 | -0.0664 | -0.0658 |
| $\theta 3$ | -0.0713 | -0.0739 | -0.0747 | -0.0759 | -0.0745 | -0.0745 |
| $\theta 4$ | -0.1548 | -0.1575 | -0.1585 | -0.1594 | -0.1577 | -0.1588 |
| $\theta 5$ | -0.1799 | -0.1820 | -0.1829 | -0.1837 | -0.1820 | -0.1829 |
| $\theta 6$ | -0.4278 | -0.4295 | -0.4317 | -0.4324 | -0.4291 | -0.4320 |
| $\theta 7$ | -0.1981 | -0.2008 | -0.2024 | -0.2030 | -0.2011 | -0.2029 |
| $\theta 8$ | -0.1026 | -0.1054 | -0.1072 | -0.1081 | -0.1058 | -0.1075 |
| $\theta 9$ | -0.3223 | -0.3248 | -0.3262 | -0.3270 | -0.3248 | -0.3276 |
| $\theta 10$ | -0.4212 | -0.4235 | -0.4251 | -0.4258 | -0.4232 | -0.4262 |
| $\theta 11$ | -0.4645 | -0.4664 | -0.4684 | -0.4692 | -0.4661 | -0.4690 |
| $\theta 12$ | -0.4967 | -0.4982 | -0.5008 | -0.5015 | -0.4976 | -0.5012 |
| $\theta 13$ | -0.4953 | -0.4969 | -0.5001 | -0.5002 | -0.4964 | -0.5005 |
| $\theta 14$ | -0.4704 | -0.4722 | -0.4766 | -0.4752 | -0.4721 | -0.4773 |

Table 5.123: Estimation Results of case 0, small noise, 14-bus DC system, meidan redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2438 | 0.2478 | 0.2490 | 0.2515 | 0.2484 | 0.2487 |
| P2 | 0.1500 | 0.1483 | 0.1503 | 0.1501 | 0.1518 | 0.1478 | 0.1486 | 0.1530 |
| P3 | 0.0750 | 0.0755 | 0.0761 | 0.0755 | 0.0752 | 0.0754 | 0.0752 | 0.0755 |
| P4 | 0.0600 | 0.0599 | 0.0614 | 0.0599 | 0.0596 | 0.0604 | 0.0602 | 0.0599 |
| P5 | -0.0750 | -0.0751 | -0.0730 | -0.0751 | -0.0754 | -0.0752 | -0.0749 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0750 | -0.0744 | -0.0746 | -0.0749 | -0.0741 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0146 | -0.0148 | -0.0151 | -0.0145 | -0.0150 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.0955 | 0.0954 | 0.0951 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0447 | -0.0452 | -0.0423 | -0.0447 | -0.0451 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0555 | -0.0558 | -0.0555 | -0.0553 | -0.0556 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0800 | -0.0798 | -0.0801 | -0.0803 | -0.0799 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.0703 | -0.0700 | -0.0697 | -0.0705 | -0.0698 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0908 | -0.0909 | -0.0912 | -0.0914 | -0.0905 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1232 | -0.1227 | -0.1269 | -0.1232 | -0.1230 | -0.1265 |
| P1-2 | 0.0669 | 0.0658 | 0.0639 | 0.0658 | 0.0661 | 0.0678 | 0.0664 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1799 | 0.1820 | 0.1829 | 0.1837 | 0.1820 | 0.1829 |
| P2-3 | 0.0086 | 0.0086 | 0.0074 | 0.0081 | 0.0086 | 0.0081 | 0.0081 | 0.0087 |
| P2-4 | 0.0922 | N/A | 0.0909 | 0.0917 | 0.0925 | 0.0916 | 0.0913 | 0.0930 |
| P2-5 | 0.1162 | N/A | 0.1160 | 0.1162 | 0.1169 | 0.1159 | 0.1156 | 0.1171 |
| P3-4 | 0.0836 | 0.0823 | 0.0835 | 0.0836 | 0.0838 | 0.0835 | 0.0832 | 0.0842 |
| P4-5 | 0.0240 | N/A | 0.0251 | 0.0245 | 0.0244 | 0.0243 | 0.0243 | 0.0242 |
| P4-7 | 0.0439 | 0.0441 | 0.0432 | 0.0434 | 0.0438 | 0.0436 | 0.0434 | 0.0441 |
| P4-9 | 0.1678 | N/A | 0.1675 | 0.1673 | 0.1677 | 0.1676 | 0.1671 | 0.1688 |
| P5-6 | 0.2483 | 0.2520 | 0.2479 | 0.2475 | 0.2488 | 0.2487 | 0.2471 | 0.2491 |
| P6-11 | 0.0368 | N/A | 0.0367 | 0.0369 | 0.0367 | 0.0368 | 0.0370 | 0.0370 |
| P6-12 | 0.0688 | 0.0688 | 0.0688 | 0.0687 | 0.0691 | 0.0691 | 0.0686 | 0.0692 |
| P6-13 | 0.0676 | N/A | 0.0674 | 0.0675 | 0.0684 | 0.0678 | 0.0674 | 0.0685 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P7-8 | -0.0950 | -0.0954 | -0.0955 | -0.0954 | -0.0951 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1242 | 0.1240 | 0.1239 | 0.1240 | 0.1237 | 0.1247 |
| P9-10 | 0.0982 | 0.0987 | 0.0989 | 0.0987 | 0.0989 | 0.0988 | 0.0984 | 0.0986 |
| P9-14 | 0.1486 | N/A | 0.1481 | 0.1474 | 0.1504 | 0.1482 | 0.1473 | 0.1497 |
| P10-11 | 0.0432 | 0.0435 | 0.0433 | 0.0429 | 0.0433 | 0.0435 | 0.0429 | 0.0428 |
| P12-13 | -0.0012 | N/A | -0.0014 | -0.0013 | -0.0007 | -0.0014 | -0.0012 | -0.0008 |
| P13-14 | -0.0236 | -0.0232 | -0.0248 | -0.0247 | -0.0235 | -0.0250 | -0.0243 | -0.0232 |

Table 5.124: Estimation errors of case 0, small noise, 14-bus DC system, meidan redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0062 | 0.0022 | 0.0010 | 0.0015 | 0.0016 | 0.0013 |
| P2 | 0.0003 | 0.0001 | 0.0018 | 0.0022 | 0.0014 | 0.0030 |
| P3 | 0.0011 | 0.0005 | 0.0002 | 0.0004 | 0.0002 | 0.0005 |
| P4 | 0.0014 | 0.0001 | 0.0004 | 0.0004 | 0.0002 | 0.0001 |
| P5 | 0.0020 | 0.0001 | 0.0004 | 0.0002 | 0.0001 | 0.0001 |
| P6 | 0.0000 | 0.0006 | 0.0004 | 0.0001 | 0.0009 | 0.0006 |
| P7 | 0.0004 | 0.0002 | 0.0001 | 0.0005 | 0.0000 | 0.0002 |
| P8 | 0.0005 | 0.0004 | 0.0001 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0003 | 0.0002 | 0.0027 | 0.0003 | 0.0001 | 0.0002 |
| P10 | 0.0005 | 0.0008 | 0.0005 | 0.0003 | 0.0006 | 0.0008 |
| P11 | 0.0000 | 0.0002 | 0.0001 | 0.0003 | 0.0001 | 0.0002 |
| P12 | 0.0003 | 0.0000 | 0.0003 | 0.0005 | 0.0002 | 0.0000 |
| P13 | 0.0008 | 0.0009 | 0.0012 | 0.0014 | 0.0005 | 0.0009 |
| P14 | 0.0018 | 0.0023 | 0.0019 | 0.0018 | 0.0020 | 0.0015 |
| P1-2 | 0.0030 | 0.0011 | 0.0008 | 0.0009 | 0.0005 | 0.0011 |
| P1-5 | 0.0032 | 0.0011 | 0.0002 | 0.0006 | 0.0011 | 0.0002 |
| P2-3 | 0.0012 | 0.0005 | 0.0000 | 0.0005 | 0.0005 | 0.0001 |
| P2-4 | 0.0013 | 0.0005 | 0.0003 | 0.0006 | 0.0008 | 0.0008 |
| P2-5 | 0.0002 | 0.0000 | 0.0007 | 0.0003 | 0.0006 | 0.0009 |
| P3-4 | 0.0001 | 0.0000 | 0.0003 | 0.0001 | 0.0003 | 0.0006 |
| P4-5 | 0.0011 | 0.0005 | 0.0004 | 0.0003 | 0.0003 | 0.0001 |
| P4-7 | 0.0007 | 0.0005 | 0.0001 | 0.0003 | 0.0005 | 0.0002 |
| P4-9 | 0.0003 | 0.0005 | 0.0001 | 0.0002 | 0.0007 | 0.0010 |
| P5-6 | 0.0003 | 0.0008 | 0.0005 | 0.0004 | 0.0012 | 0.0008 |
| P6-11 | 0.0002 | 0.0001 | 0.0001 | 0.0000 | 0.0002 | 0.0001 |
| P6-12 | 0.0000 | 0.0001 | 0.0002 | 0.0003 | 0.0002 | 0.0004 |
| P6-13 | 0.0002 | 0.0002 | 0.0007 | 0.0002 | 0.0003 | 0.0008 |
| P7-8 | 0.0005 | 0.0004 | 0.0001 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0001 | 0.0000 | 0.0001 | 0.0002 | 0.0008 |
| P9-10 | 0.0007 | 0.0005 | 0.0007 | 0.0006 | 0.0003 | 0.0005 |
| P9-14 | 0.0005 | 0.0011 | 0.0018 | 0.0004 | 0.0013 | 0.0011 |
| P10-11 | 0.0002 | 0.0003 | 0.0002 | 0.0003 | 0.0003 | 0.0003 |
| P12-13 | 0.0003 | 0.0001 | 0.0005 | 0.0002 | 0.0000 | 0.0004 |
| P13-14 | 0.0013 | 0.0012 | 0.0001 | 0.0014 | 0.0008 | 0.0004 |
| **SUM** | **0.0310** | **0.0181** | **0.0190** | **0.0177** | **0.0186** | **0.0211** |

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.125. LAV and LMS are different with others, they generate better results in median case especially for LTS.

Table 5.125: Estimation errors of case 0, small noise, 14-bus DC system

| | Meas. | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.0335** | **0.0288** | **0.0193** | **0.0272** | **0.0156** | **0.0160** | **0.0157** |
| **SUM** | **N/A** | **0.0310** | **0.0181** | **0.0190** | **0.0177** | **0.0186** | **0.0211** |

#### 5.2.1.2 Case 1: Single bad data

Two types of bad data are conducted here for performance evaluation, reverse and large bad data. Full and median redundancy system are used for testing.

**Reverse bad data**

A reverse bad data $P_4 = -0.0599$ are used to replace the original one to simulate reverse bad data in both cases.

**Full redundancy**

Table 5.126 presents the state variables, Table 5.127 shows the estimated value and Table 5.128 shows the errors between each estimator and actual value.

Table 5.126: State Variable of case 1, reverse, 14-bus DC system, full redundancy

| | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0682 | -0.0661 | -0.0660 | -0.0670 | -0.0666 | -0.0675 |
| $\theta 3$ | -0.0769 | -0.0743 | -0.0739 | -0.0751 | -0.0747 | -0.0756 |
| $\theta 4$ | -0.1753 | -0.1580 | -0.1566 | -0.1583 | -0.1575 | -0.1593 |
| $\theta 5$ | -0.1831 | -0.1820 | -0.1813 | -0.1827 | -0.1825 | -0.1832 |
| $\theta 6$ | -0.4269 | -0.4298 | -0.4279 | -0.4305 | -0.4307 | -0.4312 |
| $\theta 7$ | -0.2021 | -0.2014 | -0.2007 | -0.2023 | -0.2014 | -0.2032 |
| $\theta 8$ | -0.1022 | -0.1060 | -0.1059 | -0.1073 | -0.1061 | -0.1078 |
| $\theta 9$ | -0.3257 | -0.3253 | -0.3242 | -0.3260 | -0.3256 | -0.3268 |
| $\theta 10$ | -0.4204 | -0.4240 | -0.4222 | -0.4247 | -0.4244 | -0.4255 |
| $\theta 11$ | -0.4630 | -0.4669 | -0.4651 | -0.4677 | -0.4677 | -0.4683 |
| $\theta 12$ | -0.4949 | -0.4986 | -0.4963 | -0.4990 | -0.4996 | -0.4999 |
| $\theta 13$ | -0.4938 | -0.4973 | -0.4953 | -0.4976 | -0.4984 | -0.4987 |
| $\theta 14$ | -0.4704 | -0.4727 | -0.4714 | -0.4729 | -0.4743 | -0.4741 |

Table 5.127: Estimation Results of case 1, reverse, 14-bus DC system, full redundancy

| | Actual | Meas. | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2512 | 0.2481 | 0.2473 | 0.2497 | 0.2490 | 0.2507 |
| P2 | 0.1500 | 0.1483 | 0.1626 | 0.1499 | 0.1477 | 0.1482 | 0.1483 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0897 | 0.0755 | 0.0749 | 0.0750 | 0.0747 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **-0.0207** | **0.0590** | **0.0629** | **0.0617** | **0.0634** | **0.0599** |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P5 | -0.0750 | -0.0751 | -0.0619 | -0.0739 | -0.0744 | -0.0752 | -0.0751 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0728 | -0.0744 | -0.0738 | -0.0749 | -0.0748 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0032 | -0.0147 | -0.0154 | -0.0153 | -0.0151 | -0.0156 |
| P8 | 0.0950 | 0.0954 | 0.0999 | 0.0954 | 0.0948 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0344 | -0.0452 | -0.0457 | -0.0457 | -0.0448 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0522 | -0.0558 | -0.0552 | -0.0557 | -0.0556 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0787 | -0.0800 | -0.0800 | -0.0803 | -0.0802 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0691 | -0.0700 | -0.0694 | -0.0699 | -0.0701 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0892 | -0.0909 | -0.0902 | -0.0904 | -0.0906 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1213 | -0.1227 | -0.1234 | -0.1222 | -0.1246 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0682 | 0.0661 | 0.0660 | 0.0670 | 0.0666 | 0.0675 |
| P1-5 | 0.1831 | 0.1832 | 0.1831 | 0.1820 | 0.1813 | 0.1827 | 0.1825 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | 0.0087 | 0.0082 | 0.0079 | 0.0081 | 0.0081 | 0.0082 |
| P2-4 | 0.0922 | 0.0900 | 0.1071 | 0.0919 | 0.0906 | 0.0913 | 0.0909 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1149 | 0.1159 | 0.1152 | 0.1158 | 0.1159 | 0.1158 |
| P3-4 | 0.0836 | 0.0823 | 0.0984 | 0.0837 | 0.0827 | 0.0832 | 0.0828 | 0.0837 |
| P4-5 | 0.0240 | 0.0240 | 0.0077 | 0.0239 | 0.0246 | 0.0245 | 0.0250 | 0.0240 |
| P4-7 | 0.0439 | 0.0441 | 0.0268 | 0.0433 | 0.0441 | 0.0440 | 0.0440 | 0.0439 |
| P4-9 | 0.1678 | 0.1682 | 0.1503 | 0.1673 | 0.1675 | 0.1677 | 0.1681 | 0.1675 |
| P5-6 | 0.2483 | 0.2520 | 0.2438 | 0.2478 | 0.2467 | 0.2477 | 0.2483 | 0.2479 |
| P6-11 | 0.0368 | 0.0372 | 0.0361 | 0.0371 | 0.0372 | 0.0372 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.0680 | 0.0688 | 0.0684 | 0.0685 | 0.0689 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.0669 | 0.0675 | 0.0674 | 0.0671 | 0.0677 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.0999 | -0.0954 | -0.0948 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1235 | 0.1240 | 0.1235 | 0.1237 | 0.1242 | 0.1236 |
| P9-10 | 0.0982 | 0.0987 | 0.0948 | 0.0987 | 0.0981 | 0.0987 | 0.0988 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1447 | 0.1474 | 0.1472 | 0.1469 | 0.1487 | 0.1473 |
| P10-11 | 0.0432 | 0.0435 | 0.0426 | 0.0429 | 0.0429 | 0.0430 | 0.0433 | 0.0428 |
| P12-13 | -0.0012 | -0.0012 | -0.0011 | -0.0013 | -0.0010 | -0.0014 | -0.0012 | -0.0012 |
| P13-14 | -0.0236 | -0.0232 | -0.0234 | -0.0246 | -0.0239 | -0.0247 | -0.0241 | -0.0246 |

Table 5.128: Estimation errors of case 1, reverse, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0012 | 0.0019 | 0.0027 | 0.0003 | 0.0010 | 0.0007 |
| P2 | 0.0017 | 0.0126 | 0.0001 | 0.0023 | 0.0018 | 0.0017 | 0.0017 |
| P3 | 0.0005 | 0.0147 | 0.0005 | 0.0001 | 0.0000 | 0.0003 | 0.0005 |
| **P4** | **0.1199** | **0.0807** | **0.0010** | **0.0028** | **0.0017** | **0.0034** | **0.0001** |
| P5 | 0.0001 | 0.0131 | 0.0011 | 0.0006 | 0.0002 | 0.0000 | 0.0001 |
| P6 | 0.0006 | 0.0022 | 0.0006 | 0.0012 | 0.0001 | 0.0002 | 0.0006 |
| P7 | 0.0002 | 0.0118 | 0.0002 | 0.0004 | 0.0003 | 0.0001 | 0.0006 |
| P8 | 0.0004 | 0.0049 | 0.0004 | 0.0002 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.0106 | 0.0002 | 0.0007 | 0.0007 | 0.0002 | 0.0002 |
| P10 | 0.0008 | 0.0028 | 0.0008 | 0.0002 | 0.0007 | 0.0006 | 0.0008 |
| P11 | 0.0002 | 0.0013 | 0.0000 | 0.0000 | 0.0003 | 0.0002 | 0.0000 |
| P12 | 0.0000 | 0.0009 | 0.0000 | 0.0006 | 0.0001 | 0.0001 | 0.0000 |
| P13 | 0.0009 | 0.0008 | 0.0009 | 0.0002 | 0.0004 | 0.0006 | 0.0009 |
| P14 | 0.0023 | 0.0037 | 0.0023 | 0.0016 | 0.0028 | 0.0004 | 0.0023 |
| P1-2 | 0.0011 | 0.0013 | 0.0008 | 0.0009 | 0.0001 | 0.0003 | 0.0005 |
| P1-5 | 0.0002 | 0.0000 | 0.0011 | 0.0018 | 0.0003 | 0.0006 | 0.0002 |
| P2-3 | 0.0001 | 0.0001 | 0.0004 | 0.0007 | 0.0004 | 0.0005 | 0.0004 |
| P2-4 | 0.0022 | 0.0150 | 0.0002 | 0.0015 | 0.0009 | 0.0013 | 0.0003 |
| P2-5 | 0.0003 | 0.0013 | 0.0003 | 0.0010 | 0.0004 | 0.0003 | 0.0004 |
| P3-4 | 0.0012 | 0.0149 | 0.0001 | 0.0008 | 0.0004 | 0.0008 | 0.0001 |
| P4-5 | 0.0001 | 0.0163 | 0.0001 | 0.0006 | 0.0004 | 0.0010 | 0.0001 |
| P4-7 | 0.0002 | 0.0171 | 0.0006 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| P4-9 | 0.0004 | 0.0175 | 0.0005 | 0.0003 | 0.0001 | 0.0003 | 0.0003 |
| P5-6 | 0.0037 | 0.0045 | 0.0005 | 0.0016 | 0.0005 | 0.0000 | 0.0004 |
| P6-11 | 0.0003 | 0.0007 | 0.0003 | 0.0003 | 0.0004 | 0.0001 | 0.0003 |
| P6-12 | 0.0000 | 0.0008 | 0.0000 | 0.0004 | 0.0003 | 0.0001 | 0.0000 |
| P6-13 | 0.0009 | 0.0007 | 0.0001 | 0.0003 | 0.0005 | 0.0001 | 0.0001 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P7-8 | 0.0004 | 0.0049 | 0.0004 | 0.0002 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0004 | 0.0001 | 0.0005 | 0.0002 | 0.0003 | 0.0003 |
| P9-10 | 0.0006 | 0.0034 | 0.0006 | 0.0001 | 0.0006 | 0.0007 | 0.0005 |
| P9-14 | 0.0020 | 0.0038 | 0.0012 | 0.0013 | 0.0016 | 0.0002 | 0.0013 |
| P10-11 | 0.0003 | 0.0006 | 0.0003 | 0.0003 | 0.0002 | 0.0001 | 0.0003 |
| P12-13 | 0.0000 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0000 | 0.0001 |
| P13-14 | 0.0004 | 0.0001 | 0.0011 | 0.0003 | 0.0011 | 0.0006 | 0.0010 |
| **SUM** | **0.1533** | **0.2648** | **0.0188** | **0.0272** | **0.0184** | **0.0166** | **0.0158** |

WLS generate worse results than measurements though it has better bad data estimation, other estimator performs good both at bad data and overall summation estimation.

### Median redundancy

Table 5.129 shows the state variables, Table 5.130 presents the estimated value and Table 5.131 contains the errors between each estimator and actual value.

Table 5.129: State Variable of case 1, reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0708 | -0.0658 | -0.0661 | -0.0681 | -0.0887 | -0.0658 |
| $\theta 3$ | -0.0828 | -0.0744 | -0.0744 | -0.0765 | -0.1057 | -0.0729 |
| $\theta 4$ | -0.1818 | -0.1585 | -0.1565 | -0.1610 | -0.2030 | -0.1554 |
| $\theta 5$ | -0.1827 | -0.1824 | -0.1817 | -0.1837 | -0.1992 | -0.1832 |
| $\theta 6$ | -0.4204 | -0.4301 | -0.4296 | -0.4311 | -0.4414 | -0.4297 |
| $\theta 7$ | -0.2070 | -0.2018 | -0.2004 | -0.2046 | -0.2259 | -0.1995 |
| $\theta 8$ | -0.1074 | -0.1064 | -0.1052 | -0.1092 | -0.1246 | -0.1041 |
| $\theta 9$ | -0.3256 | -0.3257 | -0.3242 | -0.3282 | -0.3471 | -0.3236 |
| $\theta 10$ | -0.4169 | -0.4243 | -0.4232 | -0.4265 | -0.4391 | -0.4228 |
| $\theta 11$ | -0.4571 | -0.4671 | -0.4664 | -0.4685 | -0.4792 | -0.4661 |
| $\theta 12$ | -0.4862 | -0.4988 | -0.4984 | -0.4997 | -0.5081 | -0.4983 |
| $\theta 13$ | -0.4850 | -0.4976 | -0.4970 | -0.4988 | -0.5072 | -0.4968 |
| $\theta 14$ | -0.4625 | -0.4730 | -0.4718 | -0.4751 | -0.4849 | -0.4716 |

Table 5.130: Estimation Results of case 1, reverse, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2535 | 0.2482 | 0.2478 | 0.2518 | 0.2880 | 0.2490 |
| P2 | 0.1500 | 0.1483 | 0.1643 | 0.1521 | 0.1484 | 0.1488 | 0.1531 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0869 | 0.0755 | 0.0738 | 0.0760 | 0.0804 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **-0.0400** | **0.0576** | **0.0642** | **0.0563** | **-0.0484** | **0.0679** |
| P5 | -0.0750 | -0.0751 | -0.0580 | -0.0751 | -0.0748 | -0.0746 | -0.0639 | -0.0819 |
| P6 | -0.0750 | -0.0744 | -0.0703 | -0.0744 | -0.0747 | -0.0739 | -0.0716 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0063 | -0.0148 | -0.0151 | -0.0153 | -0.0030 | -0.0154 |
| P8 | 0.0950 | 0.0954 | 0.0997 | 0.0954 | 0.0951 | 0.0953 | 0.1013 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0342 | -0.0452 | -0.0450 | -0.0457 | -0.0355 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0510 | -0.0558 | -0.0557 | -0.0563 | -0.0518 | -0.0558 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P11 | -0.0800 | -0.0798 | -0.0770 | -0.0798 | -0.0801 | -0.0793 | -0.0781 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.0670 | -0.0700 | -0.0703 | -0.0695 | -0.0678 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0860 | -0.0909 | -0.0912 | -0.0904 | -0.0871 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1144 | -0.1227 | -0.1224 | -0.1232 | -0.1155 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0708 | 0.0658 | 0.0661 | 0.0681 | 0.0887 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1827 | 0.1824 | 0.1817 | 0.1837 | 0.1992 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | 0.0121 | 0.0086 | 0.0083 | 0.0084 | 0.0170 | 0.0071 |
| P2-4 | 0.0922 | N/A | 0.1110 | 0.0927 | 0.0905 | 0.0929 | 0.1143 | 0.0896 |
| P2-5 | 0.1162 | N/A | 0.1120 | 0.1166 | 0.1157 | 0.1156 | 0.1105 | 0.1174 |
| P3-4 | 0.0836 | 0.0823 | 0.0990 | 0.0841 | 0.0821 | 0.0844 | 0.0973 | 0.0826 |
| P4-5 | 0.0240 | N/A | 0.0009 | 0.0239 | 0.0252 | 0.0227 | -0.0038 | 0.0278 |
| P4-7 | 0.0439 | 0.0441 | 0.0252 | 0.0433 | 0.0438 | 0.0436 | 0.0229 | 0.0441 |
| P4-9 | 0.1678 | N/A | 0.1438 | 0.1672 | 0.1677 | 0.1672 | 0.1441 | 0.1682 |
| P5-6 | 0.2483 | 0.2520 | 0.2376 | 0.2477 | 0.2478 | 0.2474 | 0.2421 | 0.2465 |
| P6-11 | 0.0368 | N/A | 0.0367 | 0.0370 | 0.0369 | 0.0373 | 0.0379 | 0.0364 |
| P6-12 | 0.0688 | 0.0688 | 0.0659 | 0.0688 | 0.0689 | 0.0686 | 0.0668 | 0.0686 |
| P6-13 | 0.0676 | N/A | 0.0647 | 0.0675 | 0.0674 | 0.0677 | 0.0658 | 0.0671 |
| P7-8 | -0.0950 | -0.0954 | -0.0997 | -0.0954 | -0.0951 | -0.0953 | -0.1013 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1186 | 0.1239 | 0.1239 | 0.1236 | 0.1212 | 0.1241 |
| P9-10 | 0.0982 | 0.0987 | 0.0912 | 0.0986 | 0.0990 | 0.0983 | 0.0920 | 0.0992 |
| P9-14 | 0.1486 | N/A | 0.1369 | 0.1473 | 0.1476 | 0.1469 | 0.1378 | 0.1479 |
| P10-11 | 0.0432 | 0.0435 | 0.0402 | 0.0428 | 0.0432 | 0.0420 | 0.0402 | 0.0434 |
| P12-13 | -0.0012 | N/A | -0.0012 | -0.0012 | -0.0014 | -0.0009 | -0.0010 | -0.0014 |
| P13-14 | -0.0236 | -0.0232 | -0.0225 | -0.0246 | -0.0252 | -0.0237 | -0.0223 | -0.0252 |

Table 5.131: Estimation errors of case 1, reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0035 | 0.0018 | 0.0022 | 0.0018 | 0.0380 | 0.0010 |
| P2 | 0.0143 | 0.0021 | 0.0016 | 0.0012 | 0.0031 | 0.0017 |
| P3 | 0.0119 | 0.0005 | 0.0012 | 0.0010 | 0.0054 | 0.0005 |
| **P4** | **0.1000** | **0.0024** | **0.0042** | **0.0037** | **0.1084** | **0.0079** |
| P5 | 0.0170 | 0.0001 | 0.0002 | 0.0004 | 0.0111 | 0.0069 |
| P6 | 0.0047 | 0.0006 | 0.0003 | 0.0011 | 0.0034 | 0.0006 |
| P7 | 0.0087 | 0.0002 | 0.0001 | 0.0003 | 0.0120 | 0.0004 |
| P8 | 0.0047 | 0.0004 | 0.0001 | 0.0003 | 0.0063 | 0.0004 |
| P9 | 0.0108 | 0.0002 | 0.0000 | 0.0007 | 0.0095 | 0.0002 |
| P10 | 0.0040 | 0.0008 | 0.0007 | 0.0013 | 0.0032 | 0.0008 |
| P11 | 0.0030 | 0.0002 | 0.0001 | 0.0007 | 0.0019 | 0.0002 |
| P12 | 0.0030 | 0.0000 | 0.0003 | 0.0005 | 0.0022 | 0.0000 |
| P13 | 0.0040 | 0.0009 | 0.0012 | 0.0004 | 0.0029 | 0.0009 |
| P14 | 0.0106 | 0.0023 | 0.0026 | 0.0018 | 0.0095 | 0.0023 |
| P1-2 | 0.0038 | 0.0011 | 0.0008 | 0.0012 | 0.0218 | 0.0011 |
| P1-5 | 0.0004 | 0.0007 | 0.0013 | 0.0006 | 0.0161 | 0.0001 |
| P2-3 | 0.0035 | 0.0000 | 0.0003 | 0.0001 | 0.0084 | 0.0015 |
| P2-4 | 0.0189 | 0.0005 | 0.0017 | 0.0007 | 0.0222 | 0.0025 |
| P2-5 | 0.0042 | 0.0004 | 0.0005 | 0.0006 | 0.0057 | 0.0012 |
| P3-4 | 0.0154 | 0.0005 | 0.0015 | 0.0009 | 0.0138 | 0.0010 |
| P4-5 | 0.0231 | 0.0001 | 0.0012 | 0.0013 | 0.0278 | 0.0037 |
| P4-7 | 0.0187 | 0.0006 | 0.0001 | 0.0003 | 0.0210 | 0.0002 |
| P4-9 | 0.0240 | 0.0006 | 0.0001 | 0.0006 | 0.0237 | 0.0004 |
| P5-6 | 0.0107 | 0.0006 | 0.0005 | 0.0008 | 0.0062 | 0.0018 |
| P6-11 | 0.0001 | 0.0002 | 0.0000 | 0.0005 | 0.0011 | 0.0004 |
| P6-12 | 0.0030 | 0.0000 | 0.0000 | 0.0002 | 0.0020 | 0.0003 |
| P6-13 | 0.0029 | 0.0001 | 0.0002 | 0.0000 | 0.0018 | 0.0005 |
| P7-8 | 0.0047 | 0.0004 | 0.0001 | 0.0003 | 0.0063 | 0.0004 |
| P7-9 | 0.0053 | 0.0000 | 0.0000 | 0.0003 | 0.0027 | 0.0002 |
| P9-10 | 0.0069 | 0.0004 | 0.0008 | 0.0001 | 0.0062 | 0.0010 |
| P9-14 | 0.0116 | 0.0013 | 0.0010 | 0.0017 | 0.0108 | 0.0006 |
| P10-11 | 0.0029 | 0.0004 | 0.0001 | 0.0012 | 0.0030 | 0.0002 |
| P12-13 | 0.0000 | 0.0000 | 0.0002 | 0.0003 | 0.0002 | 0.0003 |

| P13-14 | 0.0011 | 0.0010 | 0.0016 | 0.0001 | 0.0013 | 0.0017 |
|---|---|---|---|---|---|---|
| **SUM** | **0.3611** | **0.0217** | **0.0268** | **0.0270** | **0.4188** | **0.0429** |

LTS generate bad results, since LTS relay on WLS estimation and WLS doesn't correctly tell the bad data, making LTS doesn't trimmed the correct data and generates bad estimation.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.132. Almost all estimators have better results with full redundancy case, the only exception LMS has only a little difference.

Table 5.132: Estimation errors of case 1, single reverse, 14-bus DC system

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.1533** | **0.2648** | **0.0188** | **0.0272** | **0.0184** | **0.0166** | **0.0158** |
| **SUM** | **N/A** | **0.3611** | **0.0217** | **0.0268** | **0.0270** | **0.4188** | **0.0429** |

## Large bad data

A large error at the same location as reverse case are used in this section, all data keep the same except $P_4 = 0.9599$, the bad data is applied to both full and median case.

### Full redundancy

State variables are presented in Table 5.133, Table 5.134 shows the estimated value and Table 5.135 presents the errors between each estimator and actual value.

Table 5.133: State Variable of case 1, large, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0362 | -0.0651 | -0.0665 | -0.0671 | -0.0666 | -0.0675 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta 3$ | -0.0359 | -0.0728 | -0.0754 | -0.0763 | -0.0747 | -0.0756 |
| $\theta 4$ | -0.0067 | -0.1551 | -0.1571 | -0.1583 | -0.1575 | -0.1593 |
| $\theta 5$ | -0.1604 | -0.1810 | -0.1817 | -0.1827 | -0.1825 | -0.1832 |
| $\theta 6$ | -0.4471 | -0.4287 | -0.4284 | -0.4304 | -0.4307 | -0.4312 |
| $\theta 7$ | -0.1753 | -0.1992 | -0.2012 | -0.2023 | -0.2014 | -0.2032 |
| $\theta 8$ | -0.1130 | -0.1038 | -0.1064 | -0.1073 | -0.1061 | -0.1078 |
| $\theta 9$ | -0.3065 | -0.3235 | -0.3246 | -0.3260 | -0.3256 | -0.3268 |
| $\theta 10$ | -0.4376 | -0.4224 | -0.4227 | -0.4247 | -0.4244 | -0.4255 |
| $\theta 11$ | -0.4888 | -0.4654 | -0.4655 | -0.4677 | -0.4677 | -0.4683 |
| $\theta 12$ | -0.5233 | -0.4975 | -0.4967 | -0.4988 | -0.4996 | -0.4999 |
| $\theta 13$ | -0.5202 | -0.4963 | -0.4957 | -0.4976 | -0.4984 | -0.4987 |
| $\theta 14$ | -0.4877 | -0.4719 | -0.4718 | -0.4729 | -0.4743 | -0.4741 |

Table 5.134: Estimation Results of case 1, large, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.1967 | 0.2461 | 0.2482 | 0.2499 | 0.2490 | 0.2507 |
| P2 | 0.1500 | 0.1483 | 0.0581 | 0.1483 | 0.1483 | 0.1488 | 0.1483 | 0.1483 |
| P3 | 0.0750 | 0.0755 | -0.0289 | 0.0747 | 0.0728 | 0.0729 | 0.0747 | 0.0755 |
| **P4** | **0.0600** | **0.9599** | **0.6811** | **0.0660** | **0.0639** | **0.0629** | **0.0634** | **0.0599** |
| P5 | -0.0750 | -0.0751 | -0.1518 | -0.0751 | -0.0748 | -0.0751 | -0.0751 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0957 | -0.0744 | -0.0738 | -0.0749 | -0.0748 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0997 | -0.0152 | -0.0154 | -0.0152 | -0.0151 | -0.0156 |
| P8 | 0.0950 | 0.0954 | 0.0623 | 0.0954 | 0.0948 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.1188 | -0.0452 | -0.0457 | -0.0457 | -0.0448 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0799 | -0.0558 | -0.0552 | -0.0557 | -0.0556 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0929 | -0.0798 | -0.0800 | -0.0803 | -0.0802 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0793 | -0.0700 | -0.0694 | -0.0695 | -0.0701 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1025 | -0.0909 | -0.0902 | -0.0908 | -0.0906 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1487 | -0.1241 | -0.1234 | -0.1222 | -0.1246 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0362 | 0.0651 | 0.0665 | 0.0671 | 0.0666 | 0.0675 |
| P1-5 | 0.1831 | 0.1832 | 0.1604 | 0.1810 | 0.1817 | 0.1827 | 0.1825 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0003 | 0.0076 | 0.0089 | 0.0091 | 0.0081 | 0.0082 |
| P2-4 | 0.0922 | 0.0900 | -0.0296 | 0.0900 | 0.0906 | 0.0912 | 0.0909 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1242 | 0.1159 | 0.1152 | 0.1156 | 0.1159 | 0.1158 |
| P3-4 | 0.0836 | 0.0823 | -0.0292 | 0.0823 | 0.0817 | 0.0820 | 0.0828 | 0.0837 |
| P4-5 | 0.0240 | 0.0240 | 0.1538 | 0.0259 | 0.0246 | 0.0245 | 0.0250 | 0.0240 |
| P4-7 | 0.0439 | 0.0441 | 0.1686 | 0.0441 | 0.0441 | 0.0440 | 0.0440 | 0.0439 |
| P4-9 | 0.1678 | 0.1682 | 0.2999 | 0.1684 | 0.1675 | 0.1677 | 0.1681 | 0.1675 |
| P5-6 | 0.2483 | 0.2520 | 0.2867 | 0.2477 | 0.2467 | 0.2477 | 0.2483 | 0.2479 |
| P6-11 | 0.0368 | 0.0372 | 0.0417 | 0.0368 | 0.0372 | 0.0372 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.0762 | 0.0688 | 0.0684 | 0.0684 | 0.0689 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.0731 | 0.0677 | 0.0674 | 0.0672 | 0.0677 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.0623 | -0.0954 | -0.0948 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1313 | 0.1242 | 0.1235 | 0.1237 | 0.1242 | 0.1236 |
| P9-10 | 0.0982 | 0.0987 | 0.1311 | 0.0989 | 0.0981 | 0.0987 | 0.0988 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1812 | 0.1485 | 0.1472 | 0.1469 | 0.1487 | 0.1473 |
| P10-11 | 0.0432 | 0.0435 | 0.0512 | 0.0431 | 0.0429 | 0.0430 | 0.0433 | 0.0428 |
| P12-13 | -0.0012 | -0.0012 | -0.0031 | -0.0012 | -0.0010 | -0.0012 | -0.0012 | -0.0012 |
| P13-14 | -0.0236 | -0.0232 | -0.0325 | -0.0244 | -0.0239 | -0.0247 | -0.0241 | -0.0246 |

Table 5.135: Estimation errors of case 1, large, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0533 | 0.0039 | 0.0018 | 0.0001 | 0.0010 | 0.0007 |
| P2 | 0.0017 | 0.0919 | 0.0017 | 0.0017 | 0.0012 | 0.0017 | 0.0017 |
| P3 | 0.0005 | 0.1039 | 0.0003 | 0.0022 | 0.0021 | 0.0003 | 0.0005 |
| **P4** | **0.8999** | **0.6211** | **0.0060** | **0.0039** | **0.0029** | **0.0034** | **0.0001** |
| P5 | 0.0001 | 0.0768 | 0.0001 | 0.0002 | 0.0001 | 0.0000 | 0.0001 |
| P6 | 0.0006 | 0.0207 | 0.0006 | 0.0012 | 0.0001 | 0.0002 | 0.0006 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P7 | 0.0002 | 0.0847 | 0.0002 | 0.0004 | 0.0003 | 0.0001 | 0.0006 |
| P8 | 0.0004 | 0.0327 | 0.0004 | 0.0002 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.0738 | 0.0002 | 0.0007 | 0.0007 | 0.0002 | 0.0002 |
| P10 | 0.0008 | 0.0249 | 0.0008 | 0.0002 | 0.0007 | 0.0006 | 0.0008 |
| P11 | 0.0002 | 0.0129 | 0.0002 | 0.0000 | 0.0003 | 0.0002 | 0.0000 |
| P12 | 0.0000 | 0.0093 | 0.0000 | 0.0006 | 0.0005 | 0.0001 | 0.0000 |
| P13 | 0.0009 | 0.0125 | 0.0009 | 0.0002 | 0.0008 | 0.0006 | 0.0009 |
| P14 | 0.0023 | 0.0237 | 0.0009 | 0.0016 | 0.0028 | 0.0004 | 0.0023 |
| P1-2 | 0.0011 | 0.0307 | 0.0018 | 0.0004 | 0.0002 | 0.0003 | 0.0005 |
| P1-5 | 0.0002 | 0.0226 | 0.0021 | 0.0014 | 0.0003 | 0.0006 | 0.0002 |
| P2-3 | 0.0001 | 0.0089 | 0.0010 | 0.0003 | 0.0006 | 0.0005 | 0.0004 |
| P2-4 | 0.0022 | 0.1217 | 0.0022 | 0.0015 | 0.0010 | 0.0013 | 0.0003 |
| P2-5 | 0.0003 | 0.0080 | 0.0003 | 0.0010 | 0.0006 | 0.0003 | 0.0004 |
| P3-4 | 0.0012 | 0.1128 | 0.0012 | 0.0019 | 0.0015 | 0.0008 | 0.0001 |
| P4-5 | 0.0001 | 0.1298 | 0.0019 | 0.0006 | 0.0004 | 0.0010 | 0.0001 |
| P4-7 | 0.0002 | 0.1247 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| P4-9 | 0.0004 | 0.1321 | 0.0005 | 0.0003 | 0.0001 | 0.0003 | 0.0003 |
| P5-6 | 0.0037 | 0.0384 | 0.0006 | 0.0016 | 0.0006 | 0.0000 | 0.0004 |
| P6-11 | 0.0003 | 0.0049 | 0.0001 | 0.0004 | 0.0004 | 0.0001 | 0.0003 |
| P6-12 | 0.0000 | 0.0074 | 0.0000 | 0.0004 | 0.0004 | 0.0001 | 0.0000 |
| P6-13 | 0.0009 | 0.0055 | 0.0000 | 0.0003 | 0.0004 | 0.0001 | 0.0001 |
| P7-8 | 0.0004 | 0.0327 | 0.0004 | 0.0002 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0073 | 0.0003 | 0.0005 | 0.0002 | 0.0003 | 0.0003 |
| P9-10 | 0.0006 | 0.0329 | 0.0007 | 0.0001 | 0.0005 | 0.0007 | 0.0005 |
| P9-14 | 0.0020 | 0.0326 | 0.0001 | 0.0013 | 0.0016 | 0.0002 | 0.0013 |
| P10-11 | 0.0003 | 0.0080 | 0.0001 | 0.0003 | 0.0002 | 0.0001 | 0.0003 |
| P12-13 | 0.0000 | 0.0019 | 0.0000 | 0.0002 | 0.0000 | 0.0000 | 0.0001 |
| P13-14 | 0.0004 | 0.0089 | 0.0008 | 0.0003 | 0.0012 | 0.0006 | 0.0010 |
| **SUM** | **0.9333** | **2.1142** | **0.0306** | **0.0283** | **0.0230** | **0.0166** | **0.0158** |

WLS have worst estimation, results from other estimators are satisfied, LTAV have best estimation.

### Median redundancy

State variables are presented in Table 5.136, Table 5.137 shows the estimated value and Table 5.138 shows the errors between each estimator and actual value.

Table 5.136: State Variable of case 1, large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0127 | -0.0615 | -0.0673 | -0.0681 | -0.0666 | -0.0658 |
| $\theta 3$ | 0.0150 | -0.0675 | -0.0760 | -0.0765 | -0.0747 | -0.0729 |
| $\theta 4$ | 0.0477 | -0.1491 | -0.1581 | -0.1610 | -0.1578 | -0.1554 |
| $\theta 5$ | -0.1586 | -0.1777 | -0.1829 | -0.1837 | -0.1824 | -0.1832 |
| $\theta 6$ | -0.4841 | -0.4251 | -0.4309 | -0.4311 | -0.4300 | -0.4297 |
| $\theta 7$ | -0.1309 | -0.1936 | -0.2019 | -0.2048 | -0.2016 | -0.1995 |
| $\theta 8$ | -0.0664 | -0.0982 | -0.1068 | -0.1099 | -0.1064 | -0.1041 |
| $\theta 9$ | -0.2974 | -0.3186 | -0.3258 | -0.3283 | -0.3255 | -0.3236 |
| $\theta 10$ | -0.4536 | -0.4179 | -0.4246 | -0.4265 | -0.4241 | -0.4228 |
| $\theta 11$ | -0.5203 | -0.4614 | -0.4678 | -0.4685 | -0.4671 | -0.4661 |
| $\theta 12$ | -0.5754 | -0.4941 | -0.4999 | -0.4997 | -0.4988 | -0.4983 |
| $\theta 13$ | -0.5719 | -0.4930 | -0.4985 | -0.4988 | -0.4975 | -0.4968 |
| $\theta 14$ | -0.5294 | -0.4688 | -0.4736 | -0.4751 | -0.4731 | -0.4716 |

Table 5.137: Estimation Results of case 1, large, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.1713 | 0.2392 | 0.2502 | 0.2518 | 0.2490 | 0.2490 |
| P2 | 0.1500 | 0.1483 | 0.0452 | 0.1483 | 0.1480 | 0.1488 | 0.1485 | 0.1483 |
| P3 | 0.0750 | 0.0755 | -0.0050 | 0.0755 | 0.0734 | 0.0760 | 0.0750 | 0.0755 |
| **P4** | **0.0600** | **0.9599** | **0.8232** | **0.0736** | **0.0633** | **0.0566** | **0.0619** | **0.0679** |
| P5 | -0.0750 | -0.0751 | -0.1855 | -0.0751 | -0.0754 | -0.0746 | -0.0752 | -0.0819 |
| P6 | -0.0750 | -0.0744 | -0.1102 | -0.0744 | -0.0747 | -0.0739 | -0.0744 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0766 | -0.0148 | -0.0151 | -0.0153 | -0.0151 | -0.0154 |
| P8 | 0.0950 | 0.0954 | 0.0645 | 0.0954 | 0.0951 | 0.0949 | 0.0952 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.1236 | -0.0452 | -0.0451 | -0.0457 | -0.0453 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0894 | -0.0558 | -0.0555 | -0.0562 | -0.0557 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.1029 | -0.0798 | -0.0801 | -0.0793 | -0.0800 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.0947 | -0.0700 | -0.0703 | -0.0695 | -0.0700 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1269 | -0.0909 | -0.0912 | -0.0904 | -0.0907 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1895 | -0.1260 | -0.1228 | -0.1232 | -0.1232 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0127 | 0.0615 | 0.0673 | 0.0681 | 0.0666 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1586 | 0.1777 | 0.1829 | 0.1837 | 0.1824 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0277 | 0.0060 | 0.0087 | 0.0084 | 0.0081 | 0.0071 |
| P2-4 | 0.0922 | N/A | -0.0604 | 0.0876 | 0.0909 | 0.0929 | 0.0912 | 0.0896 |
| P2-5 | 0.1162 | N/A | 0.1460 | 0.1162 | 0.1157 | 0.1156 | 0.1158 | 0.1174 |
| P3-4 | 0.0836 | 0.0823 | -0.0327 | 0.0815 | 0.0821 | 0.0844 | 0.0831 | 0.0826 |
| P4-5 | 0.0240 | N/A | 0.2063 | 0.0286 | 0.0248 | 0.0227 | 0.0246 | 0.0278 |
| P4-7 | 0.0439 | 0.0441 | 0.1786 | 0.0445 | 0.0438 | 0.0439 | 0.0438 | 0.0441 |
| P4-9 | 0.1678 | N/A | 0.3451 | 0.1696 | 0.1677 | 0.1673 | 0.1677 | 0.1682 |
| P5-6 | 0.2483 | 0.2520 | 0.3255 | 0.2474 | 0.2480 | 0.2474 | 0.2476 | 0.2465 |
| P6-11 | 0.0368 | N/A | 0.0362 | 0.0363 | 0.0369 | 0.0373 | 0.0371 | 0.0364 |
| P6-12 | 0.0688 | 0.0688 | 0.0913 | 0.0689 | 0.0689 | 0.0686 | 0.0687 | 0.0686 |
| P6-13 | 0.0676 | N/A | 0.0878 | 0.0678 | 0.0676 | 0.0677 | 0.0675 | 0.0671 |
| P7-8 | -0.0950 | -0.0954 | -0.0645 | -0.0954 | -0.0951 | -0.0949 | -0.0952 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1665 | 0.1251 | 0.1239 | 0.1235 | 0.1239 | 0.1241 |
| P9-10 | 0.0982 | 0.0987 | 0.1561 | 0.0993 | 0.0988 | 0.0982 | 0.0987 | 0.0992 |
| P9-14 | 0.1486 | N/A | 0.2320 | 0.1502 | 0.1477 | 0.1469 | 0.1476 | 0.1479 |
| P10-11 | 0.0432 | 0.0435 | 0.0667 | 0.0435 | 0.0432 | 0.0420 | 0.0430 | 0.0434 |
| P12-13 | -0.0012 | N/A | -0.0034 | -0.0011 | -0.0014 | -0.0009 | -0.0012 | -0.0014 |
| P13-14 | -0.0236 | -0.0232 | -0.0425 | -0.0242 | -0.0250 | -0.0237 | -0.0244 | -0.0252 |

Table 5.138: Estimation errors of case 1, large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0787 | 0.0108 | 0.0002 | 0.0018 | 0.0010 | 0.0010 |
| P2 | 0.1048 | 0.0017 | 0.0020 | 0.0012 | 0.0015 | 0.0017 |
| P3 | 0.0800 | 0.0005 | 0.0016 | 0.0010 | 0.0000 | 0.0005 |
| **P4** | **0.7632** | **0.0136** | **0.0033** | **0.0034** | **0.0019** | **0.0079** |
| P5 | 0.1105 | 0.0001 | 0.0004 | 0.0004 | 0.0002 | 0.0069 |
| P6 | 0.0352 | 0.0006 | 0.0003 | 0.0011 | 0.0006 | 0.0006 |
| P7 | 0.0616 | 0.0002 | 0.0001 | 0.0003 | 0.0001 | 0.0004 |
| P8 | 0.0305 | 0.0004 | 0.0001 | 0.0001 | 0.0002 | 0.0004 |
| P9 | 0.0786 | 0.0002 | 0.0001 | 0.0007 | 0.0003 | 0.0002 |
| P10 | 0.0344 | 0.0008 | 0.0005 | 0.0012 | 0.0007 | 0.0008 |
| P11 | 0.0229 | 0.0002 | 0.0001 | 0.0007 | 0.0000 | 0.0002 |
| P12 | 0.0247 | 0.0000 | 0.0003 | 0.0005 | 0.0000 | 0.0000 |
| P13 | 0.0369 | 0.0009 | 0.0012 | 0.0004 | 0.0007 | 0.0009 |
| P14 | 0.0645 | 0.0010 | 0.0022 | 0.0018 | 0.0018 | 0.0023 |
| P1-2 | 0.0542 | 0.0054 | 0.0003 | 0.0012 | 0.0003 | 0.0011 |
| P1-5 | 0.0245 | 0.0054 | 0.0002 | 0.0006 | 0.0007 | 0.0001 |
| P2-3 | 0.0363 | 0.0025 | 0.0002 | 0.0001 | 0.0005 | 0.0015 |
| P2-4 | 0.1525 | 0.0046 | 0.0013 | 0.0007 | 0.0010 | 0.0025 |
| P2-5 | 0.0298 | 0.0000 | 0.0005 | 0.0006 | 0.0004 | 0.0012 |

| | | | | | | |
|---|---|---|---|---|---|---|
| P3-4 | 0.1163 | 0.0020 | 0.0015 | 0.0009 | 0.0005 | 0.0010 |
| P4-5 | 0.1823 | 0.0046 | 0.0008 | 0.0013 | 0.0006 | 0.0037 |
| P4-7 | 0.1347 | 0.0006 | 0.0001 | 0.0001 | 0.0001 | 0.0002 |
| P4-9 | 0.1773 | 0.0018 | 0.0001 | 0.0005 | 0.0001 | 0.0004 |
| P5-6 | 0.0772 | 0.0008 | 0.0003 | 0.0008 | 0.0007 | 0.0018 |
| P6-11 | 0.0007 | 0.0005 | 0.0000 | 0.0005 | 0.0002 | 0.0004 |
| P6-12 | 0.0225 | 0.0001 | 0.0001 | 0.0002 | 0.0001 | 0.0003 |
| P6-13 | 0.0202 | 0.0002 | 0.0001 | 0.0000 | 0.0001 | 0.0005 |
| P7-8 | 0.0305 | 0.0004 | 0.0001 | 0.0001 | 0.0002 | 0.0004 |
| P7-9 | 0.0426 | 0.0012 | 0.0000 | 0.0005 | 0.0000 | 0.0002 |
| P9-10 | 0.0580 | 0.0011 | 0.0006 | 0.0000 | 0.0005 | 0.0010 |
| P9-14 | 0.0835 | 0.0016 | 0.0008 | 0.0017 | 0.0009 | 0.0006 |
| P10-11 | 0.0235 | 0.0003 | 0.0001 | 0.0012 | 0.0002 | 0.0002 |
| P12-13 | 0.0022 | 0.0001 | 0.0002 | 0.0003 | 0.0000 | 0.0003 |
| P13-14 | 0.0189 | 0.0006 | 0.0014 | 0.0001 | 0.0008 | 0.0017 |
| **SUM** | **2.8140** | **0.0651** | **0.0209** | **0.0259** | **0.0173** | **0.0429** |

**Comparison**

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.139. Except LMS, other estimators perform better for full redundancy case.

Table 5.139: Estimation errors of case 1, single large, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.9333** | **2.1142** | **0.0306** | **0.0283** | **0.0230** | **0.0166** | **0.0158** |
| **SUM** | **N/A** | **2.8140** | **0.0651** | **0.0209** | **0.0259** | **0.0173** | **0.0429** |

#### 5.2.1.3 Case 2: Multiple non-interacting bad data

Three different bad data are selected for the performance evaluation in this part. Regarding non-interacting consideration, $P_4$, $P_{9-10}$ and $P_{13-14}$ are selected because there's no direct connection between these five buses.

For each bad data, reverse and large bad data are the two types that are used here. They induce four major cases: three reverse, three large, two reverse one large and two large one reverse. These four cases are conducted with full & median redundancy case.

**Three reverse bad data**

The three bad data are: $P_4 = -0.0599$, $P_{9-10} = -0.0987$ and $P_{13-14} = 0.0232$. Other data remain the same as Table 5.117 for full and Table 5.121 for median redundancy case.

**Full Redundancy**

Table 5.140 presents the state variables generated by the estimation of each estimator; The estimated system results and the error between them and the actual value are illustrated in Table 5.141 and Table 5.142 respectively.

Table 5.140: State Variable of case 2, multi reverse, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0715 | -0.0662 | -0.0667 | -0.0663 | -0.0642 | -0.0658 |
| $\theta 3$ | -0.0843 | -0.0744 | -0.0748 | -0.0747 | -0.0717 | -0.0734 |
| $\theta 4$ | -0.1826 | -0.1582 | -0.1575 | -0.1565 | -0.1543 | -0.1565 |
| $\theta 5$ | -0.1783 | -0.1821 | -0.1824 | -0.1819 | -0.1803 | -0.1817 |
| $\theta 6$ | -0.3955 | -0.4298 | -0.4308 | -0.4293 | -0.4298 | -0.4294 |
| $\theta 7$ | -0.2178 | -0.2015 | -0.2023 | -0.2001 | -0.1985 | -0.2006 |
| $\theta 8$ | -0.1238 | -0.1061 | -0.1077 | -0.1044 | -0.1032 | -0.1052 |
| $\theta 9$ | -0.3287 | -0.3254 | -0.3260 | -0.3242 | -0.3231 | -0.3244 |
| $\theta 10$ | -0.3514 | -0.4240 | -0.4240 | -0.4235 | -0.4229 | -0.4234 |
| $\theta 11$ | -0.4008 | -0.4668 | -0.4671 | -0.4666 | -0.4666 | -0.4665 |
| $\theta 12$ | -0.4656 | -0.4986 | -0.5000 | -0.4977 | -0.4989 | -0.4981 |
| $\theta 13$ | -0.4718 | -0.4973 | -0.4984 | -0.4962 | -0.4974 | -0.4967 |
| $\theta 14$ | -0.4767 | -0.4727 | -0.4740 | -0.4713 | -0.4722 | -0.4719 |

Table 5.141: Estimation Results of case 2, multi reverse, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2498 | 0.2484 | 0.2491 | 0.2483 | 0.2445 | 0.2475 |
| P2 | 0.1500 | 0.1483 | 0.1591 | 0.1497 | 0.1479 | 0.1478 | 0.1497 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0855 | 0.0755 | 0.0746 | 0.0735 | 0.0751 | 0.0754 |
| **P4** | **0.0600** | **-0.0599** | **-0.0323** | **0.0588** | **0.0644** | **0.0647** | **0.0662** | **0.0635** |
| P5 | -0.0750 | -0.0751 | -0.0636 | -0.0742 | -0.0744 | -0.0756 | -0.0730 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0657 | -0.0744 | -0.0753 | -0.0749 | -0.0758 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0184 | -0.0148 | -0.0156 | -0.0152 | -0.0150 | -0.0157 |
| P8 | 0.0950 | 0.0954 | 0.0940 | 0.0954 | 0.0946 | 0.0957 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0862 | -0.0452 | -0.0461 | -0.0453 | -0.0444 | -0.0452 |
| P10 | -0.0550 | -0.0558 | 0.0266 | -0.0558 | -0.0550 | -0.0563 | -0.0560 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0546 | -0.0798 | -0.0794 | -0.0803 | -0.0806 | -0.0803 |
| P12 | -0.0700 | -0.0700 | -0.0639 | -0.0700 | -0.0709 | -0.0698 | -0.0706 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0776 | -0.0909 | -0.0903 | -0.0904 | -0.0915 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1529 | -0.1227 | -0.1236 | -0.1222 | -0.1238 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0715 | 0.0662 | 0.0667 | 0.0663 | 0.0642 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1783 | 0.1821 | 0.1824 | 0.1819 | 0.1803 | 0.1817 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P2-3 | 0.0086 | 0.0086 | 0.0128 | 0.0082 | 0.0081 | 0.0083 | 0.0075 | 0.0076 |
| P2-4 | 0.0922 | 0.0900 | 0.1111 | 0.0919 | 0.0908 | 0.0902 | 0.0902 | 0.0907 |
| P2-5 | 0.1162 | 0.1159 | 0.1068 | 0.1159 | 0.1157 | 0.1156 | 0.1161 | 0.1159 |
| P3-4 | 0.0836 | 0.0823 | 0.0983 | 0.0837 | 0.0827 | 0.0818 | 0.0826 | 0.0830 |
| P4-5 | 0.0240 | 0.0240 | -0.0043 | 0.0240 | 0.0248 | 0.0254 | 0.0260 | 0.0252 |
| P4-7 | 0.0439 | 0.0441 | 0.0352 | 0.0433 | 0.0447 | 0.0436 | 0.0442 | 0.0441 |
| P4-9 | 0.1678 | 0.1682 | 0.1461 | 0.1672 | 0.1684 | 0.1677 | 0.1688 | 0.1679 |
| P5-6 | 0.2483 | 0.2520 | 0.2172 | 0.2477 | 0.2484 | 0.2474 | 0.2494 | 0.2477 |
| P6-11 | 0.0368 | 0.0372 | 0.0052 | 0.0370 | 0.0363 | 0.0372 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.0701 | 0.0688 | 0.0692 | 0.0683 | 0.0691 | 0.0687 |
| P6-13 | 0.0676 | 0.0667 | 0.0763 | 0.0675 | 0.0676 | 0.0669 | 0.0677 | 0.0674 |
| P7-8 | -0.0950 | -0.0954 | -0.0940 | -0.0954 | -0.0946 | -0.0957 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1109 | 0.1239 | 0.1237 | 0.1241 | 0.1245 | 0.1238 |
| **P9-10** | **0.0982** | **-0.0987** | **0.0227** | **0.0986** | **0.0980** | **0.0994** | **0.0998** | **0.0990** |
| P9-14 | 0.1486 | 0.1506 | 0.1480 | 0.1473 | 0.1480 | 0.1471 | 0.1491 | 0.1475 |
| P10-11 | 0.0432 | 0.0435 | 0.0493 | 0.0428 | 0.0431 | 0.0430 | 0.0437 | 0.0432 |
| P12-13 | -0.0012 | -0.0012 | 0.0062 | -0.0013 | -0.0017 | -0.0014 | -0.0015 | -0.0013 |
| **P13-14** | **-0.0236** | **0.0232** | **0.0049** | **-0.0246** | **-0.0244** | **-0.0249** | **-0.0253** | **-0.0248** |

Table 5.142: Estimation errors of case 2, multi reverse, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0002 | 0.0016 | 0.0009 | 0.0017 | 0.0055 | 0.0025 |
| P2 | 0.0017 | 0.0091 | 0.0003 | 0.0021 | 0.0022 | 0.0003 | 0.0017 |
| P3 | 0.0005 | 0.0105 | 0.0005 | 0.0004 | 0.0015 | 0.0001 | 0.0004 |
| **P4** | **0.1199** | **0.0923** | **0.0012** | **0.0044** | **0.0047** | **0.0062** | **0.0035** |
| P5 | 0.0001 | 0.0114 | 0.0008 | 0.0006 | 0.0006 | 0.0020 | 0.0001 |
| P6 | 0.0006 | 0.0093 | 0.0006 | 0.0003 | 0.0001 | 0.0008 | 0.0006 |
| P7 | 0.0002 | 0.0034 | 0.0002 | 0.0006 | 0.0003 | 0.0000 | 0.0007 |
| P8 | 0.0004 | 0.0010 | 0.0004 | 0.0004 | 0.0007 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.0412 | 0.0002 | 0.0011 | 0.0003 | 0.0006 | 0.0002 |
| P10 | 0.0008 | 0.0816 | 0.0008 | 0.0000 | 0.0013 | 0.0010 | 0.0008 |
| P11 | 0.0002 | 0.0254 | 0.0002 | 0.0006 | 0.0003 | 0.0006 | 0.0003 |
| P12 | 0.0000 | 0.0061 | 0.0000 | 0.0009 | 0.0002 | 0.0006 | 0.0000 |
| P13 | 0.0009 | 0.0124 | 0.0009 | 0.0003 | 0.0004 | 0.0015 | 0.0009 |
| P14 | 0.0023 | 0.0279 | 0.0023 | 0.0014 | 0.0028 | 0.0012 | 0.0023 |
| P1-2 | 0.0011 | 0.0046 | 0.0007 | 0.0002 | 0.0006 | 0.0027 | 0.0011 |
| P1-5 | 0.0002 | 0.0048 | 0.0010 | 0.0007 | 0.0012 | 0.0028 | 0.0014 |
| P2-3 | 0.0001 | 0.0042 | 0.0004 | 0.0005 | 0.0002 | 0.0011 | 0.0010 |
| P2-4 | 0.0022 | 0.0189 | 0.0002 | 0.0013 | 0.0020 | 0.0020 | 0.0015 |
| P2-5 | 0.0003 | 0.0094 | 0.0003 | 0.0005 | 0.0006 | 0.0000 | 0.0003 |
| P3-4 | 0.0012 | 0.0147 | 0.0001 | 0.0008 | 0.0017 | 0.0009 | 0.0005 |
| P4-5 | 0.0001 | 0.0283 | 0.0001 | 0.0008 | 0.0014 | 0.0020 | 0.0012 |
| P4-7 | 0.0002 | 0.0087 | 0.0006 | 0.0008 | 0.0003 | 0.0003 | 0.0002 |
| P4-9 | 0.0004 | 0.0217 | 0.0006 | 0.0006 | 0.0001 | 0.0010 | 0.0001 |
| P5-6 | 0.0037 | 0.0310 | 0.0006 | 0.0001 | 0.0009 | 0.0012 | 0.0006 |
| P6-11 | 0.0003 | 0.0316 | 0.0001 | 0.0005 | 0.0004 | 0.0000 | 0.0003 |
| P6-12 | 0.0000 | 0.0012 | 0.0000 | 0.0004 | 0.0005 | 0.0003 | 0.0001 |
| P6-13 | 0.0009 | 0.0086 | 0.0001 | 0.0001 | 0.0007 | 0.0001 | 0.0002 |
| P7-8 | 0.0004 | 0.0010 | 0.0004 | 0.0004 | 0.0007 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0131 | 0.0000 | 0.0002 | 0.0002 | 0.0006 | 0.0001 |
| **P9-10** | **0.1969** | **0.0754** | **0.0004** | **0.0001** | **0.0012** | **0.0016** | **0.0008** |
| P9-14 | 0.0020 | 0.0005 | 0.0012 | 0.0006 | 0.0014 | 0.0005 | 0.0010 |
| P10-11 | 0.0003 | 0.0062 | 0.0004 | 0.0001 | 0.0001 | 0.0006 | 0.0000 |
| P12-13 | 0.0000 | 0.0074 | 0.0001 | 0.0005 | 0.0003 | 0.0003 | 0.0001 |
| **P13-14** | **0.0468** | **0.0285** | **0.0010** | **0.0008** | **0.0014** | **0.0017** | **0.0012** |
| **SUM** | **0.3961** | **0.6519** | **0.0185** | **0.0243** | **0.0327** | **0.0407** | **0.0267** |

For WLS, though it generate better estimation on bad data, they influence other locations, making the overall summation error larger than measurements. Other estimators performs good with much better performance both on bad data and overall estimation.

### Median Redundancy

The state variables, estimation results and error with actual value are presented in the following three tables.

Table 5.143: State Variable of case 2, multi reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0742 | -0.0658 | -0.0663 | -0.0667 | -0.0711 | -0.0658 |
| $\theta 3$ | -0.0905 | -0.0744 | -0.0743 | -0.0748 | -0.0846 | -0.0730 |
| $\theta 4$ | -0.1874 | -0.1585 | -0.1572 | -0.1579 | -0.1795 | -0.1554 |
| $\theta 5$ | -0.1776 | -0.1821 | -0.1827 | -0.1829 | -0.1771 | -0.1832 |
| $\theta 6$ | -0.3912 | -0.4291 | -0.4316 | -0.4314 | -0.3914 | -0.4296 |
| $\theta 7$ | -0.2186 | -0.2013 | -0.2018 | -0.2021 | -0.2040 | -0.1991 |
| $\theta 8$ | -0.1258 | -0.1059 | -0.1069 | -0.1072 | -0.1076 | -0.1037 |
| $\theta 9$ | -0.3227 | -0.3248 | -0.3259 | -0.3259 | -0.3118 | -0.3234 |
| $\theta 10$ | -0.3493 | -0.4219 | -0.4254 | -0.4247 | -0.3645 | -0.4226 |
| $\theta 11$ | -0.3910 | -0.4654 | -0.4687 | -0.4682 | -0.4135 | -0.4660 |
| $\theta 12$ | -0.4611 | -0.4979 | -0.5007 | -0.5006 | -0.4546 | -0.4981 |
| $\theta 13$ | -0.4681 | -0.4967 | -0.4993 | -0.4992 | -0.4579 | -0.4967 |
| $\theta 14$ | -0.4671 | -0.4721 | -0.4742 | -0.4741 | -0.4510 | -0.4714 |

Table 5.144: Estimation Results of case 2, multi reverse, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2518 | 0.2479 | 0.2490 | 0.2496 | 0.2482 | 0.2490 |
| P2 | 0.1500 | 0.1483 | 0.1588 | 0.1518 | 0.1488 | 0.1488 | 0.1570 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0806 | 0.0755 | 0.0750 | 0.0750 | 0.0814 | 0.0752 |
| **P4** | **0.0600** | **-0.0599** | **-0.0535** | **0.0559** | **0.0650** | **0.0629** | **-0.0491** | **0.0677** |
| P5 | -0.0750 | -0.0751 | -0.0577 | -0.0751 | -0.0756 | -0.0756 | -0.0665 | -0.0820 |
| P6 | -0.0750 | -0.0744 | -0.0670 | -0.0744 | -0.0749 | -0.0749 | -0.0626 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0199 | -0.0148 | -0.0153 | -0.0153 | -0.0129 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.0928 | 0.0954 | 0.0949 | 0.0949 | 0.0963 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0685 | -0.0452 | -0.0450 | -0.0447 | -0.0483 | -0.0452 |
| P10 | -0.0550 | -0.0558 | 0.0152 | -0.0537 | -0.0563 | -0.0553 | -0.0036 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0415 | -0.0798 | -0.0803 | -0.0803 | -0.0711 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.0628 | -0.0700 | -0.0705 | -0.0705 | -0.0599 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0849 | -0.0909 | -0.0914 | -0.0914 | -0.0767 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1434 | -0.1227 | -0.1232 | -0.1232 | -0.1322 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0742 | 0.0658 | 0.0663 | 0.0667 | 0.0711 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1776 | 0.1821 | 0.1827 | 0.1829 | 0.1771 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | 0.0163 | 0.0086 | 0.0079 | 0.0081 | 0.0135 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.1133 | 0.0927 | 0.0909 | 0.0912 | 0.1085 | 0.0896 |
| P2-5 | 0.1162 | N/A | 0.1034 | 0.1163 | 0.1163 | 0.1162 | 0.1061 | 0.1174 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P3-4 | 0.0836 | 0.0823 | 0.0969 | 0.0841 | 0.0829 | 0.0831 | 0.0949 | 0.0824 |
| P4-5 | 0.0240 | N/A | -0.0098 | 0.0236 | 0.0255 | 0.0250 | -0.0024 | 0.0278 |
| P4-7 | 0.0439 | 0.0441 | 0.0312 | 0.0428 | 0.0446 | 0.0442 | 0.0244 | 0.0437 |
| P4-9 | 0.1678 | N/A | 0.1353 | 0.1663 | 0.1687 | 0.1680 | 0.1323 | 0.1681 |
| P5-6 | 0.2483 | 0.2520 | 0.2136 | 0.2470 | 0.2489 | 0.2485 | 0.2143 | 0.2464 |
| P6-11 | 0.0368 | N/A | -0.0002 | 0.0363 | 0.0371 | 0.0368 | 0.0221 | 0.0364 |
| P6-12 | 0.0688 | 0.0688 | 0.0699 | 0.0688 | 0.0691 | 0.0691 | 0.0632 | 0.0685 |
| P6-13 | 0.0676 | N/A | 0.0769 | 0.0675 | 0.0677 | 0.0677 | 0.0665 | 0.0671 |
| P7-8 | -0.0950 | -0.0954 | -0.0928 | -0.0954 | -0.0949 | -0.0949 | -0.0963 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1041 | 0.1234 | 0.1241 | 0.1238 | 0.1079 | 0.1243 |
| **P9-10** | **0.0982** | **-0.0987** | **0.0265** | **0.0972** | **0.0995** | **0.0988** | **0.0526** | **0.0992** |
| P9-14 | 0.1486 | N/A | 0.1444 | 0.1473 | 0.1483 | 0.1482 | 0.1392 | 0.1480 |
| P10-11 | 0.0432 | 0.0435 | 0.0417 | 0.0435 | 0.0432 | 0.0435 | 0.0490 | 0.0434 |
| P12-13 | -0.0012 | N/A | 0.0070 | -0.0012 | -0.0014 | -0.0014 | 0.0033 | -0.0015 |
| **P13-14** | **-0.0236** | **0.0232** | **-0.0010** | **-0.0246** | **-0.0251** | **-0.0250** | **-0.0069** | **-0.0253** |

Table 5.145: Estimation errors of case 2, multi reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0018 | 0.0021 | 0.0010 | 0.0004 | 0.0018 | 0.0010 |
| P2 | 0.0088 | 0.0018 | 0.0012 | 0.0012 | 0.0070 | 0.0017 |
| P3 | 0.0056 | 0.0005 | 0.0000 | 0.0000 | 0.0064 | 0.0003 |
| **P4** | **0.1135** | **0.0041** | **0.0050** | **0.0029** | **0.1091** | **0.0077** |
| P5 | 0.0173 | 0.0001 | 0.0006 | 0.0006 | 0.0085 | 0.0070 |
| P6 | 0.0080 | 0.0006 | 0.0001 | 0.0001 | 0.0124 | 0.0006 |
| P7 | 0.0049 | 0.0002 | 0.0003 | 0.0003 | 0.0021 | 0.0002 |
| P8 | 0.0022 | 0.0004 | 0.0001 | 0.0001 | 0.0013 | 0.0004 |
| P9 | 0.0235 | 0.0002 | 0.0000 | 0.0003 | 0.0033 | 0.0002 |
| P10 | 0.0702 | 0.0013 | 0.0013 | 0.0003 | 0.0514 | 0.0008 |
| P11 | 0.0385 | 0.0002 | 0.0003 | 0.0003 | 0.0089 | 0.0002 |
| P12 | 0.0072 | 0.0000 | 0.0005 | 0.0005 | 0.0101 | 0.0000 |
| P13 | 0.0051 | 0.0009 | 0.0014 | 0.0014 | 0.0133 | 0.0009 |
| P14 | 0.0184 | 0.0023 | 0.0018 | 0.0018 | 0.0072 | 0.0023 |
| P1-2 | 0.0073 | 0.0011 | 0.0006 | 0.0002 | 0.0042 | 0.0011 |
| P1-5 | 0.0055 | 0.0010 | 0.0004 | 0.0002 | 0.0059 | 0.0001 |
| P2-3 | 0.0078 | 0.0000 | 0.0006 | 0.0005 | 0.0049 | 0.0014 |
| P2-4 | 0.0211 | 0.0005 | 0.0013 | 0.0010 | 0.0163 | 0.0026 |
| P2-5 | 0.0127 | 0.0002 | 0.0002 | 0.0000 | 0.0101 | 0.0012 |
| P3-4 | 0.0133 | 0.0005 | 0.0007 | 0.0005 | 0.0114 | 0.0012 |
| P4-5 | 0.0338 | 0.0004 | 0.0015 | 0.0010 | 0.0264 | 0.0038 |
| P4-7 | 0.0127 | 0.0011 | 0.0007 | 0.0003 | 0.0195 | 0.0002 |
| P4-9 | 0.0325 | 0.0016 | 0.0009 | 0.0002 | 0.0355 | 0.0002 |
| P5-6 | 0.0347 | 0.0013 | 0.0006 | 0.0003 | 0.0340 | 0.0019 |
| P6-11 | 0.0371 | 0.0005 | 0.0003 | 0.0001 | 0.0148 | 0.0004 |
| P6-12 | 0.0011 | 0.0000 | 0.0003 | 0.0003 | 0.0056 | 0.0003 |
| P6-13 | 0.0093 | 0.0001 | 0.0001 | 0.0001 | 0.0011 | 0.0006 |
| P7-8 | 0.0022 | 0.0004 | 0.0001 | 0.0001 | 0.0013 | 0.0004 |
| P7-9 | 0.0198 | 0.0005 | 0.0002 | 0.0001 | 0.0160 | 0.0004 |
| **P9-10** | **0.0716** | **0.0010** | **0.0014** | **0.0007** | **0.0455** | **0.0010** |
| P9-14 | 0.0042 | 0.0013 | 0.0002 | 0.0003 | 0.0094 | 0.0006 |
| P10-11 | 0.0014 | 0.0003 | 0.0001 | 0.0004 | 0.0059 | 0.0002 |
| P12-13 | 0.0082 | 0.0000 | 0.0002 | 0.0002 | 0.0045 | 0.0003 |
| **P13-14** | **0.0226** | **0.0010** | **0.0015** | **0.0015** | **0.0166** | **0.0017** |
| **SUM** | **0.6839** | **0.0276** | **0.0256** | **0.0179** | **0.5319** | **0.0430** |

Compared with full redundancy case, LTS performs bad in this case. The reason with fewer reference, the algorithm cannot trim the bad data correctly and

induce bad results.

### Comparison

The comparison between full and median redundancy are presented in Table 5.146, only summation of errors are shown here. Except LMR, other estimators generate better results in full redundancy case.

Table 5.146: Estimation errors of case 2, multi reverse, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.3961** | **0.6519** | **0.0185** | **0.0243** | **0.0327** | **0.0407** | **0.0267** |
| **SUM** | **N/A** | **0.6839** | **0.0276** | **0.0256** | **0.0179** | **0.5319** | **0.0430** |

## Two reverse one large bad data

In the non-conforming case, two different types are divided: two reverse one large and one reverse two large. Here, $P_4 = -0.0599$, $P_{9-10} = -0.0987$ and $P_{13-14} = -0.9232$ are used to replace the original data. As presented, the last one is large error.

### Full redundancy

The five system state variables are presented in Table 5.147, Table 5.148 shows the results and Table 5.149 illustrates the error between estimation and actual value.

Table 5.147: State Variable of case 2, two reverse, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0541 | -0.0658 | -0.0667 | -0.0672 | -0.0642 | -0.0658 |
| $\theta 3$ | -0.0495 | -0.0735 | -0.0748 | -0.0754 | -0.0717 | -0.0734 |
| $\theta 4$ | -0.1341 | -0.1568 | -0.1575 | -0.1583 | -0.1543 | -0.1565 |
| $\theta 5$ | -0.1925 | -0.1817 | -0.1824 | -0.1827 | -0.1803 | -0.1817 |
| $\theta 6$ | -0.5069 | -0.4291 | -0.4304 | -0.4309 | -0.4298 | -0.4294 |
| $\theta 7$ | -0.1284 | -0.2002 | -0.2018 | -0.2019 | -0.1985 | -0.2006 |
| $\theta 8$ | -0.0264 | -0.1048 | -0.1073 | -0.1070 | -0.1032 | -0.1052 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta 9$ | -0.2269 | -0.3242 | -0.3250 | -0.3260 | -0.3231 | -0.3244 |
| $\theta 10$ | -0.3156 | -0.4230 | -0.4233 | -0.4249 | -0.4229 | -0.4234 |
| $\theta 11$ | -0.4467 | -0.4659 | -0.4667 | -0.4680 | -0.4666 | -0.4665 |
| $\theta 12$ | -0.6399 | -0.4978 | -0.4995 | -0.4997 | -0.4989 | -0.4981 |
| $\theta 13$ | -0.6084 | -0.4965 | -0.4976 | -0.4981 | -0.4974 | -0.4967 |
| $\theta 14$ | -0.3163 | -0.4717 | -0.4731 | -0.4731 | -0.4722 | -0.4719 |

Table 5.148: Estimation Results of case 2, two reverse, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2467 | 0.2475 | 0.2491 | 0.2499 | 0.2445 | 0.2475 |
| P2 | 0.1500 | 0.1483 | 0.1596 | 0.1487 | 0.1479 | 0.1478 | 0.1497 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0893 | 0.0755 | 0.0746 | 0.0746 | 0.0751 | 0.0754 |
| **P4** | **0.0600** | **-0.0599** | **-0.0192** | **0.0615** | **0.0630** | **0.0618** | **0.0662** | **0.0635** |
| P5 | -0.0750 | -0.0751 | -0.0750 | -0.0751 | -0.0749 | -0.0747 | -0.0730 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.1399 | -0.0744 | -0.0753 | -0.0749 | -0.0758 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.0022 | -0.0147 | -0.0156 | -0.0145 | -0.0150 | -0.0157 |
| P8 | 0.0950 | 0.0954 | 0.1020 | 0.0954 | 0.0946 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.0133 | -0.0452 | -0.0443 | -0.0457 | -0.0444 | -0.0452 |
| P10 | -0.0550 | -0.0558 | 0.0424 | -0.0558 | -0.0550 | -0.0558 | -0.0560 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0710 | -0.0798 | -0.0796 | -0.0803 | -0.0806 | -0.0803 |
| P12 | -0.0700 | -0.0700 | -0.1646 | -0.0700 | -0.0709 | -0.0705 | -0.0706 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.3621 | -0.0909 | -0.0900 | -0.0905 | -0.0915 | -0.0909 |
| P14 | -0.1250 | -0.1227 | 0.2028 | -0.1227 | -0.1236 | -0.1222 | -0.1238 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0541 | 0.0658 | 0.0667 | 0.0672 | 0.0642 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1925 | 0.1817 | 0.1824 | 0.1827 | 0.1803 | 0.1817 |
| P2-3 | 0.0086 | 0.0086 | -0.0047 | 0.0077 | 0.0081 | 0.0083 | 0.0075 | 0.0076 |
| P2-4 | 0.0922 | 0.0900 | 0.0800 | 0.0910 | 0.0908 | 0.0911 | 0.0902 | 0.0907 |
| P2-5 | 0.1162 | 0.1159 | 0.1384 | 0.1159 | 0.1157 | 0.1156 | 0.1161 | 0.1159 |
| P3-4 | 0.0836 | 0.0823 | 0.0847 | 0.0832 | 0.0827 | 0.0829 | 0.0826 | 0.0830 |
| P4-5 | 0.0240 | 0.0240 | 0.0584 | 0.0249 | 0.0248 | 0.0244 | 0.0260 | 0.0252 |
| P4-7 | 0.0439 | 0.0441 | -0.0057 | 0.0434 | 0.0443 | 0.0436 | 0.0442 | 0.0441 |
| P4-9 | 0.1678 | 0.1682 | 0.0928 | 0.1674 | 0.1675 | 0.1677 | 0.1688 | 0.1679 |
| P5-6 | 0.2483 | 0.2520 | 0.3143 | 0.2474 | 0.2480 | 0.2481 | 0.2494 | 0.2477 |
| P6-11 | 0.0368 | 0.0372 | -0.0601 | 0.0368 | 0.0363 | 0.0371 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.1330 | 0.0687 | 0.0691 | 0.0689 | 0.0691 | 0.0687 |
| P6-13 | 0.0676 | 0.0667 | 0.1015 | 0.0674 | 0.0673 | 0.0672 | 0.0677 | 0.0674 |
| P7-8 | -0.0950 | -0.0954 | -0.1020 | -0.0954 | -0.0946 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.0985 | 0.1240 | 0.1232 | 0.1241 | 0.1245 | 0.1238 |
| **P9-10** | **0.0982** | **-0.0987** | **0.0887** | **0.0988** | **0.0983** | **0.0989** | **0.0998** | **0.0990** |
| P9-14 | 0.1486 | 0.1506 | 0.0893 | 0.1475 | 0.1481 | 0.1472 | 0.1491 | 0.1475 |
| P10-11 | 0.0432 | 0.0435 | 0.1311 | 0.0429 | 0.0433 | 0.0431 | 0.0437 | 0.0432 |
| P12-13 | -0.0012 | -0.0012 | -0.0315 | -0.0013 | -0.0018 | -0.0017 | -0.0015 | -0.0013 |
| **P13-14** | **-0.0236** | **-0.9232** | **-0.2921** | **-0.0248** | **-0.0245** | **-0.0249** | **-0.0253** | **-0.0248** |

Table 5.149: Estimation errors of case 2, two reverse, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0033 | 0.0025 | 0.0009 | 0.0001 | 0.0055 | 0.0025 |
| P2 | 0.0017 | 0.0096 | 0.0013 | 0.0021 | 0.0022 | 0.0003 | 0.0017 |
| P3 | 0.0005 | 0.0143 | 0.0005 | 0.0004 | 0.0004 | 0.0001 | 0.0004 |
| **P4** | **0.1199** | **0.0792** | **0.0015** | **0.0030** | **0.0018** | **0.0062** | **0.0035** |
| P5 | 0.0001 | 0.0000 | 0.0001 | 0.0001 | 0.0003 | 0.0020 | 0.0001 |
| P6 | 0.0006 | 0.0649 | 0.0006 | 0.0003 | 0.0001 | 0.0008 | 0.0006 |
| P7 | 0.0002 | 0.0172 | 0.0002 | 0.0006 | 0.0005 | 0.0000 | 0.0007 |
| P8 | 0.0004 | 0.0070 | 0.0004 | 0.0004 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.0317 | 0.0002 | 0.0007 | 0.0007 | 0.0006 | 0.0002 |
| P10 | 0.0008 | 0.0974 | 0.0008 | 0.0000 | 0.0008 | 0.0010 | 0.0008 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P11 | 0.0002 | 0.0090 | 0.0002 | 0.0004 | 0.0003 | 0.0006 | 0.0003 |
| P12 | 0.0000 | 0.0946 | 0.0000 | 0.0009 | 0.0005 | 0.0006 | 0.0000 |
| P13 | 0.0009 | 0.2721 | 0.0009 | 0.0000 | 0.0005 | 0.0015 | 0.0009 |
| P14 | 0.0023 | 0.3278 | 0.0023 | 0.0014 | 0.0028 | 0.0012 | 0.0023 |
| P1-2 | 0.0011 | 0.0128 | 0.0011 | 0.0002 | 0.0003 | 0.0027 | 0.0011 |
| P1-5 | 0.0002 | 0.0095 | 0.0014 | 0.0007 | 0.0003 | 0.0028 | 0.0014 |
| P2-3 | 0.0001 | 0.0132 | 0.0009 | 0.0005 | 0.0003 | 0.0011 | 0.0010 |
| P2-4 | 0.0022 | 0.0122 | 0.0012 | 0.0013 | 0.0010 | 0.0020 | 0.0015 |
| P2-5 | 0.0003 | 0.0222 | 0.0003 | 0.0005 | 0.0006 | 0.0000 | 0.0003 |
| P3-4 | 0.0012 | 0.0011 | 0.0003 | 0.0008 | 0.0007 | 0.0009 | 0.0005 |
| P4-5 | 0.0001 | 0.0344 | 0.0009 | 0.0008 | 0.0004 | 0.0020 | 0.0012 |
| P4-7 | 0.0002 | 0.0496 | 0.0005 | 0.0004 | 0.0003 | 0.0003 | 0.0002 |
| P4-9 | 0.0004 | 0.0750 | 0.0004 | 0.0003 | 0.0001 | 0.0010 | 0.0001 |
| P5-6 | 0.0037 | 0.0660 | 0.0009 | 0.0003 | 0.0002 | 0.0012 | 0.0006 |
| P6-11 | 0.0003 | 0.0970 | 0.0000 | 0.0005 | 0.0003 | 0.0000 | 0.0003 |
| P6-12 | 0.0000 | 0.0642 | 0.0001 | 0.0003 | 0.0001 | 0.0003 | 0.0001 |
| P6-13 | 0.0009 | 0.0339 | 0.0002 | 0.0003 | 0.0004 | 0.0001 | 0.0002 |
| P7-8 | 0.0004 | 0.0070 | 0.0004 | 0.0004 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0254 | 0.0001 | 0.0007 | 0.0002 | 0.0006 | 0.0001 |
| **P9-10** | **0.1969** | **0.0095** | **0.0006** | **0.0001** | **0.0007** | **0.0016** | **0.0008** |
| P9-14 | 0.0020 | 0.0592 | 0.0011 | 0.0005 | 0.0014 | 0.0005 | 0.0010 |
| P10-11 | 0.0003 | 0.0880 | 0.0002 | 0.0002 | 0.0000 | 0.0006 | 0.0000 |
| P12-13 | 0.0000 | 0.0303 | 0.0001 | 0.0006 | 0.0005 | 0.0003 | 0.0001 |
| **P13-14** | **0.8996** | **0.2685** | **0.0012** | **0.0009** | **0.0014** | **0.0017** | **0.0012** |
| **SUM** | **1.2490** | **2.0072** | **0.0235** | **0.0217** | **0.0202** | **0.0407** | **0.0267** |

### Median redundancy

The same bad data as in full redundancy case, the following three tables shows the results and error with actual value of the system.

Table 5.150: State Variable of case 2, two reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0512 | -0.0658 | -0.0663 | -0.0667 | -0.0708 | -0.0658 |
| $\theta 3$ | -0.0458 | -0.0744 | -0.0743 | -0.0748 | -0.0832 | -0.0730 |
| $\theta 4$ | -0.1245 | -0.1585 | -0.1572 | -0.1579 | -0.1806 | -0.1554 |
| $\theta 5$ | -0.1852 | -0.1821 | -0.1827 | -0.1828 | -0.1814 | -0.1832 |
| $\theta 6$ | -0.4694 | -0.4291 | -0.4316 | -0.4312 | -0.4147 | -0.4296 |
| $\theta 7$ | -0.1054 | -0.2013 | -0.2018 | -0.2015 | -0.2053 | -0.1991 |
| $\theta 8$ | 0.0009 | -0.1059 | -0.1069 | -0.1066 | -0.1072 | -0.1037 |
| $\theta 9$ | -0.1999 | -0.3248 | -0.3259 | -0.3256 | -0.3187 | -0.3234 |
| $\theta 10$ | -0.2715 | -0.4219 | -0.4254 | -0.4250 | -0.3866 | -0.4226 |
| $\theta 11$ | -0.3867 | -0.4654 | -0.4687 | -0.4683 | -0.4382 | -0.4660 |
| $\theta 12$ | -0.6001 | -0.4979 | -0.5007 | -0.5002 | -0.4831 | -0.4981 |
| $\theta 13$ | -0.5546 | -0.4967 | -0.4993 | -0.4987 | -0.4826 | -0.4967 |
| $\theta 14$ | -0.3060 | -0.4721 | -0.4742 | -0.4732 | -0.4607 | -0.4714 |

Table 5.151: Estimation Results of case 2, two reverse, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2364 | 0.2479 | 0.2490 | 0.2495 | 0.2522 | 0.2490 |
| P2 | 0.1500 | 0.1483 | 0.1507 | 0.1518 | 0.1488 | 0.1488 | 0.1621 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0840 | 0.0755 | 0.0750 | 0.0750 | 0.0850 | 0.0752 |

| **P4** | **0.0600** | **-0.0599** | **-0.0349** | **0.0559** | **0.0650** | **0.0620** | **-0.0435** | **0.0677** |
|---|---|---|---|---|---|---|---|---|
| P5 | -0.0750 | -0.0751 | -0.0957 | -0.0751 | -0.0756 | -0.0756 | -0.0598 | -0.0820 |
| P6 | -0.0750 | -0.0744 | -0.1510 | -0.0744 | -0.0749 | -0.0749 | -0.0733 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.0072 | -0.0148 | -0.0153 | -0.0144 | -0.0094 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.1064 | 0.0954 | 0.0949 | 0.0949 | 0.0981 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.0078 | -0.0452 | -0.0450 | -0.0447 | -0.0417 | -0.0452 |
| P10 | -0.0550 | -0.0558 | 0.0436 | -0.0537 | -0.0563 | -0.0562 | -0.0162 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0325 | -0.0798 | -0.0803 | -0.0803 | -0.0752 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.1761 | -0.0700 | -0.0705 | -0.0705 | -0.0689 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.2883 | -0.0909 | -0.0914 | -0.0914 | -0.0893 | -0.0909 |
| P14 | -0.1250 | -0.1227 | 0.1424 | -0.1227 | -0.1232 | -0.1222 | -0.1201 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0512 | 0.0658 | 0.0663 | 0.0667 | 0.0708 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1852 | 0.1821 | 0.1827 | 0.1828 | 0.1814 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0054 | 0.0086 | 0.0079 | 0.0081 | 0.0124 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.0733 | 0.0927 | 0.0909 | 0.0912 | 0.1098 | 0.0896 |
| P2-5 | 0.1162 | N/A | 0.1340 | 0.1163 | 0.1163 | 0.1162 | 0.1107 | 0.1174 |
| P3-4 | 0.0836 | 0.0823 | 0.0787 | 0.0841 | 0.0829 | 0.0831 | 0.0974 | 0.0824 |
| P4-5 | 0.0240 | N/A | 0.0607 | 0.0236 | 0.0255 | 0.0250 | 0.0009 | 0.0278 |
| P4-7 | 0.0439 | 0.0441 | -0.0191 | 0.0428 | 0.0446 | 0.0436 | 0.0247 | 0.0437 |
| P4-9 | 0.1678 | N/A | 0.0754 | 0.1663 | 0.1687 | 0.1677 | 0.1381 | 0.1681 |
| P5-6 | 0.2483 | 0.2520 | 0.2842 | 0.2470 | 0.2489 | 0.2484 | 0.2332 | 0.2464 |
| P6-11 | 0.0368 | N/A | -0.0827 | 0.0363 | 0.0371 | 0.0370 | 0.0235 | 0.0364 |
| P6-12 | 0.0688 | 0.0688 | 0.1307 | 0.0688 | 0.0691 | 0.0690 | 0.0684 | 0.0685 |
| P6-13 | 0.0676 | N/A | 0.0852 | 0.0675 | 0.0677 | 0.0675 | 0.0679 | 0.0671 |
| P7-8 | -0.0950 | -0.0954 | -0.1064 | -0.0954 | -0.0949 | -0.0949 | -0.0981 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.0945 | 0.1234 | 0.1241 | 0.1241 | 0.1134 | 0.1243 |
| **P9-10** | **0.0982** | **-0.0987** | **0.0716** | **0.0972** | **0.0995** | **0.0994** | **0.0679** | **0.0992** |
| P9-14 | 0.1486 | N/A | 0.1061 | 0.1473 | 0.1483 | 0.1477 | 0.1420 | 0.1480 |
| P10-11 | 0.0432 | 0.0435 | 0.1152 | 0.0435 | 0.0432 | 0.0433 | 0.0516 | 0.0434 |
| P12-13 | -0.0012 | N/A | -0.0455 | -0.0012 | -0.0014 | -0.0015 | -0.0005 | -0.0015 |
| **P13-14** | **-0.0236** | **-0.9232** | **-0.2486** | **-0.0246** | **-0.0251** | **-0.0255** | **-0.0219** | **-0.0253** |

Table 5.152: Estimation errors of case 2, two reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0136 | 0.0021 | 0.0010 | 0.0005 | 0.0022 | 0.0010 |
| P2 | 0.0007 | 0.0018 | 0.0012 | 0.0012 | 0.0121 | 0.0017 |
| P3 | 0.0090 | 0.0005 | 0.0000 | 0.0000 | 0.0100 | 0.0003 |
| **P4** | **0.0949** | **0.0041** | **0.0050** | **0.0020** | **0.1035** | **0.0077** |
| P5 | 0.0207 | 0.0001 | 0.0006 | 0.0006 | 0.0152 | 0.0070 |
| P6 | 0.0760 | 0.0006 | 0.0001 | 0.0001 | 0.0017 | 0.0006 |
| P7 | 0.0222 | 0.0002 | 0.0003 | 0.0006 | 0.0056 | 0.0002 |
| P8 | 0.0114 | 0.0004 | 0.0001 | 0.0001 | 0.0031 | 0.0004 |
| P9 | 0.0528 | 0.0002 | 0.0000 | 0.0003 | 0.0033 | 0.0002 |
| P10 | 0.0986 | 0.0013 | 0.0013 | 0.0012 | 0.0388 | 0.0008 |
| P11 | 0.0475 | 0.0002 | 0.0003 | 0.0003 | 0.0048 | 0.0002 |
| P12 | 0.1061 | 0.0000 | 0.0005 | 0.0005 | 0.0011 | 0.0000 |
| P13 | 0.1983 | 0.0009 | 0.0014 | 0.0014 | 0.0007 | 0.0009 |
| P14 | 0.2674 | 0.0023 | 0.0018 | 0.0028 | 0.0049 | 0.0023 |
| P1-2 | 0.0157 | 0.0011 | 0.0006 | 0.0002 | 0.0038 | 0.0011 |
| P1-5 | 0.0021 | 0.0010 | 0.0004 | 0.0002 | 0.0016 | 0.0001 |
| P2-3 | 0.0140 | 0.0000 | 0.0006 | 0.0005 | 0.0038 | 0.0014 |
| P2-4 | 0.0189 | 0.0005 | 0.0013 | 0.0010 | 0.0177 | 0.0026 |
| P2-5 | 0.0178 | 0.0002 | 0.0002 | 0.0000 | 0.0055 | 0.0012 |
| P3-4 | 0.0049 | 0.0005 | 0.0007 | 0.0005 | 0.0138 | 0.0012 |
| P4-5 | 0.0367 | 0.0004 | 0.0015 | 0.0010 | 0.0232 | 0.0038 |
| P4-7 | 0.0630 | 0.0011 | 0.0007 | 0.0003 | 0.0192 | 0.0002 |
| P4-9 | 0.0924 | 0.0016 | 0.0009 | 0.0001 | 0.0297 | 0.0002 |
| P5-6 | 0.0359 | 0.0013 | 0.0006 | 0.0001 | 0.0151 | 0.0019 |
| P6-11 | 0.1195 | 0.0005 | 0.0003 | 0.0002 | 0.0133 | 0.0004 |
| P6-12 | 0.0619 | 0.0000 | 0.0003 | 0.0002 | 0.0004 | 0.0003 |

| | | | | | | |
|---|---|---|---|---|---|---|
| P6-13 | 0.0176 | 0.0001 | 0.0001 | 0.0002 | 0.0003 | 0.0006 |
| P7-8 | 0.0114 | 0.0004 | 0.0001 | 0.0001 | 0.0031 | 0.0004 |
| P7-9 | 0.0294 | 0.0005 | 0.0002 | 0.0002 | 0.0105 | 0.0004 |
| **P9-10** | **0.0266** | **0.0010** | **0.0014** | **0.0013** | **0.0303** | **0.0010** |
| P9-14 | 0.0424 | 0.0013 | 0.0002 | 0.0009 | 0.0065 | 0.0006 |
| P10-11 | 0.0720 | 0.0003 | 0.0001 | 0.0001 | 0.0085 | 0.0002 |
| P12-13 | 0.0443 | 0.0000 | 0.0002 | 0.0003 | 0.0007 | 0.0003 |
| **P13-14** | **0.2250** | **0.0010** | **0.0015** | **0.0019** | **0.0017** | **0.0017** |
| **SUM** | **1.9707** | **0.0276** | **0.0256** | **0.0208** | **0.4157** | **0.0430** |

LTS generate the same bad data as before.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.153.Full redundancy case has better estimation results for all estimators. LTS generate bad results because with fewer reference, it cannot detect the bad data correctly.

Table 5.153: Estimation errors of case 2, two reverse, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.2490** | **2.0072** | **0.0235** | **0.0217** | **0.0202** | **0.0407** | **0.0267** |
| **SUM** | **N/A** | **1.9707** | **0.0276** | **0.0256** | **0.0208** | **0.4157** | **0.0430** |

## Two large one reverse bad data

Compared with the former case, another reverse bad data are changed to large error, $P_4 = -0.0599$, $P_{9-10} = 0.9987$ and $P_{13-14} = -0.9232$, others keep the same.

### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.154 gives the state variables, and the later two tables shows the estimation results and error with actual values.

Table 5.154: State variable of case 2, two large, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0404 | -0.0658 | -0.0667 | -0.0663 | -0.0642 | -0.0658 |
| $\theta 3$ | -0.0178 | -0.0734 | -0.0748 | -0.0745 | -0.0717 | -0.0734 |
| $\theta 4$ | -0.1069 | -0.1565 | -0.1575 | -0.1576 | -0.1543 | -0.1565 |
| $\theta 5$ | -0.2151 | -0.1817 | -0.1824 | -0.1820 | -0.1803 | -0.1817 |
| $\theta 6$ | -0.6507 | -0.4293 | -0.4303 | -0.4296 | -0.4298 | -0.4294 |
| $\theta 7$ | -0.0655 | -0.2001 | -0.2022 | -0.2016 | -0.1985 | -0.2006 |
| $\theta 8$ | 0.0674 | -0.1047 | -0.1076 | -0.1067 | -0.1032 | -0.1052 |
| $\theta 9$ | -0.2379 | -0.3242 | -0.3257 | -0.3253 | -0.3231 | -0.3244 |
| $\theta 10$ | -0.7090 | -0.4233 | -0.4253 | -0.4238 | -0.4229 | -0.4234 |
| $\theta 11$ | -0.7801 | -0.4665 | -0.4681 | -0.4668 | -0.4666 | -0.4665 |
| $\theta 12$ | -0.7552 | -0.4980 | -0.4995 | -0.4979 | -0.4989 | -0.4981 |
| $\theta 13$ | -0.6934 | -0.4967 | -0.4978 | -0.4968 | -0.4974 | -0.4967 |
| $\theta 14$ | -0.3249 | -0.4718 | -0.4736 | -0.4724 | -0.4722 | -0.4719 |

Table 5.155: Estimation Results of case 2, two large, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2555 | 0.2475 | 0.2491 | 0.2483 | 0.2445 | 0.2475 |
| P2 | 0.1500 | 0.1483 | 0.1784 | 0.1483 | 0.1479 | 0.1488 | 0.1497 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.1118 | 0.0755 | 0.0746 | 0.0750 | 0.0751 | 0.0754 |
| **P4** | **0.0600** | **-0.0599** | **0.0419** | **0.0626** | **0.0640** | **0.0616** | **0.0662** | **0.0635** |
| P5 | -0.0750 | -0.0751 | -0.0625 | -0.0751 | -0.0749 | -0.0746 | -0.0730 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.1590 | -0.0744 | -0.0735 | -0.0747 | -0.0758 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.0811 | -0.0148 | -0.0156 | -0.0153 | -0.0150 | -0.0157 |
| P8 | 0.0950 | 0.0954 | 0.1328 | 0.0954 | 0.0946 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.2547 | -0.0452 | -0.0443 | -0.0457 | -0.0444 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.4000 | -0.0558 | -0.0567 | -0.0555 | -0.0560 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.2005 | -0.0804 | -0.0806 | -0.0803 | -0.0806 | -0.0803 |
| P12 | -0.0700 | -0.0700 | -0.1664 | -0.0700 | -0.0709 | -0.0695 | -0.0706 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.3494 | -0.0909 | -0.0900 | -0.0904 | -0.0915 | -0.0909 |
| P14 | -0.1250 | -0.1227 | 0.2816 | -0.1227 | -0.1236 | -0.1228 | -0.1238 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0404 | 0.0658 | 0.0667 | 0.0663 | 0.0642 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.2151 | 0.1817 | 0.1824 | 0.1820 | 0.1803 | 0.1817 |
| P2-3 | 0.0086 | 0.0086 | -0.0226 | 0.0076 | 0.0081 | 0.0081 | 0.0075 | 0.0076 |
| P2-4 | 0.0922 | 0.0900 | 0.0666 | 0.0907 | 0.0908 | 0.0913 | 0.0902 | 0.0907 |
| P2-5 | 0.1162 | 0.1159 | 0.1748 | 0.1159 | 0.1157 | 0.1157 | 0.1161 | 0.1159 |
| P3-4 | 0.0836 | 0.0823 | 0.0892 | 0.0831 | 0.0827 | 0.0832 | 0.0826 | 0.0830 |
| P4-5 | 0.0240 | 0.0240 | 0.1082 | 0.0252 | 0.0248 | 0.0244 | 0.0260 | 0.0252 |
| P4-7 | 0.0439 | 0.0441 | -0.0415 | 0.0435 | 0.0446 | 0.0440 | 0.0442 | 0.0441 |
| P4-9 | 0.1678 | 0.1682 | 0.1309 | 0.1677 | 0.1682 | 0.1677 | 0.1688 | 0.1679 |
| P5-6 | 0.2483 | 0.2520 | 0.4356 | 0.2476 | 0.2480 | 0.2475 | 0.2494 | 0.2477 |
| P6-11 | 0.0368 | 0.0372 | 0.1294 | 0.0372 | 0.0378 | 0.0373 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.1045 | 0.0687 | 0.0692 | 0.0684 | 0.0691 | 0.0687 |
| P6-13 | 0.0676 | 0.0667 | 0.0427 | 0.0673 | 0.0675 | 0.0672 | 0.0677 | 0.0674 |
| P7-8 | -0.0950 | -0.0954 | -0.1328 | -0.0954 | -0.0946 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1724 | 0.1242 | 0.1236 | 0.1237 | 0.1245 | 0.1238 |
| **P9-10** | **0.0982** | **0.9987** | **0.4711** | **0.0991** | **0.0996** | **0.0985** | **0.0998** | **0.0990** |
| P9-14 | 0.1486 | 0.1506 | 0.0870 | 0.1476 | 0.1478 | 0.1471 | 0.1491 | 0.1475 |
| P10-11 | 0.0432 | 0.0435 | 0.0711 | 0.0432 | 0.0429 | 0.0430 | 0.0437 | 0.0432 |
| P12-13 | -0.0012 | -0.0012 | -0.0618 | -0.0013 | -0.0017 | -0.0012 | -0.0015 | -0.0013 |
| **P13-14** | **-0.0236** | **-0.9232** | **-0.3685** | **-0.0249** | **-0.0242** | **-0.0243** | **-0.0253** | **-0.0248** |

Table 5.156: Estimation errors of case 2, two large, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0055 | 0.0025 | 0.0009 | 0.0017 | 0.0055 | 0.0025 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P2 | 0.0017 | 0.0284 | 0.0017 | 0.0021 | 0.0012 | 0.0003 | 0.0017 |
| P3 | 0.0005 | 0.0368 | 0.0005 | 0.0004 | 0.0000 | 0.0001 | 0.0004 |
| **P4** | **0.1199** | **0.0181** | **0.0026** | **0.0040** | **0.0016** | **0.0062** | **0.0035** |
| P5 | 0.0001 | 0.0125 | 0.0001 | 0.0001 | 0.0004 | 0.0020 | 0.0001 |
| P6 | 0.0006 | 0.0840 | 0.0006 | 0.0015 | 0.0003 | 0.0008 | 0.0006 |
| P7 | 0.0002 | 0.0961 | 0.0002 | 0.0006 | 0.0003 | 0.0000 | 0.0007 |
| P8 | 0.0004 | 0.0378 | 0.0004 | 0.0004 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.2997 | 0.0002 | 0.0007 | 0.0007 | 0.0006 | 0.0002 |
| P10 | 0.0008 | 0.3450 | 0.0008 | 0.0017 | 0.0005 | 0.0010 | 0.0008 |
| P11 | 0.0002 | 0.1205 | 0.0004 | 0.0006 | 0.0003 | 0.0006 | 0.0003 |
| P12 | 0.0000 | 0.0964 | 0.0000 | 0.0009 | 0.0005 | 0.0006 | 0.0000 |
| P13 | 0.0009 | 0.2594 | 0.0009 | 0.0000 | 0.0004 | 0.0015 | 0.0009 |
| P14 | 0.0023 | 0.4066 | 0.0023 | 0.0014 | 0.0022 | 0.0012 | 0.0023 |
| P1-2 | 0.0011 | 0.0266 | 0.0011 | 0.0002 | 0.0006 | 0.0027 | 0.0011 |
| P1-5 | 0.0002 | 0.0320 | 0.0014 | 0.0007 | 0.0011 | 0.0028 | 0.0014 |
| P2-3 | 0.0001 | 0.0312 | 0.0010 | 0.0005 | 0.0004 | 0.0011 | 0.0010 |
| P2-4 | 0.0022 | 0.0256 | 0.0015 | 0.0013 | 0.0009 | 0.0020 | 0.0015 |
| P2-5 | 0.0003 | 0.0586 | 0.0003 | 0.0005 | 0.0005 | 0.0000 | 0.0003 |
| P3-4 | 0.0012 | 0.0056 | 0.0005 | 0.0008 | 0.0004 | 0.0009 | 0.0005 |
| P4-5 | 0.0001 | 0.0842 | 0.0011 | 0.0008 | 0.0004 | 0.0020 | 0.0012 |
| P4-7 | 0.0002 | 0.0854 | 0.0004 | 0.0007 | 0.0001 | 0.0003 | 0.0002 |
| P4-9 | 0.0004 | 0.0369 | 0.0001 | 0.0004 | 0.0001 | 0.0010 | 0.0001 |
| P5-6 | 0.0037 | 0.1873 | 0.0006 | 0.0003 | 0.0007 | 0.0012 | 0.0006 |
| P6-11 | 0.0003 | 0.0926 | 0.0003 | 0.0009 | 0.0004 | 0.0000 | 0.0003 |
| P6-12 | 0.0000 | 0.0357 | 0.0001 | 0.0004 | 0.0004 | 0.0003 | 0.0001 |
| P6-13 | 0.0009 | 0.0250 | 0.0003 | 0.0002 | 0.0004 | 0.0001 | 0.0002 |
| P7-8 | 0.0004 | 0.0378 | 0.0004 | 0.0004 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0485 | 0.0003 | 0.0003 | 0.0002 | 0.0006 | 0.0001 |
| **P9-10** | **0.9006** | **0.3730** | **0.0009** | **0.0014** | **0.0004** | **0.0016** | **0.0008** |
| P9-14 | 0.0020 | 0.0616 | 0.0010 | 0.0007 | 0.0014 | 0.0005 | 0.0010 |
| P10-11 | 0.0003 | 0.0280 | 0.0001 | 0.0003 | 0.0002 | 0.0006 | 0.0000 |
| P12-13 | 0.0000 | 0.0607 | 0.0002 | 0.0005 | 0.0000 | 0.0003 | 0.0001 |
| **P13-14** | **0.8996** | **0.3450** | **0.0013** | **0.0007** | **0.0008** | **0.0017** | **0.0012** |
| **SUM** | **1.9526** | **3.5278** | **0.0259** | **0.0275** | **0.0195** | **0.0407** | **0.0267** |

Except WLS, all other estimators have good estimation.

### Median redundancy

Table 5.157 presents the state variables, and the later two tables shows the estimation results and error with actual values.

Table 5.157: State variable of case 2, two large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0384 | -0.0658 | -0.0663 | -0.0656 | -0.0705 | -0.0658 |
| $\theta 3$ | -0.0153 | -0.0744 | -0.0743 | -0.0730 | -0.0828 | -0.0730 |
| $\theta 4$ | -0.1103 | -0.1585 | -0.1572 | -0.1555 | -0.1798 | -0.1554 |
| $\theta 5$ | -0.2116 | -0.1824 | -0.1827 | -0.1827 | -0.1800 | -0.1832 |
| $\theta 6$ | -0.6102 | -0.4301 | -0.4316 | -0.4342 | -0.4065 | -0.4296 |
| $\theta 7$ | -0.0717 | -0.2018 | -0.2018 | -0.2000 | -0.2038 | -0.1991 |
| $\theta 8$ | 0.0688 | -0.1064 | -0.1069 | -0.1041 | -0.1050 | -0.1037 |
| $\theta 9$ | -0.2494 | -0.3257 | -0.3259 | -0.3252 | -0.3186 | -0.3234 |
| $\theta 10$ | -0.6685 | -0.4243 | -0.4254 | -0.4255 | -0.4022 | -0.4226 |
| $\theta 11$ | -0.7555 | -0.4671 | -0.4687 | -0.4695 | -0.4405 | -0.4660 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta 12$ | -0.7020 | -0.4988 | -0.5007 | -0.5028 | -0.4703 | -0.4981 |
| $\theta 13$ | -0.6252 | -0.4976 | -0.4993 | -0.5008 | -0.4705 | -0.4967 |
| $\theta 14$ | -0.3244 | -0.4730 | -0.4742 | -0.4741 | -0.4518 | -0.4714 |

Table 5.158: Estimation Results of case 2, two large, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2500 | 0.2482 | 0.2490 | 0.2483 | 0.2505 | 0.2490 |
| P2 | 0.1500 | 0.1483 | 0.1834 | 0.1521 | 0.1488 | 0.1488 | 0.1607 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.1182 | 0.0755 | 0.0750 | 0.0750 | 0.0846 | 0.0752 |
| **P4** | **0.0600** | **-0.0599** | **0.0349** | **0.0576** | **0.0650** | **0.0693** | **-0.0433** | **0.0677** |
| P5 | -0.0750 | -0.0751 | -0.0875 | -0.0751 | -0.0756 | -0.0755 | -0.0634 | -0.0820 |
| P6 | -0.0750 | -0.0744 | -0.1464 | -0.0744 | -0.0749 | -0.0810 | -0.0647 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.0756 | -0.0148 | -0.0153 | -0.0153 | -0.0080 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.1406 | 0.0954 | 0.0949 | 0.0959 | 0.0988 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.1775 | -0.0452 | -0.0450 | -0.0457 | -0.0368 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.3322 | -0.0558 | -0.0563 | -0.0563 | -0.0453 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.2323 | -0.0798 | -0.0803 | -0.0793 | -0.0723 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.1686 | -0.0700 | -0.0705 | -0.0705 | -0.0637 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.2391 | -0.0909 | -0.0914 | -0.0914 | -0.0828 | -0.0909 |
| P14 | -0.1250 | -0.1227 | 0.2258 | -0.1227 | -0.1232 | -0.1222 | -0.1145 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0384 | 0.0658 | 0.0663 | 0.0656 | 0.0705 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.2116 | 0.1824 | 0.1827 | 0.1827 | 0.1800 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0232 | 0.0086 | 0.0079 | 0.0074 | 0.0123 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.0719 | 0.0927 | 0.0909 | 0.0899 | 0.1093 | 0.0896 |
| P2-5 | 0.1162 | N/A | 0.1732 | 0.1166 | 0.1163 | 0.1171 | 0.1096 | 0.1174 |
| P3-4 | 0.0836 | 0.0823 | 0.0950 | 0.0841 | 0.0829 | 0.0824 | 0.0970 | 0.0824 |
| P4-5 | 0.0240 | N/A | 0.1013 | 0.0239 | 0.0255 | 0.0272 | 0.0002 | 0.0278 |
| P4-7 | 0.0439 | 0.0441 | -0.0386 | 0.0433 | 0.0446 | 0.0446 | 0.0240 | 0.0437 |
| P4-9 | 0.1678 | N/A | 0.1391 | 0.1672 | 0.1687 | 0.1697 | 0.1388 | 0.1681 |
| P5-6 | 0.2483 | 0.2520 | 0.3986 | 0.2477 | 0.2489 | 0.2515 | 0.2265 | 0.2464 |
| P6-11 | 0.0368 | N/A | 0.1453 | 0.0370 | 0.0371 | 0.0353 | 0.0340 | 0.0364 |
| P6-12 | 0.0688 | 0.0688 | 0.0918 | 0.0688 | 0.0691 | 0.0686 | 0.0638 | 0.0685 |
| P6-13 | 0.0676 | N/A | 0.0151 | 0.0675 | 0.0677 | 0.0666 | 0.0640 | 0.0671 |
| P7-8 | -0.0950 | -0.0954 | -0.1406 | -0.0954 | -0.0949 | -0.0959 | -0.0988 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1776 | 0.1239 | 0.1241 | 0.1252 | 0.1148 | 0.1243 |
| **P9-10** | **0.0982** | **0.9987** | **0.4191** | **0.0986** | **0.0995** | **0.1003** | **0.0836** | **0.0992** |
| P9-14 | 0.1486 | N/A | 0.0751 | 0.1473 | 0.1483 | 0.1489 | 0.1332 | 0.1480 |
| P10-11 | 0.0432 | 0.0435 | 0.0870 | 0.0428 | 0.0432 | 0.0440 | 0.0383 | 0.0434 |
| P12-13 | -0.0012 | N/A | -0.0768 | -0.0012 | -0.0014 | -0.0019 | 0.0002 | -0.0015 |
| **P13-14** | **-0.0236** | **-0.9232** | **-0.3008** | **-0.0246** | **-0.0251** | **-0.0267** | **-0.0187** | **-0.0253** |

Table 5.159: Estimation errors of case 2, two large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0000 | 0.0018 | 0.0010 | 0.0017 | 0.0005 | 0.0010 |
| P2 | 0.0334 | 0.0021 | 0.0012 | 0.0012 | 0.0107 | 0.0017 |
| P3 | 0.0432 | 0.0005 | 0.0000 | 0.0000 | 0.0096 | 0.0003 |
| **P4** | **0.0251** | **0.0024** | **0.0050** | **0.0093** | **0.1033** | **0.0077** |
| P5 | 0.0125 | 0.0001 | 0.0006 | 0.0005 | 0.0116 | 0.0070 |
| P6 | 0.0714 | 0.0006 | 0.0001 | 0.0060 | 0.0103 | 0.0006 |
| P7 | 0.0906 | 0.0002 | 0.0003 | 0.0003 | 0.0070 | 0.0002 |
| P8 | 0.0456 | 0.0004 | 0.0001 | 0.0009 | 0.0038 | 0.0004 |
| P9 | 0.2225 | 0.0002 | 0.0000 | 0.0007 | 0.0082 | 0.0002 |
| P10 | 0.2772 | 0.0008 | 0.0013 | 0.0013 | 0.0097 | 0.0008 |
| P11 | 0.1523 | 0.0002 | 0.0003 | 0.0007 | 0.0077 | 0.0002 |
| P12 | 0.0986 | 0.0000 | 0.0005 | 0.0005 | 0.0063 | 0.0000 |
| P13 | 0.1491 | 0.0009 | 0.0014 | 0.0014 | 0.0072 | 0.0009 |

| | | | | | | |
|---|---|---|---|---|---|---|
| P14 | 0.3508 | 0.0023 | 0.0018 | 0.0028 | 0.0105 | 0.0023 |
| P1-2 | 0.0285 | 0.0011 | 0.0006 | 0.0013 | 0.0036 | 0.0011 |
| P1-5 | 0.0285 | 0.0007 | 0.0004 | 0.0004 | 0.0030 | 0.0001 |
| P2-3 | 0.0318 | 0.0000 | 0.0006 | 0.0011 | 0.0038 | 0.0014 |
| P2-4 | 0.0203 | 0.0005 | 0.0013 | 0.0023 | 0.0172 | 0.0026 |
| P2-5 | 0.0570 | 0.0004 | 0.0002 | 0.0009 | 0.0066 | 0.0012 |
| P3-4 | 0.0115 | 0.0005 | 0.0007 | 0.0011 | 0.0134 | 0.0012 |
| P4-5 | 0.0773 | 0.0001 | 0.0015 | 0.0032 | 0.0238 | 0.0038 |
| P4-7 | 0.0825 | 0.0006 | 0.0007 | 0.0007 | 0.0199 | 0.0002 |
| P4-9 | 0.0287 | 0.0006 | 0.0009 | 0.0019 | 0.0290 | 0.0002 |
| P5-6 | 0.1503 | 0.0006 | 0.0006 | 0.0032 | 0.0218 | 0.0019 |
| P6-11 | 0.1085 | 0.0002 | 0.0003 | 0.0015 | 0.0029 | 0.0004 |
| P6-12 | 0.0230 | 0.0000 | 0.0003 | 0.0003 | 0.0050 | 0.0003 |
| P6-13 | 0.0526 | 0.0001 | 0.0001 | 0.0010 | 0.0037 | 0.0006 |
| P7-8 | 0.0456 | 0.0004 | 0.0001 | 0.0009 | 0.0038 | 0.0004 |
| P7-9 | 0.0537 | 0.0000 | 0.0002 | 0.0013 | 0.0091 | 0.0004 |
| **P9-10** | **0.3210** | **0.0004** | **0.0014** | **0.0021** | **0.0146** | **0.0010** |
| P9-14 | 0.0735 | 0.0013 | 0.0002 | 0.0004 | 0.0154 | 0.0006 |
| P10-11 | 0.0438 | 0.0004 | 0.0001 | 0.0008 | 0.0049 | 0.0002 |
| P12-13 | 0.0756 | 0.0000 | 0.0002 | 0.0008 | 0.0013 | 0.0003 |
| **P13-14** | **0.2773** | **0.0010** | **0.0015** | **0.0032** | **0.0049** | **0.0017** |
| **SUM** | **3.1630** | **0.0217** | **0.0256** | **0.0558** | **0.4141** | **0.0430** |

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.160.

Table 5.160: Estimation errors of case 2, two large, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.9526** | **3.5278** | **0.0259** | **0.0275** | **0.0195** | **0.0407** | **0.0267** |
| **SUM** | **N/A** | **3.1630** | **0.0217** | **0.0256** | **0.0558** | **0.4141** | **0.0430** |

## Three large bad data

Compared with the base cases, $P_4 = 0.9599$, $P_{9-10} = 0.9987$ and $P_{13-14} = -0.9232$ are introduced to simulate the three large bad data in both full and median redundancy case. The results are presented below.

### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.161 gives the state variables, and the later two tables shows the estimation results and

error with actual values.

Table 5.161: State variable of case 2, multi large, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0084 | -0.0645 | -0.0667 | -0.0674 | -0.0642 | -0.0658 |
| $\theta 3$ | 0.0232 | -0.0716 | -0.0748 | -0.0764 | -0.0717 | -0.0734 |
| $\theta 4$ | 0.0617 | -0.1540 | -0.1575 | -0.1583 | -0.1543 | -0.1565 |
| $\theta 5$ | -0.1925 | -0.1807 | -0.1824 | -0.1827 | -0.1803 | -0.1817 |
| $\theta 6$ | -0.6710 | -0.4292 | -0.4306 | -0.4307 | -0.4298 | -0.4294 |
| $\theta 7$ | -0.0386 | -0.1981 | -0.2018 | -0.2019 | -0.1985 | -0.2006 |
| $\theta 8$ | 0.0565 | -0.1027 | -0.1072 | -0.1070 | -0.1032 | -0.1052 |
| $\theta 9$ | -0.2188 | -0.3228 | -0.3263 | -0.3260 | -0.3231 | -0.3244 |
| $\theta 10$ | -0.7262 | -0.4229 | -0.4257 | -0.4249 | -0.4229 | -0.4234 |
| $\theta 11$ | -0.8060 | -0.4664 | -0.4685 | -0.4679 | -0.4666 | -0.4665 |
| $\theta 12$ | -0.7837 | -0.4980 | -0.4998 | -0.4995 | -0.4989 | -0.4981 |
| $\theta 13$ | -0.7198 | -0.4968 | -0.4981 | -0.4979 | -0.4974 | -0.4967 |
| $\theta 14$ | -0.3422 | -0.4712 | -0.4740 | -0.4730 | -0.4722 | -0.4719 |

Table 5.162: Estimation Results of case 2, multi large, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2009 | 0.2452 | 0.2491 | 0.2501 | 0.2445 | 0.2475 |
| P2 | 0.1500 | 0.1483 | 0.0739 | 0.1483 | 0.1479 | 0.1480 | 0.1497 | 0.1483 |
| P3 | 0.0750 | 0.0755 | -0.0069 | 0.0752 | 0.0746 | 0.0728 | 0.0751 | 0.0754 |
| **P4** | **0.0600** | **0.9599** | **0.7437** | **0.0678** | **0.0642** | **0.0630** | **0.0662** | **0.0635** |
| P5 | -0.0750 | -0.0751 | -0.1524 | -0.0751 | -0.0746 | -0.0746 | -0.0730 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.1819 | -0.0748 | -0.0735 | -0.0746 | -0.0758 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0154 | -0.0148 | -0.0143 | -0.0145 | -0.0150 | -0.0157 |
| P8 | 0.0950 | 0.0954 | 0.0952 | 0.0954 | 0.0946 | 0.0949 | 0.0953 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.1703 | -0.0452 | -0.0461 | -0.0457 | -0.0444 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.4277 | -0.0565 | -0.0567 | -0.0560 | -0.0560 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.2147 | -0.0807 | -0.0806 | -0.0803 | -0.0806 | -0.0803 |
| P12 | -0.0700 | -0.0700 | -0.1765 | -0.0700 | -0.0709 | -0.0705 | -0.0706 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.3626 | -0.0921 | -0.0900 | -0.0904 | -0.0915 | -0.0909 |
| P14 | -0.1250 | -0.1227 | 0.2542 | -0.1227 | -0.1236 | -0.1222 | -0.1238 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0084 | 0.0645 | 0.0667 | 0.0674 | 0.0642 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1925 | 0.1807 | 0.1824 | 0.1827 | 0.1803 | 0.1817 |
| P2-3 | 0.0086 | 0.0086 | -0.0316 | 0.0071 | 0.0081 | 0.0091 | 0.0075 | 0.0076 |
| P2-4 | 0.0922 | 0.0900 | -0.0701 | 0.0895 | 0.0908 | 0.0909 | 0.0902 | 0.0907 |
| P2-5 | 0.1162 | 0.1159 | 0.1841 | 0.1162 | 0.1157 | 0.1154 | 0.1161 | 0.1159 |
| P3-4 | 0.0836 | 0.0823 | -0.0385 | 0.0823 | 0.0827 | 0.0818 | 0.0826 | 0.0830 |
| P4-5 | 0.0240 | 0.0240 | 0.2542 | 0.0267 | 0.0248 | 0.0244 | 0.0260 | 0.0252 |
| P4-7 | 0.0439 | 0.0441 | 0.1003 | 0.0441 | 0.0442 | 0.0436 | 0.0442 | 0.0441 |
| P4-9 | 0.1678 | 0.1682 | 0.2805 | 0.1688 | 0.1687 | 0.1677 | 0.1688 | 0.1679 |
| P5-6 | 0.2483 | 0.2520 | 0.4785 | 0.2485 | 0.2482 | 0.2479 | 0.2494 | 0.2477 |
| P6-11 | 0.0368 | 0.0372 | 0.1350 | 0.0372 | 0.0379 | 0.0373 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.1127 | 0.0688 | 0.0692 | 0.0689 | 0.0691 | 0.0687 |
| P6-13 | 0.0676 | 0.0667 | 0.0489 | 0.0676 | 0.0675 | 0.0672 | 0.0677 | 0.0674 |
| P7-8 | -0.0950 | -0.0954 | -0.0952 | -0.0954 | -0.0946 | -0.0949 | -0.0953 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.1801 | 0.1247 | 0.1245 | 0.1241 | 0.1245 | 0.1238 |
| **P9-10** | **0.0982** | **0.9987** | **0.5074** | **0.1000** | **0.0994** | **0.0990** | **0.0998** | **0.0990** |
| P9-14 | 0.1486 | 0.1506 | 0.1235 | 0.1484 | 0.1477 | 0.1471 | 0.1491 | 0.1475 |
| P10-11 | 0.0432 | 0.0435 | 0.0797 | 0.0435 | 0.0427 | 0.0430 | 0.0437 | 0.0432 |
| P12-13 | -0.0012 | -0.0012 | -0.0638 | -0.0012 | -0.0017 | -0.0017 | -0.0015 | -0.0013 |
| **P13-14** | **-0.0236** | **-0.9232** | **-0.3776** | **-0.0256** | **-0.0241** | **-0.0248** | **-0.0253** | **-0.0248** |

Table 5.163: Estimation errors of case 2, multi large, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0491 | 0.0048 | 0.0009 | 0.0001 | 0.0055 | 0.0025 |
| P2 | 0.0017 | 0.0761 | 0.0017 | 0.0021 | 0.0020 | 0.0003 | 0.0017 |
| P3 | 0.0005 | 0.0819 | 0.0002 | 0.0004 | 0.0022 | 0.0001 | 0.0004 |
| **P4** | **0.8999** | **0.6837** | **0.0078** | **0.0042** | **0.0030** | **0.0062** | **0.0035** |
| P5 | 0.0001 | 0.0774 | 0.0001 | 0.0004 | 0.0004 | 0.0020 | 0.0001 |
| P6 | 0.0006 | 0.1069 | 0.0002 | 0.0015 | 0.0004 | 0.0008 | 0.0006 |
| P7 | 0.0002 | 0.0004 | 0.0002 | 0.0007 | 0.0005 | 0.0000 | 0.0007 |
| P8 | 0.0004 | 0.0002 | 0.0004 | 0.0004 | 0.0001 | 0.0003 | 0.0004 |
| P9 | 0.0002 | 0.2153 | 0.0002 | 0.0011 | 0.0007 | 0.0006 | 0.0002 |
| P10 | 0.0008 | 0.3727 | 0.0015 | 0.0017 | 0.0010 | 0.0010 | 0.0008 |
| P11 | 0.0002 | 0.1347 | 0.0007 | 0.0006 | 0.0003 | 0.0006 | 0.0003 |
| P12 | 0.0000 | 0.1065 | 0.0000 | 0.0009 | 0.0005 | 0.0006 | 0.0000 |
| P13 | 0.0009 | 0.2726 | 0.0021 | 0.0000 | 0.0004 | 0.0015 | 0.0009 |
| P14 | 0.0023 | 0.3792 | 0.0023 | 0.0014 | 0.0028 | 0.0012 | 0.0023 |
| P1-2 | 0.0011 | 0.0585 | 0.0024 | 0.0002 | 0.0005 | 0.0027 | 0.0011 |
| P1-5 | 0.0002 | 0.0094 | 0.0024 | 0.0007 | 0.0003 | 0.0028 | 0.0014 |
| P2-3 | 0.0001 | 0.0402 | 0.0014 | 0.0005 | 0.0005 | 0.0011 | 0.0010 |
| P2-4 | 0.0022 | 0.1623 | 0.0027 | 0.0013 | 0.0012 | 0.0020 | 0.0015 |
| P2-5 | 0.0003 | 0.0679 | 0.0000 | 0.0005 | 0.0008 | 0.0000 | 0.0003 |
| P3-4 | 0.0012 | 0.1221 | 0.0012 | 0.0008 | 0.0017 | 0.0009 | 0.0005 |
| P4-5 | 0.0001 | 0.2302 | 0.0027 | 0.0008 | 0.0004 | 0.0020 | 0.0012 |
| P4-7 | 0.0002 | 0.0564 | 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| P4-9 | 0.0004 | 0.1127 | 0.0010 | 0.0009 | 0.0001 | 0.0010 | 0.0001 |
| P5-6 | 0.0037 | 0.2302 | 0.0002 | 0.0001 | 0.0003 | 0.0012 | 0.0006 |
| P6-11 | 0.0003 | 0.0981 | 0.0003 | 0.0011 | 0.0004 | 0.0000 | 0.0003 |
| P6-12 | 0.0000 | 0.0439 | 0.0000 | 0.0004 | 0.0000 | 0.0003 | 0.0001 |
| P6-13 | 0.0009 | 0.0188 | 0.0000 | 0.0001 | 0.0004 | 0.0001 | 0.0002 |
| P7-8 | 0.0004 | 0.0002 | 0.0004 | 0.0004 | 0.0001 | 0.0003 | 0.0004 |
| P7-9 | 0.0003 | 0.0562 | 0.0008 | 0.0006 | 0.0002 | 0.0006 | 0.0001 |
| **P9-10** | **0.9006** | **0.4093** | **0.0019** | **0.0013** | **0.0008** | **0.0016** | **0.0008** |
| P9-14 | 0.0020 | 0.0251 | 0.0002 | 0.0008 | 0.0015 | 0.0005 | 0.0010 |
| P10-11 | 0.0003 | 0.0366 | 0.0003 | 0.0004 | 0.0002 | 0.0006 | 0.0000 |
| P12-13 | 0.0000 | 0.0626 | 0.0000 | 0.0005 | 0.0005 | 0.0003 | 0.0001 |
| **P13-14** | **0.8996** | **0.3541** | **0.0021** | **0.0006** | **0.0013** | **0.0017** | **0.0012** |
| **SUM** | **2.7326** | **4.7513** | **0.0425** | **0.0287** | **0.0259** | **0.0407** | **0.0267** |

### Median redundancy

The following three tables show the results and errors in detail.

Table 5.164: State Variable of case 2, multi large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | 0.0196 | -0.0608 | -0.0663 | -0.0656 | -0.0633 | -0.0658 |
| $\theta 3$ | 0.0826 | -0.0662 | -0.0743 | -0.0730 | -0.0702 | -0.0730 |
| $\theta 4$ | 0.1192 | -0.1471 | -0.1572 | -0.1555 | -0.1523 | -0.1554 |
| $\theta 5$ | -0.1875 | -0.1784 | -0.1827 | -0.1827 | -0.1797 | -0.1832 |
| $\theta 6$ | -0.6739 | -0.4304 | -0.4316 | -0.4342 | -0.4289 | -0.4296 |
| $\theta 7$ | 0.0044 | -0.1940 | -0.2018 | -0.2000 | -0.1969 | -0.1991 |
| $\theta 8$ | 0.1098 | -0.0986 | -0.1069 | -0.1041 | -0.1017 | -0.1037 |
| $\theta 9$ | -0.2212 | -0.3214 | -0.3259 | -0.3252 | -0.3217 | -0.3234 |
| $\theta 10$ | -0.7052 | -0.4289 | -0.4254 | -0.4255 | -0.4218 | -0.4226 |
| $\theta 11$ | -0.8187 | -0.4724 | -0.4687 | -0.4695 | -0.4656 | -0.4660 |
| $\theta 12$ | -0.7912 | -0.4993 | -0.5007 | -0.5028 | -0.4981 | -0.4981 |
| $\theta 13$ | -0.7121 | -0.4970 | -0.4993 | -0.5008 | -0.4965 | -0.4967 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta 14$ | -0.3913 | -0.4706 | -0.4742 | -0.4741 | -0.4707 | -0.4714 |

Table 5.165: Estimation Results of case 2, multi large, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.1679 | 0.2392 | 0.2490 | 0.2483 | 0.2430 | 0.2490 |
| P2 | 0.1500 | 0.1483 | 0.0643 | 0.1483 | 0.1488 | 0.1488 | 0.1491 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0263 | 0.0755 | 0.0750 | 0.0750 | 0.0752 | 0.0752 |
| **P4** | **0.0600** | **0.9599** | **0.8981** | **0.0853** | **0.0650** | **0.0693** | **0.0705** | **0.0677** |
| P5 | -0.0750 | -0.0751 | -0.2149 | -0.0751 | -0.0756 | -0.0755 | -0.0744 | -0.0820 |
| P6 | -0.0750 | -0.0744 | -0.1863 | -0.0744 | -0.0749 | -0.0810 | -0.0756 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.0053 | -0.0148 | -0.0153 | -0.0153 | -0.0151 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.1055 | 0.0954 | 0.0949 | 0.0959 | 0.0952 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.0882 | -0.0452 | -0.0450 | -0.0457 | -0.0452 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.3706 | -0.0639 | -0.0563 | -0.0563 | -0.0563 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.2582 | -0.0855 | -0.0803 | -0.0793 | -0.0805 | -0.0798 |
| P12 | -0.0700 | -0.0700 | -0.1963 | -0.0711 | -0.0705 | -0.0705 | -0.0709 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.2800 | -0.0909 | -0.0914 | -0.0914 | -0.0917 | -0.0909 |
| P14 | -0.1250 | -0.1227 | 0.1507 | -0.1227 | -0.1232 | -0.1222 | -0.1231 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | -0.0196 | 0.0608 | 0.0663 | 0.0656 | 0.0633 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1875 | 0.1784 | 0.1827 | 0.1827 | 0.1797 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0629 | 0.0054 | 0.0079 | 0.0074 | 0.0069 | 0.0072 |
| P2-4 | 0.0922 | N/A | -0.0996 | 0.0863 | 0.0909 | 0.0899 | 0.0890 | 0.0896 |
| P2-5 | 0.1162 | N/A | 0.2071 | 0.1175 | 0.1163 | 0.1171 | 0.1165 | 0.1174 |
| P3-4 | 0.0836 | 0.0823 | -0.0366 | 0.0809 | 0.0829 | 0.0824 | 0.0821 | 0.0824 |
| P4-5 | 0.0240 | N/A | 0.3067 | 0.0312 | 0.0255 | 0.0272 | 0.0274 | 0.0278 |
| P4-7 | 0.0439 | 0.0441 | 0.1148 | 0.0469 | 0.0446 | 0.0446 | 0.0447 | 0.0437 |
| P4-9 | 0.1678 | N/A | 0.3404 | 0.1743 | 0.1687 | 0.1697 | 0.1695 | 0.1681 |
| P5-6 | 0.2483 | 0.2520 | 0.4864 | 0.2520 | 0.2489 | 0.2515 | 0.2492 | 0.2464 |
| P6-11 | 0.0368 | N/A | 0.1447 | 0.0420 | 0.0371 | 0.0353 | 0.0367 | 0.0364 |
| P6-12 | 0.0688 | 0.0688 | 0.1172 | 0.0689 | 0.0691 | 0.0686 | 0.0692 | 0.0685 |
| P6-13 | 0.0676 | N/A | 0.0382 | 0.0667 | 0.0677 | 0.0666 | 0.0676 | 0.0671 |
| P7-8 | -0.0950 | -0.0954 | -0.1055 | -0.0954 | -0.0949 | -0.0959 | -0.0952 | -0.0954 |
| P7-9 | 0.1239 | 0.1236 | 0.2256 | 0.1275 | 0.1241 | 0.1252 | 0.1248 | 0.1243 |
| **P9-10** | **0.0982** | **0.9987** | **0.4840** | **0.1074** | **0.0995** | **0.1003** | **0.1001** | **0.0992** |
| P9-14 | 0.1486 | N/A | 0.1701 | 0.1491 | 0.1483 | 0.1489 | 0.1489 | 0.1480 |
| P10-11 | 0.0432 | 0.0435 | 0.1134 | 0.0435 | 0.0432 | 0.0440 | 0.0438 | 0.0434 |
| P12-13 | -0.0012 | N/A | -0.0790 | -0.0022 | -0.0014 | -0.0019 | -0.0016 | -0.0015 |
| **P13-14** | **-0.0236** | **-0.9232** | **-0.3208** | **-0.0264** | **-0.0251** | **-0.0267** | **-0.0258** | **-0.0253** |

Table 5.166: Estimation errors of case 2, multi large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0821 | 0.0108 | 0.0010 | 0.0017 | 0.0070 | 0.0010 |
| P2 | 0.0857 | 0.0017 | 0.0012 | 0.0012 | 0.0009 | 0.0017 |
| P3 | 0.0487 | 0.0005 | 0.0000 | 0.0000 | 0.0002 | 0.0003 |
| **P4** | **0.8381** | **0.0253** | **0.0050** | **0.0093** | **0.0105** | **0.0077** |
| P5 | 0.1399 | 0.0001 | 0.0006 | 0.0005 | 0.0006 | 0.0070 |
| P6 | 0.1113 | 0.0006 | 0.0001 | 0.0060 | 0.0006 | 0.0006 |
| P7 | 0.0203 | 0.0002 | 0.0003 | 0.0003 | 0.0001 | 0.0002 |
| P8 | 0.0105 | 0.0004 | 0.0001 | 0.0009 | 0.0002 | 0.0004 |
| P9 | 0.1332 | 0.0002 | 0.0000 | 0.0007 | 0.0002 | 0.0002 |
| P10 | 0.3156 | 0.0089 | 0.0013 | 0.0013 | 0.0013 | 0.0008 |
| P11 | 0.1782 | 0.0055 | 0.0003 | 0.0007 | 0.0005 | 0.0002 |
| P12 | 0.1263 | 0.0011 | 0.0005 | 0.0005 | 0.0009 | 0.0000 |
| P13 | 0.1900 | 0.0009 | 0.0014 | 0.0014 | 0.0017 | 0.0009 |
| P14 | 0.2757 | 0.0023 | 0.0018 | 0.0028 | 0.0019 | 0.0023 |
| P1-2 | 0.0865 | 0.0061 | 0.0006 | 0.0013 | 0.0037 | 0.0011 |

| | | | | | | |
|---|---|---|---|---|---|---|
| P1-5 | 0.0044 | 0.0047 | 0.0004 | 0.0004 | 0.0034 | 0.0001 |
| P2-3 | 0.0715 | 0.0032 | 0.0006 | 0.0011 | 0.0017 | 0.0014 |
| P2-4 | 0.1917 | 0.0059 | 0.0013 | 0.0023 | 0.0031 | 0.0026 |
| P2-5 | 0.0910 | 0.0013 | 0.0002 | 0.0009 | 0.0003 | 0.0012 |
| P3-4 | 0.1202 | 0.0027 | 0.0007 | 0.0011 | 0.0015 | 0.0012 |
| P4-5 | 0.2827 | 0.0072 | 0.0015 | 0.0032 | 0.0034 | 0.0038 |
| P4-7 | 0.0709 | 0.0030 | 0.0007 | 0.0007 | 0.0008 | 0.0002 |
| P4-9 | 0.1726 | 0.0065 | 0.0009 | 0.0019 | 0.0017 | 0.0002 |
| P5-6 | 0.2381 | 0.0037 | 0.0006 | 0.0032 | 0.0009 | 0.0019 |
| P6-11 | 0.1079 | 0.0052 | 0.0003 | 0.0015 | 0.0001 | 0.0004 |
| P6-12 | 0.0484 | 0.0001 | 0.0003 | 0.0003 | 0.0004 | 0.0003 |
| P6-13 | 0.0294 | 0.0010 | 0.0001 | 0.0010 | 0.0000 | 0.0006 |
| P7-8 | 0.0105 | 0.0004 | 0.0001 | 0.0009 | 0.0002 | 0.0004 |
| P7-9 | 0.1017 | 0.0036 | 0.0002 | 0.0013 | 0.0009 | 0.0004 |
| **P9-10** | **0.3859** | **0.0093** | **0.0014** | **0.0021** | **0.0019** | **0.0010** |
| P9-14 | 0.0216 | 0.0006 | 0.0002 | 0.0004 | 0.0004 | 0.0006 |
| P10-11 | 0.0703 | 0.0003 | 0.0001 | 0.0008 | 0.0006 | 0.0002 |
| P12-13 | 0.0779 | 0.0010 | 0.0002 | 0.0008 | 0.0005 | 0.0003 |
| **P13-14** | **0.2972** | **0.0029** | **0.0015** | **0.0032** | **0.0022** | **0.0017** |
| **SUM** | **5.0357** | **0.1272** | **0.0256** | **0.0558** | **0.0544** | **0.0430** |

**Comparison**

Full redundancy and median redundancy estimation with the same condition of error are presented in Table 5.167. From the results, it is very easy to find that full redundancy case has better estimation results for most estimators. For LMS, median redundancy have better results.

Table 5.167: Estimation errors of case 2, multi large, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **2.7326** | **4.7513** | **0.0425** | **0.0287** | **0.0259** | **0.0407** | **0.0267** |
| **SUM** | **N/A** | **5.0357** | **0.1272** | **0.0256** | **0.0558** | **0.0544** | **0.0430** |

#### 5.2.1.4 Case 3: Multiple interacting bad data

For interacting bad data, three location connected with each other are selected. The three locations to put bad data are: $P_4$, $P_{4-7}$ and $P_{7-9}$. Various values are applied to them considering different kinds of cases for full and median redundancy.

**Two reverse bad data**

The first case is three reverse bad data: $P_4 = -0.0599$, $P_{4-7} = -0.0441$ and $P_{7-9} = -0.1236$.

**Full redundancy**

Full redundancy take all measurements as available value, the system state variables are presented in Table 5.168, Table 5.169 and Table 5.170 gives the estimation results and error with actual value.

Table 5.168: State Variable of case 3, multi reverse, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0703 | -0.0661 | -0.0667 | -0.0682 | -0.0677 | -0.0674 |
| $\theta 3$ | -0.0827 | -0.0743 | -0.0748 | -0.0773 | -0.0765 | -0.0756 |
| $\theta 4$ | -0.1766 | -0.1580 | -0.1575 | -0.1595 | -0.1721 | -0.1592 |
| $\theta 5$ | -0.1773 | -0.1820 | -0.1824 | -0.1837 | -0.1821 | -0.1832 |
| $\theta 6$ | -0.3949 | -0.4296 | -0.4306 | -0.4317 | -0.4244 | -0.4310 |
| $\theta 7$ | -0.2196 | -0.2013 | -0.2002 | -0.2035 | -0.1916 | -0.2026 |
| $\theta 8$ | -0.1391 | -0.1058 | -0.1039 | -0.1086 | -0.0902 | -0.1072 |
| $\theta 9$ | -0.2903 | -0.3252 | -0.3252 | -0.3272 | -0.3135 | -0.3265 |
| $\theta 10$ | -0.3713 | -0.4238 | -0.4238 | -0.4259 | -0.4142 | -0.4253 |
| $\theta 11$ | -0.4196 | -0.4666 | -0.4675 | -0.4689 | -0.4593 | -0.4681 |
| $\theta 12$ | -0.4578 | -0.4984 | -0.4989 | -0.5002 | -0.4923 | -0.4998 |
| $\theta 13$ | -0.4534 | -0.4972 | -0.4981 | -0.4989 | -0.4904 | -0.4985 |
| $\theta 14$ | -0.4229 | -0.4725 | -0.4749 | -0.4742 | -0.4643 | -0.4739 |

Table 5.169: Estimation Results of case 3, multi reverse, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2476 | 0.2481 | 0.2491 | 0.2519 | 0.2498 | 0.2506 |
| P2 | 0.1500 | 0.1483 | 0.1554 | 0.1499 | 0.1479 | 0.1478 | 0.1599 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0815 | 0.0755 | 0.0746 | 0.0731 | 0.0867 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **-0.0428** | **0.0588** | **0.0616** | **0.0624** | **-0.0288** | **0.0591** |
| P5 | -0.0750 | -0.0751 | -0.0675 | -0.0741 | -0.0747 | -0.0756 | -0.0642 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0716 | -0.0744 | -0.0753 | -0.0749 | -0.0737 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0528 | -0.0148 | -0.0139 | -0.0153 | 0.0008 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.0805 | 0.0954 | 0.0963 | 0.0949 | 0.1015 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.0291 | -0.0452 | -0.0443 | -0.0457 | -0.0119 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.0328 | -0.0558 | -0.0550 | -0.0557 | -0.0556 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0729 | -0.0798 | -0.0806 | -0.0803 | -0.0799 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0674 | -0.0700 | -0.0692 | -0.0699 | -0.0697 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0844 | -0.0909 | -0.0900 | -0.0906 | -0.0903 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1021 | -0.1227 | -0.1265 | -0.1222 | -0.1246 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0703 | 0.0661 | 0.0667 | 0.0682 | 0.0677 | 0.0674 |
| P1-5 | 0.1831 | 0.1832 | 0.1773 | 0.1820 | 0.1824 | 0.1837 | 0.1821 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | 0.0124 | 0.0082 | 0.0081 | 0.0091 | 0.0088 | 0.0081 |
| P2-4 | 0.0922 | 0.0900 | 0.1063 | 0.0919 | 0.0908 | 0.0913 | 0.1044 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1070 | 0.1159 | 0.1157 | 0.1155 | 0.1144 | 0.1158 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P3-4 | 0.0836 | 0.0823 | 0.0939 | 0.0837 | 0.0827 | 0.0822 | 0.0955 | 0.0837 |
| P4-5 | 0.0240 | 0.0240 | 0.0007 | 0.0240 | 0.0248 | 0.0242 | 0.0100 | 0.0240 |
| **P4-7** | **0.0439** | **-0.0441** | **0.0430** | **0.0433** | **0.0426** | **0.0440** | **0.0196** | **0.0433** |
| P4-9 | 0.1678 | 0.1682 | 0.1137 | 0.1672 | 0.1677 | 0.1677 | 0.1415 | 0.1673 |
| P5-6 | 0.2483 | 0.2520 | 0.2176 | 0.2477 | 0.2482 | 0.2479 | 0.2423 | 0.2478 |
| P6-11 | 0.0368 | 0.0372 | 0.0247 | 0.0370 | 0.0370 | 0.0373 | 0.0348 | 0.0371 |
| P6-12 | 0.0688 | 0.0688 | 0.0629 | 0.0688 | 0.0684 | 0.0686 | 0.0678 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.0584 | 0.0675 | 0.0676 | 0.0672 | 0.0660 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.0805 | -0.0954 | -0.0963 | -0.0949 | -0.1015 | -0.0954 |
| **P7-9** | **0.1239** | **-0.1236** | **0.0707** | **0.1239** | **0.1250** | **0.1237** | **0.1219** | **0.1240** |
| P9-10 | 0.0982 | 0.0987 | 0.0810 | 0.0986 | 0.0986 | 0.0987 | 0.1007 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1326 | 0.1473 | 0.1497 | 0.1469 | 0.1508 | 0.1474 |
| P10-11 | 0.0432 | 0.0435 | 0.0482 | 0.0428 | 0.0437 | 0.0430 | 0.0451 | 0.0429 |
| P12-13 | -0.0012 | -0.0012 | -0.0045 | -0.0013 | -0.0008 | -0.0014 | -0.0018 | -0.0013 |
| P13-14 | -0.0236 | -0.0232 | -0.0305 | -0.0246 | -0.0232 | -0.0247 | -0.0261 | -0.0246 |

Table 5.170: Estimation errors of case 3, multi reverse, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0024 | 0.0019 | 0.0009 | 0.0019 | 0.0002 | 0.0006 |
| P2 | 0.0017 | 0.0054 | 0.0001 | 0.0021 | 0.0022 | 0.0099 | 0.0017 |
| P3 | 0.0005 | 0.0065 | 0.0005 | 0.0004 | 0.0019 | 0.0117 | 0.0005 |
| **P4** | **0.1199** | **0.1028** | **0.0012** | **0.0015** | **0.0024** | **0.0888** | **0.0009** |
| P5 | 0.0001 | 0.0075 | 0.0009 | 0.0003 | 0.0006 | 0.0108 | 0.0001 |
| P6 | 0.0006 | 0.0034 | 0.0006 | 0.0003 | 0.0001 | 0.0013 | 0.0006 |
| P7 | 0.0002 | 0.0378 | 0.0002 | 0.0011 | 0.0003 | 0.0158 | 0.0002 |
| P8 | 0.0004 | 0.0145 | 0.0004 | 0.0013 | 0.0001 | 0.0065 | 0.0004 |
| P9 | 0.0002 | 0.0741 | 0.0002 | 0.0007 | 0.0007 | 0.0331 | 0.0002 |
| P10 | 0.0008 | 0.0222 | 0.0008 | 0.0000 | 0.0007 | 0.0006 | 0.0008 |
| P11 | 0.0002 | 0.0071 | 0.0002 | 0.0006 | 0.0003 | 0.0001 | 0.0000 |
| P12 | 0.0000 | 0.0026 | 0.0000 | 0.0008 | 0.0001 | 0.0003 | 0.0000 |
| P13 | 0.0009 | 0.0056 | 0.0009 | 0.0000 | 0.0006 | 0.0003 | 0.0009 |
| P14 | 0.0023 | 0.0229 | 0.0023 | 0.0015 | 0.0028 | 0.0004 | 0.0023 |
| P1-2 | 0.0011 | 0.0034 | 0.0008 | 0.0002 | 0.0013 | 0.0008 | 0.0005 |
| P1-5 | 0.0002 | 0.0057 | 0.0011 | 0.0007 | 0.0007 | 0.0010 | 0.0001 |
| P2-3 | 0.0001 | 0.0038 | 0.0004 | 0.0005 | 0.0006 | 0.0002 | 0.0004 |
| P2-4 | 0.0022 | 0.0142 | 0.0002 | 0.0013 | 0.0008 | 0.0122 | 0.0003 |
| P2-5 | 0.0003 | 0.0091 | 0.0003 | 0.0005 | 0.0006 | 0.0018 | 0.0004 |
| P3-4 | 0.0012 | 0.0103 | 0.0001 | 0.0008 | 0.0014 | 0.0120 | 0.0001 |
| P4-5 | 0.0001 | 0.0233 | 0.0001 | 0.0008 | 0.0002 | 0.0140 | 0.0001 |
| **P4-7** | **0.0880** | **0.0009** | **0.0006** | **0.0013** | **0.0001** | **0.0243** | **0.0006** |
| P4-9 | 0.0004 | 0.0541 | 0.0006 | 0.0001 | 0.0001 | 0.0264 | 0.0005 |
| P5-6 | 0.0037 | 0.0307 | 0.0006 | 0.0001 | 0.0003 | 0.0059 | 0.0005 |
| P6-11 | 0.0003 | 0.0122 | 0.0001 | 0.0001 | 0.0004 | 0.0020 | 0.0003 |
| P6-12 | 0.0000 | 0.0059 | 0.0000 | 0.0004 | 0.0002 | 0.0010 | 0.0000 |
| P6-13 | 0.0009 | 0.0092 | 0.0001 | 0.0001 | 0.0004 | 0.0016 | 0.0001 |
| P7-8 | 0.0004 | 0.0145 | 0.0004 | 0.0013 | 0.0001 | 0.0065 | 0.0004 |
| **P7-9** | **0.2475** | **0.0532** | **0.0000** | **0.0011** | **0.0002** | **0.0020** | **0.0001** |
| P9-10 | 0.0006 | 0.0172 | 0.0005 | 0.0005 | 0.0005 | 0.0025 | 0.0006 |
| P9-14 | 0.0020 | 0.0160 | 0.0012 | 0.0012 | 0.0016 | 0.0022 | 0.0012 |
| P10-11 | 0.0003 | 0.0051 | 0.0004 | 0.0005 | 0.0002 | 0.0019 | 0.0003 |
| P12-13 | 0.0000 | 0.0033 | 0.0001 | 0.0004 | 0.0002 | 0.0007 | 0.0001 |
| P13-14 | 0.0004 | 0.0069 | 0.0010 | 0.0004 | 0.0012 | 0.0026 | 0.0011 |
| **SUM** | **0.4884** | **0.6137** | **0.0193** | **0.0239** | **0.0256** | **0.3013** | **0.0167** |

LTS's estimation are not that good in estimation connected with bus 4, others estimator generate good results except WLS.

**Median redundancy**

System state variables, estimation results and error are presented in the following three tables:

Table 5.171: State Variable of case 3, multi reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0732 | -0.0659 | -0.0664 | -0.0663 | -0.0698 | -0.0658 |
| $\theta 3$ | -0.0896 | -0.0762 | -0.0744 | -0.0740 | -0.0818 | -0.0730 |
| $\theta 4$ | -0.1804 | -0.1621 | -0.1574 | -0.1569 | -0.1765 | -0.1557 |
| $\theta 5$ | -0.1743 | -0.1733 | -0.1826 | -0.1831 | -0.1792 | -0.1829 |
| $\theta 6$ | -0.3799 | -0.3903 | -0.4311 | -0.4346 | -0.4048 | -0.4349 |
| $\theta 7$ | -0.2148 | -0.1723 | -0.2023 | -0.2045 | -0.1936 | -0.4570 |
| $\theta 8$ | -0.1342 | -0.0769 | -0.1075 | -0.1096 | -0.0914 | -0.3616 |
| $\theta 9$ | -0.2852 | -0.2630 | -0.3267 | -0.3317 | -0.3116 | -0.3334 |
| $\theta 10$ | -0.3593 | -0.3617 | -0.4256 | -0.4309 | -0.4001 | -0.4321 |
| $\theta 11$ | -0.4027 | -0.4052 | -0.4685 | -0.4748 | -0.4396 | -0.4750 |
| $\theta 12$ | -0.4388 | -0.4562 | -0.5000 | -0.5033 | -0.4680 | -0.5040 |
| $\theta 13$ | -0.4319 | -0.4521 | -0.4994 | -0.5023 | -0.4667 | -0.5031 |
| $\theta 14$ | -0.4022 | -0.4189 | -0.4768 | -0.4786 | -0.4447 | -0.4796 |

Table 5.172: Estimation Results of case 3, multi reverse, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2474 | 0.2392 | 0.2490 | 0.2494 | 0.2490 | 0.2487 |
| P2 | 0.1500 | 0.1483 | 0.1516 | 0.1483 | 0.1489 | 0.1488 | 0.1584 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0744 | 0.0755 | 0.0749 | 0.0752 | 0.0827 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **-0.0649** | **-0.0599** | **0.0656** | **0.0751** | **-0.0466** | **0.3338** |
| P5 | -0.0750 | -0.0751 | -0.0636 | -0.0751 | -0.0757 | -0.0746 | -0.0657 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0721 | -0.0744 | -0.0739 | -0.0749 | -0.0658 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0445 | -0.0148 | -0.0154 | -0.0153 | -0.0013 | -0.5204 |
| P8 | 0.0950 | 0.0954 | 0.0805 | 0.0954 | 0.0948 | 0.0949 | 0.1022 | 0.0954 |
| P9 | -0.0450 | -0.0452 | 0.0159 | 0.0630 | -0.0446 | -0.0559 | -0.0314 | 0.1907 |
| P10 | -0.0550 | -0.0558 | -0.0307 | -0.0552 | -0.0560 | -0.0553 | -0.0491 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0661 | -0.0584 | -0.0804 | -0.0841 | -0.0742 | -0.0831 |
| P12 | -0.0700 | -0.0700 | -0.0658 | -0.0700 | -0.0694 | -0.0697 | -0.0644 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.0748 | -0.0909 | -0.0903 | -0.0904 | -0.0827 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.0873 | -0.1227 | -0.1275 | -0.1232 | -0.1111 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0732 | 0.0659 | 0.0664 | 0.0663 | 0.0698 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1743 | 0.1733 | 0.1826 | 0.1831 | 0.1792 | 0.1829 |
| P2-3 | 0.0086 | 0.0086 | 0.0164 | 0.0104 | 0.0080 | 0.0077 | 0.0120 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.1072 | 0.0963 | 0.0910 | 0.0906 | 0.1067 | 0.0899 |
| P2-5 | 0.1162 | N/A | 0.1011 | 0.1075 | 0.1163 | 0.1168 | 0.1094 | 0.1171 |
| P3-4 | 0.0836 | 0.0823 | 0.0908 | 0.0859 | 0.0829 | 0.0829 | 0.0947 | 0.0827 |
| P4-5 | 0.0240 | N/A | -0.0061 | 0.0112 | 0.0253 | 0.0262 | 0.0027 | 0.0272 |
| **P4-7** | **0.0439** | **-0.0441** | **0.0344** | **0.0101** | **0.0449** | **0.0476** | **0.0171** | **0.3014** |
| P4-9 | 0.1678 | N/A | 0.1048 | 0.1009 | 0.1693 | 0.1748 | 0.1351 | 0.1778 |
| P5-6 | 0.2483 | 0.2520 | 0.2057 | 0.2170 | 0.2485 | 0.2515 | 0.2256 | 0.2520 |
| P6-11 | 0.0368 | N/A | 0.0227 | 0.0149 | 0.0374 | 0.0402 | 0.0347 | 0.0402 |
| P6-12 | 0.0688 | 0.0688 | 0.0589 | 0.0659 | 0.0689 | 0.0687 | 0.0632 | 0.0691 |
| P6-13 | 0.0676 | N/A | 0.0520 | 0.0618 | 0.0683 | 0.0677 | 0.0619 | 0.0683 |
| P7-8 | -0.0950 | -0.0954 | -0.0805 | -0.0954 | -0.0948 | -0.0949 | -0.1022 | -0.0954 |
| **P7-9** | **0.1239** | **-0.1236** | **0.0704** | **0.0907** | **0.1244** | **0.1272** | **0.1180** | **-0.1236** |
| P9-10 | 0.0982 | 0.0987 | 0.0741 | 0.0987 | 0.0990 | 0.0992 | 0.0886 | 0.0987 |
| P9-14 | 0.1486 | N/A | 0.1170 | 0.1559 | 0.1501 | 0.1469 | 0.1331 | 0.1462 |
| P10-11 | 0.0432 | 0.0435 | 0.0434 | 0.0435 | 0.0429 | 0.0439 | 0.0394 | 0.0429 |
| P12-13 | -0.0012 | N/A | -0.0069 | -0.0041 | -0.0006 | -0.0010 | -0.0013 | -0.0009 |

Table 5.173: Estimation errors of case 3, multi reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0026 | 0.0108 | 0.0010 | 0.0006 | 0.0010 | 0.0013 |
| P2 | 0.0016 | 0.0017 | 0.0011 | 0.0012 | 0.0084 | 0.0017 |
| P3 | 0.0006 | 0.0005 | 0.0001 | 0.0002 | 0.0077 | 0.0005 |
| **P4** | **0.1249** | **0.1199** | **0.0056** | **0.0151** | **0.1066** | **0.2738** |
| P5 | 0.0114 | 0.0001 | 0.0007 | 0.0004 | 0.0093 | 0.0001 |
| P6 | 0.0029 | 0.0006 | 0.0011 | 0.0001 | 0.0092 | 0.0006 |
| P7 | 0.0295 | 0.0002 | 0.0004 | 0.0003 | 0.0137 | 0.5054 |
| P8 | 0.0145 | 0.0004 | 0.0002 | 0.0001 | 0.0072 | 0.0004 |
| P9 | 0.0609 | 0.1080 | 0.0004 | 0.0109 | 0.0136 | 0.2357 |
| P10 | 0.0243 | 0.0002 | 0.0010 | 0.0003 | 0.0059 | 0.0008 |
| P11 | 0.0139 | 0.0216 | 0.0004 | 0.0041 | 0.0058 | 0.0031 |
| P12 | 0.0042 | 0.0000 | 0.0006 | 0.0003 | 0.0056 | 0.0000 |
| P13 | 0.0152 | 0.0009 | 0.0003 | 0.0004 | 0.0073 | 0.0009 |
| P14 | 0.0377 | 0.0023 | 0.0025 | 0.0018 | 0.0139 | 0.0023 |
| P1-2 | 0.0062 | 0.0011 | 0.0005 | 0.0006 | 0.0029 | 0.0011 |
| P1-5 | 0.0088 | 0.0097 | 0.0005 | 0.0000 | 0.0039 | 0.0002 |
| P2-3 | 0.0078 | 0.0018 | 0.0006 | 0.0009 | 0.0034 | 0.0014 |
| P2-4 | 0.0151 | 0.0041 | 0.0012 | 0.0016 | 0.0146 | 0.0023 |
| P2-5 | 0.0151 | 0.0087 | 0.0001 | 0.0006 | 0.0068 | 0.0009 |
| P3-4 | 0.0072 | 0.0023 | 0.0006 | 0.0007 | 0.0111 | 0.0009 |
| P4-5 | 0.0301 | 0.0128 | 0.0013 | 0.0022 | 0.0213 | 0.0032 |
| **P4-7** | **0.0095** | **0.0338** | **0.0010** | **0.0037** | **0.0268** | **0.2575** |
| P4-9 | 0.0630 | 0.0669 | 0.0015 | 0.0070 | 0.0327 | 0.0100 |
| P5-6 | 0.0426 | 0.0313 | 0.0002 | 0.0032 | 0.0227 | 0.0037 |
| P6-11 | 0.0141 | 0.0220 | 0.0006 | 0.0034 | 0.0021 | 0.0034 |
| P6-12 | 0.0099 | 0.0029 | 0.0000 | 0.0001 | 0.0056 | 0.0003 |
| P6-13 | 0.0157 | 0.0058 | 0.0006 | 0.0001 | 0.0057 | 0.0006 |
| P7-8 | 0.0145 | 0.0004 | 0.0002 | 0.0001 | 0.0072 | 0.0004 |
| **P7-9** | **0.0535** | **0.0332** | **0.0005** | **0.0033** | **0.0059** | **0.2475** |
| P9-10 | 0.0241 | 0.0005 | 0.0008 | 0.0010 | 0.0096 | 0.0005 |
| P9-14 | 0.0315 | 0.0074 | 0.0015 | 0.0017 | 0.0154 | 0.0024 |
| P10-11 | 0.0002 | 0.0003 | 0.0002 | 0.0007 | 0.0037 | 0.0003 |
| P12-13 | 0.0057 | 0.0029 | 0.0006 | 0.0002 | 0.0001 | 0.0003 |
| P13-14 | 0.0061 | 0.0097 | 0.0009 | 0.0001 | 0.0015 | 0.0001 |
| **SUM** | **0.7251** | **0.5248** | **0.0287** | **0.0670** | **0.4184** | **1.5636** |

Problems emerge. The estimation of LTAV are much worse than others and even the always worst WLS, though in this case only LMS and LMR performs much better. Regarding the overall performance downgrade, it simple to explain. Since the three large error are strongly connected with each other, the local redundancy is low, it's hard for the estimator to detect the bad data.

As for LTAV, which always has good performance in other cases, a further investigation is conducted. During the algorithm, $P_1$, $P_4$, $P_7$, $P_9$, $P_{11}$ and $P_{4-7}$ are trimmed before final estimation, making raw data connected with them have

low redundancy. Take bus 4 for example, 6 value are connected with it while only one correct raw data available. Others such as bus 9, 5 value are connected, one correct data and one bad data are available for estimation, which will definitely generate bad results. Besides, $P_4$, $P_7$ and $P_{4-7}$ are all trimmed, making $P_{4-7}$ very hard to estimate.

But this just a particular case, when changing to other location, problems solved. Take another two sets for example: set 1: $P_{4-7} = -0.0441$, $P_{7-9} = -0.1236$ and $P_{9-10} = -0.0987$, set 2: $P_9 = 0.0452$, $P_{9-10} = -0.0987$ and $P_{10-11} = -0.0435$. The state variables are presented in Table 5.174, the estimation results and errors are shown in Table. 5.175. $SV1$, $R1$ and $E1$ corresponding to state variables, estimation and error of set 1, the other corresponding to set 2. For both cases, no matter it's the same pattern with original error, i.e. one injection two power flow or different one, three power flow, they all generate very good estimation. So the bad estimation is just a particular case from data itself, when the location, LTAV can get satisfied estimation.

Table 5.174: State variables of another two case

| | SV1 | SV2 |
|---|---|---|
| $\theta 2$ | -0.0675 | -0.0674 |
| $\theta 3$ | -0.0761 | -0.0756 |
| $\theta 4$ | -0.1592 | -0.1592 |
| $\theta 5$ | -0.1831 | -0.1832 |
| $\theta 6$ | -0.4308 | -0.4311 |
| $\theta 7$ | -0.2025 | -0.2028 |
| $\theta 8$ | -0.1071 | -0.1074 |
| $\theta 9$ | -0.3264 | -0.3270 |
| $\theta 10$ | -0.4250 | -0.4255 |
| $\theta 11$ | -0.4678 | -0.4682 |
| $\theta 12$ | -0.4996 | -0.4999 |
| $\theta 13$ | -0.4983 | -0.4987 |
| $\theta 14$ | -0.4737 | -0.4742 |

Table 5.175: Estimation and errors of another two case

| | **Actual** | **R1** | **E1** | **R2** | **E2** |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2507 | 0.0007 | 0.2506 | 0.0006 |
| P2 | 0.1500 | 0.1483 | 0.0017 | 0.1483 | 0.0017 |
| P3 | 0.0750 | 0.0744 | 0.0006 | 0.0755 | 0.0005 |
| P4 | 0.0600 | 0.0599 | 0.0001 | 0.0599 | 0.0001 |
| P5 | -0.0750 | -0.0751 | 0.0001 | -0.0751 | 0.0001 |
| P6 | -0.0750 | -0.0744 | 0.0006 | -0.0744 | 0.0006 |
| P7 | -0.0150 | -0.0148 | 0.0002 | -0.0148 | 0.0002 |
| P8 | 0.0950 | 0.0954 | 0.0004 | 0.0954 | 0.0004 |
| P9 | -0.0450 | -0.0452 | 0.0002 | -0.0462 | 0.0012 |
| P10 | -0.0550 | -0.0558 | 0.0008 | -0.0558 | 0.0008 |
| P11 | -0.0800 | -0.0798 | 0.0002 | -0.0798 | 0.0002 |
| P12 | -0.0700 | -0.0700 | 0.0000 | -0.0700 | 0.0000 |
| P13 | -0.0900 | -0.0909 | 0.0009 | -0.0909 | 0.0009 |
| P14 | -0.1250 | -0.1227 | 0.0023 | -0.1227 | 0.0023 |
| P1-2 | 0.0669 | 0.0675 | 0.0006 | 0.0674 | 0.0005 |
| P1-5 | 0.1831 | 0.1831 | 0.0001 | 0.1832 | 0.0001 |
| P2-3 | 0.0086 | 0.0086 | 0.0000 | 0.0081 | 0.0004 |
| P2-4 | 0.0922 | 0.0916 | 0.0005 | 0.0918 | 0.0004 |
| P2-5 | 0.1162 | 0.1156 | 0.0006 | 0.1158 | 0.0004 |
| P3-4 | 0.0836 | 0.0830 | 0.0006 | 0.0836 | 0.0001 |
| P4-5 | 0.0240 | 0.0240 | 0.0000 | 0.0240 | 0.0000 |
| P4-7 | 0.0439 | 0.0433 | 0.0006 | 0.0436 | 0.0003 |
| P4-9 | 0.1678 | 0.1672 | 0.0006 | 0.1678 | 0.0001 |
| P5-6 | 0.2483 | 0.2477 | 0.0006 | 0.2479 | 0.0004 |
| P6-11 | 0.0368 | 0.0370 | 0.0002 | 0.0371 | 0.0003 |
| P6-12 | 0.0688 | 0.0688 | 0.0001 | 0.0688 | 0.0000 |
| P6-13 | 0.0676 | 0.0675 | 0.0001 | 0.0676 | 0.0000 |
| P7-8 | -0.0950 | -0.0954 | 0.0004 | -0.0954 | 0.0004 |
| P7-9 | 0.1239 | 0.1239 | 0.0000 | 0.1242 | 0.0003 |
| P9-10 | 0.0982 | 0.0986 | 0.0004 | 0.0985 | 0.0003 |
| P9-14 | 0.1486 | 0.1473 | 0.0012 | 0.1472 | 0.0013 |
| P10-11 | 0.0432 | 0.0428 | 0.0004 | 0.0427 | 0.0005 |
| P12-13 | -0.0012 | -0.0012 | 0.0001 | -0.0012 | 0.0000 |
| P13-14 | -0.0236 | -0.0246 | 0.0011 | -0.0245 | 0.0010 |
| **Sum** | | | **0.0168** | | **0.0164** |

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.176. Full redundancy case has better estimation results for all estimators.

Table 5.176: Estimation errors of case 3, multi reverse, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **0.4884** | **0.6137** | **0.0193** | **0.0239** | **0.0256** | **0.3013** | **0.0167** |
| **SUM** | **N/A** | **0.7251** | **0.5248** | **0.0287** | **0.0670** | **0.4184** | **1.5636** |

### Two reverse one large bad data

In the non-conforming case, two reverse one large bad data, $P_4 = -0.0599$, $P_{4-7} = -0.0441$ and $P_{7-9} = 0.9236$ are used to replace the original data. As presented, the last one is large error.

**Full redundancy**

The five system state variables are presented in Table 5.177, Table 5.178 shows the results and Table 5.179 illustrates the error between estimation and actual value.

Table 5.177: State Variable of case 2, two reverse, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0580 | -0.0658 | -0.0667 | -0.0656 | -0.0655 | -0.0674 |
| $\theta 3$ | -0.0573 | -0.0745 | -0.0748 | -0.0733 | -0.0744 | -0.0756 |
| $\theta 4$ | -0.1427 | -0.1587 | -0.1575 | -0.1561 | -0.1586 | -0.1592 |
| $\theta 5$ | -0.1910 | -0.1784 | -0.1824 | -0.1818 | -0.1791 | -0.1832 |
| $\theta 6$ | -0.4971 | -0.4139 | -0.4306 | -0.4301 | -0.4193 | -0.4310 |
| $\theta 7$ | -0.0097 | -0.1226 | -0.2002 | -0.1995 | -0.1185 | -0.2026 |
| $\theta 8$ | 0.2007 | -0.0272 | -0.1039 | -0.1037 | -0.0231 | -0.1072 |
| $\theta 9$ | -0.3669 | -0.2923 | -0.3252 | -0.3246 | -0.2965 | -0.3265 |
| $\theta 10$ | -0.5184 | -0.3967 | -0.4238 | -0.4234 | -0.4031 | -0.4253 |
| $\theta 11$ | -0.5568 | -0.4452 | -0.4675 | -0.4669 | -0.4517 | -0.4681 |
| $\theta 12$ | -0.5776 | -0.4805 | -0.4989 | -0.4988 | -0.4868 | -0.4998 |
| $\theta 13$ | -0.5818 | -0.4771 | -0.4981 | -0.4980 | -0.4837 | -0.4985 |
| $\theta 14$ | -0.5650 | -0.4461 | -0.4749 | -0.4747 | -0.4535 | -0.4739 |

Table 5.178: Estimation Results of case 3, two reverse, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2490 | 0.2442 | 0.2491 | 0.2475 | 0.2446 | 0.2506 |
| P2 | 0.1500 | 0.1483 | 0.1590 | 0.1483 | 0.1479 | 0.1488 | 0.1499 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0860 | 0.0755 | 0.0746 | 0.0750 | 0.0754 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **-0.0304** | **-0.0599** | **0.0615** | **0.0645** | **-0.0592** | **0.0591** |
| P5 | -0.0750 | -0.0751 | -0.0662 | -0.0751 | -0.0747 | -0.0756 | -0.0728 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0813 | -0.0744 | -0.0753 | -0.0749 | -0.0758 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.2798 | 0.1104 | -0.0139 | -0.0143 | 0.1227 | -0.0148 |
| P8 | 0.0950 | 0.0954 | 0.2104 | 0.0954 | 0.0963 | 0.0959 | 0.0954 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.2317 | -0.0452 | -0.0443 | -0.0447 | -0.0522 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.1131 | -0.0558 | -0.0550 | -0.0553 | -0.0579 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0982 | -0.0798 | -0.0806 | -0.0803 | -0.0811 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0762 | -0.0700 | -0.0692 | -0.0695 | -0.0706 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1058 | -0.0909 | -0.0900 | -0.0904 | -0.0915 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1813 | -0.1227 | -0.1265 | -0.1268 | -0.1269 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0580 | 0.0658 | 0.0667 | 0.0656 | 0.0655 | 0.0674 |
| P1-5 | 0.1831 | 0.1832 | 0.1910 | 0.1784 | 0.1824 | 0.1818 | 0.1791 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0007 | 0.0087 | 0.0081 | 0.0077 | 0.0088 | 0.0081 |
| P2-4 | 0.0922 | 0.0900 | 0.0846 | 0.0929 | 0.0908 | 0.0905 | 0.0931 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1330 | 0.1126 | 0.1157 | 0.1162 | 0.1135 | 0.1158 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P3-4 | 0.0836 | 0.0823 | 0.0853 | 0.0842 | 0.0827 | 0.0827 | 0.0842 | 0.0837 |
| P4-5 | 0.0240 | 0.0240 | 0.0483 | 0.0197 | 0.0248 | 0.0258 | 0.0204 | 0.0240 |
| **P4-7** | **0.0439** | **-0.0441** | **-0.1330** | **-0.0361** | **0.0426** | **0.0435** | **-0.0401** | **0.0433** |
| P4-9 | 0.1678 | 0.1682 | 0.2242 | 0.1336 | 0.1677 | 0.1685 | 0.1378 | 0.1673 |
| P5-6 | 0.2483 | 0.2520 | 0.3061 | 0.2355 | 0.2482 | 0.2483 | 0.2402 | 0.2478 |
| P6-11 | 0.0368 | 0.0372 | 0.0597 | 0.0313 | 0.0370 | 0.0368 | 0.0325 | 0.0371 |
| P6-12 | 0.0688 | 0.0688 | 0.0804 | 0.0666 | 0.0684 | 0.0687 | 0.0675 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.0847 | 0.0632 | 0.0676 | 0.0679 | 0.0644 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.2104 | -0.0954 | -0.0963 | -0.0959 | -0.0954 | -0.0954 |
| **P7-9** | **0.1239** | **0.9236** | **0.3572** | **0.1697** | **0.1250** | **0.1251** | **0.1780** | **0.1240** |
| P9-10 | 0.0982 | 0.0987 | 0.1515 | 0.1043 | 0.0986 | 0.0988 | 0.1066 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1982 | 0.1538 | 0.1497 | 0.1501 | 0.1571 | 0.1474 |
| P10-11 | 0.0432 | 0.0435 | 0.0385 | 0.0485 | 0.0437 | 0.0435 | 0.0487 | 0.0429 |
| P12-13 | -0.0012 | -0.0012 | 0.0043 | -0.0034 | -0.0008 | -0.0008 | -0.0031 | -0.0013 |
| P13-14 | -0.0236 | -0.0232 | -0.0168 | -0.0310 | -0.0232 | -0.0233 | -0.0302 | -0.0246 |

Table 5.179: Estimation errors of case 3, two reverse, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0010 | 0.0058 | 0.0009 | 0.0025 | 0.0054 | 0.0006 |
| P2 | 0.0017 | 0.0090 | 0.0017 | 0.0021 | 0.0012 | 0.0001 | 0.0017 |
| P3 | 0.0005 | 0.0110 | 0.0005 | 0.0004 | 0.0000 | 0.0004 | 0.0005 |
| **P4** | **0.1199** | **0.0904** | **0.1199** | **0.0015** | **0.0045** | **0.1192** | **0.0009** |
| P5 | 0.0001 | 0.0088 | 0.0001 | 0.0003 | 0.0006 | 0.0022 | 0.0001 |
| P6 | 0.0006 | 0.0063 | 0.0006 | 0.0003 | 0.0001 | 0.0008 | 0.0006 |
| P7 | 0.0002 | 0.2948 | 0.1254 | 0.0011 | 0.0007 | 0.1377 | 0.0002 |
| P8 | 0.0004 | 0.1154 | 0.0004 | 0.0013 | 0.0009 | 0.0004 | 0.0004 |
| P9 | 0.0002 | 0.1867 | 0.0002 | 0.0007 | 0.0003 | 0.0072 | 0.0002 |
| P10 | 0.0008 | 0.0581 | 0.0008 | 0.0000 | 0.0003 | 0.0029 | 0.0008 |
| P11 | 0.0002 | 0.0182 | 0.0002 | 0.0006 | 0.0003 | 0.0011 | 0.0000 |
| P12 | 0.0000 | 0.0062 | 0.0000 | 0.0008 | 0.0005 | 0.0006 | 0.0000 |
| P13 | 0.0009 | 0.0158 | 0.0009 | 0.0000 | 0.0004 | 0.0015 | 0.0009 |
| P14 | 0.0023 | 0.0563 | 0.0023 | 0.0015 | 0.0018 | 0.0019 | 0.0023 |
| P1-2 | 0.0011 | 0.0089 | 0.0011 | 0.0002 | 0.0013 | 0.0014 | 0.0005 |
| P1-5 | 0.0002 | 0.0079 | 0.0047 | 0.0007 | 0.0012 | 0.0040 | 0.0001 |
| P2-3 | 0.0001 | 0.0093 | 0.0001 | 0.0005 | 0.0009 | 0.0003 | 0.0004 |
| P2-4 | 0.0022 | 0.0075 | 0.0007 | 0.0013 | 0.0017 | 0.0009 | 0.0003 |
| P2-5 | 0.0003 | 0.0168 | 0.0036 | 0.0005 | 0.0001 | 0.0027 | 0.0004 |
| P3-4 | 0.0012 | 0.0017 | 0.0006 | 0.0008 | 0.0008 | 0.0007 | 0.0001 |
| P4-5 | 0.0001 | 0.0243 | 0.0044 | 0.0008 | 0.0017 | 0.0036 | 0.0001 |
| **P4-7** | **0.0880** | **0.1769** | **0.0800** | **0.0013** | **0.0005** | **0.0840** | **0.0006** |
| P4-9 | 0.0004 | 0.0564 | 0.0342 | 0.0001 | 0.0007 | 0.0300 | 0.0005 |
| P5-6 | 0.0037 | 0.0579 | 0.0128 | 0.0001 | 0.0000 | 0.0081 | 0.0005 |
| P6-11 | 0.0003 | 0.0229 | 0.0056 | 0.0001 | 0.0001 | 0.0044 | 0.0003 |
| P6-12 | 0.0000 | 0.0116 | 0.0022 | 0.0004 | 0.0001 | 0.0013 | 0.0000 |
| P6-13 | 0.0009 | 0.0171 | 0.0044 | 0.0001 | 0.0003 | 0.0032 | 0.0001 |
| P7-8 | 0.0004 | 0.1154 | 0.0004 | 0.0013 | 0.0009 | 0.0004 | 0.0004 |
| **P7-9** | **0.7997** | **0.2333** | **0.0458** | **0.0011** | **0.0012** | **0.0541** | **0.0001** |
| P9-10 | 0.0006 | 0.0534 | 0.0062 | 0.0005 | 0.0007 | 0.0084 | 0.0006 |
| P9-14 | 0.0020 | 0.0496 | 0.0052 | 0.0012 | 0.0015 | 0.0085 | 0.0012 |
| P10-11 | 0.0003 | 0.0047 | 0.0053 | 0.0005 | 0.0003 | 0.0055 | 0.0003 |
| P12-13 | 0.0000 | 0.0055 | 0.0022 | 0.0004 | 0.0004 | 0.0019 | 0.0001 |
| P13-14 | 0.0004 | 0.0067 | 0.0075 | 0.0004 | 0.0003 | 0.0066 | 0.0011 |
| **SUM** | **1.0406** | **1.7657** | **0.4859** | **0.0239** | **0.0286** | **0.5113** | **0.0167** |

### Median redundancy

The same bad data as in full redundancy case, the following three tables shows the results and error with actual value of the system.

Table 5.180: State Variable of case 3, two reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0562 | -0.0658 | -0.0664 | -0.0663 | -0.0649 | -0.0658 |
| $\theta 3$ | -0.0529 | -0.0745 | -0.0744 | -0.0740 | -0.0737 | -0.0730 |
| $\theta 4$ | -0.1464 | -0.1587 | -0.1574 | -0.1569 | -0.1566 | -0.1557 |
| $\theta 5$ | -0.1956 | -0.1784 | -0.1826 | -0.1831 | -0.1769 | -0.1829 |
| $\theta 6$ | -0.5107 | -0.4139 | -0.4311 | -0.4346 | -0.4098 | -0.4349 |
| $\theta 7$ | -0.0443 | -0.1225 | -0.2013 | -0.1123 | -0.1165 | 0.5902 |
| $\theta 8$ | 0.1660 | -0.0271 | -0.1053 | -0.0174 | -0.0211 | 0.6856 |
| $\theta 9$ | -0.3675 | -0.2922 | -0.3258 | -0.3317 | -0.2892 | -0.3334 |
| $\theta 10$ | -0.5209 | -0.3966 | -0.4241 | -0.4309 | -0.3920 | -0.4321 |
| $\theta 11$ | -0.5681 | -0.4451 | -0.4672 | -0.4748 | -0.4394 | -0.4750 |
| $\theta 12$ | -0.5969 | -0.4805 | -0.5001 | -0.5033 | -0.4764 | -0.5040 |
| $\theta 13$ | -0.6051 | -0.4771 | -0.4995 | -0.5023 | -0.4733 | -0.5031 |
| $\theta 14$ | -0.5860 | -0.4460 | -0.4757 | -0.4786 | -0.4443 | -0.4796 |

Table 5.181: Estimation Results of case 3, two reverse, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2518 | 0.2442 | 0.2490 | 0.2494 | 0.2418 | 0.2487 |
| P2 | 0.1500 | 0.1483 | 0.1700 | 0.1483 | 0.1489 | 0.1488 | 0.1476 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0969 | 0.0755 | 0.0749 | 0.0752 | 0.0741 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **-0.0157** | **-0.0599** | **0.0637** | **-0.0171** | **-0.0617** | **-0.7134** |
| P5 | -0.0750 | -0.0751 | -0.0689 | -0.0751 | -0.0757 | -0.0746 | -0.0763 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0773 | -0.0744 | -0.0750 | -0.0749 | -0.0730 | -0.0744 |
| P7 | -0.0150 | -0.0147 | 0.2150 | 0.1104 | -0.0154 | 0.1691 | 0.1173 | 1.5740 |
| P8 | 0.0950 | 0.0954 | 0.2103 | 0.0954 | 0.0960 | 0.0949 | 0.0954 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.1723 | -0.0452 | -0.0446 | -0.1481 | -0.0474 | -0.8565 |
| P10 | -0.0550 | -0.0558 | -0.1062 | -0.0558 | -0.0552 | -0.0553 | -0.0554 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.1046 | -0.0798 | -0.0792 | -0.0841 | -0.0770 | -0.0831 |
| P12 | -0.0700 | -0.0700 | -0.0781 | -0.0700 | -0.0697 | -0.0697 | -0.0697 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1215 | -0.0909 | -0.0915 | -0.0904 | -0.0895 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1995 | -0.1227 | -0.1262 | -0.1232 | -0.1260 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0562 | 0.0658 | 0.0664 | 0.0663 | 0.0649 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.1956 | 0.1784 | 0.1826 | 0.1831 | 0.1769 | 0.1829 |
| P2-3 | 0.0086 | 0.0086 | -0.0034 | 0.0087 | 0.0080 | 0.0077 | 0.0088 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.0902 | 0.0929 | 0.0910 | 0.0906 | 0.0917 | 0.0899 |
| P2-5 | 0.1162 | N/A | 0.1394 | 0.1126 | 0.1163 | 0.1168 | 0.1120 | 0.1171 |
| P3-4 | 0.0836 | 0.0823 | 0.0936 | 0.0842 | 0.0830 | 0.0829 | 0.0829 | 0.0827 |
| P4-5 | 0.0240 | N/A | 0.0491 | 0.0197 | 0.0253 | 0.0262 | 0.0203 | 0.0272 |
| **P4-7** | **0.0439** | **-0.0441** | **-0.1021** | **-0.0361** | **0.0439** | **-0.0446** | **-0.0401** | **-0.7458** |
| P4-9 | 0.1678 | N/A | 0.2211 | 0.1336 | 0.1684 | 0.1748 | 0.1326 | 0.1778 |
| P5-6 | 0.2483 | 0.2520 | 0.3152 | 0.2355 | 0.2485 | 0.2515 | 0.2329 | 0.2520 |
| P6-11 | 0.0368 | N/A | 0.0574 | 0.0313 | 0.0361 | 0.0402 | 0.0296 | 0.0402 |
| P6-12 | 0.0688 | 0.0688 | 0.0862 | 0.0666 | 0.0690 | 0.0687 | 0.0667 | 0.0691 |
| P6-13 | 0.0676 | N/A | 0.0943 | 0.0632 | 0.0684 | 0.0677 | 0.0636 | 0.0683 |
| P7-8 | -0.0950 | -0.0954 | -0.2103 | -0.0954 | -0.0960 | -0.0949 | -0.0954 | -0.0954 |
| **P7-9** | **0.1239** | **0.9236** | **0.3232** | **0.1697** | **0.1245** | **0.2194** | **0.1727** | **0.9236** |
| P9-10 | 0.0982 | 0.0987 | 0.1534 | 0.1043 | 0.0983 | 0.0992 | 0.1028 | 0.0987 |
| P9-14 | 0.1486 | N/A | 0.2185 | 0.1538 | 0.1499 | 0.1469 | 0.1551 | 0.1462 |
| P10-11 | 0.0432 | 0.0435 | 0.0472 | 0.0485 | 0.0431 | 0.0439 | 0.0474 | 0.0429 |
| P12-13 | -0.0012 | N/A | 0.0081 | -0.0034 | -0.0007 | -0.0010 | -0.0031 | -0.0009 |
| P13-14 | -0.0236 | -0.0232 | -0.0191 | -0.0311 | -0.0238 | -0.0237 | -0.0290 | -0.0235 |

Table 5.182: Estimation errors of case 3, two reverse, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0018 | 0.0058 | 0.0010 | 0.0006 | 0.0082 | 0.0013 |
| P2 | 0.0200 | 0.0017 | 0.0011 | 0.0012 | 0.0024 | 0.0017 |
| P3 | 0.0219 | 0.0005 | 0.0001 | 0.0002 | 0.0009 | 0.0005 |
| **P4** | **0.0757** | **0.1199** | **0.0037** | **0.0771** | **0.1217** | **0.7734** |
| P5 | 0.0061 | 0.0001 | 0.0007 | 0.0004 | 0.0013 | 0.0001 |
| P6 | 0.0023 | 0.0006 | 0.0000 | 0.0001 | 0.0020 | 0.0006 |
| P7 | 0.2300 | 0.1254 | 0.0004 | 0.1841 | 0.1323 | 1.5890 |
| P8 | 0.1153 | 0.0004 | 0.0010 | 0.0001 | 0.0004 | 0.0004 |
| P9 | 0.1273 | 0.0002 | 0.0004 | 0.1031 | 0.0024 | 0.8115 |
| P10 | 0.0512 | 0.0008 | 0.0002 | 0.0003 | 0.0004 | 0.0008 |
| P11 | 0.0246 | 0.0002 | 0.0008 | 0.0041 | 0.0030 | 0.0031 |
| P12 | 0.0081 | 0.0000 | 0.0003 | 0.0003 | 0.0003 | 0.0000 |
| P13 | 0.0315 | 0.0009 | 0.0015 | 0.0004 | 0.0005 | 0.0009 |
| P14 | 0.0745 | 0.0023 | 0.0012 | 0.0018 | 0.0010 | 0.0023 |
| P1-2 | 0.0107 | 0.0011 | 0.0005 | 0.0006 | 0.0020 | 0.0011 |
| P1-5 | 0.0125 | 0.0047 | 0.0005 | 0.0000 | 0.0062 | 0.0002 |
| P2-3 | 0.0119 | 0.0001 | 0.0006 | 0.0009 | 0.0002 | 0.0014 |
| P2-4 | 0.0020 | 0.0007 | 0.0012 | 0.0016 | 0.0005 | 0.0023 |
| P2-5 | 0.0232 | 0.0036 | 0.0001 | 0.0006 | 0.0042 | 0.0009 |
| P3-4 | 0.0100 | 0.0006 | 0.0006 | 0.0007 | 0.0007 | 0.0009 |
| P4-5 | 0.0251 | 0.0043 | 0.0013 | 0.0022 | 0.0037 | 0.0032 |
| **P4-7** | **0.1460** | **0.0800** | **0.0000** | **0.0885** | **0.0840** | **0.7897** |
| P4-9 | 0.0533 | 0.0342 | 0.0006 | 0.0070 | 0.0352 | 0.0100 |
| P5-6 | 0.0669 | 0.0128 | 0.0002 | 0.0032 | 0.0154 | 0.0037 |
| P6-11 | 0.0205 | 0.0056 | 0.0007 | 0.0034 | 0.0072 | 0.0034 |
| P6-12 | 0.0174 | 0.0022 | 0.0002 | 0.0001 | 0.0022 | 0.0003 |
| P6-13 | 0.0267 | 0.0044 | 0.0007 | 0.0001 | 0.0041 | 0.0006 |
| P7-8 | 0.1153 | 0.0004 | 0.0010 | 0.0001 | 0.0004 | 0.0004 |
| **P7-9** | **0.1993** | **0.0458** | **0.0006** | **0.0955** | **0.0488** | **0.7997** |
| P9-10 | 0.0553 | 0.0062 | 0.0002 | 0.0010 | 0.0046 | 0.0005 |
| P9-14 | 0.0700 | 0.0052 | 0.0014 | 0.0017 | 0.0065 | 0.0024 |
| P10-11 | 0.0040 | 0.0054 | 0.0000 | 0.0007 | 0.0042 | 0.0003 |
| P12-13 | 0.0093 | 0.0022 | 0.0005 | 0.0002 | 0.0019 | 0.0003 |
| P13-14 | 0.0045 | 0.0075 | 0.0002 | 0.0001 | 0.0055 | 0.0001 |
| **SUM** | **1.6740** | **0.4860** | **0.0233** | **0.5820** | **0.5143** | **4.8070** |

Since it's same location with previous case, LTAV still generate extremely large error. But as the same analysis with previous case, LTAV can still generate good estimation with different locations of the same type of error.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.183.Compared with full redundancy case, median redundancy case generate much worse estimation. For those

with a little better estimation, the difference is neglectable.

Table 5.183: Estimation errors of case 3, two reverse, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.0406** | **1.7657** | **0.4859** | **0.0239** | **0.0286** | **0.5113** | **0.0167** |
| **SUM** | **N/A** | **1.6740** | **0.4860** | **0.0233** | **0.5820** | **0.5143** | **4.8070** |

### Two large one reverse bad data

Compared with the former case, another reverse bad data are changed to large error, $P_4 = -0.0599$, $P_{4-7} = 0.9441$ and $P_{7-9} = 0.9236$, others keep the same.

### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.154 gives the state variables, and the later two tables shows the estimation results and error with actual values.

Table 5.184: State variable of case 3, two large, 14-bus DC system, full redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0667 | -0.0658 | -0.0667 | -0.0679 | -0.0642 | -0.0674 |
| $\theta 3$ | -0.0593 | -0.0734 | -0.0748 | -0.0771 | -0.0718 | -0.0756 |
| $\theta 4$ | -0.2180 | -0.1565 | -0.1575 | -0.1590 | -0.1545 | -0.1592 |
| $\theta 5$ | -0.2189 | -0.1817 | -0.1824 | -0.1834 | -0.1803 | -0.1832 |
| $\theta 6$ | -0.5847 | -0.4294 | -0.4306 | -0.4323 | -0.4295 | -0.4310 |
| $\theta 7$ | -0.3691 | -0.2001 | -0.2002 | -0.2024 | -0.1986 | -0.2026 |
| $\theta 8$ | -0.2850 | -0.1047 | -0.1039 | -0.1065 | -0.1031 | -0.1072 |
| $\theta 9$ | -0.5602 | -0.3244 | -0.3252 | -0.3274 | -0.3234 | -0.3265 |
| $\theta 10$ | -0.6795 | -0.4232 | -0.4238 | -0.4260 | -0.4227 | -0.4253 |
| $\theta 11$ | -0.6803 | -0.4662 | -0.4675 | -0.4693 | -0.4663 | -0.4681 |
| $\theta 12$ | -0.6759 | -0.4982 | -0.4989 | -0.5011 | -0.4985 | -0.4998 |
| $\theta 13$ | -0.6950 | -0.4970 | -0.4981 | -0.5004 | -0.4972 | -0.4985 |
| $\theta 14$ | -0.7209 | -0.4726 | -0.4749 | -0.4775 | -0.4727 | -0.4739 |

Table 5.185: Estimation Results of case 2, two large, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2856 | 0.2475 | 0.2491 | 0.2514 | 0.2445 | 0.2506 |
| P2 | 0.1500 | 0.1483 | 0.2293 | 0.1483 | 0.1479 | 0.1478 | 0.1497 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.1660 | 0.0755 | 0.0746 | 0.0728 | 0.0751 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **0.1843** | **0.0629** | **0.0615** | **0.0632** | **0.0659** | **0.0591** |
| P5 | -0.0750 | -0.0751 | -0.0061 | -0.0750 | -0.0747 | -0.0746 | -0.0730 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0690 | -0.0744 | -0.0753 | -0.0749 | -0.0756 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0439 | -0.0147 | -0.0139 | -0.0143 | -0.0147 | -0.0147 |
| P8 | 0.0950 | 0.0954 | 0.0840 | 0.0954 | 0.0963 | 0.0959 | 0.0954 | 0.0954 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P9 | -0.0450 | -0.0452 | -0.2533 | -0.0452 | -0.0443 | -0.0447 | -0.0451 | -0.0452 |
| P10 | -0.0550 | -0.0558 | -0.1186 | -0.0558 | -0.0550 | -0.0553 | -0.0557 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.0963 | -0.0798 | -0.0806 | -0.0803 | -0.0805 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0720 | -0.0700 | -0.0692 | -0.0695 | -0.0704 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1034 | -0.0909 | -0.0900 | -0.0904 | -0.0909 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.1867 | -0.1238 | -0.1265 | -0.1272 | -0.1248 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0667 | 0.0658 | 0.0667 | 0.0679 | 0.0642 | 0.0674 |
| P1-5 | 0.1831 | 0.1832 | 0.2189 | 0.1817 | 0.1824 | 0.1834 | 0.1803 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0074 | 0.0076 | 0.0081 | 0.0091 | 0.0076 | 0.0081 |
| P2-4 | 0.0922 | 0.0900 | 0.1513 | 0.0907 | 0.0908 | 0.0911 | 0.0902 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1522 | 0.1159 | 0.1157 | 0.1155 | 0.1161 | 0.1158 |
| P3-4 | 0.0836 | 0.0823 | 0.1586 | 0.0831 | 0.0827 | 0.0819 | 0.0827 | 0.0837 |
| P4-5 | 0.0240 | 0.0240 | 0.0009 | 0.0252 | 0.0248 | 0.0244 | 0.0258 | 0.0240 |
| **P4-7** | **0.0439** | **0.9441** | **0.1511** | **0.0436** | **0.0426** | **0.0434** | **0.0441** | **0.0433** |
| P4-9 | 0.1678 | 0.1682 | 0.3422 | 0.1679 | 0.1677 | 0.1684 | 0.1689 | 0.1673 |
| P5-6 | 0.2483 | 0.2520 | 0.3658 | 0.2477 | 0.2482 | 0.2489 | 0.2492 | 0.2478 |
| P6-11 | 0.0368 | 0.0372 | 0.0955 | 0.0368 | 0.0370 | 0.0370 | 0.0369 | 0.0371 |
| P6-12 | 0.0688 | 0.0688 | 0.0911 | 0.0688 | 0.0684 | 0.0688 | 0.0690 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.1102 | 0.0677 | 0.0676 | 0.0681 | 0.0677 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.0840 | -0.0954 | -0.0963 | -0.0959 | -0.0954 | -0.0954 |
| **P7-9** | **0.1239** | **0.9236** | **0.1911** | **0.1243** | **0.1250** | **0.1250** | **0.1248** | **0.1240** |
| P9-10 | 0.0982 | 0.0987 | 0.1193 | 0.0988 | 0.0986 | 0.0986 | 0.0993 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1607 | 0.1482 | 0.1497 | 0.1501 | 0.1493 | 0.1474 |
| P10-11 | 0.0432 | 0.0435 | 0.0007 | 0.0430 | 0.0437 | 0.0433 | 0.0436 | 0.0429 |
| P12-13 | -0.0012 | -0.0012 | 0.0191 | -0.0012 | -0.0008 | -0.0007 | -0.0013 | -0.0013 |
| P13-14 | -0.0236 | -0.0232 | 0.0260 | -0.0244 | -0.0232 | -0.0229 | -0.0245 | -0.0246 |

Table 5.186: Estimation errors of case 2, two large, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0356 | 0.0025 | 0.0009 | 0.0014 | 0.0055 | 0.0006 |
| P2 | 0.0017 | 0.0793 | 0.0017 | 0.0021 | 0.0022 | 0.0003 | 0.0017 |
| P3 | 0.0005 | 0.0910 | 0.0005 | 0.0004 | 0.0022 | 0.0001 | 0.0005 |
| **P4** | **0.1199** | **0.1243** | **0.0029** | **0.0015** | **0.0032** | **0.0059** | **0.0009** |
| P5 | 0.0001 | 0.0689 | 0.0000 | 0.0003 | 0.0004 | 0.0020 | 0.0001 |
| P6 | 0.0006 | 0.0060 | 0.0006 | 0.0003 | 0.0001 | 0.0006 | 0.0006 |
| P7 | 0.0002 | 0.0290 | 0.0002 | 0.0011 | 0.0007 | 0.0003 | 0.0002 |
| P8 | 0.0004 | 0.0110 | 0.0004 | 0.0013 | 0.0009 | 0.0004 | 0.0004 |
| P9 | 0.0002 | 0.2083 | 0.0002 | 0.0007 | 0.0003 | 0.0001 | 0.0002 |
| P10 | 0.0008 | 0.0636 | 0.0008 | 0.0000 | 0.0003 | 0.0007 | 0.0008 |
| P11 | 0.0002 | 0.0163 | 0.0002 | 0.0006 | 0.0003 | 0.0005 | 0.0000 |
| P12 | 0.0000 | 0.0020 | 0.0000 | 0.0008 | 0.0005 | 0.0004 | 0.0000 |
| P13 | 0.0009 | 0.0134 | 0.0009 | 0.0000 | 0.0004 | 0.0009 | 0.0009 |
| P14 | 0.0023 | 0.0617 | 0.0012 | 0.0015 | 0.0022 | 0.0002 | 0.0023 |
| P1-2 | 0.0011 | 0.0002 | 0.0011 | 0.0002 | 0.0010 | 0.0027 | 0.0005 |
| P1-5 | 0.0002 | 0.0358 | 0.0014 | 0.0007 | 0.0004 | 0.0028 | 0.0001 |
| P2-3 | 0.0001 | 0.0160 | 0.0010 | 0.0005 | 0.0006 | 0.0010 | 0.0004 |
| P2-4 | 0.0022 | 0.0591 | 0.0015 | 0.0013 | 0.0011 | 0.0019 | 0.0003 |
| P2-5 | 0.0003 | 0.0360 | 0.0003 | 0.0005 | 0.0007 | 0.0001 | 0.0004 |
| P3-4 | 0.0012 | 0.0751 | 0.0005 | 0.0008 | 0.0016 | 0.0009 | 0.0001 |
| P4-5 | 0.0001 | 0.0231 | 0.0011 | 0.0008 | 0.0004 | 0.0018 | 0.0001 |
| **P4-7** | **0.9002** | **0.1072** | **0.0003** | **0.0013** | **0.0005** | **0.0002** | **0.0006** |
| P4-9 | 0.0004 | 0.1744 | 0.0001 | 0.0001 | 0.0006 | 0.0011 | 0.0005 |
| P5-6 | 0.0037 | 0.1176 | 0.0006 | 0.0001 | 0.0006 | 0.0009 | 0.0005 |
| P6-11 | 0.0003 | 0.0587 | 0.0000 | 0.0001 | 0.0001 | 0.0000 | 0.0003 |
| P6-12 | 0.0000 | 0.0223 | 0.0000 | 0.0004 | 0.0000 | 0.0002 | 0.0000 |
| P6-13 | 0.0009 | 0.0426 | 0.0000 | 0.0001 | 0.0005 | 0.0001 | 0.0001 |
| P7-8 | 0.0004 | 0.0110 | 0.0004 | 0.0013 | 0.0009 | 0.0004 | 0.0004 |
| **P7-9** | **0.7997** | **0.0672** | **0.0004** | **0.0011** | **0.0011** | **0.0009** | **0.0001** |
| P9-10 | 0.0006 | 0.0212 | 0.0006 | 0.0005 | 0.0004 | 0.0012 | 0.0006 |
| P9-14 | 0.0020 | 0.0122 | 0.0003 | 0.0012 | 0.0015 | 0.0007 | 0.0012 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P10-11 | 0.0003 | 0.0424 | 0.0002 | 0.0005 | 0.0001 | 0.0005 | 0.0003 |
| P12-13 | 0.0000 | 0.0203 | 0.0000 | 0.0004 | 0.0005 | 0.0001 | 0.0001 |
| P13-14 | 0.0004 | 0.0495 | 0.0008 | 0.0004 | 0.0006 | 0.0010 | 0.0011 |
| **SUM** | **1.8528** | **1.8021** | **0.0230** | **0.0239** | **0.0283** | **0.0367** | **0.0167** |

### Median redundancy

Table 5.187 presents the state variables, and the later two tables shows the estimation results and error with actual values.

Table 5.187: State variable of case 3, two large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0742 | -0.0658 | -0.0664 | -0.0663 | -0.0698 | -0.0658 |
| $\theta 3$ | -0.0743 | -0.0744 | -0.0744 | -0.0742 | -0.0818 | -0.0730 |
| $\theta 4$ | -0.2522 | -0.1585 | -0.1574 | -0.1571 | -0.1765 | -0.1557 |
| $\theta 5$ | -0.2343 | -0.1824 | -0.1826 | -0.1827 | -0.1792 | -0.1829 |
| $\theta 6$ | -0.6177 | -0.4301 | -0.4311 | -0.4318 | -0.4048 | -0.4349 |
| $\theta 7$ | -0.4085 | -0.2018 | -0.2013 | -0.2021 | -0.1936 | -1.0998 |
| $\theta 8$ | -0.3273 | -0.1064 | -0.1065 | -0.1072 | -0.0914 | -1.0044 |
| $\theta 9$ | -0.6029 | -0.3257 | -0.3258 | -0.3268 | -0.3116 | -0.3334 |
| $\theta 10$ | -0.7388 | -0.4243 | -0.4241 | -0.4260 | -0.4001 | -0.4321 |
| $\theta 11$ | -0.7405 | -0.4671 | -0.4672 | -0.4690 | -0.4396 | -0.4750 |
| $\theta 12$ | -0.7098 | -0.4988 | -0.5006 | -0.5006 | -0.4680 | -0.5040 |
| $\theta 13$ | -0.7424 | -0.4976 | -0.4997 | -0.4999 | -0.4667 | -0.5031 |
| $\theta 14$ | -0.7759 | -0.4730 | -0.4759 | -0.4762 | -0.4447 | -0.4796 |

Table 5.188: Estimation Results of case 3, two large, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.3084 | 0.2482 | 0.2490 | 0.2490 | 0.2490 | 0.2487 |
| P2 | 0.1500 | 0.1483 | 0.2641 | 0.1521 | 0.1489 | 0.1488 | 0.1584 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.1778 | 0.0755 | 0.0749 | 0.0750 | 0.0827 | 0.0755 |
| **P4** | **0.0600** | **-0.0599** | **0.1330** | **0.0576** | **0.0638** | **0.0666** | **-0.0466** | **0.9765** |
| P5 | -0.0750 | -0.0751 | 0.0070 | -0.0751 | -0.0757 | -0.0756 | -0.0657 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0437 | -0.0744 | -0.0743 | -0.0749 | -0.0658 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.0432 | -0.0148 | -0.0142 | -0.0153 | -0.0013 | -1.8058 |
| P8 | 0.0950 | 0.0954 | 0.0812 | 0.0954 | 0.0948 | 0.0949 | 0.1022 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.2362 | -0.0452 | -0.0446 | -0.0457 | -0.0314 | 0.8334 |
| P10 | -0.0550 | -0.0558 | -0.1342 | -0.0558 | -0.0552 | -0.0561 | -0.0491 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.1245 | -0.0798 | -0.0792 | -0.0803 | -0.0742 | -0.0831 |
| P12 | -0.0700 | -0.0700 | -0.0595 | -0.0700 | -0.0704 | -0.0695 | -0.0644 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1239 | -0.0909 | -0.0915 | -0.0912 | -0.0827 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.2064 | -0.1227 | -0.1263 | -0.1257 | -0.1111 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0742 | 0.0658 | 0.0664 | 0.0663 | 0.0698 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.2343 | 0.1824 | 0.1826 | 0.1827 | 0.1792 | 0.1829 |
| P2-3 | 0.0086 | 0.0086 | 0.0001 | 0.0086 | 0.0080 | 0.0079 | 0.0120 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.1781 | 0.0927 | 0.0910 | 0.0908 | 0.1067 | 0.0899 |
| P2-5 | 0.1162 | N/A | 0.1601 | 0.1166 | 0.1163 | 0.1164 | 0.1094 | 0.1171 |
| P3-4 | 0.0836 | 0.0823 | 0.1780 | 0.0841 | 0.0830 | 0.0829 | 0.0947 | 0.0827 |
| P4-5 | 0.0240 | N/A | -0.0180 | 0.0239 | 0.0253 | 0.0256 | 0.0027 | 0.0272 |
| **P4-7** | **0.0439** | **0.9441** | **0.1563** | **0.0433** | **0.0439** | **0.0450** | **0.0171** | **0.9441** |
| P4-9 | 0.1678 | N/A | 0.3507 | 0.1672 | 0.1685 | 0.1697 | 0.1351 | 0.1778 |
| P5-6 | 0.2483 | 0.2520 | 0.3834 | 0.2477 | 0.2485 | 0.2491 | 0.2256 | 0.2520 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P6-11 | 0.0368 | N/A | 0.1228 | 0.0370 | 0.0361 | 0.0372 | 0.0347 | 0.0402 |
| P6-12 | 0.0688 | 0.0688 | 0.0921 | 0.0688 | 0.0695 | 0.0688 | 0.0632 | 0.0691 |
| P6-13 | 0.0676 | N/A | 0.1248 | 0.0675 | 0.0686 | 0.0681 | 0.0619 | 0.0683 |
| P7-8 | -0.0950 | -0.0954 | -0.0812 | -0.0954 | -0.0948 | -0.0949 | -0.1022 | -0.0954 |
| **P7-9** | **0.1239** | **0.9236** | **0.1944** | **0.1239** | **0.1245** | **0.1246** | **0.1180** | **-0.7663** |
| P9-10 | 0.0982 | 0.0987 | 0.1359 | 0.0986 | 0.0983 | 0.0992 | 0.0886 | 0.0987 |
| P9-14 | 0.1486 | N/A | 0.1730 | 0.1473 | 0.1501 | 0.1494 | 0.1331 | 0.1462 |
| P10-11 | 0.0432 | 0.0435 | 0.0017 | 0.0428 | 0.0431 | 0.0431 | 0.0394 | 0.0429 |
| P12-13 | -0.0012 | N/A | 0.0326 | -0.0012 | -0.0009 | -0.0007 | -0.0013 | -0.0009 |
| P13-14 | -0.0236 | -0.0232 | 0.0334 | -0.0246 | -0.0238 | -0.0237 | -0.0220 | -0.0235 |

Table 5.189: Estimation errors of case 3, two large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0584 | 0.0018 | 0.0010 | 0.0010 | 0.0010 | 0.0013 |
| P2 | 0.1141 | 0.0021 | 0.0011 | 0.0012 | 0.0084 | 0.0017 |
| P3 | 0.1028 | 0.0005 | 0.0001 | 0.0000 | 0.0077 | 0.0005 |
| **P4** | **0.0730** | **0.0024** | **0.0038** | **0.0066** | **0.1066** | **0.9165** |
| P5 | 0.0820 | 0.0001 | 0.0007 | 0.0006 | 0.0093 | 0.0001 |
| P6 | 0.0313 | 0.0006 | 0.0007 | 0.0001 | 0.0092 | 0.0006 |
| P7 | 0.0282 | 0.0002 | 0.0008 | 0.0003 | 0.0137 | 1.7908 |
| P8 | 0.0138 | 0.0004 | 0.0002 | 0.0001 | 0.0072 | 0.0004 |
| P9 | 0.1912 | 0.0002 | 0.0004 | 0.0007 | 0.0136 | 0.8784 |
| P10 | 0.0792 | 0.0008 | 0.0002 | 0.0011 | 0.0059 | 0.0008 |
| P11 | 0.0445 | 0.0002 | 0.0008 | 0.0003 | 0.0058 | 0.0031 |
| P12 | 0.0105 | 0.0000 | 0.0004 | 0.0005 | 0.0056 | 0.0000 |
| P13 | 0.0339 | 0.0009 | 0.0015 | 0.0012 | 0.0073 | 0.0009 |
| P14 | 0.0814 | 0.0023 | 0.0013 | 0.0007 | 0.0139 | 0.0023 |
| P1-2 | 0.0073 | 0.0011 | 0.0005 | 0.0006 | 0.0029 | 0.0011 |
| P1-5 | 0.0512 | 0.0007 | 0.0005 | 0.0004 | 0.0039 | 0.0002 |
| P2-3 | 0.0085 | 0.0000 | 0.0006 | 0.0007 | 0.0034 | 0.0014 |
| P2-4 | 0.0859 | 0.0005 | 0.0012 | 0.0014 | 0.0146 | 0.0023 |
| P2-5 | 0.0439 | 0.0004 | 0.0001 | 0.0002 | 0.0068 | 0.0009 |
| P3-4 | 0.0944 | 0.0005 | 0.0006 | 0.0007 | 0.0111 | 0.0009 |
| P4-5 | 0.0420 | 0.0001 | 0.0013 | 0.0016 | 0.0213 | 0.0032 |
| **P4-7** | **0.1124** | **0.0006** | **0.0000** | **0.0011** | **0.0268** | **0.9002** |
| P4-9 | 0.1828 | 0.0006 | 0.0007 | 0.0019 | 0.0327 | 0.0100 |
| P5-6 | 0.1351 | 0.0006 | 0.0002 | 0.0008 | 0.0227 | 0.0037 |
| P6-11 | 0.0860 | 0.0002 | 0.0007 | 0.0004 | 0.0021 | 0.0034 |
| P6-12 | 0.0233 | 0.0000 | 0.0007 | 0.0000 | 0.0056 | 0.0003 |
| P6-13 | 0.0571 | 0.0001 | 0.0010 | 0.0005 | 0.0057 | 0.0006 |
| P7-8 | 0.0138 | 0.0004 | 0.0002 | 0.0001 | 0.0072 | 0.0004 |
| **P7-9** | **0.0705** | **0.0000** | **0.0006** | **0.0007** | **0.0059** | **0.8902** |
| P9-10 | 0.0377 | 0.0004 | 0.0002 | 0.0010 | 0.0096 | 0.0005 |
| P9-14 | 0.0244 | 0.0013 | 0.0015 | 0.0009 | 0.0154 | 0.0024 |
| P10-11 | 0.0415 | 0.0004 | 0.0001 | 0.0001 | 0.0037 | 0.0003 |
| P12-13 | 0.0338 | 0.0000 | 0.0003 | 0.0005 | 0.0001 | 0.0003 |
| P13-14 | 0.0570 | 0.0010 | 0.0002 | 0.0001 | 0.0015 | 0.0001 |
| **SUM** | **2.1529** | **0.0217** | **0.0238** | **0.0283** | **0.4184** | **5.4199** |

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.190. For LTS and LTAV, median

redundancy have much worse results than full redundancy case, this is because that with fewer available data, when some more data are trimmed, the local redundancy are very small and generate bad estimation.

Table 5.190: Estimation errors of case 3, two large, 14-bus DC system, comparison

| | Meas. | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|---|
| **SUM** | **1.8528** | **1.8021** | **0.0230** | **0.0239** | **0.0283** | **0.0367** | **0.0167** |
| **SUM** | **N/A** | **2.1529** | **0.0217** | **0.0238** | **0.0283** | **0.4184** | **5.4199** |

### Three large bad data

Compared with the base cases, $P_4 = 0.9599$, $P_{4-7} = 0.9441$ and $P_{7-9} = 0.9236$ are introduced to simulate the three large bad data in both full and median redundancy case. The results are presented below.

#### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.191 gives the state variables, and the later two tables shows the estimation results and error with actual values.

Table 5.191: State variable of case 3, multi large, 14-bus DC system, full redundancy

| | WLS | LAV | LMS | LMR | LTS | LTAV |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0348 | -0.0655 | -0.0667 | -0.0679 | -0.0642 | -0.0675 |
| $\theta 3$ | -0.0183 | -0.0731 | -0.0748 | -0.0771 | -0.0718 | -0.0756 |
| $\theta 4$ | -0.0493 | -0.1555 | -0.1575 | -0.1590 | -0.1545 | -0.1593 |
| $\theta 5$ | -0.1963 | -0.1817 | -0.1824 | -0.1834 | -0.1803 | -0.1832 |
| $\theta 6$ | -0.6050 | -0.4309 | -0.4306 | -0.4323 | -0.4295 | -0.4312 |
| $\theta 7$ | -0.3422 | -0.2001 | -0.2002 | -0.2024 | -0.1986 | -1.1034 |
| $\theta 8$ | -0.2959 | -0.1047 | -0.1039 | -0.1065 | -0.1031 | -1.0080 |
| $\theta 9$ | -0.5411 | -0.3253 | -0.3252 | -0.3274 | -0.3234 | -0.3268 |
| $\theta 10$ | -0.6967 | -0.4246 | -0.4238 | -0.4260 | -0.4227 | -0.4255 |
| $\theta 11$ | -0.7061 | -0.4681 | -0.4675 | -0.4693 | -0.4663 | -0.4683 |
| $\theta 12$ | -0.7043 | -0.5001 | -0.4989 | -0.5011 | -0.4985 | -0.4999 |
| $\theta 13$ | -0.7214 | -0.4993 | -0.4981 | -0.5004 | -0.4972 | -0.4987 |
| $\theta 14$ | -0.7383 | -0.4759 | -0.4749 | -0.4775 | -0.4727 | -0.4741 |

Table 5.192: Estimation Results of case 3, multi large, 14-bus DC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2310 | 0.2472 | 0.2491 | 0.2514 | 0.2445 | 0.2507 |
| P2 | 0.1500 | 0.1483 | 0.1248 | 0.1483 | 0.1479 | 0.1478 | 0.1497 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0474 | 0.0747 | 0.0746 | 0.0728 | 0.0751 | 0.0755 |
| **P4** | **0.0600** | **0.9599** | **0.8861** | **0.0684** | **0.0615** | **0.0632** | **0.0659** | **0.9601** |
| P5 | -0.0750 | -0.0751 | -0.0960 | -0.0751 | -0.0747 | -0.0746 | -0.0730 | -0.0751 |
| P6 | -0.0750 | -0.0744 | -0.0919 | -0.0744 | -0.0753 | -0.0749 | -0.0756 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.1404 | -0.0147 | -0.0139 | -0.0143 | -0.0147 | -1.8161 |
| P8 | 0.0950 | 0.0954 | 0.0464 | 0.0954 | 0.0963 | 0.0959 | 0.0954 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.3377 | -0.0452 | -0.0443 | -0.0447 | -0.0451 | 0.8551 |
| P10 | -0.0550 | -0.0558 | -0.1463 | -0.0558 | -0.0550 | -0.0553 | -0.0557 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.1105 | -0.0806 | -0.0806 | -0.0803 | -0.0805 | -0.0800 |
| P12 | -0.0700 | -0.0700 | -0.0822 | -0.0700 | -0.0692 | -0.0695 | -0.0704 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1166 | -0.0909 | -0.0900 | -0.0904 | -0.0909 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.2141 | -0.1273 | -0.1265 | -0.1272 | -0.1248 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0348 | 0.0655 | 0.0667 | 0.0679 | 0.0642 | 0.0675 |
| P1-5 | 0.1831 | 0.1832 | 0.1963 | 0.1817 | 0.1824 | 0.1834 | 0.1803 | 0.1832 |
| P2-3 | 0.0086 | 0.0086 | -0.0164 | 0.0076 | 0.0081 | 0.0091 | 0.0076 | 0.0082 |
| P2-4 | 0.0922 | 0.0900 | 0.0145 | 0.0900 | 0.0908 | 0.0911 | 0.0902 | 0.0918 |
| P2-5 | 0.1162 | 0.1159 | 0.1615 | 0.1162 | 0.1157 | 0.1155 | 0.1161 | 0.1158 |
| P3-4 | 0.0836 | 0.0823 | 0.0310 | 0.0823 | 0.0827 | 0.0819 | 0.0827 | 0.0837 |
| P4-5 | 0.0240 | 0.0240 | 0.1470 | 0.0263 | 0.0248 | 0.0244 | 0.0258 | 0.0240 |
| **P4-7** | **0.0439** | **0.9441** | **0.2929** | **0.0446** | **0.0426** | **0.0434** | **0.0441** | **0.9441** |
| P4-9 | 0.1678 | 0.1682 | 0.4918 | 0.1699 | 0.1677 | 0.1684 | 0.1689 | 0.1675 |
| P5-6 | 0.2483 | 0.2520 | 0.4087 | 0.2492 | 0.2482 | 0.2489 | 0.2492 | 0.2479 |
| P6-11 | 0.0368 | 0.0372 | 0.1011 | 0.0372 | 0.0370 | 0.0370 | 0.0369 | 0.0372 |
| P6-12 | 0.0688 | 0.0688 | 0.0993 | 0.0692 | 0.0684 | 0.0688 | 0.0690 | 0.0688 |
| P6-13 | 0.0676 | 0.0667 | 0.1164 | 0.0684 | 0.0676 | 0.0681 | 0.0677 | 0.0675 |
| P7-8 | -0.0950 | -0.0954 | -0.0464 | -0.0954 | -0.0963 | -0.0959 | -0.0954 | -0.0954 |
| **P7-9** | **0.1239** | **0.9236** | **0.1988** | **0.1252** | **0.1250** | **0.1250** | **0.1248** | **-0.7766** |
| P9-10 | 0.0982 | 0.0987 | 0.1556 | 0.0993 | 0.0986 | 0.0986 | 0.0993 | 0.0987 |
| P9-14 | 0.1486 | 0.1506 | 0.1972 | 0.1506 | 0.1497 | 0.1501 | 0.1493 | 0.1473 |
| P10-11 | 0.0432 | 0.0435 | 0.0094 | 0.0435 | 0.0437 | 0.0433 | 0.0436 | 0.0428 |
| P12-13 | -0.0012 | -0.0012 | 0.0171 | -0.0008 | -0.0008 | -0.0007 | -0.0013 | -0.0012 |
| P13-14 | -0.0236 | -0.0232 | 0.0169 | -0.0233 | -0.0232 | -0.0229 | -0.0245 | -0.0246 |

Table 5.193: Estimation errors of case 3, multi large, 14-bus DC system, full redundancy

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| P1 | 0.0108 | 0.0190 | 0.0028 | 0.0009 | 0.0014 | 0.0055 | 0.0007 |
| P2 | 0.0017 | 0.0252 | 0.0017 | 0.0021 | 0.0022 | 0.0003 | 0.0017 |
| P3 | 0.0005 | 0.0276 | 0.0003 | 0.0004 | 0.0022 | 0.0001 | 0.0005 |
| **P4** | **0.8999** | **0.8261** | **0.0084** | **0.0015** | **0.0032** | **0.0059** | **0.9001** |
| P5 | 0.0001 | 0.0210 | 0.0001 | 0.0003 | 0.0004 | 0.0020 | 0.0001 |
| P6 | 0.0006 | 0.0169 | 0.0006 | 0.0003 | 0.0001 | 0.0006 | 0.0006 |
| P7 | 0.0002 | 0.1254 | 0.0002 | 0.0011 | 0.0007 | 0.0003 | 1.8011 |
| P8 | 0.0004 | 0.0486 | 0.0004 | 0.0013 | 0.0009 | 0.0004 | 0.0004 |
| P9 | 0.0002 | 0.2927 | 0.0002 | 0.0007 | 0.0003 | 0.0001 | 0.9001 |
| P10 | 0.0008 | 0.0913 | 0.0008 | 0.0000 | 0.0003 | 0.0007 | 0.0008 |
| P11 | 0.0002 | 0.0305 | 0.0006 | 0.0006 | 0.0003 | 0.0005 | 0.0000 |
| P12 | 0.0000 | 0.0122 | 0.0000 | 0.0008 | 0.0005 | 0.0004 | 0.0000 |
| P13 | 0.0009 | 0.0266 | 0.0009 | 0.0000 | 0.0004 | 0.0009 | 0.0009 |
| P14 | 0.0023 | 0.0891 | 0.0023 | 0.0015 | 0.0022 | 0.0002 | 0.0023 |
| P1-2 | 0.0011 | 0.0321 | 0.0014 | 0.0002 | 0.0010 | 0.0027 | 0.0005 |
| P1-5 | 0.0002 | 0.0132 | 0.0014 | 0.0007 | 0.0004 | 0.0028 | 0.0002 |
| P2-3 | 0.0001 | 0.0250 | 0.0010 | 0.0005 | 0.0006 | 0.0010 | 0.0004 |
| P2-4 | 0.0022 | 0.0776 | 0.0022 | 0.0013 | 0.0011 | 0.0019 | 0.0003 |
| P2-5 | 0.0003 | 0.0453 | 0.0001 | 0.0005 | 0.0007 | 0.0001 | 0.0004 |
| P3-4 | 0.0012 | 0.0526 | 0.0012 | 0.0008 | 0.0016 | 0.0009 | 0.0001 |
| P4-5 | 0.0001 | 0.1229 | 0.0022 | 0.0008 | 0.0004 | 0.0018 | 0.0001 |
| **P4-7** | **0.9002** | **0.2490** | **0.0007** | **0.0013** | **0.0005** | **0.0002** | **0.9002** |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| P4-9 | 0.0004 | 0.3239 | 0.0020 | 0.0001 | 0.0006 | 0.0011 | 0.0003 |
| P5-6 | 0.0037 | 0.1604 | 0.0009 | 0.0001 | 0.0006 | 0.0009 | 0.0004 |
| P6-11 | 0.0003 | 0.0643 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0003 |
| P6-12 | 0.0000 | 0.0305 | 0.0004 | 0.0004 | 0.0000 | 0.0002 | 0.0000 |
| P6-13 | 0.0009 | 0.0488 | 0.0007 | 0.0001 | 0.0005 | 0.0001 | 0.0001 |
| P7-8 | 0.0004 | 0.0486 | 0.0004 | 0.0013 | 0.0009 | 0.0004 | 0.0004 |
| **P7-9** | **0.7997** | **0.0749** | **0.0013** | **0.0011** | **0.0011** | **0.0009** | **0.9005** |
| P9-10 | 0.0006 | 0.0575 | 0.0011 | 0.0005 | 0.0004 | 0.0012 | 0.0005 |
| P9-14 | 0.0020 | 0.0487 | 0.0020 | 0.0012 | 0.0015 | 0.0007 | 0.0013 |
| P10-11 | 0.0003 | 0.0338 | 0.0003 | 0.0005 | 0.0001 | 0.0005 | 0.0003 |
| P12-13 | 0.0000 | 0.0183 | 0.0004 | 0.0004 | 0.0005 | 0.0001 | 0.0001 |
| P13-14 | 0.0004 | 0.0405 | 0.0002 | 0.0004 | 0.0006 | 0.0010 | 0.0010 |
| **SUM** | **2.6328** | **3.2203** | **0.0395** | **0.0239** | **0.0283** | **0.0367** | **5.4165** |

This is the only case LTAV generate bad estimation for full redundancy case. Same type of errors at different locations are conducted as median cases. Set 1: $P_{4-7} = 0.9441$, $P_{7-9} = 0.9236$ and $P_{9-10} = 0.9987$; Set 2: $P_9 = -0.9452$, $P_{9-10} = 0.9987$ and $P_{10-11} = 0.9435$. The two sets have the same location as that in median case. The results are presented in the following two tables, it's clear that they both generate very good results.

Table 5.194: State variables of another two case

| | SV1 | SV2 |
|---|---|---|
| $\theta 2$ | -0.0671 | -0.0674 |
| $\theta 3$ | -0.0751 | -0.0756 |
| $\theta 4$ | -0.1586 | -0.1593 |
| $\theta 5$ | -0.1828 | -0.1832 |
| $\theta 6$ | -0.4307 | -0.4312 |
| $\theta 7$ | -0.2020 | -0.2029 |
| $\theta 8$ | -0.1066 | -0.1075 |
| $\theta 9$ | -0.3260 | -0.3271 |
| $\theta 10$ | -0.4248 | -0.4256 |
| $\theta 11$ | -0.4678 | -0.4683 |
| $\theta 12$ | -0.4994 | -0.5000 |
| $\theta 13$ | -0.4981 | -0.4988 |
| $\theta 14$ | -0.4734 | -0.4743 |

Table 5.195: Estimation and errors of another two case

| | **Actual** | **R1** | **E1** | **R2** | **E2** |
|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2499 | 0.0001 | 0.2507 | 0.0007 |
| P2 | 0.1500 | 0.1483 | 0.0017 | 0.1483 | 0.0017 |
| P3 | 0.0750 | 0.0755 | 0.0005 | 0.0755 | 0.0005 |
| P4 | 0.0600 | 0.0599 | 0.0001 | 0.0599 | 0.0001 |
| P5 | -0.0750 | -0.0751 | 0.0001 | -0.0751 | 0.0001 |
| P6 | -0.0750 | -0.0744 | 0.0006 | -0.0744 | 0.0006 |
| P7 | -0.0150 | -0.0148 | 0.0002 | -0.0148 | 0.0002 |
| P8 | 0.0950 | 0.0954 | 0.0004 | 0.0954 | 0.0004 |
| P9 | -0.0450 | -0.0452 | 0.0002 | -0.0464 | 0.0014 |

| | | | | | |
|---|---|---|---|---|---|
| P10 | -0.0550 | -0.0558 | 0.0008 | -0.0558 | 0.0008 |
| P11 | -0.0800 | -0.0802 | 0.0002 | -0.0798 | 0.0002 |
| P12 | -0.0700 | -0.0700 | 0.0000 | -0.0700 | 0.0000 |
| P13 | -0.0900 | -0.0909 | 0.0009 | -0.0909 | 0.0009 |
| P14 | -0.1250 | -0.1227 | 0.0023 | -0.1227 | 0.0023 |
| P1-2 | 0.0669 | 0.0671 | 0.0001 | 0.0674 | 0.0005 |
| P1-5 | 0.1831 | 0.1828 | 0.0002 | 0.1832 | 0.0002 |
| P2-3 | 0.0086 | 0.0080 | 0.0006 | 0.0082 | 0.0004 |
| P2-4 | 0.0922 | 0.0916 | 0.0006 | 0.0918 | 0.0003 |
| P2-5 | 0.1162 | 0.1158 | 0.0004 | 0.1158 | 0.0004 |
| P3-4 | 0.0836 | 0.0835 | 0.0000 | 0.0837 | 0.0001 |
| P4-5 | 0.0240 | 0.0242 | 0.0002 | 0.0240 | 0.0000 |
| P4-7 | 0.0439 | 0.0434 | 0.0005 | 0.0436 | 0.0003 |
| P4-9 | 0.1678 | 0.1674 | 0.0004 | 0.1679 | 0.0000 |
| P5-6 | 0.2483 | 0.2478 | 0.0005 | 0.2479 | 0.0003 |
| P6-11 | 0.0368 | 0.0372 | 0.0003 | 0.0371 | 0.0003 |
| P6-12 | 0.0688 | 0.0687 | 0.0001 | 0.0688 | 0.0000 |
| P6-13 | 0.0676 | 0.0675 | 0.0002 | 0.0676 | 0.0000 |
| P7-8 | -0.0950 | -0.0954 | 0.0004 | -0.0954 | 0.0004 |
| P7-9 | 0.1239 | 0.1240 | 0.0001 | 0.1242 | 0.0003 |
| P9-10 | 0.0982 | 0.0988 | 0.0007 | 0.0985 | 0.0003 |
| P9-14 | 0.1486 | 0.1474 | 0.0011 | 0.1472 | 0.0014 |
| P10-11 | 0.0432 | 0.0430 | 0.0002 | 0.0427 | 0.0005 |
| P12-13 | -0.0012 | -0.0013 | 0.0001 | -0.0012 | 0.0000 |
| P13-14 | -0.0236 | -0.0247 | 0.0011 | -0.0245 | 0.0009 |
| **Sum** | | | **0.0158** | | **0.0166** |

### Median redundancy

The following three tables show the results and errors in detail.

Table 5.196: State Variable of case 3, multi large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0161 | -0.0331 | -0.0664 | -0.0663 | -0.0633 | -0.0658 |
| $\theta 3$ | 0.0235 | -0.0108 | -0.0744 | -0.0741 | -0.0703 | -0.0730 |
| $\theta 4$ | -0.0227 | -0.0639 | -0.1574 | -0.1568 | -0.1524 | -0.1557 |
| $\theta 5$ | -0.2102 | -0.2061 | -0.1826 | -0.1831 | -0.1797 | -0.1829 |
| $\theta 6$ | -0.6814 | -0.6522 | -0.4311 | -0.4346 | -0.4288 | -0.4349 |
| $\theta 7$ | -0.3324 | -0.3376 | -1.1009 | -0.2045 | -0.1974 | -1.0998 |
| $\theta 8$ | -0.2863 | -0.2422 | -1.0061 | -0.1096 | -0.1022 | -1.0044 |
| $\theta 9$ | -0.5747 | -0.6919 | -0.3230 | -0.3318 | -0.3223 | -0.3334 |
| $\theta 10$ | -0.7755 | -0.7906 | -0.4222 | -0.4310 | -0.4220 | -0.4321 |
| $\theta 11$ | -0.8037 | -0.8335 | -0.4663 | -0.4749 | -0.4656 | -0.4750 |
| $\theta 12$ | -0.7990 | -0.7390 | -0.5006 | -0.5032 | -0.4979 | -0.5040 |
| $\theta 13$ | -0.8293 | -0.7557 | -0.5000 | -0.5023 | -0.4965 | -0.5031 |
| $\theta 14$ | -0.8428 | -0.7852 | -0.4767 | -0.4786 | -0.4715 | -0.4796 |

Table 5.197: Estimation Results of case 3, multi large, 14-bus DC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| P1 | 0.2500 | 0.2392 | 0.2263 | 0.2392 | 0.2490 | 0.2494 | 0.2430 | 0.2487 |
| P2 | 0.1500 | 0.1483 | 0.1450 | 0.1483 | 0.1489 | 0.1488 | 0.1492 | 0.1483 |
| P3 | 0.0750 | 0.0755 | 0.0859 | 0.0755 | 0.0749 | 0.0750 | 0.0752 | 0.0755 |
| **P4** | **0.0600** | **0.9599** | **0.9961** | **0.9599** | **0.9605** | **0.0756** | **0.0708** | **0.9765** |
| P5 | -0.0750 | -0.0751 | -0.1204 | -0.0751 | -0.0757 | -0.0746 | -0.0743 | -0.0751 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P6 | -0.0750 | -0.0744 | -0.0835 | -0.0744 | -0.0750 | -0.0749 | -0.0754 | -0.0744 |
| P7 | -0.0150 | -0.0147 | -0.1134 | -0.0148 | -1.8163 | -0.0153 | -0.0152 | -1.8058 |
| P8 | 0.0950 | 0.0954 | 0.0461 | 0.0954 | 0.0948 | 0.0949 | 0.0952 | 0.0954 |
| P9 | -0.0450 | -0.0452 | -0.3255 | -0.7903 | 0.8653 | -0.0562 | -0.0460 | 0.8334 |
| P10 | -0.0550 | -0.0558 | -0.1726 | -0.0558 | -0.0552 | -0.0553 | -0.0560 | -0.0558 |
| P11 | -0.0800 | -0.0798 | -0.1504 | -0.2243 | -0.0792 | -0.0842 | -0.0804 | -0.0831 |
| P12 | -0.0700 | -0.0700 | -0.0872 | -0.0700 | -0.0701 | -0.0695 | -0.0706 | -0.0700 |
| P13 | -0.0900 | -0.0909 | -0.1648 | -0.0909 | -0.0915 | -0.0904 | -0.0913 | -0.0909 |
| P14 | -0.1250 | -0.1227 | -0.2815 | -0.1227 | -0.1305 | -0.1232 | -0.1242 | -0.1227 |
| P1-2 | 0.0669 | 0.0658 | 0.0161 | 0.0331 | 0.0664 | 0.0663 | 0.0633 | 0.0658 |
| P1-5 | 0.1831 | 0.1832 | 0.2102 | 0.2061 | 0.1826 | 0.1831 | 0.1797 | 0.1829 |
| P2-3 | 0.0086 | 0.0086 | -0.0396 | -0.0223 | 0.0080 | 0.0078 | 0.0070 | 0.0072 |
| P2-4 | 0.0922 | N/A | 0.0067 | 0.0308 | 0.0910 | 0.0905 | 0.0891 | 0.0899 |
| P2-5 | 0.1162 | N/A | 0.1941 | 0.1730 | 0.1163 | 0.1168 | 0.1164 | 0.1171 |
| P3-4 | 0.0836 | 0.0823 | 0.0463 | 0.0532 | 0.0830 | 0.0828 | 0.0822 | 0.0827 |
| P4-5 | 0.0240 | N/A | 0.1874 | 0.1422 | 0.0253 | 0.0262 | 0.0273 | 0.0272 |
| **P4-7** | **0.0439** | **0.9441** | **0.3097** | **0.2737** | **0.9435** | **0.0477** | **0.0449** | **0.9441** |
| P4-9 | 0.1678 | N/A | 0.5520 | 0.6280 | 0.1656 | 0.1750 | 0.1699 | 0.1778 |
| P5-6 | 0.2483 | 0.2520 | 0.4712 | 0.4461 | 0.2485 | 0.2515 | 0.2491 | 0.2520 |
| P6-11 | 0.0368 | N/A | 0.1222 | 0.1814 | 0.0352 | 0.0403 | 0.0368 | 0.0402 |
| P6-12 | 0.0688 | 0.0688 | 0.1176 | 0.0868 | 0.0695 | 0.0686 | 0.0692 | 0.0691 |
| P6-13 | 0.0676 | N/A | 0.1479 | 0.1036 | 0.0689 | 0.0677 | 0.0677 | 0.0683 |
| P7-8 | -0.0950 | -0.0954 | -0.0461 | -0.0954 | -0.0948 | -0.0949 | -0.0952 | -0.0954 |
| **P7-9** | **0.1239** | **0.9236** | **0.2423** | **0.3543** | **-0.7779** | **0.1273** | **0.1249** | **-0.7663** |
| P9-10 | 0.0982 | 0.0987 | 0.2008 | 0.0987 | 0.0993 | 0.0992 | 0.0996 | 0.0987 |
| P9-14 | 0.1486 | N/A | 0.2680 | 0.0933 | 0.1537 | 0.1468 | 0.1492 | 0.1462 |
| P10-11 | 0.0432 | 0.0435 | 0.0282 | 0.0429 | 0.0440 | 0.0439 | 0.0436 | 0.0429 |
| P12-13 | -0.0012 | N/A | 0.0304 | 0.0168 | -0.0006 | -0.0009 | -0.0014 | -0.0009 |
| P13-14 | -0.0236 | -0.0232 | 0.0135 | 0.0294 | -0.0232 | -0.0236 | -0.0250 | -0.0235 |

Table 5.198: Estimation errors of case 3, multi large, 14-bus DC system, median redundancy

| | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|
| P1 | 0.0237 | 0.0108 | 0.0010 | 0.0006 | 0.0070 | 0.0013 |
| P2 | 0.0050 | 0.0017 | 0.0011 | 0.0012 | 0.0008 | 0.0017 |
| P3 | 0.0109 | 0.0005 | 0.0001 | 0.0000 | 0.0002 | 0.0005 |
| **P4** | **0.9361** | **0.8999** | **0.9005** | **0.0156** | **0.0108** | **0.9165** |
| P5 | 0.0454 | 0.0001 | 0.0007 | 0.0004 | 0.0007 | 0.0001 |
| P6 | 0.0085 | 0.0006 | 0.0000 | 0.0001 | 0.0004 | 0.0006 |
| P7 | 0.0984 | 0.0002 | 1.8013 | 0.0003 | 0.0002 | 1.7908 |
| P8 | 0.0489 | 0.0004 | 0.0002 | 0.0001 | 0.0002 | 0.0004 |
| P9 | 0.2805 | 0.7453 | 0.9104 | 0.0112 | 0.0010 | 0.8784 |
| P10 | 0.1176 | 0.0008 | 0.0002 | 0.0003 | 0.0010 | 0.0008 |
| P11 | 0.0704 | 0.1443 | 0.0008 | 0.0042 | 0.0004 | 0.0031 |
| P12 | 0.0172 | 0.0000 | 0.0001 | 0.0005 | 0.0006 | 0.0000 |
| P13 | 0.0748 | 0.0009 | 0.0015 | 0.0004 | 0.0013 | 0.0009 |
| P14 | 0.1565 | 0.0023 | 0.0055 | 0.0018 | 0.0008 | 0.0023 |
| P1-2 | 0.0508 | 0.0338 | 0.0005 | 0.0006 | 0.0036 | 0.0011 |
| P1-5 | 0.0271 | 0.0230 | 0.0005 | 0.0000 | 0.0034 | 0.0002 |
| P2-3 | 0.0482 | 0.0309 | 0.0006 | 0.0008 | 0.0016 | 0.0014 |
| P2-4 | 0.0855 | 0.0614 | 0.0012 | 0.0016 | 0.0030 | 0.0023 |
| P2-5 | 0.0779 | 0.0568 | 0.0001 | 0.0006 | 0.0002 | 0.0009 |
| P3-4 | 0.0373 | 0.0304 | 0.0006 | 0.0008 | 0.0014 | 0.0009 |
| P4-5 | 0.1634 | 0.1181 | 0.0013 | 0.0022 | 0.0033 | 0.0032 |
| **P4-7** | **0.2658** | **0.2298** | **0.8996** | **0.0038** | **0.0010** | **0.9002** |
| P4-9 | 0.3842 | 0.4602 | 0.0022 | 0.0072 | 0.0021 | 0.0100 |
| P5-6 | 0.2230 | 0.1978 | 0.0002 | 0.0032 | 0.0008 | 0.0037 |
| P6-11 | 0.0854 | 0.1445 | 0.0017 | 0.0035 | 0.0001 | 0.0034 |
| P6-12 | 0.0487 | 0.0180 | 0.0007 | 0.0002 | 0.0003 | 0.0003 |
| P6-13 | 0.0803 | 0.0359 | 0.0012 | 0.0001 | 0.0001 | 0.0006 |
| P7-8 | 0.0489 | 0.0004 | 0.0002 | 0.0001 | 0.0002 | 0.0004 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **P7-9** | **0.1184** | **0.2304** | **0.9018** | **0.0034** | **0.0010** | **0.8902** |
| P9-10 | 0.1026 | 0.0005 | 0.0011 | 0.0010 | 0.0015 | 0.0005 |
| P9-14 | 0.1195 | 0.0553 | 0.0052 | 0.0017 | 0.0006 | 0.0024 |
| P10-11 | 0.0150 | 0.0003 | 0.0009 | 0.0007 | 0.0004 | 0.0003 |
| P12-13 | 0.0315 | 0.0180 | 0.0006 | 0.0003 | 0.0002 | 0.0003 |
| P13-14 | 0.0370 | 0.0530 | 0.0003 | 0.0001 | 0.0014 | 0.0001 |
| **SUM** | **3.9446** | **3.6063** | **5.4436** | **0.0686** | **0.0518** | **5.4199** |

**Comparison**

Full redundancy and median redundancy estimation with the same condition of error are presented in Table 5.199. From the results, it is very easy to find that full redundancy case has better estimation results for all estimators.

Table 5.199: Estimation errors of case 3, multi large, 14-bus DC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **SUM** | **2.6328** | **3.2203** | **0.0395** | **0.0239** | **0.0283** | **0.0367** | **5.4165** |
| **SUM** | **N/A** | **3.9446** | **3.6063** | **5.4436** | **0.0686** | **0.0518** | **5.4199** |

#### 5.2.1.5 Summary

Table 5.200 and Table 5.201 presents the comparison between each estimator for full and median redundancy cases respectively.

Table 5.200: Comparison of full redundancy 14-bus estimation between estimators

| **N.o** | **Type** | **Meas.** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|
| **Case 0** | **Random** | 0.0335 | 0.0288 | 0.0193 | 0.0272 | 0.0156 | 0.0160 | 0.0157 |
| **Case 1** | **Reverse** | 0.1533 | 0.2648 | 0.0188 | 0.0272 | 0.0184 | 0.0166 | 0.0158 |
| | **Large** | 0.9333 | 2.1142 | 0.0306 | 0.0283 | 0.0230 | 0.0166 | 0.0158 |
| **Case 2** | **Reverse** | 0.3961 | 0.6519 | 0.0185 | 0.0243 | 0.0327 | 0.0407 | 0.0267 |
| | **2Re-La** | 1.2490 | 2.0072 | 0.0235 | 0.0217 | 0.0202 | 0.0407 | 0.0267 |
| | **2La-Re** | 1.9526 | 3.5278 | 0.0259 | 0.0275 | 0.0195 | 0.0407 | 0.0267 |
| | **Large** | 2.7326 | 4.7513 | 0.0425 | 0.0287 | 0.0259 | 0.0407 | 0.0267 |
| **Case 3** | **Reverse** | 0.4884 | 0.6137 | 0.0193 | 0.0239 | 0.0256 | 0.3013 | 0.0167 |
| | **2Re-La** | 1.0406 | 1.7657 | 0.4859 | 0.0239 | 0.0286 | 0.5113 | 0.0167 |
| | **2La-Re** | 1.8528 | 1.8021 | 0.0230 | 0.0239 | 0.0283 | 0.0367 | 0.0167 |
| | **Large** | 2.6328 | 3.2203 | 0.0395 | 0.0239 | 0.0283 | 0.0367 | 5.4165 |

Table 5.201: Comparison of median redundancy 14-bus estimation between estimators

| **N.o** | **Type** | **WLS** | **LAV** | **LMS** | **LMR** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|
| **Case 0** | **Random** | 0.0310 | 0.0181 | 0.0190 | 0.0177 | 0.0186 | 0.0211 |
| **Case 1** | **Reverse** | 0.3611 | 0.0217 | 0.0268 | 0.0270 | 0.4188 | 0.0429 |
| | **Large** | 2.8140 | 0.0651 | 0.0209 | 0.0259 | 0.0173 | 0.0429 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Case 2** | **Reverse** | 0.6839 | 0.0276 | 0.0256 | 0.0179 | 0.5319 | 0.0430 |
| | **2Re-La** | 1.9707 | 0.0276 | 0.0256 | 0.0208 | 0.4157 | 0.0430 |
| | **2La-Re** | 3.1630 | 0.0217 | 0.0256 | 0.0558 | 0.4141 | 0.0430 |
| | **Large** | 5.0357 | 0.1272 | 0.0256 | 0.0558 | 0.0544 | 0.0430 |
| **Case 3** | **Reverse** | 0.7251 | 0.5248 | 0.0287 | 0.0670 | 0.4184 | 1.5636 |
| | **2Re-La** | 1.6740 | 0.4860 | 0.0233 | 0.5820 | 0.5143 | 4.8070 |
| | **2La-Re** | 2.1529 | 0.0217 | 0.0238 | 0.0283 | 0.4184 | 5.4199 |
| | **Large** | 3.9446 | 3.6063 | 5.4436 | 0.0686 | 0.0518 | 5.4199 |

Generally, except WLS, other estimators' overall performance are better measurements for all cases. Comparing estimation between full redundancy and median redundancy case, the full one win most cases but it's not a guarantee. For some estimators in some scenarios, median redundancy generate better overall results, the reason is that after successfully removing all bad data, full redundancy doesn't have advantage especially when additional data are not that accurate. But the difference are very small, full redundancy cases have more opportunity to generate better and satisfied estimation.

As for LTAV, the estimator's performance is fairly well for most cases, this can be verified in two aspects. The first one is that LTAV generate the best estimation in most cases, for cases that LTAV doesn't have best results, they are very close to best results. The second one is that within each categories, LTAV generate the same estimation, this means that no matter what kind of bad data it is, LTAV can detect and eliminate them correctly and use the same correct data for estimation.

But when come to the last category, i.e. multiple interacting cases, LTAV have very bad performance. Further investigation is made and finds that they are just a few particular cases, when the location of error are changed, LTAV can still generate good estimation. LTAV may perform bad in some specific scenarios since no estimator are perfect, it's still reliable for most cases.

### 5.2.2 14-bus AC state estimation

Now it comes to AC state estimation, which is more accurate compared with DC counterpart. AC state estimation applies all branch resistance, reactance and other parameters as they are and takes everything into consideration. All the system data are presented in the Appendix 6.2.

To keep consistency and for fair comparison, the same locations and same type of bad data categories are applied as that in DC state estimation. The four main categories are conducted.

The parameters used in the estimation are:

1. Tolerance of iterative loop for all estimators: 0.0001;
2. Fixed trimmed number for LTS in the sample precess: $nt = 0$; for LTAV: $nt = 0$;
3. Sample set for LTS: $nsample = 1$; for LTAV: $nsample = 1$;

#### 5.2.2.1 Case 0: basic case with only random noise

Raw data with no bad data are used in this section. Full and median redundancy evaluation are performed.

**Full redundancy**

Table 5.202 shows the actual values of real power injection and flow in the system from load flow. Random noised are generated using normal distribution with $\sigma = 0.01665$ for voltage measurements and $\sigma = 0.05$ for power measurements

are added to the actual value to simulate raw data. As for full redundancy case, all the data are used for estimation.

Table 5.202: Raw data for 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | Q14 | -0.0500 | -0.0652 | Q1-2 | -0.2040 | -0.2571 |
| V2 | 1.0450 | 1.0743 | P1-2 | 1.5688 | 1.5835 | Q1-5 | 0.0385 | 0.1560 |
| V3 | 1.0100 | 0.9739 | P1-5 | 0.7551 | 0.7157 | Q2-3 | 0.0356 | 0.0048 |
| V4 | 1.0180 | 1.0318 | P2-3 | 0.7324 | 0.7768 | Q2-4 | -0.0155 | 0.0219 |
| V5 | 1.0200 | 1.0251 | P2-4 | 0.5613 | 0.5039 | Q2-5 | 0.0117 | 0.0021 |
| V6 | 1.0700 | 1.0491 | P2-5 | 0.4152 | 0.3618 | Q3-4 | 0.0447 | 0.0891 |
| V7 | 1.0620 | 1.0551 | P3-4 | -0.2329 | -0.2734 | Q4-5 | 0.1582 | 0.1200 |
| V8 | 1.0900 | 1.0955 | P4-5 | -0.6116 | -0.7588 | Q4-7 | -0.0968 | -0.1669 |
| V9 | 1.0560 | 1.1131 | P4-7 | 0.2807 | 0.3526 | Q4-9 | -0.0043 | -0.0754 |
| V10 | 1.0510 | 1.0952 | P4-9 | 0.1608 | 0.1771 | Q5-6 | 0.1247 | 0.1491 |
| V11 | 1.0570 | 1.0355 | P5-6 | 0.4409 | 0.4032 | Q6-11 | 0.0356 | 0.0267 |
| V12 | 1.0550 | 1.1034 | P6-11 | 0.0735 | 0.1420 | Q6-12 | 0.0250 | 0.0152 |
| V13 | 1.0500 | 1.0616 | P6-12 | 0.0779 | -0.0077 | Q6-13 | 0.0722 | 0.1432 |
| V14 | 1.0360 | 1.0350 | P6-13 | 0.1775 | 0.1724 | Q7-8 | -0.1716 | -0.1570 |
| P1 | 2.3239 | 2.3508 | P7-8 | 0.0000 | -0.0121 | Q7-9 | 0.0578 | 0.0677 |
| P2 | 0.1830 | 0.2747 | P7-9 | 0.2807 | 0.2967 | Q9-10 | 0.0422 | 0.1216 |
| P3 | -0.9420 | -1.0549 | P9-10 | 0.0523 | 0.0679 | Q9-14 | 0.0361 | -0.0041 |
| P4 | -0.4780 | -0.4349 | P9-14 | 0.0943 | 0.0511 | Q10-11 | -0.0162 | 0.0186 |
| P5 | -0.0760 | -0.0601 | P10-11 | -0.0379 | -0.0394 | Q12-13 | 0.0075 | 0.0493 |
| P6 | -0.1120 | -0.1774 | P12-13 | 0.0161 | 0.0079 | Q13-14 | 0.0175 | 0.0053 |
| P7 | 0.0000 | -0.0217 | P13-14 | 0.0564 | 0.0878 | Q2-1 | 0.2768 | 0.2876 |
| P8 | 0.0000 | 0.0171 | P2-1 | -1.5259 | -1.4712 | Q5-1 | 0.0223 | -0.0360 |
| P9 | -0.2950 | -0.1161 | P5-1 | -0.7275 | -0.6720 | Q3-2 | 0.0160 | -0.0414 |
| P10 | -0.0900 | 0.0485 | P3-2 | -0.7091 | -0.7523 | Q4-2 | 0.0302 | 0.0354 |
| P11 | -0.0350 | -0.1025 | P4-2 | -0.5445 | -0.5406 | Q5-2 | -0.0210 | 0.0151 |
| P12 | -0.0610 | 0.0907 | P5-2 | -0.4061 | -0.4668 | Q4-3 | -0.0484 | 0.0809 |
| P13 | -0.1350 | -0.0987 | P4-3 | 0.2366 | 0.1809 | Q5-4 | 0.1420 | 0.1087 |
| P14 | -0.1490 | -0.1522 | P5-4 | 0.6167 | 0.6164 | Q7-4 | 0.1138 | 0.1232 |
| Q1 | -0.1655 | -0.1298 | P7-4 | -0.2807 | -0.2041 | Q9-4 | 0.0173 | 0.0132 |
| Q2 | 0.3086 | 0.2984 | P9-4 | -0.1608 | -0.1993 | Q6-5 | -0.0805 | -0.1772 |
| Q3 | 0.0608 | 0.0546 | P6-5 | -0.4409 | -0.4223 | Q11-6 | -0.0344 | -0.0563 |
| Q4 | 0.0390 | 0.1135 | P11-6 | -0.0730 | -0.0843 | Q12-6 | -0.0235 | -0.1132 |
| Q5 | -0.0160 | 0.0545 | P12-6 | -0.0771 | -0.0212 | Q13-6 | -0.0680 | -0.0260 |
| Q6 | 0.0523 | 0.1232 | P13-6 | -0.1754 | -0.2299 | Q8-7 | 0.1762 | 0.1318 |
| Q7 | 0.0000 | 0.0336 | P8-7 | 0.0000 | 0.0016 | Q9-7 | -0.0498 | -0.0448 |
| Q8 | 0.1762 | 0.1158 | P9-7 | -0.2807 | -0.2531 | Q10-9 | -0.0418 | -0.0690 |
| Q9 | -0.1660 | -0.1301 | P10-9 | -0.0521 | 0.0029 | Q14-9 | -0.0336 | -0.0184 |
| Q10 | -0.0580 | 0.0235 | P14-9 | -0.0931 | -0.0159 | Q11-10 | 0.0164 | -0.0136 |
| Q11 | -0.0180 | 0.0064 | P11-10 | 0.0380 | 0.0423 | Q13-12 | -0.0075 | 0.0170 |
| Q12 | -0.0160 | 0.0357 | P13-12 | -0.0161 | -0.0907 | Q14-13 | -0.0164 | 0.0206 |
| Q13 | -0.0580 | -0.0217 | P14-13 | -0.0559 | -0.0930 | | | |

Table 5.203 shows the state variables, Table 5.204 shows the original actual value, measurements, estimation results and the error between each estimator and actual value.

Table 5.203: State Variable of case 0, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0815 | -0.0851 | -0.0815 | -0.0825 | **V1** | 1.0897 | 1.0781 | 1.0897 | 1.0836 |
| $\theta 3$ | -0.2175 | -0.2234 | -0.2175 | -0.2201 | **V2** | 1.0740 | 1.0643 | 1.0740 | 1.0679 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 4$ | -0.1695 | -0.1741 | -0.1695 | -0.1720 | **V3** | 1.0285 | 1.0261 | 1.0285 | 1.0223 |
| $\theta 5$ | -0.1429 | -0.1471 | -0.1429 | -0.1453 | **V4** | 1.0416 | 1.0318 | 1.0416 | 1.0364 |
| $\theta 6$ | -0.2303 | -0.2359 | -0.2303 | -0.2344 | **V5** | 1.0466 | 1.0357 | 1.0466 | 1.0416 |
| $\theta 7$ | -0.2220 | -0.2319 | -0.2220 | -0.2253 | **V6** | 1.0628 | 1.0607 | 1.0628 | 1.0702 |
| $\theta 8$ | -0.2220 | -0.2303 | -0.2220 | -0.2254 | **V7** | 1.0687 | 1.0551 | 1.0687 | 1.0671 |
| $\theta 9$ | -0.2446 | -0.2614 | -0.2446 | -0.2480 | **V8** | 1.0882 | 1.0767 | 1.0882 | 1.0869 |
| $\theta 10$ | -0.2458 | -0.2636 | -0.2458 | -0.2495 | **V9** | 1.0616 | 1.0485 | 1.0616 | 1.0615 |
| $\theta 11$ | -0.2443 | -0.2559 | -0.2443 | -0.2485 | **V10** | 1.0537 | 1.0378 | 1.0537 | 1.0548 |
| $\theta 12$ | -0.2262 | -0.2301 | -0.2262 | -0.2316 | **V11** | 1.0453 | 1.0371 | 1.0453 | 1.0502 |
| $\theta 13$ | -0.2407 | -0.2464 | -0.2407 | -0.2453 | **V12** | 1.0535 | 1.0592 | 1.0535 | 1.0624 |
| $\theta 14$ | -0.2638 | -0.2739 | -0.2638 | -0.2679 | **V13** | 1.0407 | 1.0472 | 1.0407 | 1.0489 |
| | | | | | **V14** | 1.0325 | 1.0315 | 1.0325 | 1.0358 |

Table 5.204: Estimation Results and Errors of case 0, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0897 | 1.0781 | 1.0897 | 1.0836 | 0.0086 | 0.0297 | 0.0181 | 0.0297 | 0.0236 |
| V2 | 1.0450 | 1.0743 | 1.0740 | 1.0643 | 1.0740 | 1.0679 | 0.0293 | 0.0290 | 0.0193 | 0.0290 | 0.0229 |
| V3 | 1.0100 | 0.9739 | 1.0285 | 1.0261 | 1.0285 | 1.0223 | 0.0361 | 0.0185 | 0.0161 | 0.0185 | 0.0123 |
| V4 | 1.0180 | 1.0318 | 1.0416 | 1.0318 | 1.0416 | 1.0364 | 0.0138 | 0.0236 | 0.0138 | 0.0236 | 0.0184 |
| V5 | 1.0200 | 1.0251 | 1.0466 | 1.0357 | 1.0466 | 1.0416 | 0.0051 | 0.0266 | 0.0157 | 0.0266 | 0.0216 |
| V6 | 1.0700 | 1.0491 | 1.0628 | 1.0607 | 1.0628 | 1.0702 | 0.0209 | 0.0072 | 0.0093 | 0.0072 | 0.0002 |
| V7 | 1.0620 | 1.0551 | 1.0687 | 1.0551 | 1.0687 | 1.0671 | 0.0069 | 0.0067 | 0.0069 | 0.0067 | 0.0051 |
| V8 | 1.0900 | 1.0955 | 1.0882 | 1.0767 | 1.0882 | 1.0869 | 0.0055 | 0.0018 | 0.0133 | 0.0018 | 0.0031 |
| V9 | 1.0560 | 1.1131 | 1.0616 | 1.0485 | 1.0616 | 1.0615 | 0.0571 | 0.0056 | 0.0075 | 0.0056 | 0.0055 |
| V10 | 1.0510 | 1.0952 | 1.0537 | 1.0378 | 1.0537 | 1.0548 | 0.0442 | 0.0027 | 0.0132 | 0.0027 | 0.0038 |
| V11 | 1.0570 | 1.0355 | 1.0453 | 1.0371 | 1.0453 | 1.0502 | 0.0215 | 0.0117 | 0.0199 | 0.0117 | 0.0068 |
| V12 | 1.0550 | 1.1034 | 1.0535 | 1.0592 | 1.0535 | 1.0624 | 0.0484 | 0.0015 | 0.0042 | 0.0015 | 0.0074 |
| V13 | 1.0500 | 1.0616 | 1.0407 | 1.0472 | 1.0407 | 1.0489 | 0.0116 | 0.0093 | 0.0028 | 0.0093 | 0.0011 |
| V14 | 1.0360 | 1.0350 | 1.0325 | 1.0315 | 1.0325 | 1.0358 | 0.0010 | 0.0035 | 0.0045 | 0.0035 | 0.0002 |
| P1 | 2.3239 | 2.3508 | 2.3078 | 2.3360 | 2.3078 | 2.3115 | 0.0269 | 0.0161 | 0.0121 | 0.0161 | 0.0124 |
| P2 | 0.1830 | 0.2747 | 0.2443 | 0.2126 | 0.2443 | 0.2477 | 0.0917 | 0.0613 | 0.0296 | 0.0613 | 0.0647 |
| P3 | -0.9420 | -1.0549 | -1.0427 | -1.0261 | -1.0427 | -1.0412 | 0.1129 | 0.1007 | 0.0841 | 0.1007 | 0.0992 |
| P4 | -0.4780 | -0.4349 | -0.4796 | -0.4349 | -0.4796 | -0.4774 | 0.0431 | 0.0016 | 0.0431 | 0.0016 | 0.0006 |
| P5 | -0.0760 | -0.0601 | -0.0499 | -0.0601 | -0.0499 | -0.0473 | 0.0159 | 0.0261 | 0.0159 | 0.0261 | 0.0287 |
| P6 | -0.1120 | -0.1774 | -0.1700 | -0.1774 | -0.1700 | -0.1662 | 0.0654 | 0.0580 | 0.0654 | 0.0580 | 0.0542 |
| P7 | 0.0000 | -0.0217 | -0.0527 | -0.0217 | -0.0527 | -0.0543 | 0.0217 | 0.0527 | 0.0217 | 0.0527 | 0.0543 |
| P8 | 0.0000 | 0.0171 | -0.0001 | 0.0108 | -0.0001 | -0.0006 | 0.0171 | 0.0001 | 0.0108 | 0.0001 | 0.0006 |
| P9 | -0.2950 | -0.1161 | -0.2322 | -0.3376 | -0.2322 | -0.2373 | 0.1789 | 0.0628 | 0.0426 | 0.0628 | 0.0577 |
| P10 | -0.0900 | 0.0485 | -0.0368 | -0.1023 | -0.0368 | -0.0406 | 0.1385 | 0.0532 | 0.0123 | 0.0532 | 0.0494 |
| P11 | -0.0350 | -0.1025 | -0.1086 | -0.1025 | -0.1086 | -0.1101 | 0.0675 | 0.0736 | 0.0675 | 0.0736 | 0.0751 |
| P12 | -0.0610 | 0.0907 | 0.0690 | 0.0907 | 0.0690 | 0.0679 | 0.1517 | 0.1300 | 0.1517 | 0.1300 | 0.1289 |
| P13 | -0.1350 | -0.0987 | -0.1415 | -0.0987 | -0.1415 | -0.1407 | 0.0363 | 0.0065 | 0.0363 | 0.0065 | 0.0057 |
| P14 | -0.1490 | -0.1522 | -0.1731 | -0.1522 | -0.1731 | -0.1760 | 0.0032 | 0.0241 | 0.0032 | 0.0241 | 0.0270 |
| Q1 | -0.1655 | -0.1298 | -0.1958 | -0.2372 | -0.1958 | -0.2021 | 0.0357 | 0.0303 | 0.0717 | 0.0303 | 0.0366 |
| Q2 | 0.3086 | 0.2984 | 0.2816 | 0.2984 | 0.2816 | 0.2725 | 0.0102 | 0.0270 | 0.0102 | 0.0270 | 0.0361 |
| Q3 | 0.0608 | 0.0546 | -0.0248 | 0.0546 | -0.0248 | -0.0284 | 0.0062 | 0.0856 | 0.0062 | 0.0856 | 0.0892 |
| Q4 | 0.0390 | 0.1135 | 0.0502 | 0.0658 | 0.0502 | 0.0330 | 0.0745 | 0.0112 | 0.0268 | 0.0112 | 0.0060 |
| Q5 | -0.0160 | 0.0545 | 0.1235 | 0.0545 | 0.1235 | 0.0854 | 0.0705 | 0.1395 | 0.0705 | 0.1395 | 0.1014 |
| Q6 | 0.0523 | 0.1232 | -0.0422 | -0.0667 | -0.0422 | 0.0072 | 0.0709 | 0.0945 | 0.1190 | 0.0945 | 0.0451 |
| Q7 | 0.0000 | 0.0336 | -0.0211 | -0.0521 | -0.0211 | -0.0186 | 0.0336 | 0.0211 | 0.0521 | 0.0211 | 0.0186 |
| Q8 | 0.1762 | 0.1158 | 0.1210 | 0.1318 | 0.1210 | 0.1225 | 0.0604 | 0.0552 | 0.0444 | 0.0552 | 0.0537 |
| Q9 | -0.1660 | -0.1301 | -0.1550 | -0.1477 | -0.1550 | -0.1552 | 0.0359 | 0.0110 | 0.0183 | 0.0110 | 0.0108 |
| Q10 | -0.0580 | 0.0235 | -0.0380 | -0.0878 | -0.0380 | -0.0435 | 0.0815 | 0.0200 | 0.0298 | 0.0200 | 0.0145 |
| Q11 | -0.0180 | 0.0064 | -0.0860 | -0.0746 | -0.0860 | -0.0774 | 0.0244 | 0.0680 | 0.0566 | 0.0680 | 0.0594 |
| Q12 | -0.0160 | 0.0357 | -0.0476 | -0.0311 | -0.0476 | -0.0363 | 0.0517 | 0.0316 | 0.0151 | 0.0316 | 0.0203 |
| Q13 | -0.0580 | -0.0217 | -0.1024 | -0.0277 | -0.1024 | -0.0874 | 0.0363 | 0.0444 | 0.0303 | 0.0444 | 0.0294 |
| Q14 | -0.0500 | -0.0652 | -0.0509 | -0.0366 | -0.0509 | -0.0514 | 0.0152 | 0.0009 | 0.0134 | 0.0009 | 0.0014 |
| P1-2 | 1.5688 | 1.5835 | 1.5597 | 1.5835 | 1.5597 | 1.5604 | 0.0147 | 0.0091 | 0.0147 | 0.0091 | 0.0084 |
| P1-5 | 0.7551 | 0.7157 | 0.7482 | 0.7525 | 0.7482 | 0.7511 | 0.0394 | 0.0069 | 0.0026 | 0.0069 | 0.0040 |
| P2-3 | 0.7324 | 0.7768 | 0.7832 | 0.7780 | 0.7832 | 0.7829 | 0.0444 | 0.0508 | 0.0456 | 0.0508 | 0.0505 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P2-4 | 0.5613 | 0.5039 | 0.5687 | 0.5652 | 0.5687 | 0.5704 | 0.0574 | 0.0074 | 0.0039 | 0.0074 | 0.0091 |
| P2-5 | 0.4152 | 0.3618 | 0.4120 | 0.4104 | 0.4120 | 0.4143 | 0.0534 | 0.0032 | 0.0048 | 0.0032 | 0.0009 |
| P3-4 | -0.2329 | -0.2734 | -0.2850 | -0.2734 | -0.2850 | -0.2841 | 0.0405 | 0.0521 | 0.0405 | 0.0521 | 0.0512 |
| P4-5 | -0.6116 | -0.7588 | -0.6573 | -0.6473 | -0.6573 | -0.6559 | 0.1472 | 0.0457 | 0.0357 | 0.0457 | 0.0443 |
| P4-7 | 0.2807 | 0.3526 | 0.2857 | 0.3076 | 0.2857 | 0.2880 | 0.0719 | 0.0050 | 0.0269 | 0.0050 | 0.0073 |
| P4-9 | 0.1608 | 0.1771 | 0.1539 | 0.1751 | 0.1539 | 0.1549 | 0.0163 | 0.0069 | 0.0143 | 0.0069 | 0.0059 |
| P5-6 | 0.4409 | 0.4032 | 0.4134 | 0.4148 | 0.4134 | 0.4219 | 0.0377 | 0.0275 | 0.0261 | 0.0275 | 0.0190 |
| P6-11 | 0.0735 | 0.1420 | 0.1002 | 0.1395 | 0.1002 | 0.1070 | 0.0685 | 0.0267 | 0.0660 | 0.0267 | 0.0335 |
| P6-12 | 0.0779 | -0.0077 | 0.0005 | -0.0180 | 0.0005 | 0.0027 | 0.0856 | 0.0774 | 0.0959 | 0.0774 | 0.0752 |
| P6-13 | 0.1775 | 0.1724 | 0.1427 | 0.1159 | 0.1427 | 0.1459 | 0.0051 | 0.0348 | 0.0616 | 0.0348 | 0.0316 |
| P7-8 | 0.0000 | -0.0121 | 0.0001 | -0.0108 | 0.0001 | 0.0006 | 0.0121 | 0.0001 | 0.0108 | 0.0001 | 0.0006 |
| P7-9 | 0.2807 | 0.2967 | 0.2329 | 0.2967 | 0.2329 | 0.2332 | 0.0160 | 0.0478 | 0.0160 | 0.0478 | 0.0475 |
| P9-10 | 0.0523 | 0.0679 | 0.0466 | 0.0679 | 0.0466 | 0.0451 | 0.0156 | 0.0057 | 0.0156 | 0.0057 | 0.0072 |
| P9-14 | 0.0943 | 0.0511 | 0.1080 | 0.0663 | 0.1080 | 0.1056 | 0.0432 | 0.0137 | 0.0280 | 0.0137 | 0.0113 |
| P10-11 | -0.0379 | -0.0394 | 0.0096 | -0.0349 | 0.0096 | 0.0043 | 0.0015 | 0.0475 | 0.0030 | 0.0475 | 0.0422 |
| P12-13 | 0.0161 | 0.0079 | 0.0693 | 0.0726 | 0.0693 | 0.0705 | 0.0082 | 0.0532 | 0.0565 | 0.0532 | 0.0544 |
| P13-14 | 0.0564 | 0.0878 | 0.0676 | 0.0878 | 0.0676 | 0.0728 | 0.0314 | 0.0112 | 0.0314 | 0.0112 | 0.0164 |
| P2-1 | -1.5259 | -1.4712 | -1.5196 | -1.5411 | -1.5196 | -1.5198 | 0.0547 | 0.0063 | 0.0152 | 0.0063 | 0.0061 |
| P5-1 | -0.7275 | -0.6720 | -0.7224 | -0.7259 | -0.7224 | -0.7249 | 0.0555 | 0.0051 | 0.0016 | 0.0051 | 0.0026 |
| P3-2 | -0.7091 | -0.7523 | -0.7577 | -0.7527 | -0.7577 | -0.7571 | 0.0432 | 0.0486 | 0.0436 | 0.0486 | 0.0480 |
| P4-2 | -0.5445 | -0.5406 | -0.5523 | -0.5488 | -0.5523 | -0.5538 | 0.0039 | 0.0078 | 0.0043 | 0.0078 | 0.0093 |
| P5-2 | -0.4061 | -0.4668 | -0.4035 | -0.4018 | -0.4035 | -0.4056 | 0.0607 | 0.0026 | 0.0043 | 0.0026 | 0.0005 |
| P4-3 | 0.2366 | 0.1809 | 0.2903 | 0.2786 | 0.2903 | 0.2893 | 0.0557 | 0.0537 | 0.0420 | 0.0537 | 0.0527 |
| P5-4 | 0.6167 | 0.6164 | 0.6627 | 0.6528 | 0.6627 | 0.6613 | 0.0003 | 0.0460 | 0.0361 | 0.0460 | 0.0446 |
| P7-4 | -0.2807 | -0.2041 | -0.2857 | -0.3076 | -0.2857 | -0.2880 | 0.0766 | 0.0050 | 0.0269 | 0.0050 | 0.0073 |
| P9-4 | -0.1608 | -0.1993 | -0.1539 | -0.1751 | -0.1539 | -0.1549 | 0.0385 | 0.0069 | 0.0143 | 0.0069 | 0.0059 |
| P6-5 | -0.4409 | -0.4223 | -0.4134 | -0.4148 | -0.4134 | -0.4219 | 0.0186 | 0.0275 | 0.0261 | 0.0275 | 0.0190 |
| P11-6 | -0.0730 | -0.0843 | -0.0992 | -0.1375 | -0.0992 | -0.1058 | 0.0113 | 0.0262 | 0.0645 | 0.0262 | 0.0328 |
| P12-6 | -0.0771 | -0.0212 | -0.0003 | 0.0181 | -0.0003 | -0.0026 | 0.0559 | 0.0768 | 0.0952 | 0.0768 | 0.0745 |
| P13-6 | -0.1754 | -0.2299 | -0.1408 | -0.1150 | -0.1408 | -0.1440 | 0.0545 | 0.0346 | 0.0604 | 0.0346 | 0.0314 |
| P8-7 | 0.0000 | 0.0016 | -0.0001 | 0.0108 | -0.0001 | -0.0006 | 0.0016 | 0.0001 | 0.0108 | 0.0001 | 0.0006 |
| P9-7 | -0.2807 | -0.2531 | -0.2329 | -0.2967 | -0.2329 | -0.2332 | 0.0276 | 0.0478 | 0.0160 | 0.0478 | 0.0475 |
| P10-9 | -0.0521 | 0.0029 | -0.0464 | -0.0674 | -0.0464 | -0.0449 | 0.0550 | 0.0057 | 0.0153 | 0.0057 | 0.0072 |
| P14-9 | -0.0931 | -0.0159 | -0.1062 | -0.0656 | -0.1062 | -0.1041 | 0.0772 | 0.0131 | 0.0275 | 0.0131 | 0.0110 |
| P11-10 | 0.0380 | 0.0423 | -0.0094 | 0.0350 | -0.0094 | -0.0043 | 0.0043 | 0.0474 | 0.0030 | 0.0474 | 0.0423 |
| P13-12 | -0.0161 | -0.0907 | -0.0683 | -0.0715 | -0.0683 | -0.0695 | 0.0746 | 0.0522 | 0.0554 | 0.0522 | 0.0534 |
| P14-13 | -0.0559 | -0.0930 | -0.0669 | -0.0866 | -0.0669 | -0.0720 | 0.0371 | 0.0110 | 0.0307 | 0.0110 | 0.0161 |
| Q1-2 | -0.2040 | -0.2571 | -0.2188 | -0.2571 | -0.2188 | -0.2199 | 0.0531 | 0.0148 | 0.0531 | 0.0148 | 0.0159 |
| Q1-5 | 0.0385 | 0.1560 | 0.0230 | 0.0199 | 0.0230 | 0.0177 | 0.1175 | 0.0155 | 0.0186 | 0.0155 | 0.0208 |
| Q2-3 | 0.0356 | 0.0048 | 0.0621 | 0.0235 | 0.0621 | 0.0622 | 0.0308 | 0.0265 | 0.0121 | 0.0265 | 0.0266 |
| Q2-4 | -0.0155 | 0.0219 | -0.0045 | -0.0042 | -0.0045 | -0.0107 | 0.0374 | 0.0110 | 0.0113 | 0.0110 | 0.0048 |
| Q2-5 | 0.0117 | 0.0021 | 0.0065 | 0.0136 | 0.0065 | -0.0006 | 0.0096 | 0.0052 | 0.0019 | 0.0052 | 0.0123 |
| Q3-4 | 0.0447 | 0.0891 | 0.0267 | 0.0670 | 0.0267 | 0.0210 | 0.0444 | 0.0180 | 0.0223 | 0.0180 | 0.0237 |
| Q4-5 | 0.1582 | 0.1200 | 0.0924 | 0.1200 | 0.0924 | 0.0901 | 0.0382 | 0.0658 | 0.0382 | 0.0658 | 0.0681 |
| Q4-7 | -0.0968 | -0.1669 | -0.1304 | -0.1087 | -0.1304 | -0.1477 | 0.0701 | 0.0336 | 0.0119 | 0.0336 | 0.0509 |
| Q4-9 | -0.0043 | -0.0754 | -0.0328 | -0.0243 | -0.0328 | -0.0424 | 0.0711 | 0.0285 | 0.0200 | 0.0285 | 0.0381 |
| Q5-6 | 0.1247 | 0.1491 | -0.0541 | -0.0921 | -0.0541 | -0.1081 | 0.0244 | 0.1788 | 0.2168 | 0.1788 | 0.2328 |
| Q6-11 | 0.0356 | 0.0267 | 0.0464 | 0.0604 | 0.0464 | 0.0568 | 0.0089 | 0.0108 | 0.0248 | 0.0108 | 0.0212 |
| Q6-12 | 0.0250 | 0.0152 | 0.0388 | 0.0152 | 0.0388 | 0.0312 | 0.0098 | 0.0138 | 0.0098 | 0.0138 | 0.0062 |
| Q6-13 | 0.0722 | 0.1432 | 0.1083 | 0.0519 | 0.1083 | 0.1016 | 0.0710 | 0.0361 | 0.0203 | 0.0361 | 0.0294 |
| Q7-8 | -0.1716 | -0.1570 | -0.1188 | -0.1291 | -0.1188 | -0.1203 | 0.0146 | 0.0528 | 0.0425 | 0.0528 | 0.0513 |
| Q7-9 | 0.0578 | 0.0677 | 0.0716 | 0.0677 | 0.0716 | 0.0565 | 0.0099 | 0.0138 | 0.0099 | 0.0138 | 0.0013 |
| Q9-10 | 0.0422 | 0.1216 | 0.0808 | 0.1076 | 0.0808 | 0.0672 | 0.0794 | 0.0386 | 0.0654 | 0.0386 | 0.0250 |
| Q9-14 | 0.0361 | -0.0041 | 0.0639 | 0.0352 | 0.0639 | 0.0521 | 0.0402 | 0.0278 | 0.0009 | 0.0278 | 0.0160 |
| Q10-11 | -0.0162 | 0.0186 | 0.0421 | 0.0186 | 0.0421 | 0.0232 | 0.0348 | 0.0583 | 0.0348 | 0.0583 | 0.0394 |
| Q12-13 | 0.0075 | 0.0493 | -0.0091 | -0.0160 | -0.0091 | -0.0053 | 0.0418 | 0.0166 | 0.0235 | 0.0166 | 0.0128 |
| Q13-14 | 0.0175 | 0.0053 | -0.0078 | 0.0053 | -0.0078 | 0.0044 | 0.0122 | 0.0253 | 0.0122 | 0.0253 | 0.0131 |
| Q2-1 | 0.2768 | 0.2876 | 0.2176 | 0.2655 | 0.2176 | 0.2216 | 0.0108 | 0.0592 | 0.0113 | 0.0592 | 0.0552 |
| Q5-1 | 0.0223 | -0.0360 | -0.0289 | -0.0201 | -0.0289 | -0.0206 | 0.0583 | 0.0512 | 0.0424 | 0.0512 | 0.0429 |
| Q3-2 | 0.0160 | -0.0414 | -0.0515 | -0.0124 | -0.0515 | -0.0494 | 0.0574 | 0.0675 | 0.0284 | 0.0675 | 0.0654 |
| Q4-2 | 0.0302 | 0.0354 | -0.0220 | -0.0206 | -0.0220 | -0.0142 | 0.0052 | 0.0522 | 0.0508 | 0.0522 | 0.0444 |
| Q5-2 | -0.0210 | 0.0151 | -0.0584 | -0.0637 | -0.0584 | -0.0500 | 0.0361 | 0.0374 | 0.0427 | 0.0374 | 0.0290 |
| Q4-3 | -0.0484 | 0.0809 | -0.0407 | -0.0810 | -0.0407 | -0.0347 | 0.1293 | 0.0077 | 0.0326 | 0.0077 | 0.0137 |
| Q5-4 | 0.1420 | 0.1087 | -0.0753 | -0.1029 | -0.0753 | -0.0729 | 0.0333 | 0.2173 | 0.2449 | 0.2173 | 0.2149 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q7-4 | 0.1138 | 0.1232 | 0.1490 | 0.1291 | 0.1490 | 0.1677 | 0.0094 | 0.0352 | 0.0153 | 0.0352 | 0.0539 |
| Q9-4 | 0.0173 | 0.0132 | 0.0451 | 0.0401 | 0.0451 | 0.0553 | 0.0041 | 0.0278 | 0.0228 | 0.0278 | 0.0380 |
| Q6-5 | -0.0805 | -0.1772 | 0.0914 | 0.1316 | 0.0914 | 0.1492 | 0.0967 | 0.1719 | 0.2121 | 0.1719 | 0.2297 |
| Q11-6 | -0.0344 | -0.0563 | -0.0442 | -0.0563 | -0.0442 | -0.0542 | 0.0219 | 0.0098 | 0.0219 | 0.0098 | 0.0198 |
| Q12-6 | -0.0235 | -0.1132 | -0.0385 | -0.0151 | -0.0385 | -0.0310 | 0.0897 | 0.0150 | 0.0084 | 0.0150 | 0.0075 |
| Q13-6 | -0.0680 | -0.0260 | -0.1046 | -0.0500 | -0.1046 | -0.0980 | 0.0420 | 0.0366 | 0.0180 | 0.0366 | 0.0300 |
| Q8-7 | 0.1762 | 0.1318 | 0.1210 | 0.1318 | 0.1210 | 0.1225 | 0.0444 | 0.0552 | 0.0444 | 0.0552 | 0.0537 |
| Q9-7 | -0.0498 | -0.0448 | -0.0659 | -0.0585 | -0.0659 | -0.0510 | 0.0050 | 0.0161 | 0.0087 | 0.0161 | 0.0012 |
| Q10-9 | -0.0418 | -0.0690 | -0.0801 | -0.1064 | -0.0801 | -0.0667 | 0.0272 | 0.0383 | 0.0646 | 0.0383 | 0.0249 |
| Q14-9 | -0.0336 | -0.0184 | -0.0602 | -0.0338 | -0.0602 | -0.0487 | 0.0152 | 0.0266 | 0.0002 | 0.0266 | 0.0151 |
| Q11-10 | 0.0164 | -0.0136 | -0.0418 | -0.0183 | -0.0418 | -0.0232 | 0.0300 | 0.0582 | 0.0347 | 0.0582 | 0.0396 |
| Q13-12 | -0.0075 | 0.0170 | 0.0100 | 0.0170 | 0.0100 | 0.0062 | 0.0245 | 0.0175 | 0.0245 | 0.0175 | 0.0137 |
| Q14-13 | -0.0164 | 0.0206 | 0.0093 | -0.0028 | 0.0093 | -0.0027 | 0.0370 | 0.0257 | 0.0136 | 0.0257 | 0.0137 |
| **SUM** | | | | | | | **2.9714** | **2.1408** | **2.0361** | **2.1408** | **1.9526** |

Since the noise are small, estimators generate almost the same results as measurements in the base case, the performance between estimators also have little difference.

### Median redundancy

For median redundancy case, the same location are selected to put meters as in DC part, as shown in Figure. 5.5. Reactive counterpart are added to real power data to simulate raw data for AC median redundancy state estimation, the raw data are displayed in Table. 5.205.

Table 5.205: raw data for 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | Q4 | 0.0390 | 0.1135 | P7-8 | 0.0000 | -0.0121 |
| P1 | 2.3239 | 2.3508 | Q5 | -0.0160 | 0.0545 | P7-9 | 0.2807 | 0.2967 |
| P2 | 0.1830 | 0.2747 | Q6 | 0.0523 | 0.1232 | P9-10 | 0.0523 | 0.0679 |
| P3 | -0.9420 | -1.0549 | Q7 | 0.0000 | 0.0336 | P10-11 | -0.0379 | -0.0394 |
| P4 | -0.4780 | -0.4349 | Q8 | 0.1762 | 0.1158 | P13-14 | 0.0564 | 0.0878 |
| P5 | -0.0760 | -0.0601 | Q9 | -0.1660 | -0.1301 | Q1-2 | -0.2040 | -0.2571 |
| P6 | -0.1120 | -0.1774 | Q10 | -0.0580 | 0.0235 | Q1-5 | 0.0385 | 0.1560 |
| P7 | 0.0000 | -0.0217 | Q11 | -0.0180 | 0.0064 | Q2-3 | 0.0356 | 0.0048 |
| P8 | 0.0000 | 0.0171 | Q12 | -0.0160 | 0.0357 | Q3-4 | 0.0447 | 0.0891 |
| P9 | -0.2950 | -0.1161 | Q13 | -0.0580 | -0.0217 | Q4-7 | -0.0968 | -0.1669 |
| P10 | -0.0900 | 0.0485 | Q14 | -0.0500 | -0.0652 | Q5-6 | 0.1247 | 0.1491 |
| P11 | -0.0350 | -0.1025 | P1-2 | 1.5688 | 1.5835 | Q6-12 | 0.0250 | 0.0152 |
| P12 | -0.0610 | 0.0907 | P1-5 | 0.7551 | 0.7157 | Q7-8 | -0.1716 | -0.1570 |
| P13 | -0.1350 | -0.0987 | P2-3 | 0.7324 | 0.7768 | Q7-9 | 0.0578 | 0.0677 |
| P14 | -0.1490 | -0.1522 | P3-4 | -0.2329 | -0.2734 | Q9-10 | 0.0422 | 0.1216 |
| Q1 | -0.1655 | -0.1298 | P4-7 | 0.2807 | 0.3526 | Q10-11 | -0.0162 | 0.0186 |
| Q2 | 0.3086 | 0.2984 | P5-6 | 0.4409 | 0.4032 | Q13-14 | 0.0175 | 0.0053 |
| Q3 | 0.0608 | 0.0546 | P6-12 | 0.0779 | -0.0077 | | | |

Table 5.206 shows the system state variables generated by each estimator; Table 5.207 presents the estimation results and the summation of the absolute value of errors between estimation results, the summation of raw measurements are also presented for comparison. $N/A$ means that the data are not used in the estimation.

Table 5.206: State Variable of case 0, small noise, 14-bus AC system, meidan redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0874 | -0.0900 | -0.0874 | -0.0854 | **V1** | 1.0581 | 1.0438 | 1.0581 | 1.0705 |
| $\theta 3$ | -0.2355 | -0.2441 | -0.2355 | -0.2303 | **V2** | 1.0412 | 1.0259 | 1.0412 | 1.0539 |
| $\theta 4$ | -0.1815 | -0.1894 | -0.1815 | -0.1773 | **V3** | 1.0005 | 0.9832 | 1.0005 | 1.0144 |
| $\theta 5$ | -0.1534 | -0.1585 | -0.1534 | -0.1478 | **V4** | 1.0051 | 0.9903 | 1.0051 | 1.0178 |
| $\theta 6$ | -0.2552 | -0.2554 | -0.2552 | -0.2411 | **V5** | 1.0069 | 0.9929 | 1.0069 | 1.0169 |
| $\theta 7$ | -0.2391 | -0.2571 | -0.2391 | -0.2360 | **V6** | 1.0129 | 0.9965 | 1.0129 | 0.9938 |
| $\theta 8$ | -0.2392 | -0.2551 | -0.2392 | -0.2365 | **V7** | 1.0285 | 1.0151 | 1.0285 | 1.0477 |
| $\theta 9$ | -0.2641 | -0.2892 | -0.2641 | -0.2611 | **V8** | 1.0494 | 1.0348 | 1.0494 | 1.0695 |
| $\theta 10$ | -0.2666 | -0.2911 | -0.2666 | -0.2639 | **V9** | 1.0161 | 1.0023 | 1.0161 | 1.0365 |
| $\theta 11$ | -0.2694 | -0.2841 | -0.2694 | -0.2651 | **V10** | 1.0071 | 0.9899 | 1.0071 | 1.0271 |
| $\theta 12$ | -0.2487 | -0.2486 | -0.2487 | -0.2438 | **V11** | 0.9967 | 0.9886 | 0.9967 | 1.0081 |
| $\theta 13$ | -0.2640 | -0.2682 | -0.2640 | -0.2583 | **V12** | 0.9983 | 0.9950 | 0.9983 | 0.9994 |
| $\theta 14$ | -0.2894 | -0.3014 | -0.2894 | -0.2874 | **V13** | 0.9841 | 0.9836 | 0.9841 | 0.9860 |
| | | | | | **V14** | 0.9672 | 0.9712 | 0.9672 | 0.9867 |

Table 5.207: Estimation Results and Errors of case 0, small noise, 14-bus AC system, meidan redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0581 | 1.0438 | 1.0581 | 1.0705 | 0.0019 | 0.0162 | 0.0019 | 0.0105 |
| V2 | 1.0450 | N/A | 1.0412 | 1.0259 | 1.0412 | 1.0539 | 0.0038 | 0.0191 | 0.0038 | 0.0089 |
| V3 | 1.0100 | N/A | 1.0005 | 0.9832 | 1.0005 | 1.0144 | 0.0095 | 0.0268 | 0.0095 | 0.0044 |
| V4 | 1.0180 | N/A | 1.0051 | 0.9903 | 1.0051 | 1.0178 | 0.0129 | 0.0277 | 0.0129 | 0.0002 |
| V5 | 1.0200 | N/A | 1.0069 | 0.9929 | 1.0069 | 1.0169 | 0.0131 | 0.0271 | 0.0131 | 0.0031 |
| V6 | 1.0700 | N/A | 1.0129 | 0.9965 | 1.0129 | 0.9938 | 0.0571 | 0.0735 | 0.0571 | 0.0762 |
| V7 | 1.0620 | N/A | 1.0285 | 1.0151 | 1.0285 | 1.0477 | 0.0335 | 0.0469 | 0.0335 | 0.0143 |
| V8 | 1.0900 | N/A | 1.0494 | 1.0348 | 1.0494 | 1.0695 | 0.0406 | 0.0552 | 0.0406 | 0.0205 |
| V9 | 1.0560 | N/A | 1.0161 | 1.0023 | 1.0161 | 1.0365 | 0.0399 | 0.0537 | 0.0399 | 0.0195 |
| V10 | 1.0510 | N/A | 1.0071 | 0.9899 | 1.0071 | 1.0271 | 0.0439 | 0.0611 | 0.0439 | 0.0239 |
| V11 | 1.0570 | N/A | 0.9967 | 0.9886 | 0.9967 | 1.0081 | 0.0603 | 0.0684 | 0.0603 | 0.0489 |
| V12 | 1.0550 | N/A | 0.9983 | 0.9950 | 0.9983 | 0.9994 | 0.0567 | 0.0600 | 0.0567 | 0.0556 |
| V13 | 1.0500 | N/A | 0.9841 | 0.9836 | 0.9841 | 0.9860 | 0.0659 | 0.0664 | 0.0659 | 0.0640 |
| V14 | 1.0360 | N/A | 0.9672 | 0.9712 | 0.9672 | 0.9867 | 0.0688 | 0.0648 | 0.0688 | 0.0493 |
| P1 | 2.3239 | 2.3508 | 2.3357 | 2.3439 | 2.3357 | 2.3287 | 0.0118 | 0.0200 | 0.0118 | 0.0048 |
| P2 | 0.1830 | 0.2747 | 0.2590 | 0.2747 | 0.2590 | 0.2536 | 0.0760 | 0.0917 | 0.0760 | 0.0706 |
| P3 | -0.9420 | -1.0549 | -1.0495 | -1.0549 | -1.0495 | -1.0518 | 0.1075 | 0.1129 | 0.1075 | 0.1098 |
| P4 | -0.4780 | -0.4349 | -0.4465 | -0.4349 | -0.4465 | -0.4497 | 0.0315 | 0.0431 | 0.0315 | 0.0283 |
| P5 | -0.0760 | -0.0601 | -0.0715 | -0.0601 | -0.0715 | -0.0742 | 0.0045 | 0.0159 | 0.0045 | 0.0018 |
| P6 | -0.1120 | -0.1774 | -0.2056 | -0.1774 | -0.2056 | -0.2028 | 0.0936 | 0.0654 | 0.0936 | 0.0908 |
| P7 | 0.0000 | -0.0217 | -0.0528 | -0.0481 | -0.0528 | -0.0564 | 0.0528 | 0.0481 | 0.0528 | 0.0564 |
| P8 | 0.0000 | 0.0171 | -0.0009 | 0.0121 | -0.0009 | -0.0027 | 0.0009 | 0.0121 | 0.0009 | 0.0027 |
| P9 | -0.2950 | -0.1161 | -0.1844 | -0.3320 | -0.1844 | -0.1859 | 0.1106 | 0.0370 | 0.1106 | 0.1091 |
| P10 | -0.0900 | 0.0485 | -0.0301 | -0.0951 | -0.0301 | -0.0267 | 0.0599 | 0.0051 | 0.0599 | 0.0633 |
| P11 | -0.0350 | -0.1025 | -0.1215 | -0.1025 | -0.1215 | -0.1108 | 0.0865 | 0.0675 | 0.0865 | 0.0758 |
| P12 | -0.0610 | 0.0907 | 0.0679 | 0.0907 | 0.0679 | 0.0656 | 0.1289 | 0.1517 | 0.1289 | 0.1266 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P13 | -0.1350 | -0.0987 | -0.1335 | -0.0987 | -0.1335 | -0.1253 | 0.0015 | 0.0363 | 0.0015 | 0.0097 |
| P14 | -0.1490 | -0.1522 | -0.2166 | -0.1652 | -0.2166 | -0.2156 | 0.0676 | 0.0162 | 0.0676 | 0.0666 |
| Q1 | -0.1655 | -0.1298 | -0.1429 | -0.1298 | -0.1429 | -0.1351 | 0.0226 | 0.0357 | 0.0226 | 0.0304 |
| Q2 | 0.3086 | 0.2984 | 0.3105 | 0.2984 | 0.3105 | 0.3209 | 0.0019 | 0.0102 | 0.0019 | 0.0123 |
| Q3 | 0.0608 | 0.0546 | 0.0694 | 0.0546 | 0.0694 | 0.0780 | 0.0086 | 0.0062 | 0.0086 | 0.0172 |
| Q4 | 0.0390 | 0.1135 | 0.0930 | 0.1135 | 0.0930 | 0.1129 | 0.0540 | 0.0745 | 0.0540 | 0.0739 |
| Q5 | -0.0160 | 0.0545 | 0.0328 | 0.0545 | 0.0328 | 0.0496 | 0.0488 | 0.0705 | 0.0488 | 0.0656 |
| Q6 | 0.0523 | 0.1232 | -0.0013 | -0.2180 | -0.0013 | -0.4944 | 0.0536 | 0.2703 | 0.0536 | 0.5467 |
| Q7 | 0.0000 | 0.0336 | 0.0086 | 0.0336 | 0.0086 | 0.0240 | 0.0086 | 0.0336 | 0.0086 | 0.0240 |
| Q8 | 0.1762 | 0.1158 | 0.1250 | 0.1158 | 0.1250 | 0.1326 | 0.0512 | 0.0604 | 0.0512 | 0.0436 |
| Q9 | -0.1660 | -0.1301 | -0.1397 | -0.1301 | -0.1397 | -0.1172 | 0.0263 | 0.0359 | 0.0263 | 0.0488 |
| Q10 | -0.0580 | 0.0235 | -0.0422 | -0.1014 | -0.0422 | -0.0049 | 0.0158 | 0.0434 | 0.0158 | 0.0531 |
| Q11 | -0.0180 | 0.0064 | -0.0782 | 0.0064 | -0.0782 | 0.0252 | 0.0602 | 0.0244 | 0.0602 | 0.0432 |
| Q12 | -0.0160 | 0.0357 | -0.0612 | -0.0368 | -0.0612 | 0.0165 | 0.0452 | 0.0208 | 0.0452 | 0.0325 |
| Q13 | -0.0580 | -0.0217 | -0.1277 | -0.0217 | -0.1277 | -0.0209 | 0.0697 | 0.0363 | 0.0697 | 0.0371 |
| Q14 | -0.0500 | -0.0652 | -0.1163 | -0.0652 | -0.1163 | -0.0745 | 0.0663 | 0.0152 | 0.0663 | 0.0245 |
| P1-2 | 1.5688 | 1.5835 | 1.5781 | 1.5835 | 1.5781 | 1.5787 | 0.0093 | 0.0147 | 0.0093 | 0.0099 |
| P1-5 | 0.7551 | 0.7157 | 0.7576 | 0.7604 | 0.7576 | 0.7500 | 0.0025 | 0.0053 | 0.0025 | 0.0051 |
| P2-3 | 0.7324 | 0.7768 | 0.7960 | 0.8035 | 0.7960 | 0.7978 | 0.0636 | 0.0711 | 0.0636 | 0.0654 |
| P2-4 | 0.5613 | N/A | 0.5742 | 0.5860 | 0.5742 | 0.5751 | 0.0129 | 0.0247 | 0.0129 | 0.0138 |
| P2-5 | 0.4152 | N/A | 0.4234 | 0.4238 | 0.4234 | 0.4168 | 0.0082 | 0.0086 | 0.0082 | 0.0016 |
| P3-4 | -0.2329 | -0.2734 | -0.2813 | -0.2806 | -0.2813 | -0.2811 | 0.0484 | 0.0477 | 0.0484 | 0.0482 |
| P4-5 | -0.6116 | N/A | -0.6244 | -0.6706 | -0.6244 | -0.6489 | 0.0128 | 0.0590 | 0.0128 | 0.0373 |
| P4-7 | 0.2807 | 0.3526 | 0.2909 | 0.3327 | 0.2909 | 0.3061 | 0.0102 | 0.0520 | 0.0102 | 0.0254 |
| P4-9 | 0.1608 | N/A | 0.1563 | 0.1835 | 0.1563 | 0.1638 | 0.0045 | 0.0227 | 0.0045 | 0.0030 |
| P5-6 | 0.4409 | 0.4032 | 0.4415 | 0.4076 | 0.4415 | 0.4009 | 0.0006 | 0.0333 | 0.0006 | 0.0400 |
| P6-11 | 0.0735 | N/A | 0.0907 | 0.1321 | 0.0907 | 0.0709 | 0.0172 | 0.0586 | 0.0172 | 0.0026 |
| P6-12 | 0.0779 | -0.0077 | 0.0017 | -0.0188 | 0.0017 | 0.0001 | 0.0762 | 0.0967 | 0.0762 | 0.0778 |
| P6-13 | 0.1775 | N/A | 0.1435 | 0.1170 | 0.1435 | 0.1271 | 0.0340 | 0.0605 | 0.0340 | 0.0504 |
| P7-8 | 0.0000 | -0.0121 | 0.0009 | -0.0121 | 0.0009 | 0.0027 | 0.0009 | 0.0121 | 0.0009 | 0.0027 |
| P7-9 | 0.2807 | 0.2967 | 0.2372 | 0.2967 | 0.2372 | 0.2470 | 0.0435 | 0.0160 | 0.0435 | 0.0337 |
| P9-10 | 0.0523 | 0.0679 | 0.0624 | 0.0679 | 0.0624 | 0.0691 | 0.0101 | 0.0156 | 0.0101 | 0.0168 |
| P9-14 | 0.0943 | N/A | 0.1466 | 0.0803 | 0.1466 | 0.1557 | 0.0523 | 0.0140 | 0.0523 | 0.0614 |
| P10-11 | -0.0379 | -0.0394 | 0.0320 | -0.0278 | 0.0320 | 0.0421 | 0.0699 | 0.0101 | 0.0699 | 0.0800 |
| P12-13 | 0.0161 | N/A | 0.0692 | 0.0718 | 0.0692 | 0.0656 | 0.0531 | 0.0557 | 0.0531 | 0.0495 |
| P13-14 | 0.0564 | 0.0878 | 0.0753 | 0.0878 | 0.0753 | 0.0653 | 0.0189 | 0.0314 | 0.0189 | 0.0089 |
| P2-1 | -1.5259 | N/A | -1.5347 | -1.5386 | -1.5347 | -1.5362 | 0.0088 | 0.0127 | 0.0088 | 0.0103 |
| P5-1 | -0.7275 | N/A | -0.7292 | -0.7311 | -0.7292 | -0.7227 | 0.0017 | 0.0036 | 0.0017 | 0.0048 |
| P3-2 | -0.7091 | N/A | -0.7682 | -0.7743 | -0.7682 | -0.7707 | 0.0591 | 0.0652 | 0.0591 | 0.0616 |
| P4-2 | -0.5445 | N/A | -0.5564 | -0.5669 | -0.5564 | -0.5576 | 0.0119 | 0.0224 | 0.0119 | 0.0131 |
| P5-2 | -0.4061 | N/A | -0.4137 | -0.4138 | -0.4137 | -0.4074 | 0.0076 | 0.0077 | 0.0076 | 0.0013 |
| P4-3 | 0.2366 | N/A | 0.2871 | 0.2864 | 0.2871 | 0.2869 | 0.0505 | 0.0498 | 0.0505 | 0.0503 |
| P5-4 | 0.6167 | N/A | 0.6299 | 0.6771 | 0.6299 | 0.6551 | 0.0132 | 0.0604 | 0.0132 | 0.0384 |
| P7-4 | -0.2807 | N/A | -0.2909 | -0.3327 | -0.2909 | -0.3061 | 0.0102 | 0.0520 | 0.0102 | 0.0254 |
| P9-4 | -0.1608 | N/A | -0.1563 | -0.1835 | -0.1563 | -0.1638 | 0.0045 | 0.0227 | 0.0045 | 0.0030 |
| P6-5 | -0.4409 | N/A | -0.4415 | -0.4076 | -0.4415 | -0.4009 | 0.0006 | 0.0333 | 0.0006 | 0.0400 |
| P11-6 | -0.0730 | N/A | -0.0898 | -0.1304 | -0.0898 | -0.0694 | 0.0168 | 0.0574 | 0.0168 | 0.0036 |
| P12-6 | -0.0771 | N/A | -0.0013 | 0.0189 | -0.0013 | 0.0000 | 0.0758 | 0.0960 | 0.0758 | 0.0771 |
| P13-6 | -0.1754 | N/A | -0.1407 | -0.1160 | -0.1407 | -0.1260 | 0.0347 | 0.0594 | 0.0347 | 0.0494 |
| P8-7 | 0.0000 | N/A | -0.0009 | 0.0121 | -0.0009 | -0.0027 | 0.0009 | 0.0121 | 0.0009 | 0.0027 |
| P9-7 | -0.2807 | N/A | -0.2372 | -0.2967 | -0.2372 | -0.2470 | 0.0435 | 0.0160 | 0.0435 | 0.0337 |
| P10-9 | -0.0521 | N/A | -0.0621 | -0.0673 | -0.0621 | -0.0688 | 0.0100 | 0.0152 | 0.0100 | 0.0167 |
| P14-9 | -0.0931 | N/A | -0.1423 | -0.0787 | -0.1423 | -0.1512 | 0.0492 | 0.0144 | 0.0492 | 0.0581 |
| P11-10 | 0.0380 | N/A | -0.0318 | 0.0279 | -0.0318 | -0.0414 | 0.0698 | 0.0101 | 0.0698 | 0.0794 |
| P13-12 | -0.0161 | N/A | -0.0681 | -0.0705 | -0.0681 | -0.0646 | 0.0520 | 0.0544 | 0.0520 | 0.0485 |
| P14-13 | -0.0559 | N/A | -0.0743 | -0.0864 | -0.0743 | -0.0644 | 0.0184 | 0.0305 | 0.0184 | 0.0085 |
| Q1-2 | -0.2040 | -0.2571 | -0.2031 | -0.1884 | -0.2031 | -0.2072 | 0.0009 | 0.0156 | 0.0009 | 0.0032 |
| Q1-5 | 0.0385 | 0.1560 | 0.0602 | 0.0586 | 0.0602 | 0.0721 | 0.0217 | 0.0201 | 0.0217 | 0.0336 |
| Q2-3 | 0.0356 | 0.0048 | 0.0353 | 0.0452 | 0.0353 | 0.0284 | 0.0003 | 0.0096 | 0.0003 | 0.0072 |
| Q2-4 | -0.0155 | N/A | 0.0135 | 0.0071 | 0.0135 | 0.0139 | 0.0290 | 0.0226 | 0.0290 | 0.0294 |
| Q2-5 | 0.0117 | N/A | 0.0422 | 0.0337 | 0.0422 | 0.0608 | 0.0305 | 0.0220 | 0.0305 | 0.0491 |
| Q3-4 | 0.0447 | 0.0891 | 0.0791 | 0.0652 | 0.0791 | 0.0855 | 0.0344 | 0.0205 | 0.0344 | 0.0408 |
| Q4-5 | 0.1582 | N/A | 0.1633 | 0.1629 | 0.1633 | 0.2370 | 0.0051 | 0.0047 | 0.0051 | 0.0788 |
| Q4-7 | -0.0968 | -0.1669 | -0.1067 | -0.1091 | -0.1067 | -0.1399 | 0.0099 | 0.0123 | 0.0099 | 0.0431 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q4-9 | -0.0043 | N/A | -0.0141 | -0.0129 | -0.0141 | -0.0285 | 0.0098 | 0.0086 | 0.0098 | 0.0242 |
| Q5-6 | 0.1247 | 0.1491 | -0.0031 | 0.0042 | -0.0031 | 0.1188 | 0.1278 | 0.1205 | 0.1278 | 0.0059 |
| Q6-11 | 0.0356 | N/A | 0.0395 | -0.0212 | 0.0395 | -0.1040 | 0.0039 | 0.0568 | 0.0039 | 0.1396 |
| Q6-12 | 0.0250 | 0.0152 | 0.0569 | 0.0152 | 0.0569 | -0.0216 | 0.0319 | 0.0098 | 0.0319 | 0.0466 |
| Q6-13 | 0.0722 | N/A | 0.1509 | 0.0402 | 0.1509 | -0.0038 | 0.0787 | 0.0320 | 0.0787 | 0.0760 |
| Q7-8 | -0.1716 | -0.1570 | -0.1225 | -0.1136 | -0.1225 | -0.1299 | 0.0491 | 0.0580 | 0.0491 | 0.0417 |
| Q7-9 | 0.0578 | 0.0677 | 0.1187 | 0.1234 | 0.1187 | 0.1098 | 0.0609 | 0.0656 | 0.0609 | 0.0520 |
| Q9-10 | 0.0422 | 0.1216 | 0.0842 | 0.1216 | 0.0842 | 0.0894 | 0.0420 | 0.0794 | 0.0420 | 0.0472 |
| Q9-14 | 0.0361 | N/A | 0.1159 | 0.0778 | 0.1159 | 0.1190 | 0.0798 | 0.0417 | 0.0798 | 0.0829 |
| Q10-11 | -0.0162 | 0.0186 | 0.0410 | 0.0186 | 0.0410 | 0.0835 | 0.0572 | 0.0348 | 0.0572 | 0.0997 |
| Q12-13 | 0.0075 | N/A | -0.0051 | -0.0218 | -0.0051 | -0.0052 | 0.0126 | 0.0293 | 0.0126 | 0.0127 |
| Q13-14 | 0.0175 | 0.0053 | 0.0117 | -0.0065 | 0.0117 | -0.0329 | 0.0058 | 0.0240 | 0.0058 | 0.0504 |
| Q2-1 | 0.2768 | N/A | 0.2195 | 0.2125 | 0.2195 | 0.2179 | 0.0573 | 0.0643 | 0.0573 | 0.0589 |
| Q5-1 | 0.0223 | N/A | -0.0481 | -0.0398 | -0.0481 | -0.0667 | 0.0704 | 0.0621 | 0.0704 | 0.0890 |
| Q3-2 | 0.0160 | N/A | -0.0097 | -0.0106 | -0.0097 | -0.0076 | 0.0257 | 0.0266 | 0.0257 | 0.0236 |
| Q4-2 | 0.0302 | N/A | -0.0307 | -0.0184 | -0.0307 | -0.0339 | 0.0609 | 0.0486 | 0.0609 | 0.0641 |
| Q5-2 | -0.0210 | N/A | -0.0850 | -0.0737 | -0.0850 | -0.1062 | 0.0640 | 0.0527 | 0.0640 | 0.0852 |
| Q4-3 | -0.0484 | N/A | -0.0899 | -0.0751 | -0.0899 | -0.0972 | 0.0415 | 0.0267 | 0.0415 | 0.0488 |
| Q5-4 | 0.1420 | N/A | -0.1459 | -0.1424 | -0.1459 | -0.2176 | 0.2879 | 0.2844 | 0.2879 | 0.3596 |
| Q7-4 | 0.1138 | N/A | 0.1261 | 0.1346 | 0.1261 | 0.1622 | 0.0123 | 0.0208 | 0.0123 | 0.0484 |
| Q9-4 | 0.0173 | N/A | 0.0272 | 0.0315 | 0.0272 | 0.0429 | 0.0099 | 0.0142 | 0.0099 | 0.0256 |
| Q6-5 | -0.0805 | N/A | 0.0482 | 0.0353 | 0.0482 | -0.0791 | 0.1287 | 0.1158 | 0.1287 | 0.0014 |
| Q11-6 | -0.0344 | N/A | -0.0376 | 0.0248 | -0.0376 | 0.1071 | 0.0032 | 0.0592 | 0.0032 | 0.1415 |
| Q12-6 | -0.0235 | N/A | -0.0561 | -0.0150 | -0.0561 | 0.0217 | 0.0326 | 0.0085 | 0.0326 | 0.0452 |
| Q13-6 | -0.0680 | N/A | -0.1454 | -0.0381 | -0.1454 | 0.0060 | 0.0774 | 0.0299 | 0.0774 | 0.0740 |
| Q8-7 | 0.1762 | N/A | 0.1250 | 0.1158 | 0.1250 | 0.1326 | 0.0512 | 0.0604 | 0.0512 | 0.0436 |
| Q9-7 | -0.0498 | N/A | -0.1114 | -0.1124 | -0.1114 | -0.1024 | 0.0616 | 0.0626 | 0.0616 | 0.0526 |
| Q10-9 | -0.0418 | N/A | -0.0833 | -0.1200 | -0.0833 | -0.0884 | 0.0415 | 0.0782 | 0.0415 | 0.0466 |
| Q14-9 | -0.0336 | N/A | -0.1067 | -0.0745 | -0.1067 | -0.1093 | 0.0731 | 0.0409 | 0.0731 | 0.0757 |
| Q11-10 | 0.0164 | N/A | -0.0405 | -0.0184 | -0.0405 | -0.0819 | 0.0569 | 0.0348 | 0.0569 | 0.0983 |
| Q13-12 | -0.0075 | N/A | 0.0061 | 0.0229 | 0.0061 | 0.0061 | 0.0136 | 0.0304 | 0.0136 | 0.0136 |
| Q14-13 | -0.0164 | N/A | -0.0096 | 0.0093 | -0.0096 | 0.0348 | 0.0068 | 0.0257 | 0.0068 | 0.0512 |
| **SUM** | | | | | | | **2.5633** | **2.9987** | **2.5633** | **3.0435** |

WLS and LTS share the best results, LAV performs bad in this case, making LTAV detect wrong bad data and generate worst estimation.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.208. Full redundancy cases have advantage over median ones for all estimators.

Table 5.208: Estimation errors of case 0, small noise, 14-bus AC system

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **2.9714** | **2.1408** | **2.0361** | **2.1408** | **1.9526** |
| **SUM** | **N/A** | **2.5633** | **2.9987** | **2.5633** | **3.0435** |

#### 5.2.2.2 Case 1: Single bad data

Two types of bad data are conducted here for performance evaluation, reverse and large bad data. Full and median redundancy system are used for testing.

**Reverse bad data**

A reverse bad data $P_4 = 0.4349$ are used to replace the original one to simulate reverse bad data in both cases.

**Full redundancy**

Table 5.209 presents the state variables, Table 5.210 shows the estimated value and errors between each estimator and actual value.

Table 5.209: State Variable of case 1, reverse, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0812 | -0.0814 | -0.0816 | -0.0826 | **V1** | 1.0878 | 1.0770 | 1.0900 | 1.0835 |
| $\theta 3$ | -0.2209 | -0.2210 | -0.2171 | -0.2198 | **V2** | 1.0721 | 1.0641 | 1.0743 | 1.0678 |
| $\theta 4$ | -0.1571 | -0.1679 | -0.1711 | -0.1737 | **V3** | 1.0244 | 1.0249 | 1.0290 | 1.0225 |
| $\theta 5$ | -0.1373 | -0.1423 | -0.1437 | -0.1462 | **V4** | 1.0435 | 1.0318 | 1.0413 | 1.0357 |
| $\theta 6$ | -0.2365 | -0.2331 | -0.2295 | -0.2344 | **V5** | 1.0466 | 1.0352 | 1.0466 | 1.0412 |
| $\theta 7$ | -0.2335 | -0.2342 | -0.2205 | -0.2283 | **V6** | 1.0630 | 1.0576 | 1.0628 | 1.0709 |
| $\theta 8$ | -0.2379 | -0.2340 | -0.2199 | -0.2267 | **V7** | 1.0700 | 1.0551 | 1.0685 | 1.0659 |
| $\theta 9$ | -0.2576 | -0.2637 | -0.2428 | -0.2559 | **V8** | 1.0897 | 1.0767 | 1.0881 | 1.0859 |
| $\theta 10$ | -0.2595 | -0.2638 | -0.2440 | -0.2538 | **V9** | 1.0627 | 1.0485 | 1.0614 | 1.0600 |
| $\theta 11$ | -0.2552 | -0.2557 | -0.2429 | -0.2500 | **V10** | 1.0544 | 1.0388 | 1.0537 | 1.0547 |
| $\theta 12$ | -0.2336 | -0.2276 | -0.2252 | -0.2312 | **V11** | 1.0453 | 1.0383 | 1.0453 | 1.0509 |
| $\theta 13$ | -0.2486 | -0.2440 | -0.2396 | -0.2448 | **V12** | 1.0530 | 1.0558 | 1.0535 | 1.0634 |
| $\theta 14$ | -0.2773 | -0.2731 | -0.2620 | -0.2676 | **V13** | 1.0403 | 1.0437 | 1.0408 | 1.0500 |
| | | | | | **V14** | 1.0322 | 1.0273 | 1.0326 | 1.0377 |

Table 5.210: Estimation Results and Errors of case 1, reverse, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0878 | 1.0770 | 1.0900 | 1.0835 | 0.0086 | 0.0278 | 0.0170 | 0.0300 | 0.0235 |
| V2 | 1.0450 | 1.0743 | 1.0721 | 1.0641 | 1.0743 | 1.0678 | 0.0293 | 0.0271 | 0.0191 | 0.0293 | 0.0228 |
| V3 | 1.0100 | 0.9739 | 1.0244 | 1.0249 | 1.0290 | 1.0225 | 0.0361 | 0.0144 | 0.0149 | 0.0190 | 0.0125 |
| V4 | 1.0180 | 1.0318 | 1.0435 | 1.0318 | 1.0413 | 1.0357 | 0.0138 | 0.0255 | 0.0138 | 0.0233 | 0.0177 |
| V5 | 1.0200 | 1.0251 | 1.0466 | 1.0352 | 1.0466 | 1.0412 | 0.0051 | 0.0266 | 0.0152 | 0.0266 | 0.0212 |
| V6 | 1.0700 | 1.0491 | 1.0630 | 1.0576 | 1.0628 | 1.0709 | 0.0209 | 0.0070 | 0.0124 | 0.0072 | 0.0009 |
| V7 | 1.0620 | 1.0551 | 1.0700 | 1.0551 | 1.0685 | 1.0659 | 0.0069 | 0.0080 | 0.0069 | 0.0065 | 0.0039 |
| V8 | 1.0900 | 1.0955 | 1.0897 | 1.0767 | 1.0881 | 1.0859 | 0.0055 | 0.0003 | 0.0133 | 0.0019 | 0.0041 |
| V9 | 1.0560 | 1.1131 | 1.0627 | 1.0485 | 1.0614 | 1.0600 | 0.0571 | 0.0067 | 0.0075 | 0.0054 | 0.0040 |
| V10 | 1.0510 | 1.0952 | 1.0544 | 1.0388 | 1.0537 | 1.0547 | 0.0442 | 0.0034 | 0.0122 | 0.0027 | 0.0037 |
| V11 | 1.0570 | 1.0355 | 1.0453 | 1.0383 | 1.0453 | 1.0509 | 0.0215 | 0.0117 | 0.0187 | 0.0117 | 0.0061 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V12 | 1.0550 | 1.1034 | 1.0530 | 1.0558 | 1.0535 | 1.0634 | 0.0484 | 0.0020 | 0.0008 | 0.0015 | 0.0084 |
| V13 | 1.0500 | 1.0616 | 1.0403 | 1.0437 | 1.0408 | 1.0500 | 0.0116 | 0.0097 | 0.0063 | 0.0092 | 0.0000 |
| V14 | 1.0360 | 1.0350 | 1.0322 | 1.0273 | 1.0326 | 1.0377 | 0.0010 | 0.0038 | 0.0087 | 0.0034 | 0.0017 |
| P1 | 2.3239 | 2.3508 | 2.2650 | 2.2375 | 2.3134 | 2.3172 | 0.0269 | 0.0589 | 0.0864 | 0.0105 | 0.0067 |
| P2 | 0.1830 | 0.2747 | 0.1601 | 0.2694 | 0.2552 | 0.2584 | 0.0917 | 0.0229 | 0.0864 | 0.0722 | 0.0754 |
| P3 | -0.9420 | -1.0549 | -1.1544 | -1.0549 | -1.0283 | -1.0272 | 0.1129 | 0.2124 | 0.1129 | 0.0863 | 0.0852 |
| **P4** | **-0.4780** | **0.4349** | **0.0456** | **-0.3006** | **-0.5476** | **-0.4991** | **0.9129** | **0.5236** | **0.1774** | **0.0696** | **0.0211** |
| P5 | -0.0760 | -0.0601 | -0.1020 | -0.0601 | -0.0431 | -0.0394 | 0.0159 | 0.0260 | 0.0159 | 0.0329 | 0.0366 |
| P6 | -0.1120 | -0.1774 | -0.1842 | -0.1774 | -0.1681 | -0.1622 | 0.0654 | 0.0722 | 0.0654 | 0.0561 | 0.0502 |
| P7 | 0.0000 | -0.0217 | -0.1387 | -0.0575 | -0.0415 | -0.0215 | 0.0217 | 0.1387 | 0.0575 | 0.0415 | 0.0215 |
| P8 | 0.0000 | 0.0171 | -0.0288 | 0.0016 | 0.0037 | 0.0104 | 0.0171 | 0.0288 | 0.0016 | 0.0037 | 0.0104 |
| P9 | -0.2950 | -0.1161 | -0.2878 | -0.3862 | -0.2250 | -0.3805 | 0.1789 | 0.0072 | 0.0912 | 0.0700 | 0.0855 |
| P10 | -0.0900 | 0.0485 | -0.0584 | -0.0775 | -0.0340 | -0.0092 | 0.1385 | 0.0316 | 0.0125 | 0.0560 | 0.0808 |
| P11 | -0.0350 | -0.1025 | -0.1180 | -0.1025 | -0.1074 | -0.1012 | 0.0675 | 0.0830 | 0.0675 | 0.0724 | 0.0662 |
| P12 | -0.0610 | 0.0907 | 0.0650 | 0.0900 | 0.0695 | 0.0696 | 0.1517 | 0.1260 | 0.1510 | 0.1305 | 0.1306 |
| P13 | -0.1350 | -0.0987 | -0.1432 | -0.0987 | -0.1413 | -0.1365 | 0.0363 | 0.0082 | 0.0363 | 0.0063 | 0.0015 |
| P14 | -0.1490 | -0.1522 | -0.1900 | -0.1522 | -0.1709 | -0.1431 | 0.0032 | 0.0410 | 0.0032 | 0.0219 | 0.0059 |
| Q1 | -0.1655 | -0.1298 | -0.1995 | -0.2378 | -0.1952 | -0.2014 | 0.0357 | 0.0340 | 0.0723 | 0.0297 | 0.0359 |
| Q2 | 0.3086 | 0.2984 | 0.2812 | 0.2984 | 0.2818 | 0.2733 | 0.0102 | 0.0274 | 0.0102 | 0.0268 | 0.0353 |
| Q3 | 0.0608 | 0.0546 | -0.0211 | 0.0546 | -0.0250 | -0.0277 | 0.0062 | 0.0819 | 0.0062 | 0.0858 | 0.0885 |
| Q4 | 0.0390 | 0.1135 | 0.0559 | 0.0646 | 0.0498 | 0.0354 | 0.0745 | 0.0169 | 0.0256 | 0.0108 | 0.0036 |
| Q5 | -0.0160 | 0.0545 | 0.1243 | 0.0545 | 0.1235 | 0.0841 | 0.0705 | 0.1403 | 0.0705 | 0.1395 | 0.1001 |
| Q6 | 0.0523 | 0.1232 | -0.0468 | -0.1003 | -0.0415 | 0.0078 | 0.0709 | 0.0991 | 0.1526 | 0.0938 | 0.0445 |
| Q7 | 0.0000 | 0.0336 | -0.0153 | -0.0493 | -0.0216 | -0.0176 | 0.0336 | 0.0153 | 0.0493 | 0.0216 | 0.0176 |
| Q8 | 0.1762 | 0.1158 | 0.1223 | 0.1318 | 0.1209 | 0.1233 | 0.0604 | 0.0539 | 0.0444 | 0.0553 | 0.0529 |
| Q9 | -0.1660 | -0.1301 | -0.1462 | -0.1301 | -0.1560 | -0.1538 | 0.0359 | 0.0198 | 0.0359 | 0.0100 | 0.0122 |
| Q10 | -0.0580 | 0.0235 | -0.0323 | -0.0861 | -0.0387 | -0.0421 | 0.0815 | 0.0257 | 0.0281 | 0.0193 | 0.0159 |
| Q11 | -0.0180 | 0.0064 | -0.0851 | -0.0503 | -0.0861 | -0.0769 | 0.0244 | 0.0671 | 0.0323 | 0.0681 | 0.0589 |
| Q12 | -0.0160 | 0.0357 | -0.0493 | -0.0311 | -0.0473 | -0.0367 | 0.0517 | 0.0333 | 0.0151 | 0.0313 | 0.0207 |
| Q13 | -0.0580 | -0.0217 | -0.1050 | -0.0281 | -0.1021 | -0.0884 | 0.0363 | 0.0470 | 0.0299 | 0.0441 | 0.0304 |
| Q14 | -0.0500 | -0.0652 | -0.0476 | -0.0541 | -0.0513 | -0.0524 | 0.0152 | 0.0024 | 0.0041 | 0.0013 | 0.0024 |
| P1-2 | 1.5688 | 1.5835 | 1.5482 | 1.5099 | 1.5611 | 1.5619 | 0.0147 | 0.0206 | 0.0589 | 0.0077 | 0.0069 |
| P1-5 | 0.7551 | 0.7157 | 0.7167 | 0.7276 | 0.7522 | 0.7553 | 0.0394 | 0.0384 | 0.0275 | 0.0029 | 0.0002 |
| P2-3 | 0.7324 | 0.7768 | 0.8014 | 0.7854 | 0.7808 | 0.7807 | 0.0444 | 0.0690 | 0.0530 | 0.0484 | 0.0483 |
| P2-4 | 0.5613 | 0.5039 | 0.4911 | 0.5507 | 0.5787 | 0.5800 | 0.0574 | 0.0702 | 0.0106 | 0.0174 | 0.0187 |
| P2-5 | 0.4152 | 0.3618 | 0.3762 | 0.4046 | 0.4166 | 0.4190 | 0.0534 | 0.0390 | 0.0106 | 0.0014 | 0.0038 |
| P3-4 | -0.2329 | -0.2734 | -0.3799 | -0.2954 | -0.2728 | -0.2722 | 0.0405 | 0.1470 | 0.0625 | 0.0399 | 0.0393 |
| P4-5 | -0.6116 | -0.7588 | -0.4878 | -0.6115 | -0.6791 | -0.6755 | 0.1472 | 0.1238 | 0.0001 | 0.0675 | 0.0639 |
| P4-7 | 0.2807 | 0.3526 | 0.4167 | 0.3526 | 0.2687 | 0.2948 | 0.0719 | 0.1360 | 0.0719 | 0.0120 | 0.0141 |
| P4-9 | 0.1608 | 0.1771 | 0.2065 | 0.1920 | 0.1471 | 0.1674 | 0.0163 | 0.0457 | 0.0312 | 0.0137 | 0.0066 |
| P5-6 | 0.4409 | 0.4032 | 0.4693 | 0.4223 | 0.4061 | 0.4182 | 0.0377 | 0.0284 | 0.0186 | 0.0348 | 0.0227 |
| P6-11 | 0.0735 | 0.1420 | 0.1222 | 0.1420 | 0.0974 | 0.1139 | 0.0685 | 0.0487 | 0.0685 | 0.0239 | 0.0404 |
| P6-12 | 0.0779 | -0.0077 | 0.0059 | -0.0167 | -0.0002 | 0.0007 | 0.0856 | 0.0720 | 0.0946 | 0.0781 | 0.0772 |
| P6-13 | 0.1775 | 0.1724 | 0.1569 | 0.1196 | 0.1408 | 0.1414 | 0.0051 | 0.0206 | 0.0579 | 0.0367 | 0.0361 |
| P7-8 | 0.0000 | -0.0121 | 0.0288 | -0.0016 | -0.0037 | -0.0104 | 0.0121 | 0.0288 | 0.0016 | 0.0037 | 0.0104 |
| P7-9 | 0.2807 | 0.2967 | 0.2491 | 0.2967 | 0.2308 | 0.2837 | 0.0160 | 0.0316 | 0.0160 | 0.0499 | 0.0030 |
| P9-10 | 0.0523 | 0.0679 | 0.0560 | 0.0403 | 0.0454 | -0.0020 | 0.0156 | 0.0037 | 0.0120 | 0.0069 | 0.0543 |
| P9-14 | 0.0943 | 0.0511 | 0.1118 | 0.0622 | 0.1075 | 0.0726 | 0.0432 | 0.0175 | 0.0321 | 0.0132 | 0.0217 |
| P10-11 | -0.0379 | -0.0394 | -0.0027 | -0.0375 | 0.0112 | -0.0113 | 0.0015 | 0.0352 | 0.0004 | 0.0491 | 0.0266 |
| P12-13 | 0.0161 | 0.0079 | 0.0707 | 0.0732 | 0.0691 | 0.0701 | 0.0082 | 0.0546 | 0.0571 | 0.0530 | 0.0540 |
| P13-14 | 0.0564 | 0.0878 | 0.0813 | 0.0921 | 0.0658 | 0.0722 | 0.0314 | 0.0249 | 0.0357 | 0.0094 | 0.0158 |
| P2-1 | -1.5259 | -1.4712 | -1.5086 | -1.4712 | -1.5210 | -1.5212 | 0.0547 | 0.0173 | 0.0547 | 0.0049 | 0.0047 |
| P5-1 | -0.7275 | -0.6720 | -0.6930 | -0.7026 | -0.7262 | -0.7288 | 0.0555 | 0.0345 | 0.0249 | 0.0013 | 0.0013 |
| P3-2 | -0.7091 | -0.7523 | -0.7745 | -0.7595 | -0.7555 | -0.7550 | 0.0432 | 0.0654 | 0.0504 | 0.0464 | 0.0459 |
| P4-2 | -0.5445 | -0.5406 | -0.4789 | -0.5350 | -0.5618 | -0.5628 | 0.0039 | 0.0656 | 0.0095 | 0.0173 | 0.0183 |
| P5-2 | -0.4061 | -0.4668 | -0.3690 | -0.3962 | -0.4080 | -0.4102 | 0.0607 | 0.0371 | 0.0099 | 0.0019 | 0.0041 |
| P4-3 | 0.2366 | 0.1809 | 0.3893 | 0.3014 | 0.2776 | 0.2770 | 0.0557 | 0.1527 | 0.0648 | 0.0410 | 0.0404 |
| P5-4 | 0.6167 | 0.6164 | 0.4908 | 0.6164 | 0.6849 | 0.6813 | 0.0003 | 0.1259 | 0.0003 | 0.0682 | 0.0646 |
| P7-4 | -0.2807 | -0.2041 | -0.4167 | -0.3526 | -0.2687 | -0.2948 | 0.0766 | 0.1360 | 0.0719 | 0.0120 | 0.0141 |
| P9-4 | -0.1608 | -0.1993 | -0.2065 | -0.1920 | -0.1471 | -0.1674 | 0.0385 | 0.0457 | 0.0312 | 0.0137 | 0.0066 |
| P6-5 | -0.4409 | -0.4223 | -0.4693 | -0.4223 | -0.4061 | -0.4182 | 0.0186 | 0.0284 | 0.0186 | 0.0348 | 0.0227 |
| P11-6 | -0.0730 | -0.0843 | -0.1209 | -0.1402 | -0.0964 | -0.1126 | 0.0113 | 0.0479 | 0.0672 | 0.0234 | 0.0396 |
| P12-6 | -0.0771 | -0.0212 | -0.0057 | 0.0168 | 0.0004 | -0.0005 | 0.0559 | 0.0714 | 0.0939 | 0.0775 | 0.0766 |
| P13-6 | -0.1754 | -0.2299 | -0.1548 | -0.1186 | -0.1390 | -0.1396 | 0.0545 | 0.0206 | 0.0568 | 0.0364 | 0.0358 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P8-7 | 0.0000 | 0.0016 | -0.0288 | 0.0016 | 0.0037 | 0.0104 | 0.0016 | 0.0288 | 0.0016 | 0.0037 | 0.0104 |
| P9-7 | -0.2807 | -0.2531 | -0.2491 | -0.2967 | -0.2308 | -0.2837 | 0.0276 | 0.0316 | 0.0160 | 0.0499 | 0.0030 |
| P10-9 | -0.0521 | 0.0029 | -0.0557 | -0.0399 | -0.0452 | 0.0022 | 0.0550 | 0.0036 | 0.0122 | 0.0069 | 0.0543 |
| P14-9 | -0.0931 | -0.0159 | -0.1098 | -0.0615 | -0.1058 | -0.0716 | 0.0772 | 0.0167 | 0.0316 | 0.0127 | 0.0215 |
| P11-10 | 0.0380 | 0.0423 | 0.0029 | 0.0377 | -0.0110 | 0.0114 | 0.0043 | 0.0351 | 0.0003 | 0.0490 | 0.0266 |
| P13-12 | -0.0161 | -0.0907 | -0.0697 | -0.0721 | -0.0682 | -0.0691 | 0.0746 | 0.0536 | 0.0560 | 0.0521 | 0.0530 |
| P14-13 | -0.0559 | -0.0930 | -0.0802 | -0.0907 | -0.0651 | -0.0714 | 0.0371 | 0.0243 | 0.0348 | 0.0092 | 0.0155 |
| Q1-2 | -0.2040 | -0.2571 | -0.2165 | -0.2571 | -0.2190 | -0.2200 | 0.0531 | 0.0125 | 0.0531 | 0.0150 | 0.0160 |
| Q1-5 | 0.0385 | 0.1560 | 0.0170 | 0.0193 | 0.0238 | 0.0186 | 0.1175 | 0.0215 | 0.0192 | 0.0147 | 0.0199 |
| Q2-3 | 0.0356 | 0.0048 | 0.0716 | 0.0284 | 0.0608 | 0.0610 | 0.0308 | 0.0360 | 0.0072 | 0.0252 | 0.0254 |
| Q2-4 | -0.0155 | 0.0219 | -0.0091 | -0.0016 | -0.0038 | -0.0093 | 0.0374 | 0.0064 | 0.0139 | 0.0117 | 0.0062 |
| Q2-5 | 0.0117 | 0.0021 | 0.0044 | 0.0173 | 0.0068 | -0.0004 | 0.0096 | 0.0073 | 0.0056 | 0.0049 | 0.0121 |
| Q3-4 | 0.0447 | 0.0891 | 0.0336 | 0.0697 | 0.0260 | 0.0211 | 0.0444 | 0.0111 | 0.0250 | 0.0187 | 0.0236 |
| Q4-5 | 0.1582 | 0.1200 | 0.0841 | 0.1200 | 0.0935 | 0.0880 | 0.0382 | 0.0741 | 0.0382 | 0.0647 | 0.0702 |
| Q4-7 | -0.0968 | -0.1669 | -0.1189 | -0.1059 | -0.1317 | -0.1447 | 0.0701 | 0.0221 | 0.0091 | 0.0349 | 0.0479 |
| Q4-9 | -0.0043 | -0.0754 | -0.0267 | -0.0228 | -0.0335 | -0.0399 | 0.0711 | 0.0224 | 0.0185 | 0.0292 | 0.0356 |
| Q5-6 | 0.1247 | 0.1491 | -0.0499 | -0.0796 | -0.0547 | -0.1131 | 0.0244 | 0.1746 | 0.2043 | 0.1794 | 0.2378 |
| Q6-11 | 0.0356 | 0.0267 | 0.0374 | 0.0359 | 0.0475 | 0.0541 | 0.0089 | 0.0018 | 0.0003 | 0.0119 | 0.0185 |
| Q6-12 | 0.0250 | 0.0152 | 0.0389 | 0.0152 | 0.0388 | 0.0312 | 0.0098 | 0.0139 | 0.0098 | 0.0138 | 0.0062 |
| Q6-13 | 0.0722 | 0.1432 | 0.1064 | 0.0524 | 0.1085 | 0.1007 | 0.0710 | 0.0342 | 0.0198 | 0.0363 | 0.0285 |
| Q7-8 | -0.1716 | -0.1570 | -0.1199 | -0.1292 | -0.1188 | -0.1211 | 0.0146 | 0.0517 | 0.0424 | 0.0528 | 0.0505 |
| Q7-9 | 0.0578 | 0.0677 | 0.0736 | 0.0677 | 0.0713 | 0.0604 | 0.0099 | 0.0158 | 0.0099 | 0.0135 | 0.0026 |
| Q9-10 | 0.0422 | 0.1216 | 0.0840 | 0.1057 | 0.0803 | 0.0682 | 0.0794 | 0.0418 | 0.0635 | 0.0381 | 0.0260 |
| Q9-14 | 0.0361 | -0.0041 | 0.0683 | 0.0532 | 0.0634 | 0.0537 | 0.0402 | 0.0322 | 0.0171 | 0.0273 | 0.0176 |
| Q10-11 | -0.0162 | 0.0186 | 0.0510 | 0.0186 | 0.0410 | 0.0257 | 0.0348 | 0.0672 | 0.0348 | 0.0572 | 0.0419 |
| Q12-13 | 0.0075 | 0.0493 | -0.0107 | -0.0160 | -0.0089 | -0.0057 | 0.0418 | 0.0182 | 0.0235 | 0.0164 | 0.0132 |
| Q13-14 | 0.0175 | 0.0053 | -0.0144 | 0.0053 | -0.0069 | 0.0023 | 0.0122 | 0.0319 | 0.0122 | 0.0244 | 0.0152 |
| Q2-1 | 0.2768 | 0.2876 | 0.2144 | 0.2543 | 0.2180 | 0.2220 | 0.0108 | 0.0624 | 0.0225 | 0.0588 | 0.0548 |
| Q5-1 | 0.0223 | -0.0360 | -0.0312 | -0.0262 | -0.0287 | -0.0202 | 0.0583 | 0.0535 | 0.0485 | 0.0510 | 0.0425 |
| Q3-2 | 0.0160 | -0.0414 | -0.0547 | -0.0151 | -0.0510 | -0.0488 | 0.0574 | 0.0707 | 0.0311 | 0.0670 | 0.0648 |
| Q4-2 | 0.0302 | 0.0354 | -0.0299 | -0.0256 | -0.0210 | -0.0138 | 0.0052 | 0.0601 | 0.0558 | 0.0512 | 0.0440 |
| Q5-2 | -0.0210 | 0.0151 | -0.0603 | -0.0680 | -0.0582 | -0.0496 | 0.0361 | 0.0393 | 0.0470 | 0.0372 | 0.0286 |
| Q4-3 | -0.0484 | 0.0809 | -0.0371 | -0.0814 | -0.0412 | -0.0359 | 0.1293 | 0.0113 | 0.0330 | 0.0072 | 0.0125 |
| Q5-4 | 0.1420 | 0.1087 | -0.0746 | -0.1046 | -0.0753 | -0.0697 | 0.0333 | 0.2166 | 0.2466 | 0.2173 | 0.2117 |
| Q7-4 | 0.1138 | 0.1232 | 0.1542 | 0.1319 | 0.1486 | 0.1652 | 0.0094 | 0.0404 | 0.0181 | 0.0348 | 0.0514 |
| Q9-4 | 0.0173 | 0.0132 | 0.0482 | 0.0417 | 0.0449 | 0.0548 | 0.0041 | 0.0309 | 0.0244 | 0.0276 | 0.0375 |
| Q6-5 | -0.0805 | -0.1772 | 0.0976 | 0.1200 | 0.0907 | 0.1537 | 0.0967 | 0.1781 | 0.2005 | 0.1712 | 0.2342 |
| Q11-6 | -0.0344 | -0.0563 | -0.0345 | -0.0321 | -0.0454 | -0.0514 | 0.0219 | 0.0001 | 0.0023 | 0.0110 | 0.0170 |
| Q12-6 | -0.0235 | -0.1132 | -0.0386 | -0.0151 | -0.0385 | -0.0310 | 0.0897 | 0.0151 | 0.0084 | 0.0150 | 0.0075 |
| Q13-6 | -0.0680 | -0.0260 | -0.1023 | -0.0504 | -0.1049 | -0.0973 | 0.0420 | 0.0343 | 0.0176 | 0.0369 | 0.0293 |
| Q8-7 | 0.1762 | 0.1318 | 0.1223 | 0.1318 | 0.1209 | 0.1233 | 0.0444 | 0.0539 | 0.0444 | 0.0553 | 0.0529 |
| Q9-7 | -0.0498 | -0.0448 | -0.0671 | -0.0585 | -0.0657 | -0.0522 | 0.0050 | 0.0173 | 0.0087 | 0.0159 | 0.0024 |
| Q10-9 | -0.0418 | -0.0690 | -0.0833 | -0.1047 | -0.0797 | -0.0678 | 0.0272 | 0.0415 | 0.0629 | 0.0379 | 0.0260 |
| Q14-9 | -0.0336 | -0.0184 | -0.0642 | -0.0516 | -0.0596 | -0.0517 | 0.0152 | 0.0306 | 0.0180 | 0.0260 | 0.0181 |
| Q11-10 | 0.0164 | -0.0136 | -0.0505 | -0.0183 | -0.0407 | -0.0256 | 0.0300 | 0.0669 | 0.0347 | 0.0571 | 0.0420 |
| Q13-12 | -0.0075 | 0.0170 | 0.0116 | 0.0170 | 0.0098 | 0.0066 | 0.0245 | 0.0191 | 0.0245 | 0.0173 | 0.0141 |
| Q14-13 | -0.0164 | 0.0206 | 0.0166 | -0.0026 | 0.0084 | -0.0007 | 0.0370 | 0.0330 | 0.0138 | 0.0248 | 0.0157 |
| **SUM** | | | | | | | **3.8412** | **3.6465** | **2.6437** | **2.2273** | **2.0061** |

Compared with base case, it's clear that except WLS, other estimators have much better performance both in bad data and overall summation estimation, especially LTS and LTAV.

**Median redundancy**

Table 5.211 shows the state variables, Table 5.212 presents the estimated value and errors between each estimator and actual value.

Table 5.211: State Variable of case 1, reverse, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0858 | -0.0924 | -0.0858 | -0.0856 | **V1** | 1.0548 | 1.0300 | 1.0548 | 1.0709 |
| $\theta 3$ | -0.2330 | -0.2454 | -0.2330 | -0.2305 | **V2** | 1.0383 | 1.0120 | 1.0383 | 1.0542 |
| $\theta 4$ | -0.1609 | -0.1846 | -0.1609 | -0.1792 | **V3** | 0.9963 | 0.9697 | 0.9963 | 1.0149 |
| $\theta 5$ | -0.1458 | -0.1560 | -0.1458 | -0.1485 | **V4** | 1.0074 | 0.9786 | 1.0074 | 1.0176 |
| $\theta 6$ | -0.2707 | -0.2558 | -0.2707 | -0.2396 | **V5** | 1.0054 | 0.9805 | 1.0054 | 1.0171 |
| $\theta 7$ | -0.2457 | -0.2582 | -0.2457 | -0.2354 | **V6** | 1.0124 | 0.9830 | 1.0124 | 0.9939 |
| $\theta 8$ | -0.2507 | -0.2561 | -0.2507 | -0.2353 | **V7** | 1.0300 | 1.0020 | 1.0300 | 1.0477 |
| $\theta 9$ | -0.2756 | -0.2912 | -0.2756 | -0.2599 | **V8** | 1.0512 | 1.0220 | 1.0512 | 1.0695 |
| $\theta 10$ | -0.2807 | -0.2931 | -0.2807 | -0.2625 | **V9** | 1.0177 | 0.9886 | 1.0177 | 1.0365 |
| $\theta 11$ | -0.2861 | -0.2856 | -0.2861 | -0.2634 | **V10** | 1.0079 | 0.9760 | 1.0079 | 1.0271 |
| $\theta 12$ | -0.2699 | -0.2489 | -0.2699 | -0.2418 | **V11** | 0.9964 | 0.9748 | 0.9964 | 1.0083 |
| $\theta 13$ | -0.2848 | -0.2690 | -0.2848 | -0.2563 | **V12** | 0.9957 | 0.9814 | 0.9957 | 0.9997 |
| $\theta 14$ | -0.3113 | -0.3031 | -0.3113 | -0.2853 | **V13** | 0.9818 | 0.9699 | 0.9818 | 0.9863 |
| | | | | | **V14** | 0.9646 | 0.9573 | 0.9646 | 0.9871 |

Table 5.212: Estimation Results and Errors of case 1, reverse, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0548 | 1.0300 | 1.0548 | 1.0709 | 0.0052 | 0.0300 | 0.0052 | 0.0109 |
| V2 | 1.0450 | N/A | 1.0383 | 1.0120 | 1.0383 | 1.0542 | 0.0067 | 0.0330 | 0.0067 | 0.0092 |
| V3 | 1.0100 | N/A | 0.9963 | 0.9697 | 0.9963 | 1.0149 | 0.0137 | 0.0403 | 0.0137 | 0.0049 |
| V4 | 1.0180 | N/A | 1.0074 | 0.9786 | 1.0074 | 1.0176 | 0.0106 | 0.0394 | 0.0106 | 0.0004 |
| V5 | 1.0200 | N/A | 1.0054 | 0.9805 | 1.0054 | 1.0171 | 0.0146 | 0.0395 | 0.0146 | 0.0029 |
| V6 | 1.0700 | N/A | 1.0124 | 0.9830 | 1.0124 | 0.9939 | 0.0576 | 0.0870 | 0.0576 | 0.0761 |
| V7 | 1.0620 | N/A | 1.0300 | 1.0020 | 1.0300 | 1.0477 | 0.0320 | 0.0600 | 0.0320 | 0.0143 |
| V8 | 1.0900 | N/A | 1.0512 | 1.0220 | 1.0512 | 1.0695 | 0.0388 | 0.0680 | 0.0388 | 0.0205 |
| V9 | 1.0560 | N/A | 1.0177 | 0.9886 | 1.0177 | 1.0365 | 0.0383 | 0.0674 | 0.0383 | 0.0195 |
| V10 | 1.0510 | N/A | 1.0079 | 0.9760 | 1.0079 | 1.0271 | 0.0431 | 0.0750 | 0.0431 | 0.0239 |
| V11 | 1.0570 | N/A | 0.9964 | 0.9748 | 0.9964 | 1.0083 | 0.0606 | 0.0822 | 0.0606 | 0.0487 |
| V12 | 1.0550 | N/A | 0.9957 | 0.9814 | 0.9957 | 0.9997 | 0.0593 | 0.0736 | 0.0593 | 0.0553 |
| V13 | 1.0500 | N/A | 0.9818 | 0.9699 | 0.9818 | 0.9863 | 0.0682 | 0.0801 | 0.0682 | 0.0637 |
| V14 | 1.0360 | N/A | 0.9646 | 0.9573 | 0.9646 | 0.9871 | 0.0714 | 0.0787 | 0.0714 | 0.0489 |
| P1 | 2.3239 | 2.3508 | 2.2568 | 2.3128 | 2.2568 | 2.3364 | 0.0671 | 0.0111 | 0.0671 | 0.0125 |
| P2 | 0.1830 | 0.2747 | 0.1369 | 0.1546 | 0.1369 | 0.2656 | 0.0461 | 0.0284 | 0.0461 | 0.0826 |
| P3 | -0.9420 | -1.0549 | -1.1434 | -1.0549 | -1.1434 | -1.0426 | 0.2014 | 0.1129 | 0.2014 | 0.1006 |
| **P4** | **-0.4780** | **0.4349** | **0.2749** | **-0.2608** | **0.2749** | **-0.5203** | **0.7529** | **0.2172** | **0.7529** | **0.0423** |
| P5 | -0.0760 | -0.0601 | -0.2117 | -0.0601 | -0.2117 | -0.0603 | 0.1357 | 0.0159 | 0.1357 | 0.0157 |
| P6 | -0.1120 | -0.1774 | -0.2397 | -0.1774 | -0.2397 | -0.1995 | 0.1277 | 0.0654 | 0.1277 | 0.0875 |
| P7 | 0.0000 | -0.0217 | -0.1134 | -0.0680 | -0.1134 | -0.0503 | 0.1134 | 0.0680 | 0.1134 | 0.0503 |
| P8 | 0.0000 | 0.0171 | -0.0312 | 0.0121 | -0.0312 | 0.0003 | 0.0312 | 0.0121 | 0.0312 | 0.0003 |
| P9 | -0.2950 | -0.1161 | -0.2249 | -0.3413 | -0.2249 | -0.1818 | 0.0701 | 0.0463 | 0.0701 | 0.1132 |
| P10 | -0.0900 | 0.0485 | -0.0468 | -0.0964 | -0.0468 | -0.0250 | 0.0432 | 0.0064 | 0.0432 | 0.0650 |
| P11 | -0.0350 | -0.1025 | -0.1401 | -0.1025 | -0.1401 | -0.1090 | 0.1051 | 0.0675 | 0.1051 | 0.0740 |
| P12 | -0.0610 | 0.0907 | 0.0447 | 0.0907 | 0.0447 | 0.0677 | 0.1057 | 0.1517 | 0.1057 | 0.1287 |
| P13 | -0.1350 | -0.0987 | -0.1670 | -0.0987 | -0.1670 | -0.1221 | 0.0320 | 0.0363 | 0.0320 | 0.0129 |
| P14 | -0.1490 | -0.1522 | -0.2551 | -0.1632 | -0.2551 | -0.2119 | 0.1061 | 0.0142 | 0.1061 | 0.0629 |
| Q1 | -0.1655 | -0.1298 | -0.1458 | -0.1298 | -0.1458 | -0.1347 | 0.0197 | 0.0357 | 0.0197 | 0.0308 |
| Q2 | 0.3086 | 0.2984 | 0.3083 | 0.2984 | 0.3083 | 0.3213 | 0.0003 | 0.0102 | 0.0003 | 0.0127 |
| Q3 | 0.0608 | 0.0546 | 0.0704 | 0.0546 | 0.0704 | 0.0780 | 0.0096 | 0.0062 | 0.0096 | 0.0172 |
| Q4 | 0.0390 | 0.1135 | 0.0897 | 0.1135 | 0.0897 | 0.1135 | 0.0507 | 0.0745 | 0.0507 | 0.0745 |
| Q5 | -0.0160 | 0.0545 | 0.0279 | 0.0545 | 0.0279 | 0.0503 | 0.0439 | 0.0705 | 0.0439 | 0.0663 |
| Q6 | 0.0523 | 0.1232 | 0.0046 | -0.2145 | 0.0046 | -0.4963 | 0.0477 | 0.2668 | 0.0477 | 0.5486 |
| Q7 | 0.0000 | 0.0336 | 0.0126 | 0.0336 | 0.0126 | 0.0239 | 0.0126 | 0.0336 | 0.0126 | 0.0239 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q8 | 0.1762 | 0.1158 | 0.1271 | 0.1158 | 0.1271 | 0.1325 | 0.0491 | 0.0604 | 0.0491 | 0.0437 |
| Q9 | -0.1660 | -0.1301 | -0.1360 | -0.1301 | -0.1360 | -0.1173 | 0.0300 | 0.0359 | 0.0300 | 0.0487 |
| Q10 | -0.0580 | 0.0235 | -0.0401 | -0.1013 | -0.0401 | -0.0049 | 0.0179 | 0.0433 | 0.0179 | 0.0531 |
| Q11 | -0.0180 | 0.0064 | -0.0745 | 0.0064 | -0.0745 | 0.0253 | 0.0565 | 0.0244 | 0.0565 | 0.0433 |
| Q12 | -0.0160 | 0.0357 | -0.0587 | -0.0368 | -0.0587 | 0.0166 | 0.0427 | 0.0208 | 0.0427 | 0.0326 |
| Q13 | -0.0580 | -0.0217 | -0.1222 | -0.0217 | -0.1222 | -0.0210 | 0.0642 | 0.0363 | 0.0642 | 0.0370 |
| Q14 | -0.0500 | -0.0652 | -0.1123 | -0.0652 | -0.1123 | -0.0746 | 0.0623 | 0.0152 | 0.0623 | 0.0246 |
| P1-2 | 1.5688 | 1.5835 | 1.5394 | 1.5835 | 1.5394 | 1.5825 | 0.0294 | 0.0147 | 0.0294 | 0.0137 |
| P1-5 | 0.7551 | 0.7157 | 0.7174 | 0.7293 | 0.7174 | 0.7539 | 0.0377 | 0.0258 | 0.0377 | 0.0012 |
| P2-3 | 0.7324 | 0.7768 | 0.7877 | 0.7768 | 0.7877 | 0.7987 | 0.0553 | 0.0444 | 0.0553 | 0.0663 |
| P2-4 | 0.5613 | N/A | 0.4607 | 0.5304 | 0.4607 | 0.5862 | 0.1006 | 0.0309 | 0.1006 | 0.0249 |
| P2-5 | 0.4152 | N/A | 0.3863 | 0.3848 | 0.3863 | 0.4205 | 0.0289 | 0.0304 | 0.0289 | 0.0053 |
| P3-4 | -0.2329 | -0.2734 | -0.3831 | -0.3062 | -0.3831 | -0.2711 | 0.1502 | 0.0733 | 0.1502 | 0.0382 |
| P4-5 | -0.6116 | N/A | -0.3165 | -0.6033 | -0.3165 | -0.6789 | 0.2951 | 0.0083 | 0.2951 | 0.0673 |
| P4-7 | 0.2807 | 0.3526 | 0.4294 | 0.3526 | 0.4294 | 0.2924 | 0.1487 | 0.0719 | 0.1487 | 0.0117 |
| P4-9 | 0.1608 | N/A | 0.2176 | 0.1909 | 0.2176 | 0.1577 | 0.0568 | 0.0301 | 0.0568 | 0.0031 |
| P5-6 | 0.4409 | 0.4032 | 0.5401 | 0.4091 | 0.5401 | 0.3914 | 0.0992 | 0.0318 | 0.0992 | 0.0495 |
| P6-11 | 0.0735 | N/A | 0.0954 | 0.1335 | 0.0954 | 0.0704 | 0.0219 | 0.0600 | 0.0219 | 0.0031 |
| P6-12 | 0.0779 | -0.0077 | 0.0232 | -0.0188 | 0.0232 | -0.0019 | 0.0547 | 0.0967 | 0.0547 | 0.0798 |
| P6-13 | 0.1775 | N/A | 0.1817 | 0.1170 | 0.1817 | 0.1234 | 0.0042 | 0.0605 | 0.0042 | 0.0541 |
| P7-8 | 0.0000 | -0.0121 | 0.0312 | -0.0121 | 0.0312 | -0.0003 | 0.0312 | 0.0121 | 0.0312 | 0.0003 |
| P7-9 | 0.2807 | 0.2967 | 0.2847 | 0.2967 | 0.2847 | 0.2424 | 0.0040 | 0.0160 | 0.0040 | 0.0383 |
| P9-10 | 0.0523 | 0.0679 | 0.0932 | 0.0679 | 0.0932 | 0.0662 | 0.0409 | 0.0156 | 0.0409 | 0.0139 |
| P9-14 | 0.0943 | N/A | 0.1842 | 0.0784 | 0.1842 | 0.1521 | 0.0899 | 0.0159 | 0.0899 | 0.0578 |
| P10-11 | -0.0379 | -0.0394 | 0.0459 | -0.0291 | 0.0459 | 0.0408 | 0.0838 | 0.0088 | 0.0838 | 0.0787 |
| P12-13 | 0.0161 | N/A | 0.0675 | 0.0718 | 0.0675 | 0.0657 | 0.0514 | 0.0557 | 0.0514 | 0.0496 |
| P13-14 | 0.0564 | 0.0878 | 0.0777 | 0.0878 | 0.0777 | 0.0651 | 0.0213 | 0.0314 | 0.0213 | 0.0087 |
| P2-1 | -1.5259 | N/A | -1.4978 | -1.5374 | -1.4978 | -1.5398 | 0.0281 | 0.0115 | 0.0281 | 0.0139 |
| P5-1 | -0.7275 | N/A | -0.6918 | -0.7016 | -0.6918 | -0.7264 | 0.0357 | 0.0259 | 0.0357 | 0.0011 |
| P3-2 | -0.7091 | N/A | -0.7603 | -0.7487 | -0.7603 | -0.7715 | 0.0512 | 0.0396 | 0.0512 | 0.0624 |
| P4-2 | -0.5445 | N/A | -0.4491 | -0.5143 | -0.4491 | -0.5681 | 0.0954 | 0.0302 | 0.0954 | 0.0236 |
| P5-2 | -0.4061 | N/A | -0.3781 | -0.3763 | -0.3781 | -0.4109 | 0.0280 | 0.0298 | 0.0280 | 0.0048 |
| P4-3 | 0.2366 | N/A | 0.3937 | 0.3133 | 0.3937 | 0.2765 | 0.1571 | 0.0767 | 0.1571 | 0.0399 |
| P5-4 | 0.6167 | N/A | 0.3181 | 0.6087 | 0.3181 | 0.6855 | 0.2986 | 0.0080 | 0.2986 | 0.0688 |
| P7-4 | -0.2807 | N/A | -0.4294 | -0.3526 | -0.4294 | -0.2924 | 0.1487 | 0.0719 | 0.1487 | 0.0117 |
| P9-4 | -0.1608 | N/A | -0.2176 | -0.1909 | -0.2176 | -0.1577 | 0.0568 | 0.0301 | 0.0568 | 0.0031 |
| P6-5 | -0.4409 | N/A | -0.5401 | -0.4091 | -0.5401 | -0.3914 | 0.0992 | 0.0318 | 0.0992 | 0.0495 |
| P11-6 | -0.0730 | N/A | -0.0944 | -0.1317 | -0.0944 | -0.0689 | 0.0214 | 0.0587 | 0.0214 | 0.0041 |
| P12-6 | -0.0771 | N/A | -0.0228 | 0.0189 | -0.0228 | 0.0020 | 0.0543 | 0.0960 | 0.0543 | 0.0791 |
| P13-6 | -0.1754 | N/A | -0.1782 | -0.1160 | -0.1782 | -0.1224 | 0.0028 | 0.0594 | 0.0028 | 0.0530 |
| P8-7 | 0.0000 | N/A | -0.0312 | 0.0121 | -0.0312 | 0.0003 | 0.0312 | 0.0121 | 0.0312 | 0.0003 |
| P9-7 | -0.2807 | N/A | -0.2847 | -0.2967 | -0.2847 | -0.2424 | 0.0040 | 0.0160 | 0.0040 | 0.0383 |
| P10-9 | -0.0521 | N/A | -0.0928 | -0.0673 | -0.0928 | -0.0658 | 0.0407 | 0.0152 | 0.0407 | 0.0137 |
| P14-9 | -0.0931 | N/A | -0.1784 | -0.0768 | -0.1784 | -0.1477 | 0.0853 | 0.0163 | 0.0853 | 0.0546 |
| P11-10 | 0.0380 | N/A | -0.0456 | 0.0292 | -0.0456 | -0.0401 | 0.0836 | 0.0088 | 0.0836 | 0.0781 |
| P13-12 | -0.0161 | N/A | -0.0665 | -0.0705 | -0.0665 | -0.0648 | 0.0504 | 0.0544 | 0.0504 | 0.0487 |
| P14-13 | -0.0559 | N/A | -0.0766 | -0.0864 | -0.0766 | -0.0642 | 0.0207 | 0.0305 | 0.0207 | 0.0083 |
| Q1-2 | -0.2040 | -0.2571 | -0.2011 | -0.1845 | -0.2011 | -0.2074 | 0.0029 | 0.0195 | 0.0029 | 0.0034 |
| Q1-5 | 0.0385 | 0.1560 | 0.0553 | 0.0547 | 0.0553 | 0.0727 | 0.0168 | 0.0162 | 0.0168 | 0.0342 |
| Q2-3 | 0.0356 | 0.0048 | 0.0424 | 0.0449 | 0.0424 | 0.0277 | 0.0068 | 0.0093 | 0.0068 | 0.0079 |
| Q2-4 | -0.0155 | N/A | 0.0101 | 0.0055 | 0.0101 | 0.0144 | 0.0256 | 0.0210 | 0.0256 | 0.0299 |
| Q2-5 | 0.0117 | N/A | 0.0432 | 0.0328 | 0.0432 | 0.0608 | 0.0315 | 0.0211 | 0.0315 | 0.0491 |
| Q3-4 | 0.0447 | 0.0891 | 0.0881 | 0.0673 | 0.0881 | 0.0848 | 0.0434 | 0.0226 | 0.0434 | 0.0401 |
| Q4-5 | 0.1582 | N/A | 0.1505 | 0.1558 | 0.1505 | 0.2386 | 0.0077 | 0.0024 | 0.0077 | 0.0804 |
| Q4-7 | -0.0968 | -0.1669 | -0.0929 | -0.0990 | -0.0929 | -0.1413 | 0.0039 | 0.0022 | 0.0039 | 0.0445 |
| Q4-9 | -0.0043 | N/A | -0.0067 | -0.0079 | -0.0067 | -0.0292 | 0.0024 | 0.0036 | 0.0024 | 0.0249 |
| Q5-6 | 0.1247 | 0.1491 | 0.0038 | 0.0101 | 0.0038 | 0.1185 | 0.1209 | 0.1146 | 0.1209 | 0.0062 |
| Q6-11 | 0.0356 | N/A | 0.0364 | -0.0210 | 0.0364 | -0.1041 | 0.0008 | 0.0566 | 0.0008 | 0.1397 |
| Q6-12 | 0.0250 | 0.0152 | 0.0548 | 0.0152 | 0.0548 | -0.0217 | 0.0298 | 0.0098 | 0.0298 | 0.0467 |
| Q6-13 | 0.0722 | N/A | 0.1462 | 0.0403 | 0.1462 | -0.0040 | 0.0740 | 0.0319 | 0.0740 | 0.0762 |
| Q7-8 | -0.1716 | -0.1570 | -0.1243 | -0.1135 | -0.1243 | -0.1298 | 0.0473 | 0.0581 | 0.0473 | 0.0418 |
| Q7-9 | 0.0578 | 0.0677 | 0.1193 | 0.1275 | 0.1193 | 0.1096 | 0.0615 | 0.0697 | 0.0615 | 0.0518 |
| Q9-10 | 0.0422 | 0.1216 | 0.0823 | 0.1216 | 0.0823 | 0.0895 | 0.0401 | 0.0794 | 0.0401 | 0.0473 |
| Q9-14 | 0.0361 | N/A | 0.1155 | 0.0778 | 0.1155 | 0.1189 | 0.0794 | 0.0417 | 0.0794 | 0.0828 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q10-11 | -0.0162 | 0.0186 | 0.0409 | 0.0186 | 0.0409 | 0.0836 | 0.0571 | 0.0348 | 0.0571 | 0.0998 |
| Q12-13 | 0.0075 | N/A | -0.0047 | -0.0217 | -0.0047 | -0.0052 | 0.0122 | 0.0292 | 0.0122 | 0.0127 |
| Q13-14 | 0.0175 | 0.0053 | 0.0114 | -0.0064 | 0.0114 | -0.0331 | 0.0061 | 0.0239 | 0.0061 | 0.0506 |
| Q2-1 | 0.2768 | N/A | 0.2125 | 0.2152 | 0.2125 | 0.2185 | 0.0643 | 0.0616 | 0.0643 | 0.0583 |
| Q5-1 | 0.0223 | N/A | -0.0542 | -0.0400 | -0.0542 | -0.0662 | 0.0765 | 0.0623 | 0.0765 | 0.0885 |
| Q3-2 | 0.0160 | N/A | -0.0177 | -0.0127 | -0.0177 | -0.0068 | 0.0337 | 0.0287 | 0.0337 | 0.0228 |
| Q4-2 | 0.0302 | N/A | -0.0462 | -0.0242 | -0.0462 | -0.0324 | 0.0764 | 0.0544 | 0.0764 | 0.0626 |
| Q5-2 | -0.0210 | N/A | -0.0904 | -0.0756 | -0.0904 | -0.1058 | 0.0694 | 0.0546 | 0.0694 | 0.0848 |
| Q4-3 | -0.0484 | N/A | -0.0867 | -0.0734 | -0.0867 | -0.0974 | 0.0383 | 0.0250 | 0.0383 | 0.0490 |
| Q5-4 | 0.1420 | N/A | -0.1454 | -0.1387 | -0.1454 | -0.2175 | 0.2874 | 0.2807 | 0.2874 | 0.3595 |
| Q7-4 | 0.1138 | N/A | 0.1318 | 0.1276 | 0.1318 | 0.1622 | 0.0180 | 0.0138 | 0.0180 | 0.0484 |
| Q9-4 | 0.0173 | N/A | 0.0319 | 0.0284 | 0.0319 | 0.0426 | 0.0146 | 0.0111 | 0.0146 | 0.0253 |
| Q6-5 | -0.0805 | N/A | 0.0640 | 0.0308 | 0.0640 | -0.0806 | 0.1445 | 0.1113 | 0.1445 | 0.0001 |
| Q11-6 | -0.0344 | N/A | -0.0344 | 0.0248 | -0.0344 | 0.1073 | 0.0000 | 0.0592 | 0.0000 | 0.1417 |
| Q12-6 | -0.0235 | N/A | -0.0539 | -0.0150 | -0.0539 | 0.0218 | 0.0304 | 0.0085 | 0.0304 | 0.0453 |
| Q13-6 | -0.0680 | N/A | -0.1393 | -0.0382 | -0.1393 | 0.0060 | 0.0713 | 0.0298 | 0.0713 | 0.0740 |
| Q8-7 | 0.1762 | N/A | 0.1271 | 0.1158 | 0.1271 | 0.1325 | 0.0491 | 0.0604 | 0.0491 | 0.0437 |
| Q9-7 | -0.0498 | N/A | -0.1094 | -0.1161 | -0.1094 | -0.1025 | 0.0596 | 0.0663 | 0.0596 | 0.0527 |
| Q10-9 | -0.0418 | N/A | -0.0810 | -0.1199 | -0.0810 | -0.0885 | 0.0392 | 0.0781 | 0.0392 | 0.0467 |
| Q14-9 | -0.0336 | N/A | -0.1032 | -0.0744 | -0.1032 | -0.1096 | 0.0696 | 0.0408 | 0.0696 | 0.0760 |
| Q11-10 | 0.0164 | N/A | -0.0402 | -0.0184 | -0.0402 | -0.0820 | 0.0566 | 0.0348 | 0.0566 | 0.0984 |
| Q13-12 | -0.0075 | N/A | 0.0057 | 0.0229 | 0.0057 | 0.0061 | 0.0132 | 0.0304 | 0.0132 | 0.0136 |
| Q14-13 | -0.0164 | N/A | -0.0091 | 0.0092 | -0.0091 | 0.0350 | 0.0073 | 0.0256 | 0.0073 | 0.0514 |
| **SUM** | | | | | | | **4.7656** | **3.3894** | **4.7656** | **3.1157** |

WLS performs worst as expected, LTS's estimation are the same as WLS since it doesn't detect any bad data. LAV and LTAV have much better results.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.213. Median redundancy have much worse estimation for all estimators.

Table 5.213: Estimation errors of case 1, single reverse, 14-bus AC system

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **3.8412** | **3.6465** | **2.6437** | **2.2273** | **2.0061** |
| **SUM** | **N/A** | **4.7656** | **3.3894** | **4.7656** | **3.1157** |

## Large bad data

A large error at the same location as reverse case are used in this section, all data keep the same except $P_4 = -0.9349$, the bad data is applied to both full and median case.

**Full redundancy**

State variables are presented in Table 5.214, Table 5.215 shows the estimated value and errors between each estimator and actual value.

Table 5.214: State Variable of case 1, large, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0817 | -0.0847 | -0.0817 | -0.0827 | **V1** | 1.0909 | 1.0805 | 1.0909 | 1.0848 |
| $\theta 3$ | -0.2156 | -0.2224 | -0.2156 | -0.2181 | **V2** | 1.0751 | 1.0666 | 1.0751 | 1.0691 |
| $\theta 4$ | -0.1765 | -0.1806 | -0.1765 | -0.1791 | **V3** | 1.0308 | 1.0291 | 1.0308 | 1.0248 |
| $\theta 5$ | -0.1461 | -0.1492 | -0.1461 | -0.1485 | **V4** | 1.0404 | 1.0318 | 1.0404 | 1.0353 |
| $\theta 6$ | -0.2267 | -0.2352 | -0.2267 | -0.2305 | **V5** | 1.0466 | 1.0371 | 1.0466 | 1.0416 |
| $\theta 7$ | -0.2152 | -0.2308 | -0.2152 | -0.2184 | **V6** | 1.0627 | 1.0633 | 1.0627 | 1.0700 |
| $\theta 8$ | -0.2126 | -0.2289 | -0.2126 | -0.2160 | **V7** | 1.0679 | 1.0551 | 1.0679 | 1.0664 |
| $\theta 9$ | -0.2369 | -0.2564 | -0.2369 | -0.2402 | **V8** | 1.0875 | 1.0767 | 1.0875 | 1.0862 |
| $\theta 10$ | -0.2377 | -0.2596 | -0.2377 | -0.2414 | **V9** | 1.0610 | 1.0486 | 1.0610 | 1.0609 |
| $\theta 11$ | -0.2379 | -0.2538 | -0.2379 | -0.2420 | **V10** | 1.0534 | 1.0409 | 1.0534 | 1.0545 |
| $\theta 12$ | -0.2218 | -0.2293 | -0.2218 | -0.2270 | **V11** | 1.0454 | 1.0405 | 1.0454 | 1.0502 |
| $\theta 13$ | -0.2360 | -0.2454 | -0.2360 | -0.2404 | **V12** | 1.0537 | 1.0619 | 1.0537 | 1.0626 |
| $\theta 14$ | -0.2559 | -0.2714 | -0.2559 | -0.2599 | **V13** | 1.0410 | 1.0501 | 1.0410 | 1.0490 |
| | | | | | **V14** | 1.0328 | 1.0350 | 1.0328 | 1.0360 |

Table 5.215: Estimation Results and Errors of case 1, large, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0909 | 1.0805 | 1.0909 | 1.0848 | 0.0086 | 0.0309 | 0.0205 | 0.0309 | 0.0248 |
| V2 | 1.0450 | 1.0743 | 1.0751 | 1.0666 | 1.0751 | 1.0691 | 0.0293 | 0.0301 | 0.0216 | 0.0301 | 0.0241 |
| V3 | 1.0100 | 0.9739 | 1.0308 | 1.0291 | 1.0308 | 1.0248 | 0.0361 | 0.0208 | 0.0191 | 0.0208 | 0.0148 |
| V4 | 1.0180 | 1.0318 | 1.0404 | 1.0318 | 1.0404 | 1.0353 | 0.0138 | 0.0224 | 0.0138 | 0.0224 | 0.0173 |
| V5 | 1.0200 | 1.0251 | 1.0466 | 1.0371 | 1.0466 | 1.0416 | 0.0051 | 0.0266 | 0.0171 | 0.0266 | 0.0216 |
| V6 | 1.0700 | 1.0491 | 1.0627 | 1.0633 | 1.0627 | 1.0700 | 0.0209 | 0.0073 | 0.0067 | 0.0073 | 0.0000 |
| V7 | 1.0620 | 1.0551 | 1.0679 | 1.0551 | 1.0679 | 1.0664 | 0.0069 | 0.0059 | 0.0069 | 0.0059 | 0.0044 |
| V8 | 1.0900 | 1.0955 | 1.0875 | 1.0767 | 1.0875 | 1.0862 | 0.0055 | 0.0025 | 0.0133 | 0.0025 | 0.0038 |
| V9 | 1.0560 | 1.1131 | 1.0610 | 1.0486 | 1.0610 | 1.0609 | 0.0571 | 0.0050 | 0.0074 | 0.0050 | 0.0049 |
| V10 | 1.0510 | 1.0952 | 1.0534 | 1.0409 | 1.0534 | 1.0545 | 0.0442 | 0.0024 | 0.0101 | 0.0024 | 0.0035 |
| V11 | 1.0570 | 1.0355 | 1.0454 | 1.0405 | 1.0454 | 1.0502 | 0.0215 | 0.0116 | 0.0165 | 0.0116 | 0.0068 |
| V12 | 1.0550 | 1.1034 | 1.0537 | 1.0619 | 1.0537 | 1.0626 | 0.0484 | 0.0013 | 0.0069 | 0.0013 | 0.0076 |
| V13 | 1.0500 | 1.0616 | 1.0410 | 1.0501 | 1.0410 | 1.0490 | 0.0116 | 0.0090 | 0.0001 | 0.0090 | 0.0010 |
| V14 | 1.0360 | 1.0350 | 1.0328 | 1.0350 | 1.0328 | 1.0360 | 0.0010 | 0.0032 | 0.0010 | 0.0032 | 0.0000 |
| P1 | 2.3239 | 2.3508 | 2.3323 | 2.3500 | 2.3323 | 2.3359 | 0.0269 | 0.0084 | 0.0261 | 0.0084 | 0.0120 |
| P2 | 0.1830 | 0.2747 | 0.2925 | 0.2747 | 0.2925 | 0.2958 | 0.0917 | 0.1095 | 0.0917 | 0.1095 | 0.1128 |
| P3 | -0.9420 | -1.0549 | -0.9787 | -0.9805 | -0.9787 | -0.9772 | 0.1129 | 0.0367 | 0.0385 | 0.0367 | 0.0352 |
| **P4** | **-0.4780** | **-0.9349** | **-0.7809** | **-0.6993** | **-0.7809** | **-0.7788** | **0.4569** | **0.3029** | **0.2213** | **0.3029** | **0.3008** |
| P5 | -0.0760 | -0.0601 | -0.0198 | 0.0117 | -0.0198 | -0.0176 | 0.0159 | 0.0562 | 0.0877 | 0.0562 | 0.0584 |
| P6 | -0.1120 | -0.1774 | -0.1618 | -0.1774 | -0.1618 | -0.1576 | 0.0654 | 0.0498 | 0.0654 | 0.0498 | 0.0456 |
| P7 | 0.0000 | -0.0217 | -0.0029 | -0.0217 | -0.0029 | -0.0044 | 0.0217 | 0.0029 | 0.0217 | 0.0029 | 0.0044 |
| P8 | 0.0000 | 0.0171 | 0.0166 | 0.0121 | 0.0166 | 0.0160 | 0.0171 | 0.0166 | 0.0121 | 0.0166 | 0.0160 |
| P9 | -0.2950 | -0.1161 | -0.1999 | -0.2717 | -0.1999 | -0.2049 | 0.1789 | 0.0951 | 0.0233 | 0.0951 | 0.0901 |
| P10 | -0.0900 | 0.0485 | -0.0243 | -0.0950 | -0.0243 | -0.0281 | 0.1385 | 0.0657 | 0.0050 | 0.0657 | 0.0619 |
| P11 | -0.0350 | -0.1025 | -0.1032 | -0.1025 | -0.1032 | -0.1046 | 0.0675 | 0.0682 | 0.0675 | 0.0682 | 0.0696 |
| P12 | -0.0610 | 0.0907 | 0.0713 | 0.0907 | 0.0713 | 0.0703 | 0.1517 | 0.1323 | 0.1517 | 0.1323 | 0.1313 |
| P13 | -0.1350 | -0.0987 | -0.1406 | -0.0987 | -0.1406 | -0.1397 | 0.0363 | 0.0056 | 0.0363 | 0.0056 | 0.0047 |
| P14 | -0.1490 | -0.1522 | -0.1633 | -0.1522 | -0.1633 | -0.1662 | 0.0032 | 0.0143 | 0.0032 | 0.0143 | 0.0172 |
| Q1 | -0.1655 | -0.1298 | -0.1932 | -0.2344 | -0.1932 | -0.1994 | 0.0357 | 0.0277 | 0.0689 | 0.0277 | 0.0339 |
| Q2 | 0.3086 | 0.2984 | 0.2828 | 0.2984 | 0.2828 | 0.2738 | 0.0102 | 0.0258 | 0.0102 | 0.0258 | 0.0348 |
| Q3 | 0.0608 | 0.0546 | -0.0257 | 0.0546 | -0.0257 | -0.0293 | 0.0062 | 0.0865 | 0.0062 | 0.0865 | 0.0901 |
| Q4 | 0.0390 | 0.1135 | 0.0489 | 0.0665 | 0.0489 | 0.0318 | 0.0745 | 0.0099 | 0.0275 | 0.0099 | 0.0072 |
| Q5 | -0.0160 | 0.0545 | 0.1237 | 0.0545 | 0.1237 | 0.0861 | 0.0705 | 0.1397 | 0.0705 | 0.1397 | 0.1021 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q6 | 0.0523 | 0.1232 | -0.0394 | -0.0649 | -0.0394 | 0.0093 | 0.0709 | 0.0917 | 0.1172 | 0.0917 | 0.0430 |
| Q7 | 0.0000 | 0.0336 | -0.0232 | -0.0561 | -0.0232 | -0.0207 | 0.0336 | 0.0232 | 0.0561 | 0.0232 | 0.0207 |
| Q8 | 0.1762 | 0.1158 | 0.1207 | 0.1318 | 0.1207 | 0.1222 | 0.0604 | 0.0555 | 0.0444 | 0.0555 | 0.0540 |
| Q9 | -0.1660 | -0.1301 | -0.1592 | -0.2027 | -0.1592 | -0.1594 | 0.0359 | 0.0068 | 0.0367 | 0.0068 | 0.0066 |
| Q10 | -0.0580 | 0.0235 | -0.0410 | -0.0552 | -0.0410 | -0.0464 | 0.0815 | 0.0170 | 0.0028 | 0.0170 | 0.0116 |
| Q11 | -0.0180 | 0.0064 | -0.0864 | -0.0699 | -0.0864 | -0.0779 | 0.0244 | 0.0684 | 0.0519 | 0.0684 | 0.0599 |
| Q12 | -0.0160 | 0.0357 | -0.0465 | -0.0311 | -0.0465 | -0.0355 | 0.0517 | 0.0305 | 0.0151 | 0.0305 | 0.0195 |
| Q13 | -0.0580 | -0.0217 | -0.1009 | -0.0272 | -0.1009 | -0.0861 | 0.0363 | 0.0429 | 0.0308 | 0.0429 | 0.0281 |
| Q14 | -0.0500 | -0.0652 | -0.0526 | -0.0220 | -0.0526 | -0.0531 | 0.0152 | 0.0026 | 0.0280 | 0.0026 | 0.0031 |
| P1-2 | 1.5688 | 1.5835 | 1.5662 | 1.5835 | 1.5662 | 1.5670 | 0.0147 | 0.0026 | 0.0147 | 0.0026 | 0.0018 |
| P1-5 | 0.7551 | 0.7157 | 0.7660 | 0.7665 | 0.7660 | 0.7689 | 0.0394 | 0.0109 | 0.0114 | 0.0109 | 0.0138 |
| P2-3 | 0.7324 | 0.7768 | 0.7729 | 0.7775 | 0.7729 | 0.7726 | 0.0444 | 0.0405 | 0.0451 | 0.0405 | 0.0402 |
| P2-4 | 0.5613 | 0.5039 | 0.6131 | 0.6105 | 0.6131 | 0.6147 | 0.0574 | 0.0518 | 0.0492 | 0.0518 | 0.0534 |
| P2-5 | 0.4152 | 0.3618 | 0.4324 | 0.4280 | 0.4324 | 0.4346 | 0.0534 | 0.0172 | 0.0128 | 0.0172 | 0.0194 |
| P3-4 | -0.2329 | -0.2734 | -0.2306 | -0.2282 | -0.2306 | -0.2296 | 0.0405 | 0.0023 | 0.0047 | 0.0023 | 0.0033 |
| P4-5 | -0.6116 | -0.7588 | -0.7543 | -0.7588 | -0.7543 | -0.7531 | 0.1472 | 0.1427 | 0.1472 | 0.1427 | 0.1415 |
| P4-7 | 0.2807 | 0.3526 | 0.2099 | 0.2670 | 0.2099 | 0.2123 | 0.0719 | 0.0708 | 0.0137 | 0.0708 | 0.0684 |
| P4-9 | 0.1608 | 0.1771 | 0.1236 | 0.1520 | 0.1236 | 0.1244 | 0.0163 | 0.0372 | 0.0088 | 0.0372 | 0.0364 |
| P5-6 | 0.4409 | 0.4032 | 0.3810 | 0.4032 | 0.3810 | 0.3887 | 0.0377 | 0.0599 | 0.0377 | 0.0599 | 0.0522 |
| P6-11 | 0.0735 | 0.1420 | 0.0874 | 0.1317 | 0.0874 | 0.0942 | 0.0685 | 0.0139 | 0.0582 | 0.0139 | 0.0207 |
| P6-12 | 0.0779 | -0.0077 | -0.0026 | -0.0188 | -0.0026 | -0.0005 | 0.0856 | 0.0805 | 0.0967 | 0.0805 | 0.0784 |
| P6-13 | 0.1775 | 0.1724 | 0.1344 | 0.1129 | 0.1344 | 0.1375 | 0.0051 | 0.0431 | 0.0646 | 0.0431 | 0.0400 |
| P7-8 | 0.0000 | -0.0121 | -0.0166 | -0.0121 | -0.0166 | -0.0160 | 0.0121 | 0.0166 | 0.0121 | 0.0166 | 0.0160 |
| P7-9 | 0.2807 | 0.2967 | 0.2236 | 0.2574 | 0.2236 | 0.2239 | 0.0160 | 0.0571 | 0.0233 | 0.0571 | 0.0568 |
| P9-10 | 0.0523 | 0.0679 | 0.0413 | 0.0679 | 0.0413 | 0.0398 | 0.0156 | 0.0110 | 0.0156 | 0.0110 | 0.0125 |
| P9-14 | 0.0943 | 0.0511 | 0.1060 | 0.0698 | 0.1060 | 0.1036 | 0.0432 | 0.0117 | 0.0245 | 0.0117 | 0.0093 |
| P10-11 | -0.0379 | -0.0394 | 0.0168 | -0.0274 | 0.0168 | 0.0116 | 0.0015 | 0.0547 | 0.0105 | 0.0547 | 0.0495 |
| P12-13 | 0.0161 | 0.0079 | 0.0685 | 0.0718 | 0.0685 | 0.0697 | 0.0082 | 0.0524 | 0.0557 | 0.0524 | 0.0536 |
| P13-14 | 0.0564 | 0.0878 | 0.0596 | 0.0841 | 0.0596 | 0.0648 | 0.0314 | 0.0032 | 0.0277 | 0.0032 | 0.0084 |
| P2-1 | -1.5259 | -1.4712 | -1.5259 | -1.5412 | -1.5259 | -1.5261 | 0.0547 | 0.0000 | 0.0153 | 0.0000 | 0.0002 |
| P5-1 | -0.7275 | -0.6720 | -0.7391 | -0.7390 | -0.7391 | -0.7415 | 0.0555 | 0.0116 | 0.0115 | 0.0116 | 0.0140 |
| P3-2 | -0.7091 | -0.7523 | -0.7481 | -0.7523 | -0.7481 | -0.7476 | 0.0432 | 0.0390 | 0.0432 | 0.0390 | 0.0385 |
| P4-2 | -0.5445 | -0.5406 | -0.5941 | -0.5913 | -0.5941 | -0.5955 | 0.0039 | 0.0496 | 0.0468 | 0.0496 | 0.0510 |
| P5-2 | -0.4061 | -0.4668 | -0.4231 | -0.4187 | -0.4231 | -0.4251 | 0.0607 | 0.0170 | 0.0126 | 0.0170 | 0.0190 |
| P4-3 | 0.2366 | 0.1809 | 0.2340 | 0.2319 | 0.2340 | 0.2330 | 0.0557 | 0.0026 | 0.0047 | 0.0026 | 0.0036 |
| P5-4 | 0.6167 | 0.6164 | 0.7614 | 0.7662 | 0.7614 | 0.7603 | 0.0003 | 0.1447 | 0.1495 | 0.1447 | 0.1436 |
| P7-4 | -0.2807 | -0.2041 | -0.2099 | -0.2670 | -0.2099 | -0.2123 | 0.0766 | 0.0708 | 0.0137 | 0.0708 | 0.0684 |
| P9-4 | -0.1608 | -0.1993 | -0.1236 | -0.1520 | -0.1236 | -0.1244 | 0.0385 | 0.0372 | 0.0088 | 0.0372 | 0.0364 |
| P6-5 | -0.4409 | -0.4223 | -0.3810 | -0.4032 | -0.3810 | -0.3887 | 0.0186 | 0.0599 | 0.0377 | 0.0599 | 0.0522 |
| P11-6 | -0.0730 | -0.0843 | -0.0865 | -0.1299 | -0.0865 | -0.0931 | 0.0113 | 0.0135 | 0.0569 | 0.0135 | 0.0201 |
| P12-6 | -0.0771 | -0.0212 | 0.0028 | 0.0189 | 0.0028 | 0.0006 | 0.0559 | 0.0799 | 0.0960 | 0.0799 | 0.0777 |
| P13-6 | -0.1754 | -0.2299 | -0.1326 | -0.1120 | -0.1326 | -0.1358 | 0.0545 | 0.0428 | 0.0634 | 0.0428 | 0.0396 |
| P8-7 | 0.0000 | 0.0016 | 0.0166 | 0.0121 | 0.0166 | 0.0160 | 0.0016 | 0.0166 | 0.0121 | 0.0166 | 0.0160 |
| P9-7 | -0.2807 | -0.2531 | -0.2236 | -0.2574 | -0.2236 | -0.2239 | 0.0276 | 0.0571 | 0.0233 | 0.0571 | 0.0568 |
| P10-9 | -0.0521 | 0.0029 | -0.0411 | -0.0676 | -0.0411 | -0.0396 | 0.0550 | 0.0110 | 0.0155 | 0.0110 | 0.0125 |
| P14-9 | -0.0931 | -0.0159 | -0.1043 | -0.0692 | -0.1043 | -0.1021 | 0.0772 | 0.0112 | 0.0239 | 0.0112 | 0.0090 |
| P11-10 | 0.0380 | 0.0423 | -0.0167 | 0.0274 | -0.0167 | -0.0115 | 0.0043 | 0.0547 | 0.0106 | 0.0547 | 0.0495 |
| P13-12 | -0.0161 | -0.0907 | -0.0676 | -0.0708 | -0.0676 | -0.0688 | 0.0746 | 0.0515 | 0.0547 | 0.0515 | 0.0527 |
| P14-13 | -0.0559 | -0.0930 | -0.0590 | -0.0830 | -0.0590 | -0.0642 | 0.0371 | 0.0031 | 0.0271 | 0.0031 | 0.0083 |
| Q1-2 | -0.2040 | -0.2571 | -0.2199 | -0.2571 | -0.2199 | -0.2210 | 0.0531 | 0.0159 | 0.0531 | 0.0159 | 0.0170 |
| Q1-5 | 0.0385 | 0.1560 | 0.0267 | 0.0227 | 0.0267 | 0.0216 | 0.1175 | 0.0118 | 0.0158 | 0.0118 | 0.0169 |
| Q2-3 | 0.0356 | 0.0048 | 0.0566 | 0.0202 | 0.0566 | 0.0568 | 0.0308 | 0.0210 | 0.0154 | 0.0210 | 0.0212 |
| Q2-4 | -0.0155 | 0.0219 | -0.0011 | -0.0006 | -0.0011 | -0.0072 | 0.0374 | 0.0144 | 0.0149 | 0.0144 | 0.0083 |
| Q2-5 | 0.0117 | 0.0021 | 0.0081 | 0.0145 | 0.0081 | 0.0010 | 0.0096 | 0.0036 | 0.0028 | 0.0036 | 0.0107 |
| Q3-4 | 0.0447 | 0.0891 | 0.0238 | 0.0649 | 0.0238 | 0.0182 | 0.0444 | 0.0209 | 0.0202 | 0.0209 | 0.0265 |
| Q4-5 | 0.1582 | 0.1200 | 0.0974 | 0.1224 | 0.0974 | 0.0951 | 0.0382 | 0.0608 | 0.0358 | 0.0608 | 0.0631 |
| Q4-7 | -0.0968 | -0.1669 | -0.1359 | -0.1108 | -0.1359 | -0.1531 | 0.0701 | 0.0391 | 0.0140 | 0.0391 | 0.0563 |
| Q4-9 | -0.0043 | -0.0754 | -0.0359 | -0.0263 | -0.0359 | -0.0454 | 0.0711 | 0.0316 | 0.0220 | 0.0316 | 0.0411 |
| Q5-6 | 0.1247 | 0.1491 | -0.0564 | -0.0984 | -0.0564 | -0.1098 | 0.0244 | 0.1811 | 0.2231 | 0.1811 | 0.2345 |
| Q6-11 | 0.0356 | 0.0267 | 0.0514 | 0.0600 | 0.0514 | 0.0617 | 0.0089 | 0.0158 | 0.0244 | 0.0158 | 0.0261 |
| Q6-12 | 0.0250 | 0.0152 | 0.0387 | 0.0152 | 0.0387 | 0.0312 | 0.0098 | 0.0137 | 0.0098 | 0.0137 | 0.0062 |
| Q6-13 | 0.0722 | 0.1432 | 0.1093 | 0.0513 | 0.1093 | 0.1027 | 0.0710 | 0.0371 | 0.0209 | 0.0371 | 0.0305 |
| Q7-8 | -0.1716 | -0.1570 | -0.1185 | -0.1291 | -0.1185 | -0.1199 | 0.0146 | 0.0531 | 0.0425 | 0.0531 | 0.0517 |
| Q7-9 | 0.0578 | 0.0677 | 0.0702 | 0.0659 | 0.0702 | 0.0553 | 0.0099 | 0.0124 | 0.0081 | 0.0124 | 0.0025 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q9-10 | 0.0422 | 0.1216 | 0.0788 | 0.0697 | 0.0788 | 0.0654 | 0.0794 | 0.0366 | 0.0275 | 0.0366 | 0.0232 |
| Q9-14 | 0.0361 | -0.0041 | 0.0614 | 0.0203 | 0.0614 | 0.0497 | 0.0402 | 0.0253 | 0.0158 | 0.0253 | 0.0136 |
| Q10-11 | -0.0162 | 0.0186 | 0.0371 | 0.0138 | 0.0371 | 0.0186 | 0.0348 | 0.0533 | 0.0300 | 0.0533 | 0.0348 |
| Q12-13 | 0.0075 | 0.0493 | -0.0082 | -0.0160 | -0.0082 | -0.0045 | 0.0418 | 0.0157 | 0.0235 | 0.0157 | 0.0120 |
| Q13-14 | 0.0175 | 0.0053 | -0.0041 | 0.0053 | -0.0041 | 0.0079 | 0.0122 | 0.0216 | 0.0122 | 0.0216 | 0.0096 |
| Q2-1 | 0.2768 | 0.2876 | 0.2193 | 0.2644 | 0.2193 | 0.2232 | 0.0108 | 0.0575 | 0.0124 | 0.0575 | 0.0536 |
| Q5-1 | 0.0223 | -0.0360 | -0.0278 | -0.0196 | -0.0278 | -0.0196 | 0.0583 | 0.0501 | 0.0419 | 0.0501 | 0.0419 |
| Q3-2 | 0.0160 | -0.0414 | -0.0495 | -0.0103 | -0.0495 | -0.0475 | 0.0574 | 0.0655 | 0.0263 | 0.0655 | 0.0635 |
| Q4-2 | 0.0302 | 0.0354 | -0.0174 | -0.0162 | -0.0174 | -0.0096 | 0.0052 | 0.0476 | 0.0464 | 0.0476 | 0.0398 |
| Q5-2 | -0.0210 | 0.0151 | -0.0575 | -0.0626 | -0.0575 | -0.0491 | 0.0361 | 0.0365 | 0.0416 | 0.0365 | 0.0281 |
| Q4-3 | -0.0484 | 0.0809 | -0.0425 | -0.0827 | -0.0425 | -0.0366 | 0.1293 | 0.0059 | 0.0343 | 0.0059 | 0.0118 |
| Q5-4 | 0.1420 | 0.1087 | -0.0749 | -0.0990 | -0.0749 | -0.0724 | 0.0333 | 0.2169 | 0.2410 | 0.2169 | 0.2144 |
| Q7-4 | 0.1138 | 0.1232 | 0.1478 | 0.1269 | 0.1478 | 0.1662 | 0.0094 | 0.0340 | 0.0131 | 0.0340 | 0.0524 |
| Q9-4 | 0.0173 | 0.0132 | 0.0442 | 0.0384 | 0.0442 | 0.0542 | 0.0041 | 0.0269 | 0.0211 | 0.0269 | 0.0369 |
| Q6-5 | -0.0805 | -0.1772 | 0.0882 | 0.1360 | 0.0882 | 0.1451 | 0.0967 | 0.1687 | 0.2165 | 0.1687 | 0.2256 |
| Q11-6 | -0.0344 | -0.0563 | -0.0496 | -0.0563 | -0.0496 | -0.0595 | 0.0219 | 0.0152 | 0.0219 | 0.0152 | 0.0251 |
| Q12-6 | -0.0235 | -0.1132 | -0.0384 | -0.0151 | -0.0384 | -0.0310 | 0.0897 | 0.0149 | 0.0084 | 0.0149 | 0.0075 |
| Q13-6 | -0.0680 | -0.0260 | -0.1058 | -0.0495 | -0.1058 | -0.0993 | 0.0420 | 0.0378 | 0.0185 | 0.0378 | 0.0313 |
| Q8-7 | 0.1762 | 0.1318 | 0.1207 | 0.1318 | 0.1207 | 0.1222 | 0.0444 | 0.0555 | 0.0444 | 0.0555 | 0.0540 |
| Q9-7 | -0.0498 | -0.0448 | -0.0649 | -0.0589 | -0.0649 | -0.0502 | 0.0050 | 0.0151 | 0.0091 | 0.0151 | 0.0004 |
| Q10-9 | -0.0418 | -0.0690 | -0.0782 | -0.0690 | -0.0782 | -0.0650 | 0.0272 | 0.0364 | 0.0272 | 0.0364 | 0.0232 |
| Q14-9 | -0.0336 | -0.0184 | -0.0578 | -0.0190 | -0.0578 | -0.0465 | 0.0152 | 0.0242 | 0.0146 | 0.0242 | 0.0129 |
| Q11-10 | 0.0164 | -0.0136 | -0.0368 | -0.0136 | -0.0368 | -0.0185 | 0.0300 | 0.0532 | 0.0300 | 0.0532 | 0.0349 |
| Q13-12 | -0.0075 | 0.0170 | 0.0090 | 0.0170 | 0.0090 | 0.0054 | 0.0245 | 0.0165 | 0.0245 | 0.0165 | 0.0129 |
| Q14-13 | -0.0164 | 0.0206 | 0.0052 | -0.0031 | 0.0052 | -0.0066 | 0.0370 | 0.0216 | 0.0133 | 0.0216 | 0.0098 |
| **SUM** | | | | | | | **3.3852** | **2.6711** | **2.4469** | **2.6711** | **2.5112** |

In this case, all estimators generate almost the same level of results, WLS and LTS have the same data since LTS doesn't trim any data.

### Median redundancy

State variables are presented in Table 5.216, Table 5.217 shows the estimated value and errors between each estimator and actual value.

Table 5.216: State Variable of case 1, large, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0882 | -0.0866 | -0.0882 | -0.0862 | **V1** | 1.0606 | 1.0686 | 1.0606 | 1.0729 |
| $\theta 3$ | -0.2366 | -0.2338 | -0.2366 | -0.2315 | **V2** | 1.0435 | 1.0522 | 1.0435 | 1.0560 |
| $\theta 4$ | -0.1929 | -0.1832 | -0.1929 | -0.1885 | **V3** | 1.0034 | 1.0112 | 1.0034 | 1.0172 |
| $\theta 5$ | -0.1574 | -0.1504 | -0.1574 | -0.1518 | **V4** | 1.0043 | 1.0174 | 1.0043 | 1.0169 |
| $\theta 6$ | -0.2457 | -0.2237 | -0.2457 | -0.2320 | **V5** | 1.0084 | 1.0198 | 1.0084 | 1.0182 |
| $\theta 7$ | -0.2345 | -0.2185 | -0.2345 | -0.2318 | **V6** | 1.0142 | 1.0176 | 1.0142 | 0.9945 |
| $\theta 8$ | -0.2318 | -0.2158 | -0.2318 | -0.2295 | **V7** | 1.0285 | 1.0448 | 1.0285 | 1.0477 |
| $\theta 9$ | -0.2566 | -0.2368 | -0.2566 | -0.2541 | **V8** | 1.0494 | 1.0640 | 1.0494 | 1.0695 |
| $\theta 10$ | -0.2577 | -0.2385 | -0.2577 | -0.2555 | **V9** | 1.0162 | 1.0323 | 1.0162 | 1.0366 |
| $\theta 11$ | -0.2590 | -0.2413 | -0.2590 | -0.2553 | **V10** | 1.0077 | 1.0202 | 1.0077 | 1.0277 |
| $\theta 12$ | -0.2360 | -0.2158 | -0.2360 | -0.2317 | **V11** | 0.9980 | 1.0149 | 0.9980 | 1.0092 |
| $\theta 13$ | -0.2514 | -0.2323 | -0.2514 | -0.2463 | **V12** | 1.0010 | 1.0168 | 1.0010 | 1.0017 |
| $\theta 14$ | -0.2761 | -0.2508 | -0.2761 | -0.2749 | **V13** | 0.9867 | 1.0059 | 0.9867 | 0.9882 |
| | | | | | **V14** | 0.9700 | 0.9947 | 0.9700 | 0.9894 |

Table 5.217: Estimation Results and Errors of case 1, large, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0606 | 1.0686 | 1.0606 | 1.0729 | 0.0006 | 0.0086 | 0.0006 | 0.0129 |
| V2 | 1.0450 | N/A | 1.0435 | 1.0522 | 1.0435 | 1.0560 | 0.0015 | 0.0072 | 0.0015 | 0.0110 |
| V3 | 1.0100 | N/A | 1.0034 | 1.0112 | 1.0034 | 1.0172 | 0.0066 | 0.0012 | 0.0066 | 0.0072 |
| V4 | 1.0180 | N/A | 1.0043 | 1.0174 | 1.0043 | 1.0169 | 0.0137 | 0.0006 | 0.0137 | 0.0011 |
| V5 | 1.0200 | N/A | 1.0084 | 1.0198 | 1.0084 | 1.0182 | 0.0116 | 0.0002 | 0.0116 | 0.0018 |
| V6 | 1.0700 | N/A | 1.0142 | 1.0176 | 1.0142 | 0.9945 | 0.0558 | 0.0524 | 0.0558 | 0.0755 |
| V7 | 1.0620 | N/A | 1.0285 | 1.0448 | 1.0285 | 1.0477 | 0.0335 | 0.0172 | 0.0335 | 0.0143 |
| V8 | 1.0900 | N/A | 1.0494 | 1.0640 | 1.0494 | 1.0695 | 0.0406 | 0.0260 | 0.0406 | 0.0205 |
| V9 | 1.0560 | N/A | 1.0162 | 1.0323 | 1.0162 | 1.0366 | 0.0398 | 0.0237 | 0.0398 | 0.0194 |
| V10 | 1.0510 | N/A | 1.0077 | 1.0202 | 1.0077 | 1.0277 | 0.0433 | 0.0308 | 0.0433 | 0.0233 |
| V11 | 1.0570 | N/A | 0.9980 | 1.0149 | 0.9980 | 1.0092 | 0.0590 | 0.0421 | 0.0590 | 0.0478 |
| V12 | 1.0550 | N/A | 1.0010 | 1.0168 | 1.0010 | 1.0017 | 0.0540 | 0.0382 | 0.0540 | 0.0533 |
| V13 | 1.0500 | N/A | 0.9867 | 1.0059 | 0.9867 | 0.9882 | 0.0633 | 0.0441 | 0.0633 | 0.0618 |
| V14 | 1.0360 | N/A | 0.9700 | 0.9947 | 0.9700 | 0.9894 | 0.0660 | 0.0413 | 0.0660 | 0.0466 |
| P1 | 2.3239 | 2.3508 | 2.3804 | 2.3508 | 2.3804 | 2.3736 | 0.0565 | 0.0269 | 0.0565 | 0.0497 |
| P2 | 0.1830 | 0.2747 | 0.3285 | 0.2747 | 0.3285 | 0.3235 | 0.1455 | 0.0917 | 0.1455 | 0.1405 |
| P3 | -0.9420 | -1.0549 | -0.9959 | -1.0537 | -0.9959 | -0.9979 | 0.0539 | 0.1117 | 0.0539 | 0.0559 |
| **P4** | **-0.4780** | **-0.9349** | **-0.8598** | **-0.7604** | **-0.8598** | **-0.8624** | **0.3818** | **0.2824** | **0.3818** | **0.3844** |
| P5 | -0.0760 | -0.0601 | 0.0091 | -0.0601 | 0.0091 | 0.0069 | 0.0851 | 0.0159 | 0.0851 | 0.0829 |
| P6 | -0.1120 | -0.1774 | -0.1859 | -0.1774 | -0.1859 | -0.1834 | 0.0739 | 0.0654 | 0.0739 | 0.0714 |
| P7 | 0.0000 | -0.0217 | -0.0173 | -0.0217 | -0.0173 | -0.0207 | 0.0173 | 0.0217 | 0.0173 | 0.0207 |
| P8 | 0.0000 | 0.0171 | 0.0168 | 0.0171 | 0.0168 | 0.0151 | 0.0168 | 0.0171 | 0.0168 | 0.0151 |
| P9 | -0.2950 | -0.1161 | -0.1605 | -0.1161 | -0.1605 | -0.1619 | 0.1345 | 0.1789 | 0.1345 | 0.1331 |
| P10 | -0.0900 | 0.0485 | -0.0201 | -0.0444 | -0.0201 | -0.0167 | 0.0699 | 0.0456 | 0.0699 | 0.0733 |
| P11 | -0.0350 | -0.1025 | -0.1105 | -0.1025 | -0.1105 | -0.1002 | 0.0755 | 0.0675 | 0.0755 | 0.0652 |
| P12 | -0.0610 | 0.0907 | 0.0812 | 0.0907 | 0.0812 | 0.0783 | 0.1422 | 0.1517 | 0.1422 | 0.1393 |
| P13 | -0.1350 | -0.0987 | -0.1141 | -0.0987 | -0.1141 | -0.1066 | 0.0209 | 0.0363 | 0.0209 | 0.0284 |
| P14 | -0.1490 | -0.1522 | -0.1943 | -0.1522 | -0.1943 | -0.1937 | 0.0453 | 0.0032 | 0.0453 | 0.0447 |
| Q1 | -0.1655 | -0.1298 | -0.1405 | -0.1647 | -0.1405 | -0.1324 | 0.0250 | 0.0008 | 0.0250 | 0.0331 |
| Q2 | 0.3086 | 0.2984 | 0.3131 | 0.2984 | 0.3131 | 0.3238 | 0.0045 | 0.0102 | 0.0045 | 0.0152 |
| Q3 | 0.0608 | 0.0546 | 0.0694 | 0.0546 | 0.0694 | 0.0782 | 0.0086 | 0.0062 | 0.0086 | 0.0174 |
| Q4 | 0.0390 | 0.1135 | 0.0963 | 0.1135 | 0.0963 | 0.1167 | 0.0573 | 0.0745 | 0.0573 | 0.0777 |
| Q5 | -0.0160 | 0.0545 | 0.0373 | 0.0545 | 0.0373 | 0.0544 | 0.0533 | 0.0705 | 0.0533 | 0.0704 |
| Q6 | 0.0523 | 0.1232 | -0.0039 | -0.2598 | -0.0039 | -0.5044 | 0.0562 | 0.3121 | 0.0562 | 0.5567 |
| Q7 | 0.0000 | 0.0336 | 0.0077 | 0.0336 | 0.0077 | 0.0234 | 0.0077 | 0.0336 | 0.0077 | 0.0234 |
| Q8 | 0.1762 | 0.1158 | 0.1245 | 0.1158 | 0.1245 | 0.1323 | 0.0517 | 0.0604 | 0.0517 | 0.0439 |
| Q9 | -0.1660 | -0.1301 | -0.1408 | -0.1301 | -0.1408 | -0.1178 | 0.0252 | 0.0359 | 0.0252 | 0.0482 |
| Q10 | -0.0580 | 0.0235 | -0.0430 | -0.1015 | -0.0430 | -0.0050 | 0.0150 | 0.0435 | 0.0150 | 0.0530 |
| Q11 | -0.0180 | 0.0064 | -0.0798 | 0.0064 | -0.0798 | 0.0255 | 0.0618 | 0.0244 | 0.0618 | 0.0435 |
| Q12 | -0.0160 | 0.0357 | -0.0620 | -0.0316 | -0.0620 | 0.0171 | 0.0460 | 0.0156 | 0.0460 | 0.0331 |
| Q13 | -0.0580 | -0.0217 | -0.1299 | -0.0217 | -0.1299 | -0.0212 | 0.0719 | 0.0363 | 0.0719 | 0.0368 |
| Q14 | -0.0500 | -0.0652 | -0.1176 | -0.0964 | -0.1176 | -0.0751 | 0.0676 | 0.0464 | 0.0676 | 0.0251 |
| P1-2 | 1.5688 | 1.5835 | 1.6002 | 1.5930 | 1.6002 | 1.6008 | 0.0314 | 0.0242 | 0.0314 | 0.0320 |
| P1-5 | 0.7551 | 0.7157 | 0.7802 | 0.7578 | 0.7802 | 0.7728 | 0.0251 | 0.0027 | 0.0251 | 0.0177 |
| P2-3 | 0.7324 | 0.7768 | 0.8009 | 0.8083 | 0.8009 | 0.8028 | 0.0685 | 0.0759 | 0.0685 | 0.0704 |
| P2-4 | 0.5613 | N/A | 0.6390 | 0.5986 | 0.6390 | 0.6401 | 0.0777 | 0.0373 | 0.0777 | 0.0788 |
| P2-5 | 0.4152 | N/A | 0.4443 | 0.4173 | 0.4443 | 0.4380 | 0.0291 | 0.0021 | 0.0291 | 0.0228 |
| P3-4 | -0.2329 | -0.2734 | -0.2230 | -0.2734 | -0.2230 | -0.2225 | 0.0099 | 0.0405 | 0.0099 | 0.0104 |
| P4-5 | -0.6116 | N/A | -0.8003 | -0.7470 | -0.8003 | -0.8239 | 0.1887 | 0.1354 | 0.1887 | 0.2123 |
| P4-7 | 0.2807 | 0.3526 | 0.2102 | 0.1834 | 0.2102 | 0.2256 | 0.0705 | 0.0973 | 0.0705 | 0.0551 |
| P4-9 | 0.1608 | N/A | 0.1205 | 0.1043 | 0.1205 | 0.1282 | 0.0403 | 0.0565 | 0.0403 | 0.0326 |
| P5-6 | 0.4409 | 0.4032 | 0.3839 | 0.3234 | 0.3839 | 0.3451 | 0.0570 | 0.1175 | 0.0570 | 0.0958 |
| P6-11 | 0.0735 | N/A | 0.0874 | 0.0803 | 0.0874 | 0.0677 | 0.0139 | 0.0068 | 0.0139 | 0.0058 |
| P6-12 | 0.0779 | -0.0077 | -0.0107 | -0.0247 | -0.0107 | -0.0118 | 0.0886 | 0.1026 | 0.0886 | 0.0897 |
| P6-13 | 0.1775 | N/A | 0.1213 | 0.0905 | 0.1213 | 0.1057 | 0.0562 | 0.0870 | 0.0562 | 0.0718 |
| P7-8 | 0.0000 | -0.0121 | -0.0168 | -0.0171 | -0.0168 | -0.0151 | 0.0168 | 0.0171 | 0.0168 | 0.0151 |
| P7-9 | 0.2807 | 0.2967 | 0.2097 | 0.1788 | 0.2097 | 0.2199 | 0.0710 | 0.1019 | 0.0710 | 0.0608 |
| P9-10 | 0.0523 | 0.0679 | 0.0446 | 0.0679 | 0.0446 | 0.0516 | 0.0077 | 0.0156 | 0.0077 | 0.0007 |
| P9-14 | 0.0943 | N/A | 0.1251 | 0.0990 | 0.1251 | 0.1346 | 0.0308 | 0.0047 | 0.0308 | 0.0403 |
| P10-11 | -0.0379 | -0.0394 | 0.0242 | 0.0230 | 0.0242 | 0.0346 | 0.0621 | 0.0609 | 0.0621 | 0.0725 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P12-13 | 0.0161 | N/A | 0.0701 | 0.0659 | 0.0701 | 0.0665 | 0.0540 | 0.0498 | 0.0540 | 0.0504 |
| P13-14 | 0.0564 | 0.0878 | 0.0737 | 0.0560 | 0.0737 | 0.0639 | 0.0173 | 0.0004 | 0.0173 | 0.0075 |
| P2-1 | -1.5259 | N/A | -1.5557 | -1.5495 | -1.5557 | -1.5573 | 0.0298 | 0.0236 | 0.0298 | 0.0314 |
| P5-1 | -0.7275 | N/A | -0.7503 | -0.7301 | -0.7503 | -0.7440 | 0.0228 | 0.0026 | 0.0228 | 0.0165 |
| P3-2 | -0.7091 | N/A | -0.7729 | -0.7803 | -0.7729 | -0.7754 | 0.0638 | 0.0712 | 0.0638 | 0.0663 |
| P4-2 | -0.5445 | N/A | -0.6170 | -0.5797 | -0.6170 | -0.6186 | 0.0725 | 0.0352 | 0.0725 | 0.0741 |
| P5-2 | -0.4061 | N/A | -0.4337 | -0.4081 | -0.4337 | -0.4277 | 0.0276 | 0.0020 | 0.0276 | 0.0216 |
| P4-3 | 0.2366 | N/A | 0.2268 | 0.2787 | 0.2268 | 0.2263 | 0.0098 | 0.0421 | 0.0098 | 0.0103 |
| P5-4 | 0.6167 | N/A | 0.8091 | 0.7547 | 0.8091 | 0.8335 | 0.1924 | 0.1380 | 0.1924 | 0.2168 |
| P7-4 | -0.2807 | N/A | -0.2102 | -0.1834 | -0.2102 | -0.2256 | 0.0705 | 0.0973 | 0.0705 | 0.0551 |
| P9-4 | -0.1608 | N/A | -0.1205 | -0.1043 | -0.1205 | -0.1282 | 0.0403 | 0.0565 | 0.0403 | 0.0326 |
| P6-5 | -0.4409 | N/A | -0.3839 | -0.3234 | -0.3839 | -0.3451 | 0.0570 | 0.1175 | 0.0570 | 0.0958 |
| P11-6 | -0.0730 | N/A | -0.0865 | -0.0796 | -0.0865 | -0.0662 | 0.0135 | 0.0066 | 0.0135 | 0.0068 |
| P12-6 | -0.0771 | N/A | 0.0111 | 0.0248 | 0.0111 | 0.0119 | 0.0882 | 0.1019 | 0.0882 | 0.0890 |
| P13-6 | -0.1754 | N/A | -0.1189 | -0.0898 | -0.1189 | -0.1050 | 0.0565 | 0.0856 | 0.0565 | 0.0704 |
| P8-7 | 0.0000 | N/A | 0.0168 | 0.0171 | 0.0168 | 0.0151 | 0.0168 | 0.0171 | 0.0168 | 0.0151 |
| P9-7 | -0.2807 | N/A | -0.2097 | -0.1788 | -0.2097 | -0.2199 | 0.0710 | 0.1019 | 0.0710 | 0.0608 |
| P10-9 | -0.0521 | N/A | -0.0443 | -0.0673 | -0.0443 | -0.0513 | 0.0078 | 0.0152 | 0.0078 | 0.0008 |
| P14-9 | -0.0931 | N/A | -0.1215 | -0.0967 | -0.1215 | -0.1308 | 0.0284 | 0.0036 | 0.0284 | 0.0377 |
| P11-10 | 0.0380 | N/A | -0.0240 | -0.0229 | -0.0240 | -0.0340 | 0.0620 | 0.0609 | 0.0620 | 0.0720 |
| P13-12 | -0.0161 | N/A | -0.0690 | -0.0649 | -0.0690 | -0.0655 | 0.0529 | 0.0488 | 0.0529 | 0.0494 |
| P14-13 | -0.0559 | N/A | -0.0728 | -0.0555 | -0.0728 | -0.0630 | 0.0169 | 0.0004 | 0.0169 | 0.0071 |
| Q1-2 | -0.2040 | -0.2571 | -0.2038 | -0.2141 | -0.2038 | -0.2079 | 0.0002 | 0.0101 | 0.0002 | 0.0039 |
| Q1-5 | 0.0385 | 0.1560 | 0.0633 | 0.0494 | 0.0633 | 0.0756 | 0.0248 | 0.0109 | 0.0248 | 0.0371 |
| Q2-3 | 0.0356 | 0.0048 | 0.0316 | 0.0355 | 0.0316 | 0.0246 | 0.0040 | 0.0001 | 0.0040 | 0.0110 |
| Q2-4 | -0.0155 | N/A | 0.0170 | 0.0012 | 0.0170 | 0.0172 | 0.0325 | 0.0167 | 0.0325 | 0.0327 |
| Q2-5 | 0.0117 | N/A | 0.0419 | 0.0337 | 0.0419 | 0.0609 | 0.0302 | 0.0220 | 0.0302 | 0.0492 |
| Q3-4 | 0.0447 | 0.0891 | 0.0750 | 0.0653 | 0.0750 | 0.0816 | 0.0303 | 0.0206 | 0.0303 | 0.0369 |
| Q4-5 | 0.1582 | N/A | 0.1711 | 0.1918 | 0.1711 | 0.2463 | 0.0129 | 0.0336 | 0.0129 | 0.0881 |
| Q4-7 | -0.0968 | -0.1669 | -0.1145 | -0.1334 | -0.1145 | -0.1484 | 0.0177 | 0.0366 | 0.0177 | 0.0516 |
| Q4-9 | -0.0043 | N/A | -0.0183 | -0.0253 | -0.0183 | -0.0330 | 0.0140 | 0.0210 | 0.0140 | 0.0287 |
| Q5-6 | 0.1247 | 0.1491 | -0.0081 | 0.0212 | -0.0081 | 0.1165 | 0.1328 | 0.1035 | 0.1328 | 0.0082 |
| Q6-11 | 0.0356 | N/A | 0.0413 | -0.0235 | 0.0413 | -0.1047 | 0.0057 | 0.0591 | 0.0057 | 0.1403 |
| Q6-12 | 0.0250 | 0.0152 | 0.0577 | 0.0152 | 0.0577 | -0.0222 | 0.0327 | 0.0098 | 0.0327 | 0.0472 |
| Q6-13 | 0.0722 | N/A | 0.1528 | 0.0458 | 0.1528 | -0.0047 | 0.0806 | 0.0264 | 0.0806 | 0.0769 |
| Q7-8 | -0.1716 | -0.1570 | -0.1220 | -0.1137 | -0.1220 | -0.1296 | 0.0496 | 0.0579 | 0.0496 | 0.0420 |
| Q7-9 | 0.0578 | 0.0677 | 0.1173 | 0.1211 | 0.1173 | 0.1083 | 0.0595 | 0.0633 | 0.0595 | 0.0505 |
| Q9-10 | 0.0422 | 0.1216 | 0.0845 | 0.1216 | 0.0845 | 0.0897 | 0.0423 | 0.0794 | 0.0423 | 0.0475 |
| Q9-14 | 0.0361 | N/A | 0.1153 | 0.0971 | 0.1153 | 0.1186 | 0.0792 | 0.0610 | 0.0792 | 0.0825 |
| Q10-11 | -0.0162 | 0.0186 | 0.0407 | 0.0186 | 0.0407 | 0.0839 | 0.0569 | 0.0348 | 0.0569 | 0.1001 |
| Q12-13 | 0.0075 | N/A | -0.0052 | -0.0166 | -0.0052 | -0.0053 | 0.0127 | 0.0241 | 0.0127 | 0.0128 |
| Q13-14 | 0.0175 | 0.0053 | 0.0118 | 0.0053 | 0.0118 | -0.0336 | 0.0057 | 0.0122 | 0.0057 | 0.0511 |
| Q2-1 | 0.2768 | N/A | 0.2227 | 0.2280 | 0.2227 | 0.2211 | 0.0541 | 0.0488 | 0.0541 | 0.0557 |
| Q5-1 | 0.0223 | N/A | -0.0452 | -0.0424 | -0.0452 | -0.0641 | 0.0675 | 0.0647 | 0.0675 | 0.0864 |
| Q3-2 | 0.0160 | N/A | -0.0056 | -0.0107 | -0.0056 | -0.0034 | 0.0216 | 0.0267 | 0.0216 | 0.0194 |
| Q4-2 | 0.0302 | N/A | -0.0217 | -0.0167 | -0.0217 | -0.0250 | 0.0519 | 0.0469 | 0.0519 | 0.0552 |
| Q5-2 | -0.0210 | N/A | -0.0822 | -0.0798 | -0.0822 | -0.1039 | 0.0612 | 0.0588 | 0.0612 | 0.0829 |
| Q4-3 | -0.0484 | N/A | -0.0910 | -0.0781 | -0.0910 | -0.0984 | 0.0426 | 0.0297 | 0.0426 | 0.0500 |
| Q5-4 | 0.1420 | N/A | -0.1431 | -0.1676 | -0.1431 | -0.2162 | 0.2851 | 0.3096 | 0.2851 | 0.3582 |
| Q7-4 | 0.1138 | N/A | 0.1261 | 0.1436 | 0.1261 | 0.1628 | 0.0123 | 0.0298 | 0.0123 | 0.0490 |
| Q9-4 | 0.0173 | N/A | 0.0263 | 0.0313 | 0.0263 | 0.0421 | 0.0090 | 0.0140 | 0.0090 | 0.0248 |
| Q6-5 | -0.0805 | N/A | 0.0422 | 0.0025 | 0.0422 | -0.0865 | 0.1227 | 0.0830 | 0.1227 | 0.0060 |
| Q11-6 | -0.0344 | N/A | -0.0395 | 0.0248 | -0.0395 | 0.1079 | 0.0051 | 0.0592 | 0.0051 | 0.1423 |
| Q12-6 | -0.0235 | N/A | -0.0568 | -0.0150 | -0.0568 | 0.0224 | 0.0333 | 0.0085 | 0.0333 | 0.0459 |
| Q13-6 | -0.0680 | N/A | -0.1480 | -0.0445 | -0.1480 | 0.0062 | 0.0800 | 0.0235 | 0.0800 | 0.0742 |
| Q8-7 | 0.1762 | N/A | 0.1245 | 0.1158 | 0.1245 | 0.1323 | 0.0517 | 0.0604 | 0.0517 | 0.0439 |
| Q9-7 | -0.0498 | N/A | -0.1113 | -0.1164 | -0.1113 | -0.1023 | 0.0615 | 0.0666 | 0.0615 | 0.0525 |
| Q10-9 | -0.0418 | N/A | -0.0838 | -0.1201 | -0.0838 | -0.0889 | 0.0420 | 0.0783 | 0.0420 | 0.0471 |
| Q14-9 | -0.0336 | N/A | -0.1077 | -0.0922 | -0.1077 | -0.1105 | 0.0741 | 0.0586 | 0.0741 | 0.0769 |
| Q11-10 | 0.0164 | N/A | -0.0403 | -0.0184 | -0.0403 | -0.0824 | 0.0567 | 0.0348 | 0.0567 | 0.0988 |
| Q13-12 | -0.0075 | N/A | 0.0062 | 0.0175 | 0.0062 | 0.0062 | 0.0137 | 0.0250 | 0.0137 | 0.0137 |
| Q14-13 | -0.0164 | N/A | -0.0098 | -0.0042 | -0.0098 | 0.0354 | 0.0066 | 0.0122 | 0.0066 | 0.0518 |
| **SUM** | | | | | | | **3.6031** | **3.4331** | **3.6031** | **4.0409** |

LTAV doesn't successfully detect the correct bad data and generate worst estimation among all estimators.

**Comparison**

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.218. It's clear that full redundancy cases generate better estimation for all estimators over median counterpart.

Table 5.218: Estimation errors of case 1, single large, 14-bus AC system, comparison

| | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| SUM | 3.3852 | 2.6711 | 2.4469 | 2.6711 | 2.5112 |
| SUM | N/A | 3.6031 | 3.4331 | 3.6031 | 4.0409 |

#### 5.2.2.3 Case 2: Multiple non-interacting bad data

To keep consistency, $P_4$, $P_{9-10}$ and $P_{13-14}$ are selected as in DC state estimation.

For each bad data, reverse and large bad data are the two types that are used here. They induce four major cases: three reverse, three large, two reverse one large and two large one reverse. These four cases are conducted with full & median redundancy case.

**Three reverse bad data**

The three bad data are: $P_4 = 0.4349$, $P_{9-10} = -0.0679$ and $P_{13-14} = -0.0878$. Other data remain the same as Table 5.117 for full and Table 5.121 for median redundancy case.

**Full Redundancy**

Table 5.219 presents the state variables generated by the estimation of each estimator; The estimated system results and the error between them and the actual value are illustrated in Table 5.220.

Table 5.219: State Variable of case 2, multi reverse, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0812 | -0.0814 | -0.0816 | -0.0826 | **V1** | 1.0876 | 1.0770 | 1.0899 | 1.0834 |
| $\theta 3$ | -0.2210 | -0.2210 | -0.2171 | -0.2199 | **V2** | 1.0719 | 1.0641 | 1.0742 | 1.0677 |
| $\theta 4$ | -0.1570 | -0.1679 | -0.1711 | -0.1737 | **V3** | 1.0242 | 1.0249 | 1.0289 | 1.0224 |
| $\theta 5$ | -0.1372 | -0.1423 | -0.1437 | -0.1462 | **V4** | 1.0435 | 1.0318 | 1.0412 | 1.0356 |
| $\theta 6$ | -0.2366 | -0.2331 | -0.2295 | -0.2345 | **V5** | 1.0464 | 1.0352 | 1.0465 | 1.0411 |
| $\theta 7$ | -0.2325 | -0.2342 | -0.2193 | -0.2280 | **V6** | 1.0624 | 1.0576 | 1.0622 | 1.0704 |
| $\theta 8$ | -0.2371 | -0.2340 | -0.2190 | -0.2264 | **V7** | 1.0701 | 1.0551 | 1.0686 | 1.0658 |
| $\theta 9$ | -0.2555 | -0.2637 | -0.2406 | -0.2555 | **V8** | 1.0898 | 1.0767 | 1.0882 | 1.0858 |
| $\theta 10$ | -0.2552 | -0.2604 | -0.2395 | -0.2506 | **V9** | 1.0629 | 1.0485 | 1.0616 | 1.0599 |
| $\theta 11$ | -0.2523 | -0.2539 | -0.2399 | -0.2475 | **V10** | 1.0554 | 1.0399 | 1.0547 | 1.0557 |
| $\theta 12$ | -0.2347 | -0.2289 | -0.2262 | -0.2322 | **V11** | 1.0459 | 1.0388 | 1.0459 | 1.0515 |
| $\theta 13$ | -0.2495 | -0.2446 | -0.2404 | -0.2457 | **V12** | 1.0517 | 1.0552 | 1.0522 | 1.0622 |
| $\theta 14$ | -0.2726 | -0.2734 | -0.2572 | -0.2630 | **V13** | 1.0393 | 1.0439 | 1.0398 | 1.0491 |
| | | | | | **V14** | 1.0337 | 1.0276 | 1.0341 | 1.0396 |

Table 5.220: Estimation Results and Errors of case 2, multi reverse, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0876 | 1.0770 | 1.0899 | 1.0834 | 0.0086 | 0.0276 | 0.0170 | 0.0299 | 0.0234 |
| V2 | 1.0450 | 1.0743 | 1.0719 | 1.0641 | 1.0742 | 1.0677 | 0.0293 | 0.0269 | 0.0191 | 0.0292 | 0.0227 |
| V3 | 1.0100 | 0.9739 | 1.0242 | 1.0249 | 1.0289 | 1.0224 | 0.0361 | 0.0142 | 0.0149 | 0.0189 | 0.0124 |
| V4 | 1.0180 | 1.0318 | 1.0435 | 1.0318 | 1.0412 | 1.0356 | 0.0138 | 0.0255 | 0.0138 | 0.0232 | 0.0176 |
| V5 | 1.0200 | 1.0251 | 1.0464 | 1.0352 | 1.0465 | 1.0411 | 0.0051 | 0.0264 | 0.0152 | 0.0265 | 0.0211 |
| V6 | 1.0700 | 1.0491 | 1.0624 | 1.0576 | 1.0622 | 1.0704 | 0.0209 | 0.0076 | 0.0124 | 0.0078 | 0.0004 |
| V7 | 1.0620 | 1.0551 | 1.0701 | 1.0551 | 1.0686 | 1.0658 | 0.0069 | 0.0081 | 0.0069 | 0.0066 | 0.0038 |
| V8 | 1.0900 | 1.0955 | 1.0898 | 1.0767 | 1.0882 | 1.0858 | 0.0055 | 0.0002 | 0.0133 | 0.0018 | 0.0042 |
| V9 | 1.0560 | 1.1131 | 1.0629 | 1.0485 | 1.0616 | 1.0599 | 0.0571 | 0.0069 | 0.0075 | 0.0056 | 0.0039 |
| V10 | 1.0510 | 1.0952 | 1.0554 | 1.0399 | 1.0547 | 1.0557 | 0.0442 | 0.0044 | 0.0111 | 0.0037 | 0.0047 |
| V11 | 1.0570 | 1.0355 | 1.0459 | 1.0388 | 1.0459 | 1.0515 | 0.0215 | 0.0111 | 0.0182 | 0.0111 | 0.0055 |
| V12 | 1.0550 | 1.1034 | 1.0517 | 1.0552 | 1.0522 | 1.0622 | 0.0484 | 0.0033 | 0.0002 | 0.0028 | 0.0072 |
| V13 | 1.0500 | 1.0616 | 1.0393 | 1.0439 | 1.0398 | 1.0491 | 0.0116 | 0.0107 | 0.0061 | 0.0102 | 0.0009 |
| V14 | 1.0360 | 1.0350 | 1.0337 | 1.0276 | 1.0341 | 1.0396 | 0.0010 | 0.0023 | 0.0084 | 0.0019 | 0.0036 |
| P1 | 2.3239 | 2.3508 | 2.2643 | 2.2375 | 2.3133 | 2.3171 | 0.0269 | 0.0596 | 0.0864 | 0.0106 | 0.0068 |
| P2 | 0.1830 | 0.2747 | 0.1590 | 0.2694 | 0.2551 | 0.2584 | 0.0917 | 0.0240 | 0.0864 | 0.0721 | 0.0754 |
| P3 | -0.9420 | -1.0549 | -1.1558 | -1.0549 | -1.0283 | -1.0272 | 0.1129 | 0.2138 | 0.1129 | 0.0863 | 0.0852 |
| **P4** | **-0.4780** | **0.4349** | **0.0414** | **-0.3006** | **-0.5582** | **-0.5012** | **0.9129** | **0.5194** | **0.1774** | **0.0802** | **0.0232** |
| P5 | -0.0760 | -0.0601 | -0.1030 | -0.0601 | -0.0435 | -0.0396 | 0.0159 | 0.0270 | 0.0159 | 0.0325 | 0.0364 |
| P6 | -0.1120 | -0.1774 | -0.1896 | -0.1774 | -0.1733 | -0.1670 | 0.0654 | 0.0776 | 0.0654 | 0.0613 | 0.0550 |
| P7 | 0.0000 | -0.0217 | -0.1439 | -0.0575 | -0.0456 | -0.0211 | 0.0217 | 0.1439 | 0.0575 | 0.0456 | 0.0211 |
| P8 | 0.0000 | 0.0171 | -0.0305 | 0.0016 | 0.0023 | 0.0105 | 0.0171 | 0.0305 | 0.0016 | 0.0023 | 0.0105 |
| P9 | -0.2950 | -0.1161 | -0.3115 | -0.4284 | -0.2479 | -0.4331 | 0.1789 | 0.0165 | 0.1334 | 0.0471 | 0.1381 |
| P10 | -0.0900 | 0.0485 | -0.0219 | -0.0258 | 0.0027 | 0.0332 | 0.1385 | 0.0681 | 0.0642 | 0.0927 | 0.1232 |
| P11 | -0.0350 | -0.1025 | -0.1097 | -0.1025 | -0.0990 | -0.0914 | 0.0675 | 0.0747 | 0.0675 | 0.0640 | 0.0564 |
| P12 | -0.0610 | 0.0907 | 0.0588 | 0.0802 | 0.0635 | 0.0638 | 0.1517 | 0.1198 | 0.1412 | 0.1245 | 0.1248 |
| P13 | -0.1350 | -0.0987 | -0.1652 | -0.0987 | -0.1633 | -0.1576 | 0.0363 | 0.0302 | 0.0363 | 0.0283 | 0.0226 |
| P14 | -0.1490 | -0.1522 | -0.1634 | -0.1522 | -0.1440 | -0.1099 | 0.0032 | 0.0144 | 0.0032 | 0.0050 | 0.0391 |
| Q1 | -0.1655 | -0.1298 | -0.1994 | -0.2378 | -0.1951 | -0.2011 | 0.0357 | 0.0339 | 0.0723 | 0.0296 | 0.0356 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q2 | 0.3086 | 0.2984 | 0.2812 | 0.2984 | 0.2819 | 0.2735 | 0.0102 | 0.0274 | 0.0102 | 0.0267 | 0.0351 |
| Q3 | 0.0608 | 0.0546 | -0.0212 | 0.0546 | -0.0252 | -0.0276 | 0.0062 | 0.0820 | 0.0062 | 0.0860 | 0.0884 |
| Q4 | 0.0390 | 0.1135 | 0.0550 | 0.0646 | 0.0490 | 0.0353 | 0.0745 | 0.0160 | 0.0256 | 0.0100 | 0.0037 |
| Q5 | -0.0160 | 0.0545 | 0.1249 | 0.0545 | 0.1241 | 0.0848 | 0.0705 | 0.1409 | 0.0705 | 0.1401 | 0.1008 |
| Q6 | 0.0523 | 0.1232 | -0.0471 | -0.1020 | -0.0418 | 0.0071 | 0.0709 | 0.0994 | 0.1543 | 0.0941 | 0.0452 |
| Q7 | 0.0000 | 0.0336 | -0.0154 | -0.0493 | -0.0217 | -0.0175 | 0.0336 | 0.0154 | 0.0493 | 0.0217 | 0.0175 |
| Q8 | 0.1762 | 0.1158 | 0.1222 | 0.1318 | 0.1209 | 0.1234 | 0.0604 | 0.0540 | 0.0444 | 0.0553 | 0.0528 |
| Q9 | -0.1660 | -0.1301 | -0.1464 | -0.1301 | -0.1561 | -0.1533 | 0.0359 | 0.0196 | 0.0359 | 0.0099 | 0.0127 |
| Q10 | -0.0580 | 0.0235 | -0.0331 | -0.0877 | -0.0396 | -0.0424 | 0.0815 | 0.0249 | 0.0297 | 0.0184 | 0.0156 |
| Q11 | -0.0180 | 0.0064 | -0.0860 | -0.0524 | -0.0871 | -0.0778 | 0.0244 | 0.0680 | 0.0344 | 0.0691 | 0.0598 |
| Q12 | -0.0160 | 0.0357 | -0.0497 | -0.0312 | -0.0478 | -0.0373 | 0.0517 | 0.0337 | 0.0152 | 0.0318 | 0.0213 |
| Q13 | -0.0580 | -0.0217 | -0.1045 | -0.0248 | -0.1015 | -0.0883 | 0.0363 | 0.0465 | 0.0332 | 0.0435 | 0.0303 |
| Q14 | -0.0500 | -0.0652 | -0.0486 | -0.0527 | -0.0521 | -0.0534 | 0.0152 | 0.0014 | 0.0027 | 0.0021 | 0.0034 |
| P1-2 | 1.5688 | 1.5835 | 1.5480 | 1.5099 | 1.5611 | 1.5618 | 0.0147 | 0.0208 | 0.0589 | 0.0077 | 0.0070 |
| P1-5 | 0.7551 | 0.7157 | 0.7163 | 0.7276 | 0.7522 | 0.7553 | 0.0394 | 0.0388 | 0.0275 | 0.0029 | 0.0002 |
| P2-3 | 0.7324 | 0.7768 | 0.8016 | 0.7854 | 0.7808 | 0.7806 | 0.0444 | 0.0692 | 0.0530 | 0.0484 | 0.0482 |
| P2-4 | 0.5613 | 0.5039 | 0.4901 | 0.5507 | 0.5786 | 0.5799 | 0.0574 | 0.0712 | 0.0106 | 0.0173 | 0.0186 |
| P2-5 | 0.4152 | 0.3618 | 0.3757 | 0.4046 | 0.4166 | 0.4190 | 0.0534 | 0.0395 | 0.0106 | 0.0014 | 0.0038 |
| P3-4 | -0.2329 | -0.2734 | -0.3811 | -0.2954 | -0.2728 | -0.2722 | 0.0405 | 0.1482 | 0.0625 | 0.0399 | 0.0393 |
| P4-5 | -0.6116 | -0.7588 | -0.4855 | -0.6115 | -0.6789 | -0.6751 | 0.1472 | 0.1261 | 0.0001 | 0.0673 | 0.0635 |
| P4-7 | 0.2807 | 0.3526 | 0.4120 | 0.3526 | 0.2624 | 0.2931 | 0.0719 | 0.1313 | 0.0719 | 0.0183 | 0.0124 |
| P4-9 | 0.1608 | 0.1771 | 0.2024 | 0.1920 | 0.1424 | 0.1665 | 0.0163 | 0.0416 | 0.0312 | 0.0184 | 0.0057 |
| P5-6 | 0.4409 | 0.4032 | 0.4696 | 0.4223 | 0.4058 | 0.4184 | 0.0377 | 0.0287 | 0.0186 | 0.0351 | 0.0225 |
| P6-11 | 0.0735 | 0.1420 | 0.1059 | 0.1329 | 0.0809 | 0.0993 | 0.0685 | 0.0324 | 0.0594 | 0.0074 | 0.0258 |
| P6-12 | 0.0779 | -0.0077 | 0.0106 | -0.0110 | 0.0045 | 0.0050 | 0.0856 | 0.0673 | 0.0889 | 0.0734 | 0.0729 |
| P6-13 | 0.1775 | 0.1724 | 0.1635 | 0.1230 | 0.1472 | 0.1471 | 0.0051 | 0.0140 | 0.0545 | 0.0303 | 0.0304 |
| P7-8 | 0.0000 | -0.0121 | 0.0305 | -0.0016 | -0.0023 | -0.0105 | 0.0121 | 0.0305 | 0.0016 | 0.0023 | 0.0105 |
| P7-9 | 0.2807 | 0.2967 | 0.2376 | 0.2967 | 0.2191 | 0.2825 | 0.0160 | 0.0431 | 0.0160 | 0.0616 | 0.0018 |
| **P9-10** | **0.0523** | **-0.0679** | **0.0271** | **-0.0026** | **0.0165** | **-0.0399** | **0.1202** | **0.0252** | **0.0549** | **0.0358** | **0.0922** |
| P9-14 | 0.0943 | 0.0511 | 0.1014 | 0.0629 | 0.0971 | 0.0557 | 0.0432 | 0.0071 | 0.0314 | 0.0028 | 0.0386 |
| P10-11 | -0.0379 | -0.0394 | 0.0050 | -0.0287 | 0.0190 | -0.0068 | 0.0015 | 0.0429 | 0.0092 | 0.0569 | 0.0311 |
| P12-13 | 0.0161 | 0.0079 | 0.0693 | 0.0691 | 0.0677 | 0.0686 | 0.0082 | 0.0532 | 0.0530 | 0.0516 | 0.0525 |
| **P13-14** | **0.0564** | **-0.0878** | **0.0643** | **0.0914** | **0.0488** | **0.0553** | **0.1442** | **0.0079** | **0.0350** | **0.0076** | **0.0011** |
| P2-1 | -1.5259 | -1.4712 | -1.5084 | -1.4712 | -1.5209 | -1.5212 | 0.0547 | 0.0175 | 0.0547 | 0.0050 | 0.0047 |
| P5-1 | -0.7275 | -0.6720 | -0.6926 | -0.7026 | -0.7261 | -0.7287 | 0.0555 | 0.0349 | 0.0249 | 0.0014 | 0.0012 |
| P3-2 | -0.7091 | -0.7523 | -0.7747 | -0.7595 | -0.7555 | -0.7550 | 0.0432 | 0.0656 | 0.0504 | 0.0464 | 0.0459 |
| P4-2 | -0.5445 | -0.5406 | -0.4779 | -0.5350 | -0.5617 | -0.5627 | 0.0039 | 0.0666 | 0.0095 | 0.0172 | 0.0182 |
| P5-2 | -0.4061 | -0.4668 | -0.3686 | -0.3962 | -0.4079 | -0.4102 | 0.0607 | 0.0375 | 0.0099 | 0.0018 | 0.0041 |
| P4-3 | 0.2366 | 0.1809 | 0.3905 | 0.3014 | 0.2776 | 0.2770 | 0.0557 | 0.1539 | 0.0648 | 0.0410 | 0.0404 |
| P5-4 | 0.6167 | 0.6164 | 0.4885 | 0.6164 | 0.6847 | 0.6809 | 0.0003 | 0.1282 | 0.0003 | 0.0680 | 0.0642 |
| P7-4 | -0.2807 | -0.2041 | -0.4120 | -0.3526 | -0.2624 | -0.2931 | 0.0766 | 0.1313 | 0.0719 | 0.0183 | 0.0124 |
| P9-4 | -0.1608 | -0.1993 | -0.2024 | -0.1920 | -0.1424 | -0.1665 | 0.0385 | 0.0416 | 0.0312 | 0.0184 | 0.0057 |
| P6-5 | -0.4409 | -0.4223 | -0.4696 | -0.4223 | -0.4058 | -0.4184 | 0.0186 | 0.0287 | 0.0186 | 0.0351 | 0.0225 |
| P11-6 | -0.0730 | -0.0843 | -0.1049 | -0.1313 | -0.0801 | -0.0983 | 0.0113 | 0.0319 | 0.0583 | 0.0071 | 0.0253 |
| P12-6 | -0.0771 | -0.0212 | -0.0104 | 0.0111 | -0.0043 | -0.0049 | 0.0559 | 0.0667 | 0.0882 | 0.0728 | 0.0722 |
| P13-6 | -0.1754 | -0.2299 | -0.1613 | -0.1219 | -0.1452 | -0.1453 | 0.0545 | 0.0141 | 0.0535 | 0.0302 | 0.0301 |
| P8-7 | 0.0000 | 0.0016 | -0.0305 | 0.0016 | 0.0023 | 0.0105 | 0.0016 | 0.0305 | 0.0016 | 0.0023 | 0.0105 |
| P9-7 | -0.2807 | -0.2531 | -0.2376 | -0.2967 | -0.2191 | -0.2825 | 0.0276 | 0.0431 | 0.0160 | 0.0616 | 0.0018 |
| P10-9 | -0.0521 | 0.0029 | -0.0269 | 0.0029 | -0.0163 | 0.0400 | 0.0550 | 0.0252 | 0.0550 | 0.0358 | 0.0921 |
| P14-9 | -0.0931 | -0.0159 | -0.0997 | -0.0622 | -0.0956 | -0.0550 | 0.0772 | 0.0066 | 0.0309 | 0.0025 | 0.0381 |
| P11-10 | 0.0380 | 0.0423 | -0.0048 | 0.0288 | -0.0189 | 0.0069 | 0.0043 | 0.0428 | 0.0092 | 0.0569 | 0.0311 |
| P13-12 | -0.0161 | -0.0907 | -0.0683 | -0.0681 | -0.0668 | -0.0677 | 0.0746 | 0.0522 | 0.0520 | 0.0507 | 0.0516 |
| P14-13 | -0.0559 | -0.0930 | -0.0636 | -0.0900 | -0.0484 | -0.0548 | 0.0371 | 0.0077 | 0.0341 | 0.0075 | 0.0011 |
| Q1-2 | -0.2040 | -0.2571 | -0.2164 | -0.2571 | -0.2190 | -0.2199 | 0.0531 | 0.0124 | 0.0531 | 0.0150 | 0.0159 |
| Q1-5 | 0.0385 | 0.1560 | 0.0170 | 0.0193 | 0.0239 | 0.0188 | 0.1175 | 0.0215 | 0.0192 | 0.0146 | 0.0197 |
| Q2-3 | 0.0356 | 0.0048 | 0.0717 | 0.0284 | 0.0609 | 0.0610 | 0.0308 | 0.0361 | 0.0072 | 0.0253 | 0.0254 |
| Q2-4 | -0.0155 | 0.0219 | -0.0093 | -0.0016 | -0.0039 | -0.0093 | 0.0374 | 0.0062 | 0.0139 | 0.0116 | 0.0062 |
| Q2-5 | 0.0117 | 0.0021 | 0.0045 | 0.0173 | 0.0070 | -0.0002 | 0.0096 | 0.0072 | 0.0056 | 0.0047 | 0.0119 |
| Q3-4 | 0.0447 | 0.0891 | 0.0335 | 0.0697 | 0.0259 | 0.0212 | 0.0444 | 0.0112 | 0.0250 | 0.0188 | 0.0235 |
| Q4-5 | 0.1582 | 0.1200 | 0.0852 | 0.1200 | 0.0947 | 0.0885 | 0.0382 | 0.0730 | 0.0382 | 0.0635 | 0.0697 |
| Q4-7 | -0.0968 | -0.1669 | -0.1203 | -0.1059 | -0.1330 | -0.1450 | 0.0701 | 0.0235 | 0.0091 | 0.0362 | 0.0482 |
| Q4-9 | -0.0043 | -0.0754 | -0.0276 | -0.0228 | -0.0344 | -0.0400 | 0.0711 | 0.0233 | 0.0185 | 0.0301 | 0.0357 |
| Q5-6 | 0.1247 | 0.1491 | -0.0477 | -0.0796 | -0.0526 | -0.1114 | 0.0244 | 0.1724 | 0.2043 | 0.1773 | 0.2361 |
| Q6-11 | 0.0356 | 0.0267 | 0.0384 | 0.0374 | 0.0487 | 0.0544 | 0.0089 | 0.0028 | 0.0018 | 0.0131 | 0.0188 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q6-12 | 0.0250 | 0.0152 | 0.0394 | 0.0152 | 0.0393 | 0.0318 | 0.0098 | 0.0144 | 0.0098 | 0.0143 | 0.0068 |
| Q6-13 | 0.0722 | 0.1432 | 0.1063 | 0.0491 | 0.1084 | 0.1005 | 0.0710 | 0.0341 | 0.0231 | 0.0362 | 0.0283 |
| Q7-8 | -0.1716 | -0.1570 | -0.1198 | -0.1292 | -0.1187 | -0.1211 | 0.0146 | 0.0518 | 0.0424 | 0.0529 | 0.0505 |
| Q7-9 | 0.0578 | 0.0677 | 0.0727 | 0.0677 | 0.0704 | 0.0604 | 0.0099 | 0.0149 | 0.0099 | 0.0126 | 0.0026 |
| Q9-10 | 0.0422 | 0.1216 | 0.0840 | 0.1071 | 0.0803 | 0.0686 | 0.0794 | 0.0418 | 0.0649 | 0.0381 | 0.0264 |
| Q9-14 | 0.0361 | -0.0041 | 0.0677 | 0.0517 | 0.0627 | 0.0538 | 0.0402 | 0.0316 | 0.0156 | 0.0266 | 0.0177 |
| Q10-11 | -0.0162 | 0.0186 | 0.0503 | 0.0186 | 0.0403 | 0.0257 | 0.0348 | 0.0665 | 0.0348 | 0.0565 | 0.0419 |
| Q12-13 | 0.0075 | 0.0493 | -0.0107 | -0.0161 | -0.0089 | -0.0058 | 0.0418 | 0.0182 | 0.0236 | 0.0164 | 0.0133 |
| Q13-14 | 0.0175 | 0.0053 | -0.0141 | 0.0053 | -0.0066 | 0.0020 | 0.0122 | 0.0316 | 0.0122 | 0.0241 | 0.0155 |
| Q2-1 | 0.2768 | 0.2876 | 0.2143 | 0.2543 | 0.2180 | 0.2220 | 0.0108 | 0.0625 | 0.0225 | 0.0588 | 0.0548 |
| Q5-1 | 0.0223 | -0.0360 | -0.0313 | -0.0262 | -0.0288 | -0.0203 | 0.0583 | 0.0536 | 0.0485 | 0.0511 | 0.0426 |
| Q3-2 | 0.0160 | -0.0414 | -0.0548 | -0.0151 | -0.0510 | -0.0487 | 0.0574 | 0.0708 | 0.0311 | 0.0670 | 0.0647 |
| Q4-2 | 0.0302 | 0.0354 | -0.0298 | -0.0256 | -0.0208 | -0.0138 | 0.0052 | 0.0600 | 0.0558 | 0.0510 | 0.0440 |
| Q5-2 | -0.0210 | 0.0151 | -0.0605 | -0.0680 | -0.0583 | -0.0498 | 0.0361 | 0.0395 | 0.0470 | 0.0373 | 0.0288 |
| Q4-3 | -0.0484 | 0.0809 | -0.0369 | -0.0814 | -0.0410 | -0.0359 | 0.1293 | 0.0115 | 0.0330 | 0.0074 | 0.0125 |
| Q5-4 | 0.1420 | 0.1087 | -0.0758 | -0.1046 | -0.0764 | -0.0703 | 0.0333 | 0.2178 | 0.2466 | 0.2184 | 0.2123 |
| Q7-4 | 0.1138 | 0.1232 | 0.1549 | 0.1319 | 0.1494 | 0.1654 | 0.0094 | 0.0411 | 0.0181 | 0.0356 | 0.0516 |
| Q9-4 | 0.0173 | 0.0132 | 0.0483 | 0.0417 | 0.0451 | 0.0548 | 0.0041 | 0.0310 | 0.0244 | 0.0278 | 0.0375 |
| Q6-5 | -0.0805 | -0.1772 | 0.0955 | 0.1200 | 0.0885 | 0.1520 | 0.0967 | 0.1760 | 0.2005 | 0.1690 | 0.2325 |
| Q11-6 | -0.0344 | -0.0563 | -0.0361 | -0.0340 | -0.0471 | -0.0522 | 0.0219 | 0.0017 | 0.0004 | 0.0127 | 0.0178 |
| Q12-6 | -0.0235 | -0.1132 | -0.0391 | -0.0151 | -0.0389 | -0.0316 | 0.0897 | 0.0156 | 0.0084 | 0.0154 | 0.0081 |
| Q13-6 | -0.0680 | -0.0260 | -0.1020 | -0.0471 | -0.1046 | -0.0969 | 0.0420 | 0.0340 | 0.0209 | 0.0366 | 0.0289 |
| Q8-7 | 0.1762 | 0.1318 | 0.1222 | 0.1318 | 0.1209 | 0.1234 | 0.0444 | 0.0540 | 0.0444 | 0.0553 | 0.0528 |
| Q9-7 | -0.0498 | -0.0448 | -0.0668 | -0.0585 | -0.0653 | -0.0524 | 0.0050 | 0.0170 | 0.0087 | 0.0155 | 0.0026 |
| Q10-9 | -0.0418 | -0.0690 | -0.0834 | -0.1063 | -0.0798 | -0.0681 | 0.0272 | 0.0416 | 0.0645 | 0.0380 | 0.0263 |
| Q14-9 | -0.0336 | -0.0184 | -0.0642 | -0.0501 | -0.0595 | -0.0523 | 0.0152 | 0.0306 | 0.0165 | 0.0259 | 0.0187 |
| Q11-10 | 0.0164 | -0.0136 | -0.0498 | -0.0184 | -0.0399 | -0.0256 | 0.0300 | 0.0662 | 0.0348 | 0.0563 | 0.0420 |
| Q13-12 | -0.0075 | 0.0170 | 0.0116 | 0.0170 | 0.0097 | 0.0066 | 0.0245 | 0.0191 | 0.0245 | 0.0172 | 0.0141 |
| Q14-13 | -0.0164 | 0.0206 | 0.0155 | -0.0026 | 0.0074 | -0.0011 | 0.0370 | 0.0319 | 0.0138 | 0.0238 | 0.0153 |
| **SUM** | | | | | | | **4.0586** | **3.6727** | **2.7602** | **2.2691** | **2.1637** |

The same condition as single reverse bad data case, only WLS generate bad data.

### Median Redundancy

The state variables, estimation results and error with actual value are presented in the following two tables.

Table 5.221: State Variable of case 2, multi reverse, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0858 | -0.0923 | -0.0858 | -0.0855 | **V1** | 1.0545 | 1.0305 | 1.0545 | 1.0712 |
| $\theta 3$ | -0.2330 | -0.2452 | -0.2330 | -0.2303 | **V2** | 1.0381 | 1.0124 | 1.0381 | 1.0545 |
| $\theta 4$ | -0.1603 | -0.1844 | -0.1603 | -0.1791 | **V3** | 0.9961 | 0.9701 | 0.9961 | 1.0151 |
| $\theta 5$ | -0.1455 | -0.1556 | -0.1455 | -0.1484 | **V4** | 1.0075 | 0.9790 | 1.0075 | 1.0177 |
| $\theta 6$ | -0.2704 | -0.2540 | -0.2704 | -0.2389 | **V5** | 1.0052 | 0.9810 | 1.0052 | 1.0175 |
| $\theta 7$ | -0.2425 | -0.2580 | -0.2425 | -0.2429 | **V6** | 1.0109 | 0.9832 | 1.0109 | 0.9962 |
| $\theta 8$ | -0.2481 | -0.2559 | -0.2481 | -0.2417 | **V7** | 1.0308 | 1.0024 | 1.0308 | 1.0467 |
| $\theta 9$ | -0.2699 | -0.2909 | -0.2699 | -0.2740 | **V8** | 1.0521 | 1.0224 | 1.0521 | 1.0687 |
| $\theta 10$ | -0.2714 | -0.2821 | -0.2714 | -0.2678 | **V9** | 1.0187 | 0.9890 | 1.0187 | 1.0353 |
| $\theta 11$ | -0.2785 | -0.2791 | -0.2785 | -0.2633 | **V10** | 1.0103 | 0.9804 | 1.0103 | 1.0293 |
| $\theta 12$ | -0.2722 | -0.2500 | -0.2722 | -0.2416 | **V11** | 0.9981 | 0.9773 | 0.9981 | 1.0119 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\theta$13 | -0.2862 | -0.2681 | -0.2862 | -0.2555 | **V12** | 0.9921 | 0.9802 | 0.9921 | 1.0019 |
| $\theta$14 | -0.3029 | -0.3008 | -0.3029 | -0.2777 | **V13** | 0.9795 | 0.9701 | 0.9795 | 0.9897 |
| | | | | | **V14** | 0.9675 | 0.9585 | 0.9675 | 0.9958 |

Table 5.222: Estimation Results and Errors of case 2, multi reverse, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0545 | 1.0305 | 1.0545 | 1.0712 | 0.0055 | 0.0295 | 0.0055 | 0.0112 |
| V2 | 1.0450 | N/A | 1.0381 | 1.0124 | 1.0381 | 1.0545 | 0.0069 | 0.0326 | 0.0069 | 0.0095 |
| V3 | 1.0100 | N/A | 0.9961 | 0.9701 | 0.9961 | 1.0151 | 0.0139 | 0.0399 | 0.0139 | 0.0051 |
| V4 | 1.0180 | N/A | 1.0075 | 0.9790 | 1.0075 | 1.0177 | 0.0105 | 0.0390 | 0.0105 | 0.0003 |
| V5 | 1.0200 | N/A | 1.0052 | 0.9810 | 1.0052 | 1.0175 | 0.0148 | 0.0390 | 0.0148 | 0.0025 |
| V6 | 1.0700 | N/A | 1.0109 | 0.9832 | 1.0109 | 0.9962 | 0.0591 | 0.0868 | 0.0591 | 0.0738 |
| V7 | 1.0620 | N/A | 1.0308 | 1.0024 | 1.0308 | 1.0467 | 0.0312 | 0.0596 | 0.0312 | 0.0153 |
| V8 | 1.0900 | N/A | 1.0521 | 1.0224 | 1.0521 | 1.0687 | 0.0379 | 0.0676 | 0.0379 | 0.0213 |
| V9 | 1.0560 | N/A | 1.0187 | 0.9890 | 1.0187 | 1.0353 | 0.0373 | 0.0670 | 0.0373 | 0.0207 |
| V10 | 1.0510 | N/A | 1.0103 | 0.9804 | 1.0103 | 1.0293 | 0.0407 | 0.0706 | 0.0407 | 0.0217 |
| V11 | 1.0570 | N/A | 0.9981 | 0.9773 | 0.9981 | 1.0119 | 0.0589 | 0.0797 | 0.0589 | 0.0451 |
| V12 | 1.0550 | N/A | 0.9921 | 0.9802 | 0.9921 | 1.0019 | 0.0629 | 0.0748 | 0.0629 | 0.0531 |
| V13 | 1.0500 | N/A | 0.9795 | 0.9701 | 0.9795 | 0.9897 | 0.0705 | 0.0799 | 0.0705 | 0.0603 |
| V14 | 1.0360 | N/A | 0.9675 | 0.9585 | 0.9675 | 0.9958 | 0.0685 | 0.0775 | 0.0685 | 0.0402 |
| P1 | 2.3239 | 2.3508 | 2.2536 | 2.3119 | 2.2536 | 2.3363 | 0.0703 | 0.0120 | 0.0703 | 0.0124 |
| P2 | 0.1830 | 0.2747 | 0.1322 | 0.1535 | 0.1322 | 0.2656 | 0.0508 | 0.0295 | 0.0508 | 0.0826 |
| P3 | -0.9420 | -1.0549 | -1.1470 | -1.0549 | -1.1470 | -1.0424 | 0.2050 | 0.1129 | 0.2050 | 0.1004 |
| **P4** | **-0.4780** | **0.4349** | **0.2697** | **-0.2650** | **0.2697** | **-0.4546** | **0.7477** | **0.2130** | **0.7477** | **0.0234** |
| P5 | -0.0760 | -0.0601 | -0.2178 | -0.0601 | -0.2178 | -0.0600 | 0.1418 | 0.0159 | 0.1418 | 0.0160 |
| P6 | -0.1120 | -0.1774 | -0.2517 | -0.1774 | -0.2517 | -0.1999 | 0.1397 | 0.0654 | 0.1397 | 0.0879 |
| P7 | 0.0000 | -0.0217 | -0.1202 | -0.0680 | -0.1202 | -0.0349 | 0.1202 | 0.0680 | 0.1202 | 0.0349 |
| P8 | 0.0000 | 0.0171 | -0.0346 | 0.0121 | -0.0346 | 0.0080 | 0.0346 | 0.0121 | 0.0346 | 0.0080 |
| P9 | -0.2950 | -0.1161 | -0.2484 | -0.4713 | -0.2484 | -0.4641 | 0.0466 | 0.1763 | 0.0466 | 0.1691 |
| P10 | -0.0900 | 0.0485 | 0.0066 | 0.0485 | 0.0066 | 0.0567 | 0.0966 | 0.1385 | 0.0966 | 0.1467 |
| P11 | -0.0350 | -0.1025 | -0.1128 | -0.1025 | -0.1128 | -0.0810 | 0.0778 | 0.0675 | 0.0778 | 0.0460 |
| P12 | -0.0610 | 0.0907 | 0.0278 | 0.0717 | 0.0278 | 0.0617 | 0.0888 | 0.1327 | 0.0888 | 0.1227 |
| P13 | -0.1350 | -0.0987 | -0.2014 | -0.1009 | -0.2014 | -0.1355 | 0.0664 | 0.0341 | 0.0664 | 0.0005 |
| P14 | -0.1490 | -0.1522 | -0.2177 | -0.1522 | -0.2177 | -0.1115 | 0.0687 | 0.0032 | 0.0687 | 0.0375 |
| Q1 | -0.1655 | -0.1298 | -0.1462 | -0.1298 | -0.1462 | -0.1343 | 0.0193 | 0.0357 | 0.0193 | 0.0312 |
| Q2 | 0.3086 | 0.2984 | 0.3077 | 0.2984 | 0.3077 | 0.3221 | 0.0009 | 0.0102 | 0.0009 | 0.0135 |
| Q3 | 0.0608 | 0.0546 | 0.0702 | 0.0546 | 0.0702 | 0.0784 | 0.0094 | 0.0062 | 0.0094 | 0.0176 |
| Q4 | 0.0390 | 0.1135 | 0.0893 | 0.1135 | 0.0893 | 0.1140 | 0.0503 | 0.0745 | 0.0503 | 0.0750 |
| Q5 | -0.0160 | 0.0545 | 0.0267 | 0.0545 | 0.0267 | 0.0517 | 0.0427 | 0.0705 | 0.0427 | 0.0677 |
| Q6 | 0.0523 | 0.1232 | 0.0030 | -0.2203 | 0.0030 | -0.5036 | 0.0493 | 0.2726 | 0.0493 | 0.5559 |
| Q7 | 0.0000 | 0.0336 | 0.0127 | 0.0336 | 0.0127 | 0.0247 | 0.0127 | 0.0336 | 0.0127 | 0.0247 |
| Q8 | 0.1762 | 0.1158 | 0.1271 | 0.1158 | 0.1271 | 0.1329 | 0.0491 | 0.0604 | 0.0491 | 0.0433 |
| Q9 | -0.1660 | -0.1301 | -0.1363 | -0.1301 | -0.1363 | -0.1154 | 0.0297 | 0.0359 | 0.0297 | 0.0506 |
| Q10 | -0.0580 | 0.0235 | -0.0411 | -0.1015 | -0.0411 | -0.0028 | 0.0169 | 0.0435 | 0.0169 | 0.0552 |
| Q11 | -0.0180 | 0.0064 | -0.0764 | 0.0064 | -0.0764 | 0.0282 | 0.0584 | 0.0244 | 0.0584 | 0.0462 |
| Q12 | -0.0160 | 0.0357 | -0.0618 | -0.0352 | -0.0618 | 0.0158 | 0.0458 | 0.0192 | 0.0458 | 0.0318 |
| Q13 | -0.0580 | -0.0217 | -0.1224 | -0.0217 | -0.1224 | -0.0191 | 0.0644 | 0.0363 | 0.0644 | 0.0389 |
| Q14 | -0.0500 | -0.0652 | -0.1112 | -0.0652 | -0.1112 | -0.0740 | 0.0612 | 0.0152 | 0.0612 | 0.0240 |
| P1-2 | 1.5688 | 1.5835 | 1.5381 | 1.5835 | 1.5381 | 1.5825 | 0.0307 | 0.0147 | 0.0307 | 0.0137 |
| P1-5 | 0.7551 | 0.7157 | 0.7155 | 0.7284 | 0.7155 | 0.7539 | 0.0396 | 0.0267 | 0.0396 | 0.0012 |
| P2-3 | 0.7324 | 0.7768 | 0.7875 | 0.7768 | 0.7875 | 0.7987 | 0.0551 | 0.0444 | 0.0551 | 0.0663 |
| P2-4 | 0.5613 | N/A | 0.4568 | 0.5304 | 0.4568 | 0.5863 | 0.1045 | 0.0309 | 0.1045 | 0.0250 |
| P2-5 | 0.4152 | N/A | 0.3845 | 0.3838 | 0.3845 | 0.4203 | 0.0307 | 0.0314 | 0.0307 | 0.0051 |
| P3-4 | -0.2329 | -0.2734 | -0.3869 | -0.3061 | -0.3869 | -0.2709 | 0.1540 | 0.0732 | 0.1540 | 0.0380 |
| P4-5 | -0.6116 | N/A | -0.3077 | -0.6074 | -0.3077 | -0.6804 | 0.3039 | 0.0042 | 0.3039 | 0.0688 |
| P4-7 | 0.2807 | 0.3526 | 0.4168 | 0.3526 | 0.4168 | 0.3325 | 0.1361 | 0.0719 | 0.1361 | 0.0518 |
| P4-9 | 0.1608 | N/A | 0.2084 | 0.1909 | 0.2084 | 0.1852 | 0.0476 | 0.0301 | 0.0476 | 0.0244 |
| P5-6 | 0.4409 | 0.4032 | 0.5393 | 0.4032 | 0.5393 | 0.3901 | 0.0984 | 0.0377 | 0.0984 | 0.0508 |
| P6-11 | 0.0735 | N/A | 0.0587 | 0.1108 | 0.0587 | 0.0704 | 0.0148 | 0.0373 | 0.0148 | 0.0031 |
| P6-12 | 0.0779 | -0.0077 | 0.0347 | -0.0077 | 0.0347 | -0.0001 | 0.0432 | 0.0856 | 0.0432 | 0.0780 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P6-13 | 0.1775 | N/A | 0.1941 | 0.1227 | 0.1941 | 0.1199 | 0.0166 | 0.0548 | 0.0166 | 0.0576 |
| P7-8 | 0.0000 | -0.0121 | 0.0346 | -0.0121 | 0.0346 | -0.0080 | 0.0346 | 0.0121 | 0.0346 | 0.0080 |
| P7-9 | 0.2807 | 0.2967 | 0.2620 | 0.2967 | 0.2620 | 0.3055 | 0.0187 | 0.0160 | 0.0187 | 0.0248 |
| **P9-10** | **0.0523** | **-0.0679** | **0.0486** | **-0.0550** | **0.0486** | **-0.0435** | **0.0037** | **0.1073** | **0.0037** | **0.0958** |
| P9-14 | 0.0943 | N/A | 0.1734 | 0.0713 | 0.1734 | 0.0701 | 0.0791 | 0.0230 | 0.0791 | 0.0242 |
| P10-11 | -0.0379 | -0.0394 | 0.0550 | -0.0071 | 0.0550 | 0.0129 | 0.0929 | 0.0308 | 0.0929 | 0.0508 |
| P12-13 | 0.0161 | N/A | 0.0620 | 0.0640 | 0.0620 | 0.0615 | 0.0459 | 0.0479 | 0.0459 | 0.0454 |
| **P13-14** | **0.0564** | **-0.0878** | **0.0501** | **0.0836** | **0.0501** | **0.0442** | **0.0063** | **0.0272** | **0.0063** | **0.0122** |
| P2-1 | -1.5259 | N/A | -1.4965 | -1.5374 | -1.4965 | -1.5398 | 0.0294 | 0.0115 | 0.0294 | 0.0139 |
| P5-1 | -0.7275 | N/A | -0.6900 | -0.7009 | -0.6900 | -0.7263 | 0.0375 | 0.0266 | 0.0375 | 0.0012 |
| P3-2 | -0.7091 | N/A | -0.7601 | -0.7488 | -0.7601 | -0.7715 | 0.0510 | 0.0397 | 0.0510 | 0.0624 |
| P4-2 | -0.5445 | N/A | -0.4454 | -0.5144 | -0.4454 | -0.5682 | 0.0991 | 0.0301 | 0.0991 | 0.0237 |
| P5-2 | -0.4061 | N/A | -0.3763 | -0.3753 | -0.3763 | -0.4108 | 0.0298 | 0.0308 | 0.0298 | 0.0047 |
| P4-3 | 0.2366 | N/A | 0.3976 | 0.3133 | 0.3976 | 0.2763 | 0.1610 | 0.0767 | 0.1610 | 0.0397 |
| P5-4 | 0.6167 | N/A | 0.3093 | 0.6129 | 0.3093 | 0.6871 | 0.3074 | 0.0038 | 0.3074 | 0.0704 |
| P7-4 | -0.2807 | N/A | -0.4168 | -0.3526 | -0.4168 | -0.3325 | 0.1361 | 0.0719 | 0.1361 | 0.0518 |
| P9-4 | -0.1608 | N/A | -0.2084 | -0.1909 | -0.2084 | -0.1852 | 0.0476 | 0.0301 | 0.0476 | 0.0244 |
| P6-5 | -0.4409 | N/A | -0.5393 | -0.4032 | -0.5393 | -0.3901 | 0.0984 | 0.0377 | 0.0984 | 0.0508 |
| P11-6 | -0.0730 | N/A | -0.0583 | -0.1096 | -0.0583 | -0.0687 | 0.0147 | 0.0366 | 0.0147 | 0.0043 |
| P12-6 | -0.0771 | N/A | -0.0342 | 0.0077 | -0.0342 | 0.0002 | 0.0429 | 0.0848 | 0.0429 | 0.0773 |
| P13-6 | -0.1754 | N/A | -0.1903 | -0.1215 | -0.1903 | -0.1190 | 0.0149 | 0.0539 | 0.0149 | 0.0564 |
| P8-7 | 0.0000 | N/A | -0.0346 | 0.0121 | -0.0346 | 0.0080 | 0.0346 | 0.0121 | 0.0346 | 0.0080 |
| P9-7 | -0.2807 | N/A | -0.2620 | -0.2967 | -0.2620 | -0.3055 | 0.0187 | 0.0160 | 0.0187 | 0.0248 |
| P10-9 | -0.0521 | N/A | -0.0483 | 0.0556 | -0.0483 | 0.0438 | 0.0038 | 0.1077 | 0.0038 | 0.0959 |
| P14-9 | -0.0931 | N/A | -0.1681 | -0.0699 | -0.1681 | -0.0679 | 0.0750 | 0.0232 | 0.0750 | 0.0252 |
| P11-10 | 0.0380 | N/A | -0.0546 | 0.0071 | -0.0546 | -0.0123 | 0.0926 | 0.0309 | 0.0926 | 0.0503 |
| P13-12 | -0.0161 | N/A | -0.0611 | -0.0630 | -0.0611 | -0.0607 | 0.0450 | 0.0469 | 0.0450 | 0.0446 |
| P14-13 | -0.0559 | N/A | -0.0496 | -0.0823 | -0.0496 | -0.0436 | 0.0063 | 0.0264 | 0.0063 | 0.0123 |
| Q1-2 | -0.2040 | -0.2571 | -0.2014 | -0.1844 | -0.2014 | -0.2067 | 0.0026 | 0.0196 | 0.0026 | 0.0027 |
| Q1-5 | 0.0385 | 0.1560 | 0.0552 | 0.0546 | 0.0552 | 0.0724 | 0.0167 | 0.0161 | 0.0167 | 0.0339 |
| Q2-3 | 0.0356 | 0.0048 | 0.0424 | 0.0449 | 0.0424 | 0.0282 | 0.0068 | 0.0093 | 0.0068 | 0.0074 |
| Q2-4 | -0.0155 | N/A | 0.0090 | 0.0057 | 0.0090 | 0.0160 | 0.0245 | 0.0212 | 0.0245 | 0.0315 |
| Q2-5 | 0.0117 | N/A | 0.0436 | 0.0330 | 0.0436 | 0.0602 | 0.0319 | 0.0213 | 0.0319 | 0.0485 |
| Q3-4 | 0.0447 | 0.0891 | 0.0878 | 0.0675 | 0.0878 | 0.0858 | 0.0431 | 0.0228 | 0.0431 | 0.0411 |
| Q4-5 | 0.1582 | N/A | 0.1559 | 0.1560 | 0.1559 | 0.2298 | 0.0023 | 0.0022 | 0.0023 | 0.0716 |
| Q4-7 | -0.0968 | -0.1669 | -0.0975 | -0.0990 | -0.0975 | -0.1341 | 0.0007 | 0.0022 | 0.0007 | 0.0373 |
| Q4-9 | -0.0043 | N/A | -0.0095 | -0.0078 | -0.0095 | -0.0246 | 0.0052 | 0.0035 | 0.0052 | 0.0203 |
| Q5-6 | 0.1247 | 0.1491 | 0.0093 | 0.0107 | 0.0093 | 0.1103 | 0.1154 | 0.1140 | 0.1154 | 0.0144 |
| Q6-11 | 0.0356 | N/A | 0.0372 | -0.0223 | 0.0372 | -0.1109 | 0.0016 | 0.0579 | 0.0016 | 0.1465 |
| Q6-12 | 0.0250 | 0.0152 | 0.0575 | 0.0152 | 0.0575 | -0.0221 | 0.0325 | 0.0098 | 0.0325 | 0.0471 |
| Q6-13 | 0.0722 | N/A | 0.1459 | 0.0377 | 0.1459 | -0.0103 | 0.0737 | 0.0345 | 0.0737 | 0.0825 |
| Q7-8 | -0.1716 | -0.1570 | -0.1243 | -0.1135 | -0.1243 | -0.1302 | 0.0473 | 0.0581 | 0.0473 | 0.0414 |
| Q7-9 | 0.0578 | 0.0677 | 0.1169 | 0.1276 | 0.1169 | 0.1132 | 0.0591 | 0.0698 | 0.0591 | 0.0554 |
| Q9-10 | 0.0422 | 0.1216 | 0.0828 | 0.1216 | 0.0828 | 0.0912 | 0.0406 | 0.0794 | 0.0406 | 0.0490 |
| Q9-14 | 0.0361 | N/A | 0.1136 | 0.0782 | 0.1136 | 0.1184 | 0.0775 | 0.0421 | 0.0775 | 0.0823 |
| Q10-11 | -0.0162 | 0.0186 | 0.0410 | 0.0186 | 0.0410 | 0.0876 | 0.0572 | 0.0348 | 0.0572 | 0.1038 |
| Q12-13 | 0.0075 | N/A | -0.0054 | -0.0201 | -0.0054 | -0.0065 | 0.0129 | 0.0276 | 0.0129 | 0.0140 |
| Q13-14 | 0.0175 | 0.0053 | 0.0097 | -0.0073 | 0.0097 | -0.0384 | 0.0078 | 0.0248 | 0.0078 | 0.0559 |
| Q2-1 | 0.2768 | N/A | 0.2128 | 0.2149 | 0.2128 | 0.2176 | 0.0640 | 0.0619 | 0.0640 | 0.0592 |
| Q5-1 | 0.0223 | N/A | -0.0545 | -0.0404 | -0.0545 | -0.0661 | 0.0768 | 0.0627 | 0.0768 | 0.0884 |
| Q3-2 | 0.0160 | N/A | -0.0176 | -0.0129 | -0.0176 | -0.0074 | 0.0336 | 0.0289 | 0.0336 | 0.0234 |
| Q4-2 | 0.0302 | N/A | -0.0456 | -0.0244 | -0.0456 | -0.0341 | 0.0758 | 0.0546 | 0.0758 | 0.0643 |
| Q5-2 | -0.0210 | N/A | -0.0909 | -0.0760 | -0.0909 | -0.1054 | 0.0699 | 0.0550 | 0.0699 | 0.0844 |
| Q4-3 | -0.0484 | N/A | -0.0859 | -0.0736 | -0.0859 | -0.0984 | 0.0375 | 0.0252 | 0.0375 | 0.0500 |
| Q5-4 | 0.1420 | N/A | -0.1510 | -0.1388 | -0.1510 | -0.2088 | 0.2930 | 0.2808 | 0.2930 | 0.3508 |
| Q7-4 | 0.1138 | N/A | 0.1344 | 0.1276 | 0.1344 | 0.1595 | 0.0206 | 0.0138 | 0.0206 | 0.0457 |
| Q9-4 | 0.0173 | N/A | 0.0326 | 0.0284 | 0.0326 | 0.0428 | 0.0153 | 0.0111 | 0.0153 | 0.0255 |
| Q6-5 | -0.0805 | N/A | 0.0583 | 0.0290 | 0.0583 | -0.0731 | 0.1388 | 0.1095 | 0.1388 | 0.0074 |
| Q11-6 | -0.0344 | N/A | -0.0363 | 0.0249 | -0.0363 | 0.1143 | 0.0019 | 0.0593 | 0.0019 | 0.1487 |
| Q12-6 | -0.0235 | N/A | -0.0564 | -0.0151 | -0.0564 | 0.0223 | 0.0329 | 0.0084 | 0.0329 | 0.0458 |
| Q13-6 | -0.0680 | N/A | -0.1384 | -0.0354 | -0.1384 | 0.0122 | 0.0704 | 0.0326 | 0.0704 | 0.0802 |
| Q8-7 | 0.1762 | N/A | 0.1271 | 0.1158 | 0.1271 | 0.1329 | 0.0491 | 0.0604 | 0.0491 | 0.0433 |
| Q9-7 | -0.0498 | N/A | -0.1084 | -0.1162 | -0.1084 | -0.1026 | 0.0586 | 0.0664 | 0.0586 | 0.0528 |
| Q10-9 | -0.0418 | N/A | -0.0820 | -0.1201 | -0.0820 | -0.0904 | 0.0402 | 0.0783 | 0.0402 | 0.0486 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q14-9 | -0.0336 | N/A | -0.1024 | -0.0751 | -0.1024 | -0.1137 | 0.0688 | 0.0415 | 0.0688 | 0.0801 |
| Q11-10 | 0.0164 | N/A | -0.0401 | -0.0185 | -0.0401 | -0.0861 | 0.0565 | 0.0349 | 0.0565 | 0.1025 |
| Q13-12 | -0.0075 | N/A | 0.0062 | 0.0210 | 0.0062 | 0.0072 | 0.0137 | 0.0285 | 0.0137 | 0.0147 |
| Q14-13 | -0.0164 | N/A | -0.0088 | 0.0099 | -0.0088 | 0.0397 | 0.0076 | 0.0263 | 0.0076 | 0.0561 |
| **SUM** | | | | | | | **4.7477** | **3.6854** | **4.7477** | **3.2348** |

As full redundancy case, LTAV generate best results, WLS and LTS share the same estimation because LTS doesn't trim any data.

### Comparison

The comparison between full and median redundancy are presented in Table 5.223, only summation of errors are shown here. The same situation as before, the estimation from full redundancy are much better.

Table 5.223: Estimation errors of case 2, multi reverse, 14-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **4.0586** | **3.6727** | **2.7602** | **2.2691** | **2.1637** |
| **SUM** | **N/A** | **4.7477** | **3.6854** | **4.7477** | **3.2348** |

## Two reverse one large bad data

In the non-conforming case, two different types are divided: two reverse one large and one reverse two large. Here, $P_4 = 0.4349$, $P_{9-10} = -0.0679$ and $P_{13-14} = 0.9879$ are used to replace the original data. As presented, the last one is large error.

### Full redundancy

The five system state variables are presented in Table 5.224, Table 5.225 shows the results and errors between estimation and actual value.

Table 5.224: State Variable of case 2, two reverse, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0812 | -0.0848 | -0.0816 | -0.0826 | **V1** | 1.0879 | 1.0768 | 1.0899 | 1.0833 |
| $\theta 3$ | -0.2209 | -0.2239 | -0.2171 | -0.2199 | **V2** | 1.0722 | 1.0630 | 1.0742 | 1.0677 |
| $\theta 4$ | -0.1576 | -0.1701 | -0.1711 | -0.1737 | **V3** | 1.0246 | 1.0243 | 1.0289 | 1.0224 |
| $\theta 5$ | -0.1369 | -0.1445 | -0.1436 | -0.1461 | **V4** | 1.0432 | 1.0318 | 1.0412 | 1.0355 |
| $\theta 6$ | -0.2268 | -0.2351 | -0.2282 | -0.2331 | **V5** | 1.0471 | 1.0352 | 1.0466 | 1.0411 |
| $\theta 7$ | -0.2399 | -0.2365 | -0.2205 | -0.2293 | **V6** | 1.0678 | 1.0593 | 1.0629 | 1.0712 |
| $\theta 8$ | -0.2449 | -0.2363 | -0.2202 | -0.2278 | **V7** | 1.0674 | 1.0534 | 1.0682 | 1.0654 |
| $\theta 9$ | -0.2658 | -0.2661 | -0.2421 | -0.2573 | **V8** | 1.0872 | 1.0750 | 1.0878 | 1.0854 |
| $\theta 10$ | -0.2628 | -0.2625 | -0.2407 | -0.2519 | **V9** | 1.0594 | 1.0468 | 1.0611 | 1.0594 |
| $\theta 11$ | -0.2510 | -0.2551 | -0.2398 | -0.2474 | **V10** | 1.0528 | 1.0374 | 1.0544 | 1.0553 |
| $\theta 12$ | -0.2196 | -0.2300 | -0.2241 | -0.2300 | **V11** | 1.0473 | 1.0366 | 1.0461 | 1.0518 |
| $\theta 13$ | -0.2368 | -0.2464 | -0.2386 | -0.2439 | **V12** | 1.0614 | 1.0574 | 1.0536 | 1.0637 |
| $\theta 14$ | -0.3033 | -0.2761 | -0.2616 | -0.2676 | **V13** | 1.0463 | 1.0453 | 1.0408 | 1.0502 |
| | | | | | **V14** | 1.0202 | 1.0286 | 1.0322 | 1.0376 |

Table 5.225: Estimation Results and Errors of case 2, two reverse, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0879 | 1.0768 | 1.0899 | 1.0833 | 0.0086 | 0.0279 | 0.0168 | 0.0299 | 0.0233 |
| V2 | 1.0450 | 1.0743 | 1.0722 | 1.0630 | 1.0742 | 1.0677 | 0.0293 | 0.0272 | 0.0180 | 0.0292 | 0.0227 |
| V3 | 1.0100 | 0.9739 | 1.0246 | 1.0243 | 1.0289 | 1.0224 | 0.0361 | 0.0146 | 0.0143 | 0.0189 | 0.0124 |
| V4 | 1.0180 | 1.0318 | 1.0432 | 1.0318 | 1.0412 | 1.0355 | 0.0138 | 0.0252 | 0.0138 | 0.0232 | 0.0175 |
| V5 | 1.0200 | 1.0251 | 1.0471 | 1.0352 | 1.0466 | 1.0411 | 0.0051 | 0.0271 | 0.0152 | 0.0266 | 0.0211 |
| V6 | 1.0700 | 1.0491 | 1.0678 | 1.0593 | 1.0629 | 1.0712 | 0.0209 | 0.0022 | 0.0107 | 0.0071 | 0.0012 |
| V7 | 1.0620 | 1.0551 | 1.0674 | 1.0534 | 1.0682 | 1.0654 | 0.0069 | 0.0054 | 0.0086 | 0.0062 | 0.0034 |
| V8 | 1.0900 | 1.0955 | 1.0872 | 1.0750 | 1.0878 | 1.0854 | 0.0055 | 0.0028 | 0.0150 | 0.0022 | 0.0046 |
| V9 | 1.0560 | 1.1131 | 1.0594 | 1.0468 | 1.0611 | 1.0594 | 0.0571 | 0.0034 | 0.0092 | 0.0051 | 0.0034 |
| V10 | 1.0510 | 1.0952 | 1.0528 | 1.0374 | 1.0544 | 1.0553 | 0.0442 | 0.0018 | 0.0136 | 0.0034 | 0.0043 |
| V11 | 1.0570 | 1.0355 | 1.0473 | 1.0366 | 1.0461 | 1.0518 | 0.0215 | 0.0097 | 0.0204 | 0.0109 | 0.0052 |
| V12 | 1.0550 | 1.1034 | 1.0614 | 1.0574 | 1.0536 | 1.0637 | 0.0484 | 0.0064 | 0.0024 | 0.0014 | 0.0087 |
| V13 | 1.0500 | 1.0616 | 1.0463 | 1.0453 | 1.0408 | 1.0502 | 0.0116 | 0.0037 | 0.0047 | 0.0092 | 0.0002 |
| V14 | 1.0360 | 1.0350 | 1.0202 | 1.0286 | 1.0322 | 1.0376 | 0.0010 | 0.0158 | 0.0074 | 0.0038 | 0.0016 |
| P1 | 2.3239 | 2.3508 | 2.2635 | 2.3125 | 2.3128 | 2.3167 | 0.0269 | 0.0604 | 0.0114 | 0.0111 | 0.0072 |
| P2 | 0.1830 | 0.2747 | 0.1604 | 0.1832 | 0.2547 | 0.2580 | 0.0917 | 0.0226 | 0.0002 | 0.0717 | 0.0750 |
| P3 | -0.9420 | -1.0549 | -1.1509 | -1.0549 | -1.0284 | -1.0273 | 0.1129 | 0.2089 | 0.1129 | 0.0864 | 0.0853 |
| **P4** | **-0.4780** | **0.4349** | **0.0584** | **-0.2863** | **-0.5520** | **-0.4944** | **0.9129** | **0.5364** | **0.1917** | **0.0740** | **0.0164** |
| P5 | -0.0760 | -0.0601 | -0.1126 | -0.0601 | -0.0453 | -0.0415 | 0.0159 | 0.0366 | 0.0159 | 0.0307 | 0.0345 |
| P6 | -0.1120 | -0.1774 | -0.1499 | -0.1774 | -0.1678 | -0.1610 | 0.0654 | 0.0379 | 0.0654 | 0.0558 | 0.0490 |
| P7 | 0.0000 | -0.0217 | -0.1497 | -0.0575 | -0.0471 | -0.0225 | 0.0217 | 0.1497 | 0.0575 | 0.0471 | 0.0225 |
| P8 | 0.0000 | 0.0171 | -0.0324 | 0.0016 | 0.0018 | 0.0100 | 0.0171 | 0.0324 | 0.0016 | 0.0018 | 0.0100 |
| P9 | -0.2950 | -0.1161 | -0.3118 | -0.4315 | -0.2484 | -0.4348 | 0.1789 | 0.0168 | 0.1365 | 0.0466 | 0.1398 |
| P10 | -0.0900 | 0.0485 | -0.0385 | -0.0302 | 0.0001 | 0.0307 | 0.1385 | 0.0515 | 0.0598 | 0.0901 | 0.1207 |
| P11 | -0.0350 | -0.1025 | -0.1056 | -0.1025 | -0.0985 | -0.0908 | 0.0675 | 0.0706 | 0.0675 | 0.0635 | 0.0558 |
| P12 | -0.0610 | 0.0907 | 0.0993 | 0.0880 | 0.0692 | 0.0698 | 0.1517 | 0.1603 | 0.1490 | 0.1302 | 0.1308 |
| P13 | -0.1350 | -0.0987 | -0.0215 | -0.0987 | -0.1431 | -0.1364 | 0.0363 | 0.1135 | 0.0363 | 0.0081 | 0.0014 |
| P14 | -0.1490 | -0.1522 | -0.3702 | -0.1522 | -0.1736 | -0.1408 | 0.0032 | 0.2212 | 0.0032 | 0.0246 | 0.0082 |
| Q1 | -0.1655 | -0.1298 | -0.2012 | -0.2398 | -0.1953 | -0.2015 | 0.0357 | 0.0357 | 0.0743 | 0.0298 | 0.0360 |
| Q2 | 0.3086 | 0.2984 | 0.2807 | 0.2984 | 0.2818 | 0.2733 | 0.0102 | 0.0279 | 0.0102 | 0.0268 | 0.0353 |
| Q3 | 0.0608 | 0.0546 | -0.0192 | 0.0546 | -0.0249 | -0.0273 | 0.0062 | 0.0800 | 0.0062 | 0.0857 | 0.0881 |
| Q4 | 0.0390 | 0.1135 | 0.0638 | 0.0804 | 0.0501 | 0.0364 | 0.0745 | 0.0248 | 0.0414 | 0.0111 | 0.0026 |
| Q5 | -0.0160 | 0.0545 | 0.1156 | 0.0545 | 0.1228 | 0.0831 | 0.0705 | 0.1316 | 0.0705 | 0.1388 | 0.0991 |
| Q6 | 0.0523 | 0.1232 | -0.0384 | -0.0730 | -0.0407 | 0.0088 | 0.0709 | 0.0907 | 0.1253 | 0.0930 | 0.0435 |
| Q7 | 0.0000 | 0.0336 | -0.0176 | -0.0576 | -0.0220 | -0.0177 | 0.0336 | 0.0176 | 0.0576 | 0.0220 | 0.0177 |
| Q8 | 0.1762 | 0.1158 | 0.1227 | 0.1318 | 0.1209 | 0.1235 | 0.0604 | 0.0535 | 0.0444 | 0.0553 | 0.0527 |
| Q9 | -0.1660 | -0.1301 | -0.1528 | -0.1301 | -0.1570 | -0.1542 | 0.0359 | 0.0132 | 0.0359 | 0.0090 | 0.0118 |
| Q10 | -0.0580 | 0.0235 | -0.0335 | -0.0988 | -0.0396 | -0.0424 | 0.0815 | 0.0245 | 0.0408 | 0.0184 | 0.0156 |
| Q11 | -0.0180 | 0.0064 | -0.0837 | -0.0709 | -0.0867 | -0.0773 | 0.0244 | 0.0657 | 0.0529 | 0.0687 | 0.0593 |
| Q12 | -0.0160 | 0.0357 | -0.0452 | -0.0311 | -0.0473 | -0.0367 | 0.0517 | 0.0292 | 0.0151 | 0.0313 | 0.0207 |
| Q13 | -0.0580 | -0.0217 | -0.1016 | -0.0277 | -0.1016 | -0.0882 | 0.0363 | 0.0436 | 0.0303 | 0.0436 | 0.0302 |
| Q14 | -0.0500 | -0.0652 | -0.0367 | -0.0439 | -0.0511 | -0.0523 | 0.0152 | 0.0133 | 0.0061 | 0.0011 | 0.0023 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P1-2 | 1.5688 | 1.5835 | 1.5487 | 1.5748 | 1.5611 | 1.5619 | 0.0147 | 0.0201 | 0.0060 | 0.0077 | 0.0069 |
| P1-5 | 0.7551 | 0.7157 | 0.7148 | 0.7377 | 0.7517 | 0.7548 | 0.0394 | 0.0403 | 0.0174 | 0.0034 | 0.0003 |
| P2-3 | 0.7324 | 0.7768 | 0.8015 | 0.7804 | 0.7809 | 0.7807 | 0.0444 | 0.0691 | 0.0480 | 0.0485 | 0.0483 |
| P2-4 | 0.5613 | 0.5039 | 0.4946 | 0.5408 | 0.5787 | 0.5800 | 0.0574 | 0.0667 | 0.0205 | 0.0174 | 0.0187 |
| P2-5 | 0.4152 | 0.3618 | 0.3733 | 0.3947 | 0.4160 | 0.4184 | 0.0534 | 0.0419 | 0.0205 | 0.0008 | 0.0032 |
| P3-4 | -0.2329 | -0.2734 | -0.3763 | -0.3000 | -0.2728 | -0.2722 | 0.0405 | 0.1434 | 0.0671 | 0.0399 | 0.0393 |
| P4-5 | -0.6116 | -0.7588 | -0.5141 | -0.6115 | -0.6818 | -0.6781 | 0.1472 | 0.0975 | 0.0001 | 0.0702 | 0.0665 |
| P4-7 | 0.2807 | 0.3526 | 0.4478 | 0.3526 | 0.2684 | 0.2996 | 0.0719 | 0.1671 | 0.0719 | 0.0123 | 0.0189 |
| P4-9 | 0.1608 | 0.1771 | 0.2214 | 0.1921 | 0.1455 | 0.1699 | 0.0163 | 0.0606 | 0.0313 | 0.0153 | 0.0091 |
| P5-6 | 0.4409 | 0.4032 | 0.4275 | 0.4223 | 0.4001 | 0.4125 | 0.0377 | 0.0134 | 0.0186 | 0.0408 | 0.0284 |
| P6-11 | 0.0735 | 0.1420 | 0.1542 | 0.1376 | 0.0878 | 0.1068 | 0.0685 | 0.0807 | 0.0641 | 0.0143 | 0.0333 |
| P6-12 | 0.0779 | -0.0077 | -0.0158 | -0.0150 | 0.0007 | 0.0011 | 0.0856 | 0.0937 | 0.0929 | 0.0772 | 0.0768 |
| P6-13 | 0.1775 | 0.1724 | 0.1392 | 0.1223 | 0.1438 | 0.1436 | 0.0051 | 0.0383 | 0.0552 | 0.0337 | 0.0339 |
| P7-8 | 0.0000 | -0.0121 | 0.0324 | -0.0016 | -0.0018 | -0.0100 | 0.0121 | 0.0324 | 0.0016 | 0.0018 | 0.0100 |
| P7-9 | 0.2807 | 0.2967 | 0.2657 | 0.2967 | 0.2232 | 0.2870 | 0.0160 | 0.0150 | 0.0160 | 0.0575 | 0.0063 |
| **P9-10** | **0.0523** | **-0.0679** | **-0.0075** | **-0.0025** | **0.0117** | **-0.0452** | **0.1202** | **0.0598** | **0.0548** | **0.0406** | **0.0975** |
| P9-14 | 0.0943 | 0.0511 | 0.1828 | 0.0598 | 0.1085 | 0.0673 | 0.0432 | 0.0885 | 0.0345 | 0.0142 | 0.0270 |
| P10-11 | -0.0379 | -0.0394 | -0.0462 | -0.0331 | 0.0117 | -0.0148 | 0.0015 | 0.0083 | 0.0048 | 0.0496 | 0.0231 |
| P12-13 | 0.0161 | 0.0079 | 0.0834 | 0.0729 | 0.0697 | 0.0708 | 0.0082 | 0.0673 | 0.0568 | 0.0536 | 0.0547 |
| **P13-14** | **0.0564** | **0.9878** | **0.1979** | **0.0944** | **0.0675** | **0.0752** | **0.9314** | **0.1415** | **0.0380** | **0.0111** | **0.0188** |
| P2-1 | -1.5259 | -1.4712 | -1.5090 | -1.5327 | -1.5209 | -1.5212 | 0.0547 | 0.0169 | 0.0068 | 0.0050 | 0.0047 |
| P5-1 | -0.7275 | -0.6720 | -0.6912 | -0.7121 | -0.7257 | -0.7283 | 0.0555 | 0.0363 | 0.0154 | 0.0018 | 0.0008 |
| P3-2 | -0.7091 | -0.7523 | -0.7746 | -0.7549 | -0.7556 | -0.7551 | 0.0432 | 0.0655 | 0.0458 | 0.0465 | 0.0460 |
| P4-2 | -0.5445 | -0.5406 | -0.4822 | -0.5257 | -0.5618 | -0.5628 | 0.0039 | 0.0623 | 0.0188 | 0.0173 | 0.0183 |
| P5-2 | -0.4061 | -0.4668 | -0.3663 | -0.3867 | -0.4073 | -0.4096 | 0.0607 | 0.0398 | 0.0194 | 0.0012 | 0.0035 |
| P4-3 | 0.2366 | 0.1809 | 0.3855 | 0.3062 | 0.2776 | 0.2770 | 0.0557 | 0.1489 | 0.0696 | 0.0410 | 0.0404 |
| P5-4 | 0.6167 | 0.6164 | 0.5174 | 0.6164 | 0.6876 | 0.6839 | 0.0003 | 0.0993 | 0.0003 | 0.0709 | 0.0672 |
| P7-4 | -0.2807 | -0.2041 | -0.4478 | -0.3526 | -0.2684 | -0.2996 | 0.0766 | 0.1671 | 0.0719 | 0.0123 | 0.0189 |
| P9-4 | -0.1608 | -0.1993 | -0.2214 | -0.1921 | -0.1455 | -0.1699 | 0.0385 | 0.0606 | 0.0313 | 0.0153 | 0.0091 |
| P6-5 | -0.4409 | -0.4223 | -0.4275 | -0.4223 | -0.4001 | -0.4125 | 0.0186 | 0.0134 | 0.0186 | 0.0408 | 0.0284 |
| P11-6 | -0.0730 | -0.0843 | -0.1521 | -0.1357 | -0.0870 | -0.1056 | 0.0113 | 0.0791 | 0.0627 | 0.0140 | 0.0326 |
| P12-6 | -0.0771 | -0.0212 | 0.0159 | 0.0151 | -0.0006 | -0.0010 | 0.0559 | 0.0930 | 0.0922 | 0.0765 | 0.0761 |
| P13-6 | -0.1754 | -0.2299 | -0.1374 | -0.1213 | -0.1419 | -0.1418 | 0.0545 | 0.0380 | 0.0541 | 0.0335 | 0.0336 |
| P8-7 | 0.0000 | 0.0016 | -0.0324 | 0.0016 | 0.0018 | 0.0100 | 0.0016 | 0.0324 | 0.0016 | 0.0018 | 0.0100 |
| P9-7 | -0.2807 | -0.2531 | -0.2657 | -0.2967 | -0.2232 | -0.2870 | 0.0276 | 0.0150 | 0.0160 | 0.0575 | 0.0063 |
| P10-9 | -0.0521 | 0.0029 | 0.0077 | 0.0029 | -0.0115 | 0.0454 | 0.0550 | 0.0598 | 0.0550 | 0.0406 | 0.0975 |
| P14-9 | -0.0931 | -0.0159 | -0.1784 | -0.0592 | -0.1068 | -0.0665 | 0.0772 | 0.0853 | 0.0339 | 0.0137 | 0.0266 |
| P11-10 | 0.0380 | 0.0423 | 0.0465 | 0.0332 | -0.0115 | 0.0148 | 0.0043 | 0.0085 | 0.0048 | 0.0495 | 0.0232 |
| P13-12 | -0.0161 | -0.0907 | -0.0820 | -0.0718 | -0.0688 | -0.0698 | 0.0746 | 0.0659 | 0.0557 | 0.0527 | 0.0537 |
| P14-13 | -0.0559 | -0.0930 | -0.1918 | -0.0930 | -0.0668 | -0.0743 | 0.0371 | 0.1359 | 0.0371 | 0.0109 | 0.0184 |
| Q1-2 | -0.2040 | -0.2571 | -0.2165 | -0.2571 | -0.2190 | -0.2199 | 0.0531 | 0.0125 | 0.0531 | 0.0150 | 0.0159 |
| Q1-5 | 0.0385 | 0.1560 | 0.0153 | 0.0173 | 0.0237 | 0.0184 | 0.1175 | 0.0232 | 0.0212 | 0.0148 | 0.0201 |
| Q2-3 | 0.0356 | 0.0048 | 0.0710 | 0.0261 | 0.0608 | 0.0610 | 0.0308 | 0.0354 | 0.0095 | 0.0252 | 0.0254 |
| Q2-4 | -0.0155 | 0.0219 | -0.0075 | -0.0057 | -0.0037 | -0.0091 | 0.0374 | 0.0080 | 0.0098 | 0.0118 | 0.0064 |
| Q2-5 | 0.0117 | 0.0021 | 0.0027 | 0.0133 | 0.0067 | -0.0005 | 0.0096 | 0.0090 | 0.0016 | 0.0050 | 0.0122 |
| Q3-4 | 0.0447 | 0.0891 | 0.0350 | 0.0684 | 0.0261 | 0.0213 | 0.0444 | 0.0097 | 0.0237 | 0.0186 | 0.0234 |
| Q4-5 | 0.1582 | 0.1200 | 0.0731 | 0.1200 | 0.0931 | 0.0868 | 0.0382 | 0.0851 | 0.0382 | 0.0651 | 0.0714 |
| Q4-7 | -0.0968 | -0.1669 | -0.1045 | -0.0975 | -0.1310 | -0.1430 | 0.0701 | 0.0077 | 0.0007 | 0.0342 | 0.0462 |
| Q4-9 | -0.0043 | -0.0754 | -0.0192 | -0.0196 | -0.0333 | -0.0389 | 0.0711 | 0.0149 | 0.0153 | 0.0290 | 0.0346 |
| Q5-6 | 0.1247 | 0.1491 | -0.0731 | -0.0873 | -0.0559 | -0.1156 | 0.0244 | 0.1978 | 0.2120 | 0.1806 | 0.2403 |
| Q6-11 | 0.0356 | 0.0267 | 0.0383 | 0.0565 | 0.0484 | 0.0542 | 0.0089 | 0.0027 | 0.0209 | 0.0128 | 0.0186 |
| Q6-12 | 0.0250 | 0.0152 | 0.0343 | 0.0152 | 0.0386 | 0.0310 | 0.0098 | 0.0093 | 0.0098 | 0.0136 | 0.0060 |
| Q6-13 | 0.0722 | 0.1432 | 0.1057 | 0.0521 | 0.1084 | 0.1004 | 0.0710 | 0.0335 | 0.0201 | 0.0362 | 0.0282 |
| Q7-8 | -0.1716 | -0.1570 | -0.1203 | -0.1292 | -0.1187 | -0.1212 | 0.0146 | 0.0513 | 0.0424 | 0.0529 | 0.0504 |
| Q7-9 | 0.0578 | 0.0677 | 0.0811 | 0.0677 | 0.0716 | 0.0616 | 0.0099 | 0.0233 | 0.0099 | 0.0138 | 0.0038 |
| Q9-10 | 0.0422 | 0.1216 | 0.0847 | 0.1185 | 0.0804 | 0.0686 | 0.0794 | 0.0425 | 0.0763 | 0.0382 | 0.0264 |
| Q9-14 | 0.0361 | -0.0041 | 0.0703 | 0.0428 | 0.0632 | 0.0540 | 0.0402 | 0.0342 | 0.0067 | 0.0271 | 0.0179 |
| Q10-11 | -0.0162 | 0.0186 | 0.0506 | 0.0186 | 0.0404 | 0.0257 | 0.0348 | 0.0668 | 0.0348 | 0.0566 | 0.0419 |
| Q12-13 | 0.0075 | 0.0493 | -0.0112 | -0.0160 | -0.0090 | -0.0059 | 0.0418 | 0.0187 | 0.0235 | 0.0165 | 0.0134 |
| Q13-14 | 0.0175 | 0.0053 | -0.0119 | 0.0053 | -0.0068 | 0.0019 | 0.0122 | 0.0294 | 0.0122 | 0.0243 | 0.0156 |
| Q2-1 | 0.2768 | 0.2876 | 0.2144 | 0.2647 | 0.2180 | 0.2220 | 0.0108 | 0.0624 | 0.0121 | 0.0588 | 0.0548 |
| Q5-1 | 0.0223 | -0.0360 | -0.0302 | -0.0214 | -0.0286 | -0.0201 | 0.0583 | 0.0525 | 0.0437 | 0.0509 | 0.0424 |
| Q3-2 | 0.0160 | -0.0414 | -0.0542 | -0.0138 | -0.0510 | -0.0487 | 0.0574 | 0.0702 | 0.0298 | 0.0670 | 0.0647 |
| Q4-2 | 0.0302 | 0.0354 | -0.0309 | -0.0231 | -0.0210 | -0.0139 | 0.0052 | 0.0611 | 0.0533 | 0.0512 | 0.0441 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q5-2 | -0.0210 | 0.0151 | -0.0591 | -0.0651 | -0.0581 | -0.0495 | 0.0361 | 0.0381 | 0.0441 | 0.0371 | 0.0285 |
| Q4-3 | -0.0484 | 0.0809 | -0.0390 | -0.0797 | -0.0412 | -0.0361 | 0.1293 | 0.0094 | 0.0313 | 0.0072 | 0.0123 |
| Q5-4 | 0.1420 | 0.1087 | -0.0626 | -0.1046 | -0.0747 | -0.0684 | 0.0333 | 0.2046 | 0.2466 | 0.2167 | 0.2104 |
| Q7-4 | 0.1138 | 0.1232 | 0.1443 | 0.1232 | 0.1478 | 0.1640 | 0.0094 | 0.0305 | 0.0094 | 0.0340 | 0.0502 |
| Q9-4 | 0.0173 | 0.0132 | 0.0437 | 0.0384 | 0.0444 | 0.0542 | 0.0041 | 0.0264 | 0.0211 | 0.0271 | 0.0369 |
| Q6-5 | -0.0805 | -0.1772 | 0.1134 | 0.1280 | 0.0909 | 0.1554 | 0.0967 | 0.1939 | 0.2085 | 0.1714 | 0.2359 |
| Q11-6 | -0.0344 | -0.0563 | -0.0339 | -0.0526 | -0.0466 | -0.0517 | 0.0219 | 0.0005 | 0.0182 | 0.0122 | 0.0173 |
| Q12-6 | -0.0235 | -0.1132 | -0.0340 | -0.0151 | -0.0383 | -0.0308 | 0.0897 | 0.0105 | 0.0084 | 0.0148 | 0.0073 |
| Q13-6 | -0.0680 | -0.0260 | -0.1022 | -0.0500 | -0.1047 | -0.0969 | 0.0420 | 0.0342 | 0.0180 | 0.0367 | 0.0289 |
| Q8-7 | 0.1762 | 0.1318 | 0.1227 | 0.1318 | 0.1209 | 0.1235 | 0.0444 | 0.0535 | 0.0444 | 0.0553 | 0.0527 |
| Q9-7 | -0.0498 | -0.0448 | -0.0736 | -0.0585 | -0.0663 | -0.0532 | 0.0050 | 0.0238 | 0.0087 | 0.0165 | 0.0034 |
| Q10-9 | -0.0418 | -0.0690 | -0.0841 | -0.1174 | -0.0799 | -0.0681 | 0.0272 | 0.0423 | 0.0756 | 0.0381 | 0.0263 |
| Q14-9 | -0.0336 | -0.0184 | -0.0610 | -0.0414 | -0.0594 | -0.0522 | 0.0152 | 0.0274 | 0.0078 | 0.0258 | 0.0186 |
| Q11-10 | 0.0164 | -0.0136 | -0.0498 | -0.0183 | -0.0401 | -0.0256 | 0.0300 | 0.0662 | 0.0347 | 0.0565 | 0.0420 |
| Q13-12 | -0.0075 | 0.0170 | 0.0125 | 0.0170 | 0.0099 | 0.0068 | 0.0245 | 0.0200 | 0.0245 | 0.0174 | 0.0143 |
| Q14-13 | -0.0164 | 0.0206 | 0.0244 | -0.0025 | 0.0083 | -0.0001 | 0.0370 | 0.0408 | 0.0139 | 0.0247 | 0.0163 |
| **SUM** | | | | | | | **4.8458** | **4.2585** | **2.5858** | **2.2761** | **2.1360** |

### Median redundancy

The same bad data as in full redundancy case, the following two tables shows the results and error with actual value of the system.

Table 5.226: State Variable of case 2, two reverse, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0861 | -0.0858 | -0.0854 | -0.0853 | **V1** | 1.0545 | 1.0686 | 1.0588 | 1.0720 |
| $\theta 3$ | -0.2344 | -0.2273 | -0.2313 | -0.2297 | **V2** | 1.0377 | 1.0510 | 1.0422 | 1.0553 |
| $\theta 4$ | -0.1631 | -0.1709 | -0.1624 | -0.1783 | **V3** | 0.9950 | 1.0103 | 1.0007 | 1.0160 |
| $\theta 5$ | -0.1456 | -0.1460 | -0.1467 | -0.1486 | **V4** | 1.0050 | 1.0171 | 1.0099 | 1.0185 |
| $\theta 6$ | -0.2559 | -0.2554 | -0.2776 | -0.2454 | **V5** | 1.0057 | 1.0175 | 1.0085 | 1.0181 |
| $\theta 7$ | -0.2608 | -0.2391 | -0.2553 | -0.2418 | **V6** | 1.0211 | 1.0128 | 1.0161 | 0.9972 |
| $\theta 8$ | -0.2677 | -0.2372 | -0.2576 | -0.2408 | **V7** | 1.0215 | 1.0402 | 1.0307 | 1.0471 |
| $\theta 9$ | -0.2943 | -0.2697 | -0.2950 | -0.2720 | **V8** | 1.0424 | 1.0594 | 1.0528 | 1.0693 |
| $\theta 10$ | -0.2913 | -0.2653 | -0.2890 | -0.2665 | **V9** | 1.0076 | 1.0265 | 1.0166 | 1.0351 |
| $\theta 11$ | -0.2832 | -0.2706 | -0.2874 | -0.2636 | **V10** | 1.0013 | 1.0178 | 1.0128 | 1.0296 |
| $\theta 12$ | -0.2462 | -0.2559 | -0.2777 | -0.2485 | **V11** | 0.9976 | 1.0113 | 1.0051 | 1.0133 |
| $\theta 13$ | -0.2684 | -0.2807 | -0.3072 | -0.2721 | **V12** | 1.0145 | 1.0079 | 1.0021 | 1.0030 |
| $\theta 14$ | -0.3636 | -0.3609 | -0.4364 | -0.3606 | **V13** | 0.9906 | 0.9906 | 0.9737 | 0.9813 |
| | | | | | **V14** | 0.9366 | 0.9526 | 0.9079 | 0.9528 |

Table 5.227: Estimation Results and Errors of case 2, two reverse, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0545 | 1.0686 | 1.0588 | 1.0720 | 0.0055 | 0.0086 | 0.0012 | 0.0120 |
| V2 | 1.0450 | N/A | 1.0377 | 1.0510 | 1.0422 | 1.0553 | 0.0073 | 0.0060 | 0.0028 | 0.0103 |
| V3 | 1.0100 | N/A | 0.9950 | 1.0103 | 1.0007 | 1.0160 | 0.0150 | 0.0003 | 0.0093 | 0.0060 |
| V4 | 1.0180 | N/A | 1.0050 | 1.0171 | 1.0099 | 1.0185 | 0.0130 | 0.0009 | 0.0081 | 0.0005 |
| V5 | 1.0200 | N/A | 1.0057 | 1.0175 | 1.0085 | 1.0181 | 0.0143 | 0.0025 | 0.0115 | 0.0019 |
| V6 | 1.0700 | N/A | 1.0211 | 1.0128 | 1.0161 | 0.9972 | 0.0489 | 0.0572 | 0.0539 | 0.0728 |
| V7 | 1.0620 | N/A | 1.0215 | 1.0402 | 1.0307 | 1.0471 | 0.0405 | 0.0218 | 0.0313 | 0.0149 |
| V8 | 1.0900 | N/A | 1.0424 | 1.0594 | 1.0528 | 1.0693 | 0.0476 | 0.0306 | 0.0372 | 0.0207 |
| V9 | 1.0560 | N/A | 1.0076 | 1.0265 | 1.0166 | 1.0351 | 0.0484 | 0.0295 | 0.0394 | 0.0209 |
| V10 | 1.0510 | N/A | 1.0013 | 1.0178 | 1.0128 | 1.0296 | 0.0497 | 0.0332 | 0.0382 | 0.0214 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V11 | 1.0570 | N/A | 0.9976 | 1.0113 | 1.0051 | 1.0133 | 0.0594 | 0.0457 | 0.0519 | 0.0437 |
| V12 | 1.0550 | N/A | 1.0145 | 1.0079 | 1.0021 | 1.0030 | 0.0405 | 0.0471 | 0.0529 | 0.0520 |
| V13 | 1.0500 | N/A | 0.9906 | 0.9906 | 0.9737 | 0.9813 | 0.0594 | 0.0594 | 0.0763 | 0.0687 |
| V14 | 1.0360 | N/A | 0.9366 | 0.9526 | 0.9079 | 0.9528 | 0.0994 | 0.0834 | 0.1281 | 0.0832 |
| P1 | 2.3239 | 2.3508 | 2.2610 | 2.3213 | 2.2712 | 2.3372 | 0.0629 | 0.0026 | 0.0527 | 0.0133 |
| P2 | 0.1830 | 0.2747 | 0.1444 | 0.1651 | 0.1585 | 0.2659 | 0.0386 | 0.0179 | 0.0245 | 0.0829 |
| P3 | -0.9420 | -1.0549 | -1.1405 | -1.0549 | -1.1260 | -1.0423 | 0.1985 | 0.1129 | 0.1840 | 0.1003 |
| **P4** | **-0.4780** | **0.4349** | **0.2734** | **-0.2188** | **0.3014** | **-0.4343** | **0.7514** | **0.2592** | **0.7794** | **0.0437** |
| P5 | -0.0760 | -0.0601 | -0.1940 | -0.0601 | -0.1842 | -0.0585 | 0.1180 | 0.0159 | 0.1082 | 0.0175 |
| P6 | -0.1120 | -0.1774 | -0.1679 | -0.1774 | -0.1710 | -0.1624 | 0.0559 | 0.0654 | 0.0590 | 0.0504 |
| P7 | 0.0000 | -0.0217 | -0.1345 | -0.0680 | -0.0796 | -0.0387 | 0.1345 | 0.0680 | 0.0796 | 0.0387 |
| P8 | 0.0000 | 0.0171 | -0.0417 | 0.0121 | -0.0143 | 0.0061 | 0.0417 | 0.0121 | 0.0143 | 0.0061 |
| P9 | -0.2950 | -0.1161 | -0.2619 | -0.1161 | -0.1154 | -0.1281 | 0.0331 | 0.1789 | 0.1796 | 0.1669 |
| P10 | -0.0900 | 0.0485 | -0.0220 | 0.0485 | 0.0578 | 0.0565 | 0.0680 | 0.1385 | 0.1478 | 0.1465 |
| P11 | -0.0350 | -0.1025 | -0.1303 | -0.1025 | -0.0705 | -0.0606 | 0.0953 | 0.0675 | 0.0355 | 0.0256 |
| P12 | -0.0610 | 0.0907 | 0.1331 | 0.0907 | 0.1149 | 0.1061 | 0.1941 | 0.1517 | 0.1759 | 0.1671 |
| P13 | -0.1350 | -0.0987 | -0.0090 | -0.0987 | -0.0927 | -0.0841 | 0.1260 | 0.0363 | 0.0423 | 0.0509 |
| P14 | -0.1490 | -0.1522 | -0.5466 | -0.5768 | -0.8446 | -0.5886 | 0.3976 | 0.4278 | 0.6956 | 0.4396 |
| Q1 | -0.1655 | -0.1298 | -0.1440 | -0.1298 | -0.1424 | -0.1341 | 0.0215 | 0.0357 | 0.0231 | 0.0314 |
| Q2 | 0.3086 | 0.2984 | 0.3122 | 0.2984 | 0.3131 | 0.3225 | 0.0036 | 0.0102 | 0.0045 | 0.0139 |
| Q3 | 0.0608 | 0.0546 | 0.0720 | 0.0546 | 0.0734 | 0.0787 | 0.0112 | 0.0062 | 0.0126 | 0.0179 |
| Q4 | 0.0390 | 0.1135 | 0.0914 | 0.1135 | 0.0969 | 0.1152 | 0.0524 | 0.0745 | 0.0579 | 0.0762 |
| Q5 | -0.0160 | 0.0545 | 0.0340 | 0.0545 | 0.0351 | 0.0522 | 0.0500 | 0.0705 | 0.0511 | 0.0682 |
| Q6 | 0.0523 | 0.1232 | 0.0245 | -0.2385 | 0.0171 | -0.4638 | 0.0278 | 0.2908 | 0.0352 | 0.5161 |
| Q7 | 0.0000 | 0.0336 | 0.0068 | 0.0336 | 0.0229 | 0.0273 | 0.0068 | 0.0336 | 0.0229 | 0.0273 |
| Q8 | 0.1762 | 0.1158 | 0.1242 | 0.1158 | 0.1321 | 0.1342 | 0.0520 | 0.0604 | 0.0441 | 0.0420 |
| Q9 | -0.1660 | -0.1301 | -0.1443 | -0.1301 | -0.1068 | -0.1064 | 0.0217 | 0.0359 | 0.0592 | 0.0596 |
| Q10 | -0.0580 | 0.0235 | -0.0450 | -0.0914 | -0.0269 | -0.0015 | 0.0130 | 0.0334 | 0.0311 | 0.0565 |
| Q11 | -0.0180 | 0.0064 | -0.0715 | 0.0064 | -0.0626 | 0.0249 | 0.0535 | 0.0244 | 0.0446 | 0.0429 |
| Q12 | -0.0160 | 0.0357 | -0.0367 | -0.0369 | -0.0510 | 0.0151 | 0.0207 | 0.0209 | 0.0350 | 0.0311 |
| Q13 | -0.0580 | -0.0217 | -0.1079 | -0.0217 | -0.1117 | -0.0214 | 0.0499 | 0.0363 | 0.0537 | 0.0366 |
| Q14 | -0.0500 | -0.0652 | -0.1087 | -0.0652 | -0.0778 | -0.0621 | 0.0587 | 0.0152 | 0.0278 | 0.0121 |
| P1-2 | 1.5688 | 1.5835 | 1.5451 | 1.5835 | 1.5440 | 1.5813 | 0.0237 | 0.0147 | 0.0248 | 0.0125 |
| P1-5 | 0.7551 | 0.7157 | 0.7159 | 0.7378 | 0.7273 | 0.7558 | 0.0392 | 0.0173 | 0.0278 | 0.0007 |
| P2-3 | 0.7324 | 0.7768 | 0.7927 | 0.7768 | 0.7869 | 0.7975 | 0.0603 | 0.0444 | 0.0545 | 0.0651 |
| P2-4 | 0.5613 | N/A | 0.4731 | 0.5315 | 0.4763 | 0.5839 | 0.0882 | 0.0298 | 0.0850 | 0.0226 |
| P2-5 | 0.4152 | N/A | 0.3819 | 0.3974 | 0.3976 | 0.4232 | 0.0333 | 0.0178 | 0.0176 | 0.0080 |
| P3-4 | -0.2329 | -0.2734 | -0.3756 | -0.3041 | -0.3662 | -0.2719 | 0.1427 | 0.0712 | 0.1333 | 0.0390 |
| P4-5 | -0.6116 | N/A | -0.3867 | -0.5566 | -0.3346 | -0.6594 | 0.2249 | 0.0550 | 0.2770 | 0.0478 |
| P4-7 | 0.2807 | 0.3526 | 0.4894 | 0.3526 | 0.4722 | 0.3307 | 0.2087 | 0.0719 | 0.1915 | 0.0500 |
| P4-9 | 0.1608 | N/A | 0.2457 | 0.1911 | 0.2519 | 0.1830 | 0.0849 | 0.0303 | 0.0911 | 0.0222 |
| P5-6 | 0.4409 | 0.4032 | 0.4814 | 0.4792 | 0.5695 | 0.4176 | 0.0405 | 0.0383 | 0.1286 | 0.0233 |
| P6-11 | 0.0735 | N/A | 0.1615 | 0.0669 | 0.0631 | 0.0443 | 0.0880 | 0.0066 | 0.0104 | 0.0292 |
| P6-12 | 0.0779 | -0.0077 | -0.0217 | 0.0092 | 0.0220 | 0.0011 | 0.0996 | 0.0687 | 0.0559 | 0.0768 |
| P6-13 | 0.1775 | N/A | 0.1737 | 0.2257 | 0.3134 | 0.2098 | 0.0038 | 0.0482 | 0.1359 | 0.0323 |
| P7-8 | 0.0000 | -0.0121 | 0.0417 | -0.0121 | 0.0143 | -0.0061 | 0.0417 | 0.0121 | 0.0143 | 0.0061 |
| P7-9 | 0.2807 | 0.2967 | 0.3132 | 0.2967 | 0.3783 | 0.2980 | 0.0325 | 0.0160 | 0.0976 | 0.0173 |
| **P9-10** | **0.0523** | **-0.0679** | **-0.0063** | **-0.0119** | **-0.0497** | **-0.0382** | **0.0586** | **0.0642** | **0.1020** | **0.0905** |
| P9-14 | 0.0943 | N/A | 0.3032 | 0.3836 | 0.5644 | 0.3911 | 0.2089 | 0.2893 | 0.4701 | 0.2968 |
| P10-11 | -0.0379 | -0.0394 | -0.0285 | 0.0362 | 0.0079 | 0.0180 | 0.0094 | 0.0741 | 0.0458 | 0.0559 |
| P12-13 | 0.0161 | N/A | 0.1112 | 0.0999 | 0.1366 | 0.1071 | 0.0951 | 0.0838 | 0.1205 | 0.0910 |
| **P13-14** | **0.0564** | **0.9878** | **0.2699** | **0.2211** | **0.3449** | **0.2273** | **0.2135** | **0.1647** | **0.2885** | **0.1709** |
| P2-1 | -1.5259 | N/A | -1.5032 | -1.5407 | -1.5024 | -1.5388 | 0.0227 | 0.0148 | 0.0235 | 0.0129 |
| P5-1 | -0.7275 | N/A | -0.6904 | -0.7114 | -0.7012 | -0.7282 | 0.0371 | 0.0161 | 0.0263 | 0.0007 |
| P3-2 | -0.7091 | N/A | -0.7649 | -0.7508 | -0.7598 | -0.7704 | 0.0558 | 0.0417 | 0.0507 | 0.0613 |
| P4-2 | -0.5445 | N/A | -0.4608 | -0.5165 | -0.4641 | -0.5660 | 0.0837 | 0.0280 | 0.0804 | 0.0215 |
| P5-2 | -0.4061 | N/A | -0.3738 | -0.3889 | -0.3890 | -0.4136 | 0.0323 | 0.0172 | 0.0171 | 0.0075 |
| P4-3 | 0.2366 | N/A | 0.3859 | 0.3107 | 0.3759 | 0.2773 | 0.1493 | 0.0741 | 0.1393 | 0.0407 |
| P5-4 | 0.6167 | N/A | 0.3888 | 0.5610 | 0.3364 | 0.6656 | 0.2279 | 0.0557 | 0.2803 | 0.0489 |
| P7-4 | -0.2807 | N/A | -0.4894 | -0.3526 | -0.4722 | -0.3307 | 0.2087 | 0.0719 | 0.1915 | 0.0500 |
| P9-4 | -0.1608 | N/A | -0.2457 | -0.1911 | -0.2519 | -0.1830 | 0.0849 | 0.0303 | 0.0911 | 0.0222 |
| P6-5 | -0.4409 | N/A | -0.4814 | -0.4792 | -0.5695 | -0.4176 | 0.0405 | 0.0383 | 0.1286 | 0.0233 |
| P11-6 | -0.0730 | N/A | -0.1589 | -0.0664 | -0.0627 | -0.0431 | 0.0859 | 0.0066 | 0.0103 | 0.0299 |
| P12-6 | -0.0771 | N/A | 0.0219 | -0.0092 | -0.0217 | -0.0010 | 0.0990 | 0.0679 | 0.0554 | 0.0761 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P13-6 | -0.1754 | N/A | -0.1703 | -0.2222 | -0.3052 | -0.2068 | 0.0051 | 0.0468 | 0.1298 | 0.0314 |
| P8-7 | 0.0000 | N/A | -0.0417 | 0.0121 | -0.0143 | 0.0061 | 0.0417 | 0.0121 | 0.0143 | 0.0061 |
| P9-7 | -0.2807 | N/A | -0.3132 | -0.2967 | -0.3783 | -0.2980 | 0.0325 | 0.0160 | 0.0976 | 0.0173 |
| P10-9 | -0.0521 | N/A | 0.0065 | 0.0123 | 0.0499 | 0.0385 | 0.0586 | 0.0644 | 0.1020 | 0.0906 |
| P14-9 | -0.0931 | N/A | -0.2896 | -0.3642 | -0.5213 | -0.3705 | 0.1965 | 0.2711 | 0.4282 | 0.2774 |
| P11-10 | 0.0380 | N/A | 0.0286 | -0.0361 | -0.0078 | -0.0175 | 0.0094 | 0.0741 | 0.0458 | 0.0555 |
| P13-12 | -0.0161 | N/A | -0.1085 | -0.0976 | -0.1325 | -0.1046 | 0.0924 | 0.0815 | 0.1164 | 0.0885 |
| P14-13 | -0.0559 | N/A | -0.2570 | -0.2126 | -0.3232 | -0.2181 | 0.2011 | 0.1567 | 0.2673 | 0.1622 |
| Q1-2 | -0.2040 | -0.2571 | -0.1969 | -0.1917 | -0.2012 | -0.2071 | 0.0071 | 0.0123 | 0.0028 | 0.0031 |
| Q1-5 | 0.0385 | 0.1560 | 0.0529 | 0.0619 | 0.0588 | 0.0730 | 0.0144 | 0.0234 | 0.0203 | 0.0345 |
| Q2-3 | 0.0356 | 0.0048 | 0.0459 | 0.0373 | 0.0404 | 0.0278 | 0.0103 | 0.0017 | 0.0048 | 0.0078 |
| Q2-4 | -0.0155 | N/A | 0.0177 | 0.0117 | 0.0151 | 0.0163 | 0.0332 | 0.0272 | 0.0306 | 0.0318 |
| Q2-5 | 0.0117 | N/A | 0.0392 | 0.0455 | 0.0459 | 0.0608 | 0.0275 | 0.0338 | 0.0342 | 0.0491 |
| Q3-4 | 0.0447 | 0.0891 | 0.0913 | 0.0755 | 0.0910 | 0.0864 | 0.0466 | 0.0308 | 0.0463 | 0.0417 |
| Q4-5 | 0.1582 | N/A | 0.1085 | 0.1738 | 0.1425 | 0.2286 | 0.0497 | 0.0156 | 0.0157 | 0.0704 |
| Q4-7 | -0.0968 | -0.1669 | -0.0572 | -0.1031 | -0.0809 | -0.1322 | 0.0396 | 0.0063 | 0.0159 | 0.0354 |
| Q4-9 | -0.0043 | N/A | 0.0112 | -0.0084 | 0.0041 | -0.0228 | 0.0155 | 0.0041 | 0.0084 | 0.0185 |
| Q5-6 | 0.1247 | 0.1491 | -0.0392 | 0.0464 | 0.0047 | 0.1111 | 0.1639 | 0.0783 | 0.1200 | 0.0136 |
| Q6-11 | 0.0356 | N/A | 0.0454 | -0.0237 | 0.0264 | -0.1010 | 0.0098 | 0.0593 | 0.0092 | 0.1366 |
| Q6-12 | 0.0250 | 0.0152 | 0.0367 | 0.0152 | 0.0449 | -0.0233 | 0.0117 | 0.0098 | 0.0199 | 0.0483 |
| Q6-13 | 0.0722 | N/A | 0.1508 | 0.0608 | 0.1744 | 0.0172 | 0.0786 | 0.0114 | 0.1022 | 0.0550 |
| Q7-8 | -0.1716 | -0.1570 | -0.1214 | -0.1137 | -0.1293 | -0.1315 | 0.0502 | 0.0579 | 0.0423 | 0.0401 |
| Q7-9 | 0.0578 | 0.0677 | 0.1341 | 0.1339 | 0.1395 | 0.1194 | 0.0763 | 0.0761 | 0.0817 | 0.0616 |
| Q9-10 | 0.0422 | 0.1216 | 0.0773 | 0.1110 | 0.0648 | 0.0819 | 0.0351 | 0.0688 | 0.0226 | 0.0397 |
| Q9-14 | 0.0361 | N/A | 0.1305 | 0.1154 | 0.1775 | 0.1455 | 0.0944 | 0.0793 | 0.1414 | 0.1094 |
| Q10-11 | -0.0162 | 0.0186 | 0.0318 | 0.0186 | 0.0374 | 0.0798 | 0.0480 | 0.0348 | 0.0536 | 0.0960 |
| Q12-13 | 0.0075 | N/A | -0.0005 | -0.0218 | -0.0067 | -0.0083 | 0.0080 | 0.0293 | 0.0142 | 0.0158 |
| Q13-14 | 0.0175 | 0.0053 | 0.0335 | 0.0083 | 0.0360 | -0.0206 | 0.0160 | 0.0092 | 0.0185 | 0.0381 |
| Q2-1 | 0.2768 | N/A | 0.2094 | 0.2039 | 0.2116 | 0.2175 | 0.0674 | 0.0729 | 0.0652 | 0.0593 |
| Q5-1 | 0.0223 | N/A | -0.0522 | -0.0600 | -0.0562 | -0.0664 | 0.0745 | 0.0823 | 0.0785 | 0.0887 |
| Q3-2 | 0.0160 | N/A | -0.0193 | -0.0209 | -0.0176 | -0.0077 | 0.0353 | 0.0369 | 0.0336 | 0.0237 |
| Q4-2 | 0.0302 | N/A | -0.0515 | -0.0390 | -0.0495 | -0.0350 | 0.0817 | 0.0692 | 0.0797 | 0.0652 |
| Q5-2 | -0.0210 | N/A | -0.0870 | -0.0936 | -0.0923 | -0.1057 | 0.0660 | 0.0726 | 0.0713 | 0.0847 |
| Q4-3 | -0.0484 | N/A | -0.0906 | -0.0850 | -0.0921 | -0.0990 | 0.0422 | 0.0366 | 0.0437 | 0.0506 |
| Q5-4 | 0.1420 | N/A | -0.1018 | -0.1599 | -0.1371 | -0.2088 | 0.2438 | 0.3019 | 0.2791 | 0.3508 |
| Q7-4 | 0.1138 | N/A | 0.1063 | 0.1297 | 0.1269 | 0.1573 | 0.0075 | 0.0159 | 0.0131 | 0.0435 |
| Q9-4 | 0.0173 | N/A | 0.0210 | 0.0275 | 0.0294 | 0.0404 | 0.0037 | 0.0102 | 0.0121 | 0.0231 |
| Q6-5 | -0.0805 | N/A | 0.0933 | 0.0062 | 0.0703 | -0.0688 | 0.1738 | 0.0867 | 0.1508 | 0.0117 |
| Q11-6 | -0.0344 | N/A | -0.0400 | 0.0247 | -0.0255 | 0.1035 | 0.0056 | 0.0591 | 0.0089 | 0.1379 |
| Q12-6 | -0.0235 | N/A | -0.0363 | -0.0151 | -0.0443 | 0.0234 | 0.0128 | 0.0084 | 0.0208 | 0.0469 |
| Q13-6 | -0.0680 | N/A | -0.1442 | -0.0538 | -0.1582 | -0.0114 | 0.0762 | 0.0142 | 0.0902 | 0.0566 |
| Q8-7 | 0.1762 | N/A | 0.1242 | 0.1158 | 0.1321 | 0.1342 | 0.0520 | 0.0604 | 0.0441 | 0.0420 |
| Q9-7 | -0.0498 | N/A | -0.1219 | -0.1232 | -0.1227 | -0.1091 | 0.0721 | 0.0734 | 0.0729 | 0.0593 |
| Q10-9 | -0.0418 | N/A | -0.0768 | -0.1100 | -0.0642 | -0.0813 | 0.0350 | 0.0682 | 0.0224 | 0.0395 |
| Q14-9 | -0.0336 | N/A | -0.1015 | -0.0742 | -0.0859 | -0.1015 | 0.0679 | 0.0406 | 0.0523 | 0.0679 |
| Q11-10 | 0.0164 | N/A | -0.0314 | -0.0183 | -0.0371 | -0.0786 | 0.0478 | 0.0347 | 0.0535 | 0.0950 |
| Q13-12 | -0.0075 | N/A | 0.0029 | 0.0238 | 0.0104 | 0.0106 | 0.0104 | 0.0313 | 0.0179 | 0.0181 |
| Q14-13 | -0.0164 | N/A | -0.0072 | 0.0090 | 0.0081 | 0.0394 | 0.0092 | 0.0254 | 0.0245 | 0.0558 |
| **SUM** | | | | | | | **5.4858** | **4.1392** | **6.3330** | **4.1127** |

LTS performs relative bad because it does not trim bus 4 power injection and leads to bad estimation in all estimation relate to bus 4.

**Comparison**

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.228.Full redundancy case has better estimation results for all estimators.

Table 5.228: Estimation errors of case 2, two reverse, 14-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **4.8458** | **4.2585** | **2.5858** | **2.2761** | **2.1360** |
| **SUM** | **N/A** | **5.4858** | **4.1392** | **6.3330** | **4.1127** |

### Two large one reverse bad data

Compared with the former case, another reverse bad data are changed to large error, $P_4 = 0.4349$, $P_{9-10} = 0.9679$ and $P_{13-14} = 0.9878$, others keep the same.

#### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.229 gives the state variables, and the other table shows the estimation results and error with actual values.

Table 5.229: State variable of case 2, two large, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0811 | -0.0843 | -0.0815 | -0.0826 | **V1** | 1.0887 | 1.0768 | 1.0903 | 1.0834 |
| $\theta 3$ | -0.2203 | -0.2235 | -0.2169 | -0.2199 | **V2** | 1.0731 | 1.0632 | 1.0746 | 1.0677 |
| $\theta 4$ | -0.1578 | -0.1698 | -0.1710 | -0.1737 | **V3** | 1.0256 | 1.0244 | 1.0293 | 1.0224 |
| $\theta 5$ | -0.1377 | -0.1442 | -0.1438 | -0.1462 | **V4** | 1.0441 | 1.0318 | 1.0416 | 1.0355 |
| $\theta 6$ | -0.2382 | -0.2348 | -0.2328 | -0.2335 | **V5** | 1.0475 | 1.0352 | 1.0468 | 1.0411 |
| $\theta 7$ | -0.2383 | -0.2362 | -0.2221 | -0.2290 | **V6** | 1.0658 | 1.0590 | 1.0630 | 1.0711 |
| $\theta 8$ | -0.2409 | -0.2360 | -0.2206 | -0.2275 | **V7** | 1.0698 | 1.0534 | 1.0688 | 1.0655 |
| $\theta 9$ | -0.2693 | -0.2658 | -0.2480 | -0.2569 | **V8** | 1.0896 | 1.0750 | 1.0885 | 1.0855 |
| $\theta 10$ | -0.2864 | -0.2633 | -0.2614 | -0.2523 | **V9** | 1.0622 | 1.0468 | 1.0617 | 1.0596 |
| $\theta 11$ | -0.2748 | -0.2555 | -0.2515 | -0.2481 | **V10** | 1.0478 | 1.0370 | 1.0491 | 1.0552 |
| $\theta 12$ | -0.2298 | -0.2293 | -0.2279 | -0.2304 | **V11** | 1.0407 | 1.0364 | 1.0439 | 1.0516 |
| $\theta 13$ | -0.2458 | -0.2459 | -0.2421 | -0.2442 | **V12** | 1.0594 | 1.0573 | 1.0538 | 1.0636 |
| $\theta 14$ | -0.3013 | -0.2756 | -0.2615 | -0.2675 | **V13** | 1.0456 | 1.0450 | 1.0414 | 1.0501 |
| | | | | | **V14** | 1.0254 | 1.0283 | 1.0353 | 1.0377 |

Table 5.230: Estimation Results and Errors of case 2, two large, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0887 | 1.0768 | 1.0903 | 1.0834 | 0.0086 | 0.0287 | 0.0168 | 0.0303 | 0.0234 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V2 | 1.0450 | 1.0743 | 1.0731 | 1.0632 | 1.0746 | 1.0677 | 0.0293 | 0.0281 | 0.0182 | 0.0296 | 0.0227 |
| V3 | 1.0100 | 0.9739 | 1.0256 | 1.0244 | 1.0293 | 1.0224 | 0.0361 | 0.0156 | 0.0144 | 0.0193 | 0.0124 |
| V4 | 1.0180 | 1.0318 | 1.0441 | 1.0318 | 1.0416 | 1.0355 | 0.0138 | 0.0261 | 0.0138 | 0.0236 | 0.0175 |
| V5 | 1.0200 | 1.0251 | 1.0475 | 1.0352 | 1.0468 | 1.0411 | 0.0051 | 0.0275 | 0.0152 | 0.0268 | 0.0211 |
| V6 | 1.0700 | 1.0491 | 1.0658 | 1.0590 | 1.0630 | 1.0711 | 0.0209 | 0.0042 | 0.0110 | 0.0070 | 0.0011 |
| V7 | 1.0620 | 1.0551 | 1.0698 | 1.0534 | 1.0688 | 1.0655 | 0.0069 | 0.0078 | 0.0086 | 0.0068 | 0.0035 |
| V8 | 1.0900 | 1.0955 | 1.0896 | 1.0750 | 1.0885 | 1.0855 | 0.0055 | 0.0004 | 0.0150 | 0.0015 | 0.0045 |
| V9 | 1.0560 | 1.1131 | 1.0622 | 1.0468 | 1.0617 | 1.0596 | 0.0571 | 0.0062 | 0.0092 | 0.0057 | 0.0036 |
| V10 | 1.0510 | 1.0952 | 1.0478 | 1.0370 | 1.0491 | 1.0552 | 0.0442 | 0.0032 | 0.0140 | 0.0019 | 0.0042 |
| V11 | 1.0570 | 1.0355 | 1.0407 | 1.0364 | 1.0439 | 1.0516 | 0.0215 | 0.0163 | 0.0206 | 0.0131 | 0.0054 |
| V12 | 1.0550 | 1.1034 | 1.0594 | 1.0573 | 1.0538 | 1.0636 | 0.0484 | 0.0044 | 0.0023 | 0.0012 | 0.0086 |
| V13 | 1.0500 | 1.0616 | 1.0456 | 1.0450 | 1.0414 | 1.0501 | 0.0116 | 0.0044 | 0.0050 | 0.0086 | 0.0001 |
| V14 | 1.0360 | 1.0350 | 1.0254 | 1.0283 | 1.0353 | 1.0377 | 0.0010 | 0.0106 | 0.0077 | 0.0007 | 0.0017 |
| P1 | 2.3239 | 2.3508 | 2.2693 | 2.3017 | 2.3147 | 2.3168 | 0.0269 | 0.0546 | 0.0222 | 0.0092 | 0.0071 |
| P2 | 0.1830 | 0.2747 | 0.1673 | 0.1956 | 0.2565 | 0.2581 | 0.0917 | 0.0157 | 0.0126 | 0.0735 | 0.0751 |
| P3 | -0.9420 | -1.0549 | -1.1463 | -1.0549 | -1.0280 | -1.0273 | 0.1129 | 0.2043 | 0.1129 | 0.0860 | 0.0853 |
| **P4** | **-0.4780** | **0.4349** | **0.0697** | **-0.2883** | **-0.5210** | **-0.4957** | **0.9129** | **0.5477** | **0.1897** | **0.0430** | **0.0177** |
| P5 | -0.0760 | -0.0601 | -0.0925 | -0.0601 | -0.0373 | -0.0409 | 0.0159 | 0.0165 | 0.0159 | 0.0387 | 0.0351 |
| P6 | -0.1120 | -0.1774 | -0.1575 | -0.1774 | -0.1639 | -0.1615 | 0.0654 | 0.0455 | 0.0654 | 0.0519 | 0.0495 |
| P7 | 0.0000 | -0.0217 | -0.1029 | -0.0575 | -0.0216 | -0.0222 | 0.0217 | 0.1029 | 0.0575 | 0.0216 | 0.0222 |
| P8 | 0.0000 | 0.0171 | -0.0168 | 0.0016 | 0.0103 | 0.0101 | 0.0171 | 0.0168 | 0.0016 | 0.0103 | 0.0101 |
| P9 | -0.2950 | -0.1161 | -0.1292 | -0.4170 | -0.1323 | -0.4214 | 0.1789 | 0.1658 | 0.1220 | 0.1627 | 0.1264 |
| P10 | -0.0900 | 0.0485 | -0.2971 | -0.0473 | -0.2441 | 0.0209 | 0.1385 | 0.2071 | 0.0427 | 0.1541 | 0.1109 |
| P11 | -0.0350 | -0.1025 | -0.1738 | -0.1025 | -0.0856 | -0.0934 | 0.0675 | 0.1388 | 0.0675 | 0.0506 | 0.0584 |
| P12 | -0.0610 | 0.0907 | 0.0963 | 0.0907 | 0.0700 | 0.0696 | 0.1517 | 0.1573 | 0.1517 | 0.1310 | 0.1306 |
| P13 | -0.1350 | -0.0987 | -0.0290 | -0.0987 | -0.1431 | -0.1370 | 0.0363 | 0.1060 | 0.0363 | 0.0081 | 0.0020 |
| P14 | -0.1490 | -0.1522 | -0.3181 | -0.1522 | -0.1393 | -0.1405 | 0.0032 | 0.1691 | 0.0032 | 0.0097 | 0.0085 |
| Q1 | -0.1655 | -0.1298 | -0.1998 | -0.2395 | -0.1950 | -0.2014 | 0.0357 | 0.0343 | 0.0740 | 0.0295 | 0.0359 |
| Q2 | 0.3086 | 0.2984 | 0.2811 | 0.2984 | 0.2820 | 0.2733 | 0.0102 | 0.0275 | 0.0102 | 0.0266 | 0.0353 |
| Q3 | 0.0608 | 0.0546 | -0.0207 | 0.0546 | -0.0251 | -0.0274 | 0.0062 | 0.0815 | 0.0062 | 0.0859 | 0.0882 |
| Q4 | 0.0390 | 0.1135 | 0.0601 | 0.0798 | 0.0503 | 0.0361 | 0.0745 | 0.0211 | 0.0408 | 0.0113 | 0.0029 |
| Q5 | -0.0160 | 0.0545 | 0.1227 | 0.0545 | 0.1246 | 0.0834 | 0.0705 | 0.1387 | 0.0705 | 0.1406 | 0.0994 |
| Q6 | 0.0523 | 0.1232 | -0.0468 | -0.0751 | -0.0436 | 0.0085 | 0.0709 | 0.0991 | 0.1274 | 0.0959 | 0.0438 |
| Q7 | 0.0000 | 0.0336 | -0.0133 | -0.0576 | -0.0200 | -0.0177 | 0.0336 | 0.0133 | 0.0576 | 0.0200 | 0.0177 |
| Q8 | 0.1762 | 0.1158 | 0.1229 | 0.1318 | 0.1213 | 0.1235 | 0.0604 | 0.0533 | 0.0444 | 0.0549 | 0.0527 |
| Q9 | -0.1660 | -0.1301 | -0.1418 | -0.1301 | -0.1520 | -0.1541 | 0.0359 | 0.0242 | 0.0359 | 0.0140 | 0.0119 |
| Q10 | -0.0580 | 0.0235 | -0.0243 | -0.0976 | -0.0338 | -0.0423 | 0.0815 | 0.0337 | 0.0396 | 0.0242 | 0.0157 |
| Q11 | -0.0180 | 0.0064 | -0.0805 | -0.0689 | -0.0841 | -0.0772 | 0.0244 | 0.0625 | 0.0509 | 0.0661 | 0.0592 |
| Q12 | -0.0160 | 0.0357 | -0.0464 | -0.0311 | -0.0483 | -0.0367 | 0.0517 | 0.0304 | 0.0151 | 0.0323 | 0.0207 |
| Q13 | -0.0580 | -0.0217 | -0.1061 | -0.0286 | -0.1035 | -0.0883 | 0.0363 | 0.0481 | 0.0294 | 0.0455 | 0.0303 |
| Q14 | -0.0500 | -0.0652 | -0.0393 | -0.0450 | -0.0522 | -0.0523 | 0.0152 | 0.0107 | 0.0050 | 0.0022 | 0.0023 |
| P1-2 | 1.5688 | 1.5835 | 1.5492 | 1.5654 | 1.5613 | 1.5619 | 0.0147 | 0.0196 | 0.0034 | 0.0075 | 0.0069 |
| P1-5 | 0.7551 | 0.7157 | 0.7200 | 0.7362 | 0.7535 | 0.7550 | 0.0394 | 0.0351 | 0.0189 | 0.0016 | 0.0001 |
| P2-3 | 0.7324 | 0.7768 | 0.8002 | 0.7811 | 0.7807 | 0.7807 | 0.0444 | 0.0678 | 0.0487 | 0.0483 | 0.0483 |
| P2-4 | 0.5613 | 0.5039 | 0.4969 | 0.5422 | 0.5787 | 0.5800 | 0.0574 | 0.0644 | 0.0191 | 0.0174 | 0.0187 |
| P2-5 | 0.4152 | 0.3618 | 0.3799 | 0.3961 | 0.4182 | 0.4186 | 0.0534 | 0.0353 | 0.0191 | 0.0030 | 0.0034 |
| P3-4 | -0.2329 | -0.2734 | -0.3729 | -0.2994 | -0.2726 | -0.2722 | 0.0405 | 0.1400 | 0.0665 | 0.0397 | 0.0393 |
| P4-5 | -0.6116 | -0.7588 | -0.4965 | -0.6115 | -0.6729 | -0.6774 | 0.1472 | 0.1151 | 0.0001 | 0.0613 | 0.0658 |
| P4-7 | 0.2807 | 0.3526 | 0.4396 | 0.3526 | 0.2784 | 0.2982 | 0.0719 | 0.1589 | 0.0719 | 0.0023 | 0.0175 |
| P4-9 | 0.1608 | 0.1771 | 0.2291 | 0.1921 | 0.1580 | 0.1692 | 0.0163 | 0.0683 | 0.0313 | 0.0028 | 0.0084 |
| P5-6 | 0.4409 | 0.4032 | 0.4767 | 0.4223 | 0.4209 | 0.4141 | 0.0377 | 0.0358 | 0.0186 | 0.0200 | 0.0268 |
| P6-11 | 0.0735 | 0.1420 | 0.2200 | 0.1403 | 0.1250 | 0.1084 | 0.0685 | 0.1465 | 0.0668 | 0.0515 | 0.0349 |
| P6-12 | 0.0779 | -0.0077 | -0.0194 | -0.0166 | -0.0022 | 0.0011 | 0.0856 | 0.0973 | 0.0945 | 0.0801 | 0.0768 |
| P6-13 | 0.1775 | 0.1724 | 0.1187 | 0.1213 | 0.1342 | 0.1431 | 0.0051 | 0.0588 | 0.0562 | 0.0433 | 0.0344 |
| P7-8 | 0.0000 | -0.0121 | 0.0168 | -0.0016 | -0.0103 | -0.0101 | 0.0121 | 0.0168 | 0.0016 | 0.0103 | 0.0101 |
| P7-9 | 0.2807 | 0.2967 | 0.3198 | 0.2967 | 0.2671 | 0.2861 | 0.0160 | 0.0391 | 0.0160 | 0.0136 | 0.0054 |
| **P9-10** | **0.0523** | **0.9679** | **0.2575** | **0.0120** | **0.2077** | **-0.0345** | **0.9156** | **0.2052** | **0.0403** | **0.1554** | **0.0868** |
| P9-14 | 0.0943 | 0.0511 | 0.1621 | 0.0598 | 0.0851 | 0.0683 | 0.0432 | 0.0678 | 0.0345 | 0.0092 | 0.0260 |
| P10-11 | -0.0379 | -0.0394 | -0.0417 | -0.0357 | -0.0377 | -0.0137 | 0.0015 | 0.0038 | 0.0022 | 0.0002 | 0.0242 |
| P12-13 | 0.0161 | 0.0079 | 0.0766 | 0.0740 | 0.0676 | 0.0705 | 0.0082 | 0.0605 | 0.0579 | 0.0515 | 0.0544 |
| **P13-14** | **0.0564** | **0.9878** | **0.1637** | **0.0944** | **0.0560** | **0.0739** | **0.9314** | **0.1073** | **0.0380** | **0.0004** | **0.0175** |
| P2-1 | -1.5259 | -1.4712 | -1.5096 | -1.5238 | -1.5211 | -1.5212 | 0.0547 | 0.0163 | 0.0021 | 0.0048 | 0.0047 |
| P5-1 | -0.7275 | -0.6720 | -0.6961 | -0.7107 | -0.7274 | -0.7285 | 0.0555 | 0.0314 | 0.0168 | 0.0001 | 0.0010 |
| P3-2 | -0.7091 | -0.7523 | -0.7734 | -0.7555 | -0.7554 | -0.7551 | 0.0432 | 0.0643 | 0.0464 | 0.0463 | 0.0460 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P4-2 | -0.5445 | -0.5406 | -0.4844 | -0.5270 | -0.5618 | -0.5628 | 0.0039 | 0.0601 | 0.0175 | 0.0173 | 0.0183 |
| P5-2 | -0.4061 | -0.4668 | -0.3726 | -0.3881 | -0.4094 | -0.4098 | 0.0607 | 0.0335 | 0.0180 | 0.0033 | 0.0037 |
| P4-3 | 0.2366 | 0.1809 | 0.3819 | 0.3055 | 0.2774 | 0.2770 | 0.0557 | 0.1453 | 0.0689 | 0.0408 | 0.0404 |
| P5-4 | 0.6167 | 0.6164 | 0.4996 | 0.6164 | 0.6786 | 0.6832 | 0.0003 | 0.1171 | 0.0003 | 0.0619 | 0.0665 |
| P7-4 | -0.2807 | -0.2041 | -0.4396 | -0.3526 | -0.2784 | -0.2982 | 0.0766 | 0.1589 | 0.0719 | 0.0023 | 0.0175 |
| P9-4 | -0.1608 | -0.1993 | -0.2291 | -0.1921 | -0.1580 | -0.1692 | 0.0385 | 0.0683 | 0.0313 | 0.0028 | 0.0084 |
| P6-5 | -0.4409 | -0.4223 | -0.4767 | -0.4223 | -0.4209 | -0.4141 | 0.0186 | 0.0358 | 0.0186 | 0.0200 | 0.0268 |
| P11-6 | -0.0730 | -0.0843 | -0.2158 | -0.1384 | -0.1236 | -0.1072 | 0.0113 | 0.1428 | 0.0654 | 0.0506 | 0.0342 |
| P12-6 | -0.0771 | -0.0212 | 0.0196 | 0.0167 | 0.0024 | -0.0010 | 0.0559 | 0.0967 | 0.0938 | 0.0795 | 0.0761 |
| P13-6 | -0.1754 | -0.2299 | -0.1173 | -0.1202 | -0.1324 | -0.1414 | 0.0545 | 0.0581 | 0.0552 | 0.0430 | 0.0340 |
| P8-7 | 0.0000 | 0.0016 | -0.0168 | 0.0016 | 0.0103 | 0.0101 | 0.0016 | 0.0168 | 0.0016 | 0.0103 | 0.0101 |
| P9-7 | -0.2807 | -0.2531 | -0.3198 | -0.2967 | -0.2671 | -0.2861 | 0.0276 | 0.0391 | 0.0160 | 0.0136 | 0.0054 |
| P10-9 | -0.0521 | 0.0029 | -0.2554 | -0.0116 | -0.2063 | 0.0346 | 0.0550 | 0.2033 | 0.0405 | 0.1542 | 0.0867 |
| P14-9 | -0.0931 | -0.0159 | -0.1586 | -0.0592 | -0.0838 | -0.0675 | 0.0772 | 0.0655 | 0.0339 | 0.0093 | 0.0256 |
| P11-10 | 0.0380 | 0.0423 | 0.0421 | 0.0359 | 0.0380 | 0.0138 | 0.0043 | 0.0041 | 0.0021 | 0.0000 | 0.0242 |
| P13-12 | -0.0161 | -0.0907 | -0.0755 | -0.0729 | -0.0667 | -0.0696 | 0.0746 | 0.0594 | 0.0568 | 0.0506 | 0.0535 |
| P14-13 | -0.0559 | -0.0930 | -0.1595 | -0.0930 | -0.0555 | -0.0731 | 0.0371 | 0.1036 | 0.0371 | 0.0004 | 0.0172 |
| Q1-2 | -0.2040 | -0.2571 | -0.2168 | -0.2571 | -0.2191 | -0.2199 | 0.0531 | 0.0128 | 0.0531 | 0.0151 | 0.0159 |
| Q1-5 | 0.0385 | 0.1560 | 0.0171 | 0.0176 | 0.0241 | 0.0185 | 0.1175 | 0.0214 | 0.0209 | 0.0144 | 0.0200 |
| Q2-3 | 0.0356 | 0.0048 | 0.0707 | 0.0264 | 0.0608 | 0.0610 | 0.0308 | 0.0351 | 0.0092 | 0.0252 | 0.0254 |
| Q2-4 | -0.0155 | 0.0219 | -0.0081 | -0.0051 | -0.0037 | -0.0092 | 0.0374 | 0.0074 | 0.0104 | 0.0118 | 0.0063 |
| Q2-5 | 0.0117 | 0.0021 | 0.0041 | 0.0139 | 0.0070 | -0.0005 | 0.0096 | 0.0076 | 0.0022 | 0.0047 | 0.0122 |
| Q3-4 | 0.0447 | 0.0891 | 0.0339 | 0.0686 | 0.0260 | 0.0213 | 0.0444 | 0.0108 | 0.0239 | 0.0187 | 0.0234 |
| Q4-5 | 0.1582 | 0.1200 | 0.0796 | 0.1200 | 0.0933 | 0.0871 | 0.0382 | 0.0786 | 0.0382 | 0.0649 | 0.0711 |
| Q4-7 | -0.0968 | -0.1669 | -0.1133 | -0.0975 | -0.1316 | -0.1435 | 0.0701 | 0.0165 | 0.0007 | 0.0348 | 0.0467 |
| Q4-9 | -0.0043 | -0.0754 | -0.0223 | -0.0196 | -0.0328 | -0.0392 | 0.0711 | 0.0180 | 0.0153 | 0.0285 | 0.0349 |
| Q5-6 | 0.1247 | 0.1491 | -0.0578 | -0.0861 | -0.0535 | -0.1149 | 0.0244 | 0.1825 | 0.2108 | 0.1782 | 0.2396 |
| Q6-11 | 0.0356 | 0.0267 | 0.0332 | 0.0546 | 0.0433 | 0.0542 | 0.0089 | 0.0024 | 0.0190 | 0.0077 | 0.0186 |
| Q6-12 | 0.0250 | 0.0152 | 0.0361 | 0.0152 | 0.0396 | 0.0311 | 0.0098 | 0.0111 | 0.0098 | 0.0146 | 0.0061 |
| Q6-13 | 0.0722 | 0.1432 | 0.1056 | 0.0529 | 0.1085 | 0.1005 | 0.0710 | 0.0334 | 0.0193 | 0.0363 | 0.0283 |
| Q7-8 | -0.1716 | -0.1570 | -0.1206 | -0.1292 | -0.1191 | -0.1212 | 0.0146 | 0.0510 | 0.0424 | 0.0525 | 0.0504 |
| Q7-9 | 0.0578 | 0.0677 | 0.0785 | 0.0677 | 0.0725 | 0.0612 | 0.0099 | 0.0207 | 0.0099 | 0.0147 | 0.0034 |
| Q9-10 | 0.0422 | 0.1216 | 0.0866 | 0.1173 | 0.0820 | 0.0685 | 0.0794 | 0.0444 | 0.0751 | 0.0398 | 0.0263 |
| Q9-14 | 0.0361 | -0.0041 | 0.0705 | 0.0440 | 0.0644 | 0.0539 | 0.0402 | 0.0344 | 0.0079 | 0.0283 | 0.0178 |
| Q10-11 | -0.0162 | 0.0186 | 0.0568 | 0.0186 | 0.0445 | 0.0257 | 0.0348 | 0.0730 | 0.0348 | 0.0607 | 0.0419 |
| Q12-13 | 0.0075 | 0.0493 | -0.0106 | -0.0160 | -0.0091 | -0.0058 | 0.0418 | 0.0181 | 0.0235 | 0.0166 | 0.0133 |
| Q13-14 | 0.0175 | 0.0053 | -0.0151 | 0.0053 | -0.0084 | 0.0020 | 0.0122 | 0.0326 | 0.0122 | 0.0259 | 0.0155 |
| Q2-1 | 0.2768 | 0.2876 | 0.2145 | 0.2632 | 0.2179 | 0.2220 | 0.0108 | 0.0623 | 0.0136 | 0.0589 | 0.0548 |
| Q5-1 | 0.0223 | -0.0360 | -0.0308 | -0.0221 | -0.0287 | -0.0201 | 0.0583 | 0.0531 | 0.0444 | 0.0510 | 0.0424 |
| Q3-2 | 0.0160 | -0.0414 | -0.0546 | -0.0140 | -0.0511 | -0.0487 | 0.0574 | 0.0706 | 0.0300 | 0.0671 | 0.0647 |
| Q4-2 | 0.0302 | 0.0354 | -0.0302 | -0.0235 | -0.0211 | -0.0139 | 0.0052 | 0.0604 | 0.0537 | 0.0513 | 0.0441 |
| Q5-2 | -0.0210 | 0.0151 | -0.0598 | -0.0655 | -0.0582 | -0.0495 | 0.0361 | 0.0388 | 0.0445 | 0.0372 | 0.0285 |
| Q4-3 | -0.0484 | 0.0809 | -0.0383 | -0.0800 | -0.0412 | -0.0361 | 0.1293 | 0.0101 | 0.0316 | 0.0072 | 0.0123 |
| Q5-4 | 0.1420 | 0.1087 | -0.0698 | -0.1046 | -0.0754 | -0.0688 | 0.0333 | 0.2118 | 0.2466 | 0.2174 | 0.2108 |
| Q7-4 | 0.1138 | 0.1232 | 0.1520 | 0.1232 | 0.1495 | 0.1644 | 0.0094 | 0.0382 | 0.0094 | 0.0357 | 0.0506 |
| Q9-4 | 0.0173 | 0.0132 | 0.0485 | 0.0384 | 0.0458 | 0.0543 | 0.0041 | 0.0312 | 0.0211 | 0.0285 | 0.0370 |
| Q6-5 | -0.0805 | -0.1772 | 0.1072 | 0.1268 | 0.0921 | 0.1549 | 0.0967 | 0.1877 | 0.2073 | 0.1726 | 0.2354 |
| Q11-6 | -0.0344 | -0.0563 | -0.0245 | -0.0506 | -0.0403 | -0.0516 | 0.0219 | 0.0099 | 0.0162 | 0.0059 | 0.0172 |
| Q12-6 | -0.0235 | -0.1132 | -0.0358 | -0.0151 | -0.0392 | -0.0309 | 0.0897 | 0.0123 | 0.0084 | 0.0157 | 0.0074 |
| Q13-6 | -0.0680 | -0.0260 | -0.1027 | -0.0509 | -0.1051 | -0.0970 | 0.0420 | 0.0347 | 0.0171 | 0.0371 | 0.0290 |
| Q8-7 | 0.1762 | 0.1318 | 0.1229 | 0.1318 | 0.1213 | 0.1235 | 0.0444 | 0.0533 | 0.0444 | 0.0549 | 0.0527 |
| Q9-7 | -0.0498 | -0.0448 | -0.0681 | -0.0585 | -0.0651 | -0.0529 | 0.0050 | 0.0183 | 0.0087 | 0.0153 | 0.0031 |
| Q10-9 | -0.0418 | -0.0690 | -0.0811 | -0.1162 | -0.0783 | -0.0681 | 0.0272 | 0.0393 | 0.0744 | 0.0365 | 0.0263 |
| Q14-9 | -0.0336 | -0.0184 | -0.0630 | -0.0426 | -0.0616 | -0.0521 | 0.0152 | 0.0294 | 0.0090 | 0.0280 | 0.0185 |
| Q11-10 | 0.0164 | -0.0136 | -0.0559 | -0.0183 | -0.0439 | -0.0256 | 0.0300 | 0.0723 | 0.0347 | 0.0603 | 0.0420 |
| Q13-12 | -0.0075 | 0.0170 | 0.0117 | 0.0170 | 0.0099 | 0.0067 | 0.0245 | 0.0192 | 0.0245 | 0.0174 | 0.0142 |
| Q14-13 | -0.0164 | 0.0206 | 0.0237 | -0.0025 | 0.0094 | -0.0003 | 0.0370 | 0.0401 | 0.0139 | 0.0258 | 0.0161 |
| **SUM** | | | | | | | **5.6412** | **4.7040** | **2.5548** | **2.4080** | **2.1046** |

## Median redundancy

Table 5.231 presents the state variables, and the other table shows the estimation results and error with actual values.

Table 5.231: State variable of case 2, two large, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0861 | -0.0920 | -0.0855 | -0.0855 | **V1** | 1.0553 | 1.0323 | 1.0579 | 1.0712 |
| $\theta 3$ | -0.2332 | -0.2443 | -0.2317 | -0.2301 | **V2** | 1.0386 | 1.0143 | 1.0413 | 1.0545 |
| $\theta 4$ | -0.1648 | -0.1838 | -0.1628 | -0.1787 | **V3** | 0.9967 | 0.9722 | 0.9997 | 1.0151 |
| $\theta 5$ | -0.1483 | -0.1564 | -0.1470 | -0.1487 | **V4** | 1.0056 | 0.9811 | 1.0090 | 1.0177 |
| $\theta 6$ | -0.2773 | -0.2649 | -0.2780 | -0.2435 | **V5** | 1.0053 | 0.9824 | 1.0077 | 1.0174 |
| $\theta 7$ | -0.2635 | -0.2570 | -0.2559 | -0.2426 | **V6** | 1.0183 | 0.9840 | 1.0159 | 0.9964 |
| $\theta 8$ | -0.2646 | -0.2549 | -0.2580 | -0.2412 | **V7** | 1.0249 | 1.0052 | 1.0301 | 1.0468 |
| $\theta 9$ | -0.3087 | -0.2897 | -0.2960 | -0.2738 | **V8** | 1.0464 | 1.0251 | 1.0520 | 1.0687 |
| $\theta 10$ | -0.3401 | -0.3108 | -0.2970 | -0.2936 | **V9** | 1.0117 | 0.9922 | 1.0165 | 1.0354 |
| $\theta 11$ | -0.3372 | -0.3012 | -0.2931 | -0.2808 | **V10** | 0.9922 | 0.9729 | 1.0095 | 1.0192 |
| $\theta 12$ | -0.2627 | -0.2610 | -0.2770 | -0.2429 | **V11** | 0.9827 | 0.9726 | 1.0023 | 1.0056 |
| $\theta 13$ | -0.2816 | -0.2774 | -0.3042 | -0.2572 | **V12** | 1.0133 | 0.9811 | 1.0021 | 1.0039 |
| $\theta 14$ | -0.3571 | -0.3040 | -0.4157 | -0.2847 | **V13** | 0.9935 | 0.9716 | 0.9764 | 0.9907 |
| | | | | | **V14** | 0.9525 | 0.9611 | 0.9193 | 0.9932 |

Table 5.232: Estimation Results and Errors of case 2, two large, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0553 | 1.0323 | 1.0579 | 1.0712 | 0.0047 | 0.0277 | 0.0021 | 0.0112 |
| V2 | 1.0450 | N/A | 1.0386 | 1.0143 | 1.0413 | 1.0545 | 0.0064 | 0.0307 | 0.0037 | 0.0095 |
| V3 | 1.0100 | N/A | 0.9967 | 0.9722 | 0.9997 | 1.0151 | 0.0133 | 0.0378 | 0.0103 | 0.0051 |
| V4 | 1.0180 | N/A | 1.0056 | 0.9811 | 1.0090 | 1.0177 | 0.0124 | 0.0369 | 0.0090 | 0.0003 |
| V5 | 1.0200 | N/A | 1.0053 | 0.9824 | 1.0077 | 1.0174 | 0.0147 | 0.0376 | 0.0123 | 0.0026 |
| V6 | 1.0700 | N/A | 1.0183 | 0.9840 | 1.0159 | 0.9964 | 0.0517 | 0.0860 | 0.0541 | 0.0736 |
| V7 | 1.0620 | N/A | 1.0249 | 1.0052 | 1.0301 | 1.0468 | 0.0371 | 0.0568 | 0.0319 | 0.0152 |
| V8 | 1.0900 | N/A | 1.0464 | 1.0251 | 1.0520 | 1.0687 | 0.0436 | 0.0649 | 0.0380 | 0.0213 |
| V9 | 1.0560 | N/A | 1.0117 | 0.9922 | 1.0165 | 1.0354 | 0.0443 | 0.0638 | 0.0395 | 0.0206 |
| V10 | 1.0510 | N/A | 0.9922 | 0.9729 | 1.0095 | 1.0192 | 0.0588 | 0.0781 | 0.0415 | 0.0318 |
| V11 | 1.0570 | N/A | 0.9827 | 0.9726 | 1.0023 | 1.0056 | 0.0743 | 0.0844 | 0.0547 | 0.0514 |
| V12 | 1.0550 | N/A | 1.0133 | 0.9811 | 1.0021 | 1.0039 | 0.0417 | 0.0739 | 0.0529 | 0.0511 |
| V13 | 1.0500 | N/A | 0.9935 | 0.9716 | 0.9764 | 0.9907 | 0.0565 | 0.0784 | 0.0736 | 0.0593 |
| V14 | 1.0360 | N/A | 0.9525 | 0.9611 | 0.9193 | 0.9932 | 0.0835 | 0.0749 | 0.1167 | 0.0428 |
| P1 | 2.3239 | 2.3508 | 2.2766 | 2.3178 | 2.2716 | 2.3366 | 0.0473 | 0.0061 | 0.0523 | 0.0127 |
| P2 | 0.1830 | 0.2747 | 0.1662 | 0.1610 | 0.1589 | 0.2654 | 0.0168 | 0.0220 | 0.0241 | 0.0824 |
| P3 | -0.9420 | -1.0549 | -1.1204 | -1.0549 | -1.1258 | -1.0425 | 0.1784 | 0.1129 | 0.1838 | 0.1005 |
| **P4** | **-0.4780** | **0.4349** | **0.3095** | **-0.2356** | **0.3019** | **-0.4359** | **0.7875** | **0.2424** | **0.7799** | **0.0421** |
| P5 | -0.0760 | -0.0601 | -0.1755 | -0.0601 | -0.1844 | -0.0601 | 0.0995 | 0.0159 | 0.1084 | 0.0159 |
| P6 | -0.1120 | -0.1774 | -0.1757 | -0.1774 | -0.1773 | -0.1845 | 0.0637 | 0.0654 | 0.0653 | 0.0725 |
| P7 | 0.0000 | -0.0217 | -0.0641 | -0.0680 | -0.0773 | -0.0339 | 0.0641 | 0.0680 | 0.0773 | 0.0339 |
| P8 | 0.0000 | 0.0171 | -0.0066 | 0.0121 | -0.0132 | 0.0085 | 0.0066 | 0.0121 | 0.0132 | 0.0085 |
| P9 | -0.2950 | -0.1161 | -0.0627 | -0.1161 | -0.1088 | -0.1129 | 0.2323 | 0.1789 | 0.1862 | 0.1821 |
| P10 | -0.0900 | 0.0485 | -0.3952 | -0.3225 | -0.0420 | -0.3124 | 0.3052 | 0.2325 | 0.0480 | 0.2224 |
| P11 | -0.0350 | -0.1025 | -0.3151 | -0.1230 | -0.0849 | -0.1015 | 0.2801 | 0.0880 | 0.0499 | 0.0665 |
| P12 | -0.0610 | 0.0907 | 0.1336 | 0.0666 | 0.1073 | 0.0783 | 0.1946 | 0.1276 | 0.1683 | 0.1393 |
| P13 | -0.1350 | -0.0987 | 0.0201 | -0.0987 | -0.0978 | -0.1038 | 0.1551 | 0.0363 | 0.0372 | 0.0312 |
| P14 | -0.1490 | -0.1522 | -0.4266 | -0.1522 | -0.7401 | -0.1530 | 0.2776 | 0.0032 | 0.5911 | 0.0040 |
| Q1 | -0.1655 | -0.1298 | -0.1421 | -0.1298 | -0.1428 | -0.1343 | 0.0234 | 0.0357 | 0.0227 | 0.0312 |
| Q2 | 0.3086 | 0.2984 | 0.3131 | 0.2984 | 0.3124 | 0.3220 | 0.0045 | 0.0102 | 0.0038 | 0.0134 |
| Q3 | 0.0608 | 0.0546 | 0.0725 | 0.0546 | 0.0728 | 0.0783 | 0.0117 | 0.0062 | 0.0120 | 0.0175 |
| Q4 | 0.0390 | 0.1135 | 0.0935 | 0.1135 | 0.0956 | 0.1139 | 0.0545 | 0.0745 | 0.0566 | 0.0749 |
| Q5 | -0.0160 | 0.0545 | 0.0347 | 0.0545 | 0.0340 | 0.0516 | 0.0507 | 0.0705 | 0.0500 | 0.0676 |
| Q6 | 0.0523 | 0.1232 | 0.0114 | -0.2146 | 0.0138 | -0.4952 | 0.0409 | 0.2669 | 0.0385 | 0.5475 |
| Q7 | 0.0000 | 0.0336 | 0.0150 | 0.0336 | 0.0208 | 0.0246 | 0.0150 | 0.0336 | 0.0208 | 0.0246 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q8 | 0.1762 | 0.1158 | 0.1281 | 0.1158 | 0.1311 | 0.1329 | 0.0481 | 0.0604 | 0.0451 | 0.0433 |
| Q9 | -0.1660 | -0.1301 | -0.1288 | -0.1301 | -0.1133 | -0.1154 | 0.0372 | 0.0359 | 0.0527 | 0.0506 |
| Q10 | -0.0580 | 0.0235 | -0.0263 | -0.0947 | -0.0293 | -0.0001 | 0.0317 | 0.0367 | 0.0287 | 0.0579 |
| Q11 | -0.0180 | 0.0064 | -0.0646 | 0.0064 | -0.0644 | 0.0292 | 0.0466 | 0.0244 | 0.0464 | 0.0472 |
| Q12 | -0.0160 | 0.0357 | -0.0402 | -0.0329 | -0.0527 | 0.0176 | 0.0242 | 0.0169 | 0.0367 | 0.0336 |
| Q13 | -0.0580 | -0.0217 | -0.1145 | -0.0217 | -0.1142 | -0.0206 | 0.0565 | 0.0363 | 0.0562 | 0.0374 |
| Q14 | -0.0500 | -0.0652 | -0.1037 | -0.0652 | -0.0863 | -0.0737 | 0.0537 | 0.0152 | 0.0363 | 0.0237 |
| P1-2 | 1.5688 | 1.5835 | 1.5465 | 1.5835 | 1.5441 | 1.5815 | 0.0223 | 0.0147 | 0.0247 | 0.0127 |
| P1-5 | 0.7551 | 0.7157 | 0.7300 | 0.7343 | 0.7275 | 0.7551 | 0.0251 | 0.0208 | 0.0276 | 0.0000 |
| P2-3 | 0.7324 | 0.7768 | 0.7877 | 0.7768 | 0.7869 | 0.7978 | 0.0553 | 0.0444 | 0.0545 | 0.0654 |
| P2-4 | 0.5613 | N/A | 0.4835 | 0.5306 | 0.4766 | 0.5843 | 0.0778 | 0.0307 | 0.0847 | 0.0230 |
| P2-5 | 0.4152 | N/A | 0.3996 | 0.3912 | 0.3978 | 0.4222 | 0.0156 | 0.0240 | 0.0174 | 0.0070 |
| P3-4 | -0.2329 | -0.2734 | -0.3601 | -0.3060 | -0.3661 | -0.2719 | 0.1272 | 0.0731 | 0.1332 | 0.0390 |
| P4-5 | -0.6116 | N/A | -0.3565 | -0.5777 | -0.3350 | -0.6651 | 0.2551 | 0.0339 | 0.2766 | 0.0535 |
| P4-7 | 0.2807 | 0.3526 | 0.4965 | 0.3526 | 0.4726 | 0.3326 | 0.2158 | 0.0719 | 0.1919 | 0.0519 |
| P4-9 | 0.1608 | N/A | 0.2707 | 0.1909 | 0.2529 | 0.1856 | 0.1099 | 0.0301 | 0.0921 | 0.0248 |
| P5-6 | 0.4409 | 0.4032 | 0.5605 | 0.4461 | 0.5694 | 0.4085 | 0.1196 | 0.0052 | 0.1285 | 0.0324 |
| P6-11 | 0.0735 | N/A | 0.3196 | 0.1653 | 0.0899 | 0.1366 | 0.2461 | 0.0918 | 0.0164 | 0.0631 |
| P6-12 | 0.0779 | -0.0077 | -0.0398 | -0.0077 | 0.0181 | -0.0131 | 0.1177 | 0.0856 | 0.0598 | 0.0910 |
| P6-13 | 0.1775 | N/A | 0.1050 | 0.1111 | 0.2840 | 0.1006 | 0.0725 | 0.0664 | 0.1065 | 0.0769 |
| P7-8 | 0.0000 | -0.0121 | 0.0066 | -0.0121 | 0.0132 | -0.0085 | 0.0066 | 0.0121 | 0.0132 | 0.0085 |
| P7-9 | 0.2807 | 0.2967 | 0.4258 | 0.2967 | 0.3821 | 0.3071 | 0.1451 | 0.0160 | 0.1014 | 0.0264 |
| **P9-10** | **0.0523** | **0.9679** | **0.4056** | **0.2862** | **0.0381** | **0.2835** | **0.3533** | **0.2339** | **0.0142** | **0.2312** |
| P9-14 | 0.0943 | N/A | 0.2282 | 0.0853 | 0.4880 | 0.0963 | 0.1339 | 0.0090 | 0.3937 | 0.0020 |
| P10-11 | -0.0379 | -0.0394 | 0.0051 | -0.0394 | -0.0040 | -0.0316 | 0.0430 | 0.0015 | 0.0339 | 0.0063 |
| P12-13 | 0.0161 | N/A | 0.0933 | 0.0588 | 0.1251 | 0.0651 | 0.0772 | 0.0427 | 0.1090 | 0.0490 |
| **P13-14** | **0.0564** | **0.9878** | **0.2147** | **0.0694** | **0.3009** | **0.0602** | **0.1583** | **0.0130** | **0.2445** | **0.0038** |
| P2-1 | -1.5259 | N/A | -1.5046 | -1.5376 | -1.5024 | -1.5389 | 0.0213 | 0.0117 | 0.0235 | 0.0130 |
| P5-1 | -0.7275 | N/A | -0.7035 | -0.7063 | -0.7014 | -0.7275 | 0.0240 | 0.0212 | 0.0261 | 0.0000 |
| P3-2 | -0.7091 | N/A | -0.7603 | -0.7489 | -0.7597 | -0.7707 | 0.0512 | 0.0398 | 0.0506 | 0.0616 |
| P4-2 | -0.5445 | N/A | -0.4707 | -0.5146 | -0.4643 | -0.5663 | 0.0738 | 0.0299 | 0.0802 | 0.0218 |
| P5-2 | -0.4061 | N/A | -0.3909 | -0.3825 | -0.3891 | -0.4126 | 0.0152 | 0.0236 | 0.0170 | 0.0065 |
| P4-3 | 0.2366 | N/A | 0.3695 | 0.3131 | 0.3757 | 0.2773 | 0.1329 | 0.0765 | 0.1391 | 0.0407 |
| P5-4 | 0.6167 | N/A | 0.3584 | 0.5827 | 0.3368 | 0.6714 | 0.2583 | 0.0340 | 0.2799 | 0.0547 |
| P7-4 | -0.2807 | N/A | -0.4965 | -0.3526 | -0.4726 | -0.3326 | 0.2158 | 0.0719 | 0.1919 | 0.0519 |
| P9-4 | -0.1608 | N/A | -0.2707 | -0.1909 | -0.2529 | -0.1856 | 0.1099 | 0.0301 | 0.0921 | 0.0248 |
| P6-5 | -0.4409 | N/A | -0.5605 | -0.4461 | -0.5694 | -0.4085 | 0.1196 | 0.0052 | 0.1285 | 0.0324 |
| P11-6 | -0.0730 | N/A | -0.3102 | -0.1626 | -0.0891 | -0.1337 | 0.2372 | 0.0896 | 0.0161 | 0.0607 |
| P12-6 | -0.0771 | N/A | 0.0402 | 0.0077 | -0.0178 | 0.0132 | 0.1173 | 0.0848 | 0.0593 | 0.0903 |
| P13-6 | -0.1754 | N/A | -0.1031 | -0.1102 | -0.2771 | -0.0999 | 0.0723 | 0.0652 | 0.1017 | 0.0755 |
| P8-7 | 0.0000 | N/A | -0.0066 | 0.0121 | -0.0132 | 0.0085 | 0.0066 | 0.0121 | 0.0132 | 0.0085 |
| P9-7 | -0.2807 | N/A | -0.4258 | -0.2967 | -0.3821 | -0.3071 | 0.1451 | 0.0160 | 0.1014 | 0.0264 |
| P10-9 | -0.0521 | N/A | -0.4003 | -0.2831 | -0.0380 | -0.2809 | 0.3482 | 0.2310 | 0.0141 | 0.2288 |
| P14-9 | -0.0931 | N/A | -0.2200 | -0.0837 | -0.4556 | -0.0936 | 0.1269 | 0.0094 | 0.3625 | 0.0005 |
| P11-10 | 0.0380 | N/A | -0.0049 | 0.0396 | 0.0042 | 0.0322 | 0.0429 | 0.0016 | 0.0338 | 0.0058 |
| P13-12 | -0.0161 | N/A | -0.0915 | -0.0580 | -0.1216 | -0.0642 | 0.0754 | 0.0419 | 0.1055 | 0.0481 |
| P14-13 | -0.0559 | N/A | -0.2066 | -0.0685 | -0.2845 | -0.0594 | 0.1507 | 0.0126 | 0.2286 | 0.0035 |
| Q1-2 | -0.2040 | -0.2571 | -0.1994 | -0.1859 | -0.2012 | -0.2069 | 0.0046 | 0.0181 | 0.0028 | 0.0029 |
| Q1-5 | 0.0385 | 0.1560 | 0.0572 | 0.0561 | 0.0584 | 0.0726 | 0.0187 | 0.0176 | 0.0199 | 0.0341 |
| Q2-3 | 0.0356 | 0.0048 | 0.0421 | 0.0439 | 0.0407 | 0.0281 | 0.0065 | 0.0083 | 0.0051 | 0.0075 |
| Q2-4 | -0.0155 | N/A | 0.0166 | 0.0049 | 0.0146 | 0.0158 | 0.0321 | 0.0204 | 0.0301 | 0.0313 |
| Q2-5 | 0.0117 | N/A | 0.0426 | 0.0341 | 0.0452 | 0.0603 | 0.0309 | 0.0224 | 0.0335 | 0.0486 |
| Q3-4 | 0.0447 | 0.0891 | 0.0901 | 0.0674 | 0.0903 | 0.0858 | 0.0454 | 0.0227 | 0.0456 | 0.0411 |
| Q4-5 | 0.1582 | N/A | 0.1248 | 0.1613 | 0.1421 | 0.2294 | 0.0334 | 0.0031 | 0.0161 | 0.0712 |
| Q4-7 | -0.0968 | -0.1669 | -0.0701 | -0.1030 | -0.0818 | -0.1339 | 0.0267 | 0.0062 | 0.0150 | 0.0371 |
| Q4-9 | -0.0043 | N/A | 0.0082 | -0.0101 | 0.0030 | -0.0244 | 0.0125 | 0.0058 | 0.0073 | 0.0201 |
| Q5-6 | 0.1247 | 0.1491 | -0.0197 | 0.0173 | 0.0019 | 0.1105 | 0.1444 | 0.1074 | 0.1228 | 0.0142 |
| Q6-11 | 0.0356 | N/A | 0.0385 | -0.0189 | 0.0269 | -0.1079 | 0.0029 | 0.0545 | 0.0087 | 0.1435 |
| Q6-12 | 0.0250 | 0.0152 | 0.0394 | 0.0152 | 0.0460 | -0.0229 | 0.0144 | 0.0098 | 0.0210 | 0.0479 |
| Q6-13 | 0.0722 | N/A | 0.1409 | 0.0383 | 0.1666 | -0.0072 | 0.0687 | 0.0339 | 0.0944 | 0.0794 |
| Q7-8 | -0.1716 | -0.1570 | -0.1255 | -0.1135 | -0.1283 | -0.1302 | 0.0461 | 0.0581 | 0.0433 | 0.0414 |
| Q7-9 | 0.0578 | 0.0677 | 0.1324 | 0.1242 | 0.1352 | 0.1134 | 0.0746 | 0.0664 | 0.0774 | 0.0556 |
| Q9-10 | 0.0422 | 0.1216 | 0.0870 | 0.1216 | 0.0693 | 0.0934 | 0.0448 | 0.0794 | 0.0271 | 0.0512 |
| Q9-14 | 0.0361 | N/A | 0.1183 | 0.0744 | 0.1604 | 0.1166 | 0.0822 | 0.0383 | 0.1243 | 0.0805 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q10-11 | -0.0162 | 0.0186 | 0.0464 | 0.0186 | 0.0394 | 0.0863 | 0.0626 | 0.0348 | 0.0556 | 0.1025 |
| Q12-13 | 0.0075 | N/A | -0.0016 | -0.0178 | -0.0073 | -0.0055 | 0.0091 | 0.0253 | 0.0148 | 0.0130 |
| Q13-14 | 0.0175 | 0.0053 | 0.0192 | -0.0039 | 0.0283 | -0.0354 | 0.0017 | 0.0214 | 0.0108 | 0.0529 |
| Q2-1 | 0.2768 | N/A | 0.2118 | 0.2155 | 0.2120 | 0.2177 | 0.0650 | 0.0613 | 0.0648 | 0.0591 |
| Q5-1 | 0.0223 | N/A | -0.0525 | -0.0407 | -0.0553 | -0.0659 | 0.0748 | 0.0630 | 0.0776 | 0.0882 |
| Q3-2 | 0.0160 | N/A | -0.0176 | -0.0128 | -0.0175 | -0.0076 | 0.0336 | 0.0288 | 0.0335 | 0.0236 |
| Q4-2 | 0.0302 | N/A | -0.0490 | -0.0241 | -0.0487 | -0.0343 | 0.0792 | 0.0543 | 0.0789 | 0.0645 |
| Q5-2 | -0.0210 | N/A | -0.0881 | -0.0764 | -0.0914 | -0.1052 | 0.0671 | 0.0554 | 0.0704 | 0.0842 |
| Q4-3 | -0.0484 | N/A | -0.0916 | -0.0737 | -0.0914 | -0.0984 | 0.0432 | 0.0253 | 0.0430 | 0.0500 |
| Q5-4 | 0.1420 | N/A | -0.1189 | -0.1455 | -0.1366 | -0.2093 | 0.2609 | 0.2875 | 0.2786 | 0.3513 |
| Q7-4 | 0.1138 | N/A | 0.1210 | 0.1316 | 0.1281 | 0.1592 | 0.0072 | 0.0178 | 0.0143 | 0.0454 |
| Q9-4 | 0.0173 | N/A | 0.0309 | 0.0305 | 0.0309 | 0.0427 | 0.0136 | 0.0132 | 0.0136 | 0.0254 |
| Q6-5 | -0.0805 | N/A | 0.0928 | 0.0312 | 0.0730 | -0.0699 | 0.1733 | 0.1117 | 0.1535 | 0.0106 |
| Q11-6 | -0.0344 | N/A | -0.0186 | 0.0246 | -0.0252 | 0.1139 | 0.0158 | 0.0590 | 0.0092 | 0.1483 |
| Q12-6 | -0.0235 | N/A | -0.0386 | -0.0151 | -0.0454 | 0.0231 | 0.0151 | 0.0084 | 0.0219 | 0.0466 |
| Q13-6 | -0.0680 | N/A | -0.1370 | -0.0365 | -0.1529 | 0.0085 | 0.0690 | 0.0315 | 0.0849 | 0.0765 |
| Q8-7 | 0.1762 | N/A | 0.1281 | 0.1158 | 0.1311 | 0.1329 | 0.0481 | 0.0604 | 0.0451 | 0.0433 |
| Q9-7 | -0.0498 | N/A | -0.1116 | -0.1129 | -0.1182 | -0.1027 | 0.0618 | 0.0631 | 0.0684 | 0.0529 |
| Q10-9 | -0.0418 | N/A | -0.0728 | -0.1133 | -0.0688 | -0.0863 | 0.0310 | 0.0715 | 0.0270 | 0.0445 |
| Q14-9 | -0.0336 | N/A | -0.1008 | -0.0708 | -0.0914 | -0.1108 | 0.0672 | 0.0372 | 0.0578 | 0.0772 |
| Q11-10 | 0.0164 | N/A | -0.0460 | -0.0182 | -0.0391 | -0.0847 | 0.0624 | 0.0346 | 0.0555 | 0.1011 |
| Q13-12 | -0.0075 | N/A | 0.0033 | 0.0186 | 0.0105 | 0.0063 | 0.0108 | 0.0261 | 0.0180 | 0.0138 |
| Q14-13 | -0.0164 | N/A | -0.0029 | 0.0056 | 0.0050 | 0.0371 | 0.0135 | 0.0220 | 0.0214 | 0.0535 |
| **SUM** | | | | | | | **6.4459** | **3.8904** | **5.8919** | **3.4916** |

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.233. In this case, full redundancy generate better estimation for all estimators. LTAV performs best in both median & full redundancy.

Table 5.233: Estimation errors of case 2, two large, 14-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **5.6412** | **4.7040** | **2.5548** | **2.4080** | **2.1046** |
| **SUM** | **N/A** | **6.4459** | **3.8904** | **5.8919** | **3.4916** |

### Three large bad data

Compared with the base cases, $P_4 = -0.9349$, $P_{9-10} = 0.9679$ and $P_{13-14} = 0.9878$ are introduced to simulate the three large bad data in both full and median redundancy case. The results are presented below.

**Full redundancy**

Based on the original raw data and large bad data shown above, Table 5.234 gives the state variables, the other table shows the estimation results and error with actual values.

Table 5.234: State variable of case 2, multi large, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0816 | -0.0847 | -0.0817 | -0.0827 | **V1** | 1.0918 | 1.0804 | 1.0909 | 1.0848 |
| $\theta 3$ | -0.2151 | -0.2224 | -0.2156 | -0.2182 | **V2** | 1.0761 | 1.0665 | 1.0752 | 1.0691 |
| $\theta 4$ | -0.1771 | -0.1805 | -0.1766 | -0.1791 | **V3** | 1.0319 | 1.0290 | 1.0309 | 1.0247 |
| $\theta 5$ | -0.1465 | -0.1494 | -0.1462 | -0.1485 | **V4** | 1.0410 | 1.0318 | 1.0404 | 1.0353 |
| $\theta 6$ | -0.2285 | -0.2354 | -0.2266 | -0.2303 | **V5** | 1.0475 | 1.0370 | 1.0467 | 1.0416 |
| $\theta 7$ | -0.2200 | -0.2299 | -0.2158 | -0.2183 | **V6** | 1.0656 | 1.0631 | 1.0630 | 1.0699 |
| $\theta 8$ | -0.2157 | -0.2280 | -0.2132 | -0.2160 | **V7** | 1.0677 | 1.0551 | 1.0678 | 1.0663 |
| $\theta 9$ | -0.2486 | -0.2550 | -0.2383 | -0.2399 | **V8** | 1.0874 | 1.0767 | 1.0874 | 1.0862 |
| $\theta 10$ | -0.2647 | -0.2639 | -0.2419 | -0.2404 | **V9** | 1.0605 | 1.0487 | 1.0608 | 1.0609 |
| $\theta 11$ | -0.2576 | -0.2558 | -0.2383 | -0.2412 | **V10** | 1.0469 | 1.0388 | 1.0522 | 1.0547 |
| $\theta 12$ | -0.2183 | -0.2294 | -0.2217 | -0.2270 | **V11** | 1.0410 | 1.0393 | 1.0456 | 1.0504 |
| $\theta 13$ | -0.2333 | -0.2454 | -0.2359 | -0.2404 | **V12** | 1.0601 | 1.0617 | 1.0540 | 1.0624 |
| $\theta 14$ | -0.2800 | -0.2706 | -0.2552 | -0.2593 | **V13** | 1.0463 | 1.0500 | 1.0413 | 1.0489 |
| | | | | | **V14** | 1.0259 | 1.0350 | 1.0335 | 1.0362 |

Table 5.235: Estimation Results and Errors of case 2, multi large, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0918 | 1.0804 | 1.0909 | 1.0848 | 0.0086 | 0.0318 | 0.0204 | 0.0309 | 0.0248 |
| V2 | 1.0450 | 1.0743 | 1.0761 | 1.0665 | 1.0752 | 1.0691 | 0.0293 | 0.0311 | 0.0215 | 0.0302 | 0.0241 |
| V3 | 1.0100 | 0.9739 | 1.0319 | 1.0290 | 1.0309 | 1.0247 | 0.0361 | 0.0219 | 0.0190 | 0.0209 | 0.0147 |
| V4 | 1.0180 | 1.0318 | 1.0410 | 1.0318 | 1.0404 | 1.0353 | 0.0138 | 0.0230 | 0.0138 | 0.0224 | 0.0173 |
| V5 | 1.0200 | 1.0251 | 1.0475 | 1.0370 | 1.0467 | 1.0416 | 0.0051 | 0.0275 | 0.0170 | 0.0267 | 0.0216 |
| V6 | 1.0700 | 1.0491 | 1.0656 | 1.0631 | 1.0630 | 1.0699 | 0.0209 | 0.0044 | 0.0069 | 0.0070 | 0.0001 |
| V7 | 1.0620 | 1.0551 | 1.0677 | 1.0551 | 1.0678 | 1.0663 | 0.0069 | 0.0057 | 0.0069 | 0.0058 | 0.0043 |
| V8 | 1.0900 | 1.0955 | 1.0874 | 1.0767 | 1.0874 | 1.0862 | 0.0055 | 0.0026 | 0.0133 | 0.0026 | 0.0038 |
| V9 | 1.0560 | 1.1131 | 1.0605 | 1.0487 | 1.0608 | 1.0609 | 0.0571 | 0.0045 | 0.0073 | 0.0048 | 0.0049 |
| V10 | 1.0510 | 1.0952 | 1.0469 | 1.0388 | 1.0522 | 1.0547 | 0.0442 | 0.0041 | 0.0122 | 0.0012 | 0.0037 |
| V11 | 1.0570 | 1.0355 | 1.0410 | 1.0393 | 1.0456 | 1.0504 | 0.0215 | 0.0160 | 0.0177 | 0.0114 | 0.0066 |
| V12 | 1.0550 | 1.1034 | 1.0601 | 1.0617 | 1.0540 | 1.0624 | 0.0484 | 0.0051 | 0.0067 | 0.0010 | 0.0074 |
| V13 | 1.0500 | 1.0616 | 1.0463 | 1.0500 | 1.0413 | 1.0489 | 0.0116 | 0.0037 | 0.0000 | 0.0087 | 0.0011 |
| V14 | 1.0360 | 1.0350 | 1.0259 | 1.0350 | 1.0335 | 1.0362 | 0.0010 | 0.0101 | 0.0010 | 0.0025 | 0.0002 |
| P1 | 2.3239 | 2.3508 | 2.3367 | 2.3508 | 2.3327 | 2.3357 | 0.0269 | 0.0128 | 0.0269 | 0.0088 | 0.0118 |
| P2 | 0.1830 | 0.2747 | 0.2997 | 0.2747 | 0.2932 | 0.2955 | 0.0917 | 0.1167 | 0.0917 | 0.1102 | 0.1125 |
| P3 | -0.9420 | -1.0549 | -0.9707 | -0.9815 | -0.9778 | -0.9775 | 0.1129 | 0.0287 | 0.0395 | 0.0358 | 0.0355 |
| **P4** | **-0.4780** | **-0.9349** | **-0.7565** | **-0.6953** | **-0.7780** | **-0.7796** | **0.4569** | **0.2785** | **0.2173** | **0.3000** | **0.3016** |
| P5 | -0.0760 | -0.0601 | -0.0100 | 0.0011 | -0.0192 | -0.0181 | 0.0159 | 0.0660 | 0.0771 | 0.0568 | 0.0579 |
| P6 | -0.1120 | -0.1774 | -0.1353 | -0.1693 | -0.1592 | -0.1582 | 0.0654 | 0.0233 | 0.0573 | 0.0472 | 0.0462 |
| P7 | 0.0000 | -0.0217 | 0.0329 | -0.0217 | 0.0006 | -0.0059 | 0.0217 | 0.0329 | 0.0217 | 0.0006 | 0.0059 |
| P8 | 0.0000 | 0.0171 | 0.0285 | 0.0121 | 0.0177 | 0.0155 | 0.0171 | 0.0285 | 0.0121 | 0.0177 | 0.0155 |
| P9 | -0.2950 | -0.1161 | -0.0416 | -0.1899 | -0.1837 | -0.2113 | 0.1789 | 0.2534 | 0.1051 | 0.1113 | 0.0837 |
| P10 | -0.0900 | 0.0485 | -0.2629 | -0.1794 | -0.0806 | -0.0186 | 0.1385 | 0.1729 | 0.0894 | 0.0094 | 0.0714 |
| P11 | -0.0350 | -0.1025 | -0.1589 | -0.1005 | -0.0841 | -0.1023 | 0.0675 | 0.1239 | 0.0655 | 0.0491 | 0.0673 |
| P12 | -0.0610 | 0.0907 | 0.1023 | 0.0907 | 0.0714 | 0.0695 | 0.1517 | 0.1633 | 0.1517 | 0.1324 | 0.1305 |
| P13 | -0.1350 | -0.0987 | -0.0274 | -0.0987 | -0.1426 | -0.1427 | 0.0363 | 0.1076 | 0.0363 | 0.0076 | 0.0077 |
| P14 | -0.1490 | -0.1522 | -0.2915 | -0.1522 | -0.1531 | -0.1633 | 0.0032 | 0.1425 | 0.0032 | 0.0041 | 0.0143 |
| Q1 | -0.1655 | -0.1298 | -0.1935 | -0.2342 | -0.1932 | -0.1994 | 0.0357 | 0.0280 | 0.0687 | 0.0277 | 0.0339 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q2 | 0.3086 | 0.2984 | 0.2826 | 0.2984 | 0.2828 | 0.2738 | 0.0102 | 0.0260 | 0.0102 | 0.0258 | 0.0348 |
| Q3 | 0.0608 | 0.0546 | -0.0254 | 0.0546 | -0.0256 | -0.0293 | 0.0062 | 0.0862 | 0.0062 | 0.0864 | 0.0901 |
| Q4 | 0.0390 | 0.1135 | 0.0523 | 0.0659 | 0.0494 | 0.0318 | 0.0745 | 0.0133 | 0.0269 | 0.0104 | 0.0072 |
| Q5 | -0.0160 | 0.0545 | 0.1218 | 0.0545 | 0.1234 | 0.0861 | 0.0705 | 0.1378 | 0.0705 | 0.1394 | 0.1021 |
| Q6 | 0.0523 | 0.1232 | -0.0394 | -0.0651 | -0.0393 | 0.0093 | 0.0709 | 0.0917 | 0.1174 | 0.0916 | 0.0430 |
| Q7 | 0.0000 | 0.0336 | -0.0219 | -0.0576 | -0.0231 | -0.0208 | 0.0336 | 0.0219 | 0.0576 | 0.0231 | 0.0208 |
| Q8 | 0.1762 | 0.1158 | 0.1212 | 0.1318 | 0.1208 | 0.1222 | 0.0604 | 0.0550 | 0.0444 | 0.0554 | 0.0540 |
| Q9 | -0.1660 | -0.1301 | -0.1562 | -0.2009 | -0.1589 | -0.1595 | 0.0359 | 0.0098 | 0.0349 | 0.0071 | 0.0065 |
| Q10 | -0.0580 | 0.0235 | -0.0339 | -0.0551 | -0.0404 | -0.0466 | 0.0815 | 0.0241 | 0.0029 | 0.0176 | 0.0114 |
| Q11 | -0.0180 | 0.0064 | -0.0813 | -0.0699 | -0.0863 | -0.0781 | 0.0244 | 0.0633 | 0.0519 | 0.0683 | 0.0601 |
| Q12 | -0.0160 | 0.0357 | -0.0438 | -0.0311 | -0.0469 | -0.0356 | 0.0517 | 0.0278 | 0.0151 | 0.0309 | 0.0196 |
| Q13 | -0.0580 | -0.0217 | -0.1023 | -0.0261 | -0.1010 | -0.0860 | 0.0363 | 0.0443 | 0.0319 | 0.0430 | 0.0280 |
| Q14 | -0.0500 | -0.0652 | -0.0457 | -0.0224 | -0.0529 | -0.0532 | 0.0152 | 0.0043 | 0.0276 | 0.0029 | 0.0032 |
| P1-2 | 1.5688 | 1.5835 | 1.5673 | 1.5835 | 1.5664 | 1.5669 | 0.0147 | 0.0015 | 0.0147 | 0.0024 | 0.0019 |
| P1-5 | 0.7551 | 0.7157 | 0.7694 | 0.7673 | 0.7663 | 0.7687 | 0.0394 | 0.0143 | 0.0122 | 0.0112 | 0.0136 |
| P2-3 | 0.7324 | 0.7768 | 0.7717 | 0.7775 | 0.7728 | 0.7726 | 0.0444 | 0.0393 | 0.0451 | 0.0404 | 0.0402 |
| P2-4 | 0.5613 | 0.5039 | 0.6188 | 0.6094 | 0.6137 | 0.6146 | 0.0574 | 0.0575 | 0.0481 | 0.0524 | 0.0533 |
| P2-5 | 0.4152 | 0.3618 | 0.4361 | 0.4291 | 0.4327 | 0.4344 | 0.0534 | 0.0209 | 0.0139 | 0.0175 | 0.0192 |
| P3-4 | -0.2329 | -0.2734 | -0.2236 | -0.2292 | -0.2298 | -0.2298 | 0.0405 | 0.0093 | 0.0037 | 0.0031 | 0.0031 |
| P4-5 | -0.6116 | -0.7588 | -0.7627 | -0.7502 | -0.7559 | -0.7531 | 0.1472 | 0.1511 | 0.1386 | 0.1443 | 0.1415 |
| P4-7 | 0.2807 | 0.3526 | 0.2328 | 0.2627 | 0.2131 | 0.2117 | 0.0719 | 0.0479 | 0.0180 | 0.0676 | 0.0690 |
| P4-9 | 0.1608 | 0.1771 | 0.1462 | 0.1495 | 0.1263 | 0.1238 | 0.0163 | 0.0146 | 0.0113 | 0.0345 | 0.0370 |
| P5-6 | 0.4409 | 0.4032 | 0.3889 | 0.4032 | 0.3804 | 0.3879 | 0.0377 | 0.0520 | 0.0377 | 0.0605 | 0.0530 |
| P6-11 | 0.0735 | 0.1420 | 0.1845 | 0.1420 | 0.0895 | 0.0908 | 0.0685 | 0.1110 | 0.0685 | 0.0160 | 0.0173 |
| P6-12 | 0.0779 | -0.0077 | -0.0276 | -0.0193 | -0.0027 | 0.0002 | 0.0856 | 0.1055 | 0.0972 | 0.0806 | 0.0777 |
| P6-13 | 0.1775 | 0.1724 | 0.0968 | 0.1111 | 0.1344 | 0.1387 | 0.0051 | 0.0807 | 0.0664 | 0.0431 | 0.0388 |
| P7-8 | 0.0000 | -0.0121 | -0.0285 | -0.0121 | -0.0177 | -0.0155 | 0.0121 | 0.0285 | 0.0121 | 0.0177 | 0.0155 |
| P7-9 | 0.2807 | 0.2967 | 0.2942 | 0.2531 | 0.2315 | 0.2213 | 0.0160 | 0.0135 | 0.0276 | 0.0492 | 0.0594 |
| **P9-10** | **0.0523** | **0.9679** | **0.2423** | **0.1407** | **0.0765** | **0.0314** | **0.9156** | **0.1900** | **0.0884** | **0.0242** | **0.0209** |
| P9-14 | 0.0943 | 0.0511 | 0.1564 | 0.0720 | 0.0975 | 0.1024 | 0.0432 | 0.0621 | 0.0223 | 0.0032 | 0.0081 |
| P10-11 | -0.0379 | -0.0394 | -0.0224 | -0.0394 | -0.0044 | 0.0126 | 0.0015 | 0.0155 | 0.0015 | 0.0335 | 0.0505 |
| P12-13 | 0.0161 | 0.0079 | 0.0745 | 0.0714 | 0.0685 | 0.0696 | 0.0082 | 0.0584 | 0.0553 | 0.0524 | 0.0535 |
| **P13-14** | **0.0564** | **0.9878** | **0.1415** | **0.0819** | **0.0577** | **0.0630** | **0.9314** | **0.0851** | **0.0255** | **0.0013** | **0.0066** |
| P2-1 | -1.5259 | -1.4712 | -1.5270 | -1.5412 | -1.5260 | -1.5261 | 0.0547 | 0.0011 | 0.0153 | 0.0001 | 0.0002 |
| P5-1 | -0.7275 | -0.6720 | -0.7422 | -0.7397 | -0.7393 | -0.7413 | 0.0555 | 0.0147 | 0.0122 | 0.0118 | 0.0138 |
| P3-2 | -0.7091 | -0.7523 | -0.7471 | -0.7523 | -0.7480 | -0.7476 | 0.0432 | 0.0380 | 0.0432 | 0.0389 | 0.0385 |
| P4-2 | -0.5445 | -0.5406 | -0.5995 | -0.5903 | -0.5947 | -0.5953 | 0.0039 | 0.0550 | 0.0458 | 0.0502 | 0.0508 |
| P5-2 | -0.4061 | -0.4668 | -0.4267 | -0.4197 | -0.4234 | -0.4249 | 0.0607 | 0.0206 | 0.0136 | 0.0173 | 0.0188 |
| P4-3 | 0.2366 | 0.1809 | 0.2268 | 0.2330 | 0.2332 | 0.2333 | 0.0557 | 0.0098 | 0.0036 | 0.0034 | 0.0033 |
| P5-4 | 0.6167 | 0.6164 | 0.7700 | 0.7574 | 0.7630 | 0.7602 | 0.0003 | 0.1533 | 0.1407 | 0.1463 | 0.1435 |
| P7-4 | -0.2807 | -0.2041 | -0.2328 | -0.2627 | -0.2131 | -0.2117 | 0.0766 | 0.0479 | 0.0180 | 0.0676 | 0.0690 |
| P9-4 | -0.1608 | -0.1993 | -0.1462 | -0.1495 | -0.1263 | -0.1238 | 0.0385 | 0.0146 | 0.0113 | 0.0345 | 0.0370 |
| P6-5 | -0.4409 | -0.4223 | -0.3889 | -0.4032 | -0.3804 | -0.3879 | 0.0186 | 0.0520 | 0.0377 | 0.0605 | 0.0530 |
| P11-6 | -0.0730 | -0.0843 | -0.1814 | -0.1400 | -0.0886 | -0.0898 | 0.0113 | 0.1084 | 0.0670 | 0.0156 | 0.0168 |
| P12-6 | -0.0771 | -0.0212 | 0.0279 | 0.0193 | 0.0029 | -0.0001 | 0.0559 | 0.1050 | 0.0964 | 0.0800 | 0.0770 |
| P13-6 | -0.1754 | -0.2299 | -0.0955 | -0.1103 | -0.1327 | -0.1370 | 0.0545 | 0.0799 | 0.0651 | 0.0427 | 0.0384 |
| P8-7 | 0.0000 | 0.0016 | 0.0285 | 0.0121 | 0.0177 | 0.0155 | 0.0016 | 0.0285 | 0.0121 | 0.0177 | 0.0155 |
| P9-7 | -0.2807 | -0.2531 | -0.2942 | -0.2531 | -0.2315 | -0.2213 | 0.0276 | 0.0135 | 0.0276 | 0.0492 | 0.0594 |
| P10-9 | -0.0521 | 0.0029 | -0.2405 | -0.1400 | -0.0762 | -0.0312 | 0.0550 | 0.1884 | 0.0879 | 0.0241 | 0.0209 |
| P14-9 | -0.0931 | -0.0159 | -0.1532 | -0.0714 | -0.0960 | -0.1010 | 0.0772 | 0.0601 | 0.0217 | 0.0029 | 0.0079 |
| P11-10 | 0.0380 | 0.0423 | 0.0226 | 0.0395 | 0.0045 | -0.0126 | 0.0043 | 0.0154 | 0.0015 | 0.0335 | 0.0506 |
| P13-12 | -0.0161 | -0.0907 | -0.0734 | -0.0703 | -0.0676 | -0.0687 | 0.0746 | 0.0573 | 0.0542 | 0.0515 | 0.0526 |
| P14-13 | -0.0559 | -0.0930 | -0.1383 | -0.0808 | -0.0571 | -0.0623 | 0.0371 | 0.0824 | 0.0249 | 0.0012 | 0.0064 |
| Q1-2 | -0.2040 | -0.2571 | -0.2203 | -0.2571 | -0.2199 | -0.2210 | 0.0531 | 0.0163 | 0.0531 | 0.0159 | 0.0170 |
| Q1-5 | 0.0385 | 0.1560 | 0.0267 | 0.0229 | 0.0267 | 0.0216 | 0.1175 | 0.0118 | 0.0156 | 0.0118 | 0.0169 |
| Q2-3 | 0.0356 | 0.0048 | 0.0558 | 0.0202 | 0.0565 | 0.0568 | 0.0308 | 0.0202 | 0.0154 | 0.0209 | 0.0212 |
| Q2-4 | -0.0155 | 0.0219 | -0.0002 | -0.0008 | -0.0010 | -0.0072 | 0.0374 | 0.0153 | 0.0147 | 0.0145 | 0.0083 |
| Q2-5 | 0.0117 | 0.0021 | 0.0076 | 0.0145 | 0.0080 | 0.0010 | 0.0096 | 0.0041 | 0.0028 | 0.0037 | 0.0107 |
| Q3-4 | 0.0447 | 0.0891 | 0.0240 | 0.0650 | 0.0239 | 0.0182 | 0.0444 | 0.0207 | 0.0203 | 0.0208 | 0.0265 |
| Q4-5 | 0.1582 | 0.1200 | 0.0931 | 0.1224 | 0.0968 | 0.0951 | 0.0382 | 0.0651 | 0.0358 | 0.0614 | 0.0631 |
| Q4-7 | -0.0968 | -0.1669 | -0.1313 | -0.1111 | -0.1352 | -0.1532 | 0.0701 | 0.0345 | 0.0143 | 0.0384 | 0.0564 |
| Q4-9 | -0.0043 | -0.0754 | -0.0324 | -0.0268 | -0.0354 | -0.0455 | 0.0711 | 0.0281 | 0.0225 | 0.0311 | 0.0412 |
| Q5-6 | 0.1247 | 0.1491 | -0.0648 | -0.0979 | -0.0574 | -0.1097 | 0.0244 | 0.1895 | 0.2226 | 0.1821 | 0.2344 |
| Q6-11 | 0.0356 | 0.0267 | 0.0461 | 0.0605 | 0.0505 | 0.0619 | 0.0089 | 0.0105 | 0.0249 | 0.0149 | 0.0263 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q6-12 | 0.0250 | 0.0152 | 0.0362 | 0.0152 | 0.0389 | 0.0313 | 0.0098 | 0.0112 | 0.0098 | 0.0139 | 0.0063 |
| Q6-13 | 0.0722 | 0.1432 | 0.1089 | 0.0509 | 0.1092 | 0.1027 | 0.0710 | 0.0367 | 0.0213 | 0.0370 | 0.0305 |
| Q7-8 | -0.1716 | -0.1570 | -0.1189 | -0.1291 | -0.1186 | -0.1199 | 0.0146 | 0.0527 | 0.0425 | 0.0530 | 0.0517 |
| Q7-9 | 0.0578 | 0.0677 | 0.0749 | 0.0646 | 0.0709 | 0.0552 | 0.0099 | 0.0171 | 0.0068 | 0.0131 | 0.0026 |
| Q9-10 | 0.0422 | 0.1216 | 0.0807 | 0.0709 | 0.0792 | 0.0655 | 0.0794 | 0.0385 | 0.0287 | 0.0370 | 0.0233 |
| Q9-14 | 0.0361 | -0.0041 | 0.0639 | 0.0198 | 0.0617 | 0.0496 | 0.0402 | 0.0278 | 0.0163 | 0.0256 | 0.0135 |
| Q10-11 | -0.0162 | 0.0186 | 0.0419 | 0.0139 | 0.0379 | 0.0184 | 0.0348 | 0.0581 | 0.0301 | 0.0541 | 0.0346 |
| Q12-13 | 0.0075 | 0.0493 | -0.0081 | -0.0161 | -0.0084 | -0.0045 | 0.0418 | 0.0156 | 0.0236 | 0.0159 | 0.0120 |
| Q13-14 | 0.0175 | 0.0053 | -0.0050 | 0.0061 | -0.0045 | 0.0079 | 0.0122 | 0.0225 | 0.0114 | 0.0220 | 0.0096 |
| Q2-1 | 0.2768 | 0.2876 | 0.2194 | 0.2644 | 0.2193 | 0.2232 | 0.0108 | 0.0574 | 0.0124 | 0.0575 | 0.0536 |
| Q5-1 | 0.0223 | -0.0360 | -0.0272 | -0.0195 | -0.0278 | -0.0196 | 0.0583 | 0.0495 | 0.0418 | 0.0501 | 0.0419 |
| Q3-2 | 0.0160 | -0.0414 | -0.0494 | -0.0104 | -0.0495 | -0.0475 | 0.0574 | 0.0654 | 0.0264 | 0.0655 | 0.0635 |
| Q4-2 | 0.0302 | 0.0354 | -0.0175 | -0.0163 | -0.0174 | -0.0097 | 0.0052 | 0.0477 | 0.0465 | 0.0476 | 0.0399 |
| Q5-2 | -0.0210 | 0.0151 | -0.0567 | -0.0625 | -0.0574 | -0.0492 | 0.0361 | 0.0357 | 0.0415 | 0.0364 | 0.0282 |
| Q4-3 | -0.0484 | 0.0809 | -0.0432 | -0.0827 | -0.0426 | -0.0366 | 0.1293 | 0.0052 | 0.0343 | 0.0058 | 0.0118 |
| Q5-4 | 0.1420 | 0.1087 | -0.0702 | -0.0996 | -0.0742 | -0.0725 | 0.0333 | 0.2122 | 0.2416 | 0.2162 | 0.2145 |
| Q7-4 | 0.1138 | 0.1232 | 0.1448 | 0.1267 | 0.1473 | 0.1662 | 0.0094 | 0.0310 | 0.0129 | 0.0335 | 0.0524 |
| Q9-4 | 0.0173 | 0.0132 | 0.0436 | 0.0385 | 0.0440 | 0.0542 | 0.0041 | 0.0263 | 0.0212 | 0.0267 | 0.0369 |
| Q6-5 | -0.0805 | -0.1772 | 0.0981 | 0.1355 | 0.0892 | 0.1448 | 0.0967 | 0.1786 | 0.2160 | 0.1697 | 0.2253 |
| Q11-6 | -0.0344 | -0.0563 | -0.0398 | -0.0563 | -0.0487 | -0.0598 | 0.0219 | 0.0054 | 0.0219 | 0.0143 | 0.0254 |
| Q12-6 | -0.0235 | -0.1132 | -0.0357 | -0.0151 | -0.0385 | -0.0310 | 0.0897 | 0.0122 | 0.0084 | 0.0150 | 0.0075 |
| Q13-6 | -0.0680 | -0.0260 | -0.1065 | -0.0492 | -0.1058 | -0.0993 | 0.0420 | 0.0385 | 0.0188 | 0.0378 | 0.0313 |
| Q8-7 | 0.1762 | 0.1318 | 0.1212 | 0.1318 | 0.1208 | 0.1222 | 0.0444 | 0.0550 | 0.0444 | 0.0554 | 0.0540 |
| Q9-7 | -0.0498 | -0.0448 | -0.0660 | -0.0578 | -0.0653 | -0.0502 | 0.0050 | 0.0162 | 0.0080 | 0.0155 | 0.0004 |
| Q10-9 | -0.0418 | -0.0690 | -0.0758 | -0.0690 | -0.0783 | -0.0651 | 0.0272 | 0.0340 | 0.0272 | 0.0365 | 0.0233 |
| Q14-9 | -0.0336 | -0.0184 | -0.0570 | -0.0184 | -0.0585 | -0.0465 | 0.0152 | 0.0234 | 0.0152 | 0.0249 | 0.0129 |
| Q11-10 | 0.0164 | -0.0136 | -0.0415 | -0.0136 | -0.0376 | -0.0183 | 0.0300 | 0.0579 | 0.0300 | 0.0540 | 0.0347 |
| Q13-12 | -0.0075 | 0.0170 | 0.0091 | 0.0170 | 0.0092 | 0.0054 | 0.0245 | 0.0166 | 0.0245 | 0.0167 | 0.0129 |
| Q14-13 | -0.0164 | 0.0206 | 0.0113 | -0.0040 | 0.0056 | -0.0067 | 0.0370 | 0.0277 | 0.0124 | 0.0220 | 0.0097 |
| **SUM** | | | | | | | **5.1852** | **3.6738** | **2.6666** | **2.5734** | **2.5166** |

### Median redundancy

The following two tables show the results and errors in detail.

Table 5.236: State Variable of case 2, multi large, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0885 | -0.0861 | -0.0882 | -0.0855 | **V1** | 1.0613 | 1.0686 | 1.0610 | 1.0712 |
| $\theta 3$ | -0.2368 | -0.2324 | -0.2366 | -0.2301 | **V2** | 1.0440 | 1.0510 | 1.0438 | 1.0545 |
| $\theta 4$ | -0.1968 | -0.1813 | -0.1940 | -0.1787 | **V3** | 1.0039 | 1.0094 | 1.0037 | 1.0151 |
| $\theta 5$ | -0.1598 | -0.1512 | -0.1579 | -0.1487 | **V4** | 1.0029 | 1.0147 | 1.0041 | 1.0177 |
| $\theta 6$ | -0.2520 | -0.2431 | -0.2466 | -0.2435 | **V5** | 1.0084 | 1.0168 | 1.0086 | 1.0174 |
| $\theta 7$ | -0.2522 | -0.2374 | -0.2397 | -0.2426 | **V6** | 1.0197 | 1.0143 | 1.0162 | 0.9964 |
| $\theta 8$ | -0.2454 | -0.2346 | -0.2358 | -0.2412 | **V7** | 1.0242 | 1.0391 | 1.0276 | 1.0468 |
| $\theta 9$ | -0.2895 | -0.2667 | -0.2661 | -0.2738 | **V8** | 1.0453 | 1.0584 | 1.0485 | 1.0687 |
| $\theta 10$ | -0.3166 | -0.2832 | -0.2790 | -0.2936 | **V9** | 1.0109 | 1.0257 | 1.0151 | 1.0354 |
| $\theta 11$ | -0.3093 | -0.2737 | -0.2691 | -0.2808 | **V10** | 0.9925 | 1.0081 | 1.0020 | 1.0192 |
| $\theta 12$ | -0.2287 | -0.2362 | -0.2346 | -0.2429 | **V11** | 0.9844 | 1.0066 | 0.9965 | 1.0056 |
| $\theta 13$ | -0.2481 | -0.2532 | -0.2500 | -0.2572 | **V12** | 1.0178 | 1.0130 | 1.0045 | 1.0039 |
| $\theta 14$ | -0.3212 | -0.2768 | -0.2744 | -0.2847 | **V13** | 0.9978 | 1.0017 | 0.9902 | 0.9907 |
| | | | | | **V14** | 0.9585 | 0.9883 | 0.9743 | 0.9932 |

Table 5.237: Estimation Results and Errors of case 2, multi large, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0613 | 1.0686 | 1.0610 | 1.0712 | 0.0013 | 0.0086 | 0.0010 | 0.0112 |
| V2 | 1.0450 | N/A | 1.0440 | 1.0510 | 1.0438 | 1.0545 | 0.0010 | 0.0060 | 0.0012 | 0.0095 |
| V3 | 1.0100 | N/A | 1.0039 | 1.0094 | 1.0037 | 1.0151 | 0.0061 | 0.0006 | 0.0063 | 0.0051 |
| V4 | 1.0180 | N/A | 1.0029 | 1.0147 | 1.0041 | 1.0177 | 0.0151 | 0.0033 | 0.0139 | 0.0003 |
| V5 | 1.0200 | N/A | 1.0084 | 1.0168 | 1.0086 | 1.0174 | 0.0116 | 0.0032 | 0.0114 | 0.0026 |
| V6 | 1.0700 | N/A | 1.0197 | 1.0143 | 1.0162 | 0.9964 | 0.0503 | 0.0557 | 0.0538 | 0.0736 |
| V7 | 1.0620 | N/A | 1.0242 | 1.0391 | 1.0276 | 1.0468 | 0.0378 | 0.0229 | 0.0344 | 0.0152 |
| V8 | 1.0900 | N/A | 1.0453 | 1.0584 | 1.0485 | 1.0687 | 0.0447 | 0.0316 | 0.0415 | 0.0213 |
| V9 | 1.0560 | N/A | 1.0109 | 1.0257 | 1.0151 | 1.0354 | 0.0451 | 0.0303 | 0.0409 | 0.0206 |
| V10 | 1.0510 | N/A | 0.9925 | 1.0081 | 1.0020 | 1.0192 | 0.0585 | 0.0429 | 0.0490 | 0.0318 |
| V11 | 1.0570 | N/A | 0.9844 | 1.0066 | 0.9965 | 1.0056 | 0.0726 | 0.0504 | 0.0605 | 0.0514 |
| V12 | 1.0550 | N/A | 1.0178 | 1.0130 | 1.0045 | 1.0039 | 0.0372 | 0.0420 | 0.0505 | 0.0511 |
| V13 | 1.0500 | N/A | 0.9978 | 1.0017 | 0.9902 | 0.9907 | 0.0522 | 0.0483 | 0.0598 | 0.0593 |
| V14 | 1.0360 | N/A | 0.9585 | 0.9883 | 0.9743 | 0.9932 | 0.0775 | 0.0477 | 0.0617 | 0.0428 |
| P1 | 2.3239 | 2.3508 | 2.4000 | 2.3508 | 2.3857 | 2.3366 | 0.0761 | 0.0269 | 0.0618 | 0.0127 |
| P2 | 0.1830 | 0.2747 | 0.3576 | 0.2747 | 0.3365 | 0.2654 | 0.1746 | 0.0917 | 0.1535 | 0.0824 |
| P3 | -0.9420 | -1.0549 | -0.9731 | -1.0480 | -0.9897 | -1.0425 | 0.0311 | 0.1060 | 0.0477 | 0.1005 |
| **P4** | **-0.4780** | **-0.9349** | **-0.8239** | **-0.5213** | **-0.8501** | **-0.4359** | **0.3459** | **0.0433** | **0.3721** | **0.0421** |
| P5 | -0.0760 | -0.0601 | 0.0457 | -0.0601 | 0.0191 | -0.0601 | 0.1217 | 0.0159 | 0.0951 | 0.0159 |
| P6 | -0.1120 | -0.1774 | -0.1217 | -0.1774 | -0.1711 | -0.1845 | 0.0097 | 0.0654 | 0.0591 | 0.0725 |
| P7 | 0.0000 | -0.0217 | 0.0317 | -0.0217 | -0.0035 | -0.0339 | 0.0317 | 0.0217 | 0.0035 | 0.0339 |
| P8 | 0.0000 | 0.0171 | 0.0412 | 0.0171 | 0.0237 | 0.0085 | 0.0412 | 0.0171 | 0.0237 | 0.0085 |
| P9 | -0.2950 | -0.1161 | 0.0009 | -0.1161 | -0.1143 | -0.1129 | 0.2959 | 0.1789 | 0.1807 | 0.1821 |
| P10 | -0.0900 | 0.0485 | -0.3684 | -0.2847 | -0.2207 | -0.3124 | 0.2784 | 0.1947 | 0.1307 | 0.2224 |
| P11 | -0.0350 | -0.1025 | -0.2841 | -0.1025 | -0.0976 | -0.1015 | 0.2491 | 0.0675 | 0.0626 | 0.0665 |
| P12 | -0.0610 | 0.0907 | 0.1694 | 0.0885 | 0.0916 | 0.0783 | 0.2304 | 0.1495 | 0.1526 | 0.1393 |
| P13 | -0.1350 | -0.0987 | 0.0720 | -0.0987 | -0.0976 | -0.1038 | 0.2070 | 0.0363 | 0.0374 | 0.0312 |
| P14 | -0.1490 | -0.1522 | -0.3655 | -0.1522 | -0.1531 | -0.1530 | 0.2165 | 0.0032 | 0.0041 | 0.0040 |
| Q1 | -0.1655 | -0.1298 | -0.1372 | -0.1298 | -0.1400 | -0.1343 | 0.0283 | 0.0357 | 0.0255 | 0.0312 |
| Q2 | 0.3086 | 0.2984 | 0.3175 | 0.2984 | 0.3139 | 0.3220 | 0.0089 | 0.0102 | 0.0053 | 0.0134 |
| Q3 | 0.0608 | 0.0546 | 0.0711 | 0.0546 | 0.0696 | 0.0783 | 0.0103 | 0.0062 | 0.0088 | 0.0175 |
| Q4 | 0.0390 | 0.1135 | 0.0998 | 0.1135 | 0.0969 | 0.1139 | 0.0608 | 0.0745 | 0.0579 | 0.0749 |
| Q5 | -0.0160 | 0.0545 | 0.0434 | 0.0545 | 0.0385 | 0.0516 | 0.0594 | 0.0705 | 0.0545 | 0.0676 |
| Q6 | 0.0523 | 0.1232 | 0.0010 | -0.2554 | -0.0037 | -0.4952 | 0.0513 | 0.3077 | 0.0560 | 0.5475 |
| Q7 | 0.0000 | 0.0336 | 0.0090 | 0.0336 | 0.0077 | 0.0246 | 0.0090 | 0.0336 | 0.0077 | 0.0246 |
| Q8 | 0.1762 | 0.1158 | 0.1253 | 0.1158 | 0.1246 | 0.1329 | 0.0509 | 0.0604 | 0.0516 | 0.0433 |
| Q9 | -0.1660 | -0.1301 | -0.1357 | -0.1301 | -0.1403 | -0.1154 | 0.0303 | 0.0359 | 0.0257 | 0.0506 |
| Q10 | -0.0580 | 0.0235 | -0.0311 | -0.0903 | -0.0405 | -0.0001 | 0.0269 | 0.0323 | 0.0175 | 0.0579 |
| Q11 | -0.0180 | 0.0064 | -0.0715 | 0.0064 | -0.0772 | 0.0292 | 0.0535 | 0.0244 | 0.0592 | 0.0472 |
| Q12 | -0.0160 | 0.0357 | -0.0450 | -0.0320 | -0.0611 | 0.0176 | 0.0290 | 0.0160 | 0.0451 | 0.0336 |
| Q13 | -0.0580 | -0.0217 | -0.1237 | -0.0217 | -0.1295 | -0.0206 | 0.0657 | 0.0363 | 0.0715 | 0.0374 |
| Q14 | -0.0500 | -0.0652 | -0.1088 | -0.1010 | -0.1171 | -0.0737 | 0.0588 | 0.0510 | 0.0671 | 0.0237 |
| P1-2 | 1.5688 | 1.5835 | 1.6073 | 1.5882 | 1.6023 | 1.5815 | 0.0385 | 0.0194 | 0.0335 | 0.0127 |
| P1-5 | 0.7551 | 0.7157 | 0.7927 | 0.7626 | 0.7835 | 0.7551 | 0.0376 | 0.0075 | 0.0284 | 0.0000 |
| P2-3 | 0.7324 | 0.7768 | 0.8010 | 0.8023 | 0.8010 | 0.7978 | 0.0686 | 0.0699 | 0.0686 | 0.0654 |
| P2-4 | 0.5613 | N/A | 0.6618 | 0.5912 | 0.6456 | 0.5843 | 0.1005 | 0.0299 | 0.0843 | 0.0230 |
| P2-5 | 0.4152 | N/A | 0.4574 | 0.4263 | 0.4476 | 0.4222 | 0.0422 | 0.0111 | 0.0324 | 0.0070 |
| P3-4 | -0.2329 | -0.2734 | -0.2001 | -0.2734 | -0.2166 | -0.2719 | 0.0328 | 0.0405 | 0.0163 | 0.0390 |
| P4-5 | -0.6116 | N/A | -0.8409 | -0.6812 | -0.8138 | -0.6651 | 0.2293 | 0.0696 | 0.2022 | 0.0535 |
| P4-7 | 0.2807 | 0.3526 | 0.2779 | 0.2890 | 0.2304 | 0.3326 | 0.0028 | 0.0083 | 0.0503 | 0.0519 |
| P4-9 | 0.1608 | N/A | 0.1741 | 0.1648 | 0.1363 | 0.1856 | 0.0133 | 0.0040 | 0.0245 | 0.0248 |
| P5-6 | 0.4409 | 0.4032 | 0.4031 | 0.4032 | 0.3863 | 0.4085 | 0.0378 | 0.0377 | 0.0546 | 0.0324 |
| P6-11 | 0.0735 | N/A | 0.3093 | 0.1441 | 0.1329 | 0.1366 | 0.2358 | 0.0706 | 0.0594 | 0.0631 |
| P6-12 | 0.0779 | -0.0077 | -0.0731 | -0.0206 | -0.0206 | -0.0131 | 0.1510 | 0.0985 | 0.0985 | 0.0910 |
| P6-13 | 0.1775 | N/A | 0.0452 | 0.1024 | 0.1029 | 0.1006 | 0.1323 | 0.0751 | 0.0746 | 0.0769 |
| P7-8 | 0.0000 | -0.0121 | -0.0412 | -0.0171 | -0.0237 | -0.0085 | 0.0412 | 0.0171 | 0.0237 | 0.0085 |
| P7-9 | 0.2807 | 0.2967 | 0.3509 | 0.2844 | 0.2506 | 0.3071 | 0.0702 | 0.0037 | 0.0301 | 0.0264 |
| **P9-10** | **0.0523** | **0.9679** | **0.3566** | **0.2476** | **0.1887** | **0.2835** | **0.3043** | **0.1953** | **0.1364** | **0.2312** |
| P9-14 | 0.0943 | N/A | 0.1693 | 0.0856 | 0.0839 | 0.0963 | 0.0750 | 0.0087 | 0.0104 | 0.0020 |
| P10-11 | -0.0379 | -0.0394 | -0.0161 | -0.0394 | -0.0333 | -0.0316 | 0.0218 | 0.0015 | 0.0046 | 0.0063 |
| P12-13 | 0.0161 | N/A | 0.0953 | 0.0678 | 0.0706 | 0.0651 | 0.0792 | 0.0517 | 0.0545 | 0.0490 |
| **P13-14** | **0.0564** | **0.9878** | **0.2091** | **0.0696** | **0.0727** | **0.0602** | **0.1527** | **0.0132** | **0.0163** | **0.0038** |
| P2-1 | -1.5259 | N/A | -1.5625 | -1.5451 | -1.5577 | -1.5389 | 0.0366 | 0.0192 | 0.0318 | 0.0130 |
| P5-1 | -0.7275 | N/A | -0.7618 | -0.7344 | -0.7533 | -0.7275 | 0.0343 | 0.0069 | 0.0258 | 0.0000 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P3-2 | -0.7091 | N/A | -0.7730 | -0.7746 | -0.7731 | -0.7707 | 0.0639 | 0.0655 | 0.0640 | 0.0616 |
| P4-2 | -0.5445 | N/A | -0.6383 | -0.5727 | -0.6232 | -0.5663 | 0.0938 | 0.0282 | 0.0787 | 0.0218 |
| P5-2 | -0.4061 | N/A | -0.4461 | -0.4166 | -0.4368 | -0.4126 | 0.0400 | 0.0105 | 0.0307 | 0.0065 |
| P4-3 | 0.2366 | N/A | 0.2033 | 0.2788 | 0.2202 | 0.2773 | 0.0333 | 0.0422 | 0.0164 | 0.0407 |
| P5-4 | 0.6167 | N/A | 0.8506 | 0.6877 | 0.8230 | 0.6714 | 0.2339 | 0.0710 | 0.2063 | 0.0547 |
| P7-4 | -0.2807 | N/A | -0.2779 | -0.2890 | -0.2304 | -0.3326 | 0.0028 | 0.0083 | 0.0503 | 0.0519 |
| P9-4 | -0.1608 | N/A | -0.1741 | -0.1648 | -0.1363 | -0.1856 | 0.0133 | 0.0040 | 0.0245 | 0.0248 |
| P6-5 | -0.4409 | N/A | -0.4031 | -0.4032 | -0.3863 | -0.4085 | 0.0378 | 0.0377 | 0.0546 | 0.0324 |
| P11-6 | -0.0730 | N/A | -0.3004 | -0.1421 | -0.1311 | -0.1337 | 0.2274 | 0.0691 | 0.0581 | 0.0607 |
| P12-6 | -0.0771 | N/A | 0.0740 | 0.0207 | 0.0210 | 0.0132 | 0.1511 | 0.0978 | 0.0981 | 0.0903 |
| P13-6 | -0.1754 | N/A | -0.0437 | -0.1015 | -0.1008 | -0.0999 | 0.1317 | 0.0739 | 0.0746 | 0.0755 |
| P8-7 | 0.0000 | N/A | 0.0412 | 0.0171 | 0.0237 | 0.0085 | 0.0412 | 0.0171 | 0.0237 | 0.0085 |
| P9-7 | -0.2807 | N/A | -0.3509 | -0.2844 | -0.2506 | -0.3071 | 0.0702 | 0.0037 | 0.0301 | 0.0264 |
| P10-9 | -0.0521 | N/A | -0.3524 | -0.2453 | -0.1874 | -0.2809 | 0.3003 | 0.1932 | 0.1353 | 0.2288 |
| P14-9 | -0.0931 | N/A | -0.1640 | -0.0835 | -0.0814 | -0.0936 | 0.0709 | 0.0096 | 0.0117 | 0.0005 |
| P11-10 | 0.0380 | N/A | 0.0163 | 0.0396 | 0.0335 | 0.0322 | 0.0217 | 0.0016 | 0.0045 | 0.0058 |
| P13-12 | -0.0161 | N/A | -0.0934 | -0.0667 | -0.0695 | -0.0642 | 0.0773 | 0.0506 | 0.0534 | 0.0481 |
| P14-13 | -0.0559 | N/A | -0.2016 | -0.0687 | -0.0717 | -0.0594 | 0.1457 | 0.0128 | 0.0158 | 0.0035 |
| Q1-2 | -0.2040 | -0.2571 | -0.2023 | -0.1926 | -0.2035 | -0.2069 | 0.0017 | 0.0114 | 0.0005 | 0.0029 |
| Q1-5 | 0.0385 | 0.1560 | 0.0651 | 0.0628 | 0.0635 | 0.0726 | 0.0266 | 0.0243 | 0.0250 | 0.0341 |
| Q2-3 | 0.0356 | 0.0048 | 0.0313 | 0.0395 | 0.0315 | 0.0281 | 0.0043 | 0.0039 | 0.0041 | 0.0075 |
| Q2-4 | -0.0155 | N/A | 0.0230 | 0.0114 | 0.0184 | 0.0158 | 0.0385 | 0.0269 | 0.0339 | 0.0313 |
| Q2-5 | 0.0117 | N/A | 0.0412 | 0.0419 | 0.0414 | 0.0603 | 0.0295 | 0.0302 | 0.0297 | 0.0486 |
| Q3-4 | 0.0447 | 0.0891 | 0.0765 | 0.0704 | 0.0752 | 0.0858 | 0.0318 | 0.0257 | 0.0305 | 0.0411 |
| Q4-5 | 0.1582 | N/A | 0.1502 | 0.1765 | 0.1652 | 0.2294 | 0.0080 | 0.0183 | 0.0070 | 0.0712 |
| Q4-7 | -0.0968 | -0.1669 | -0.0969 | -0.1131 | -0.1100 | -0.1339 | 0.0001 | 0.0163 | 0.0132 | 0.0371 |
| Q4-9 | -0.0043 | N/A | -0.0070 | -0.0136 | -0.0156 | -0.0244 | 0.0027 | 0.0093 | 0.0113 | 0.0201 |
| Q5-6 | 0.1247 | 0.1491 | -0.0299 | 0.0294 | -0.0153 | 0.1105 | 0.1546 | 0.0953 | 0.1400 | 0.0142 |
| Q6-11 | 0.0356 | N/A | 0.0418 | -0.0270 | 0.0384 | -0.1079 | 0.0062 | 0.0626 | 0.0028 | 0.1435 |
| Q6-12 | 0.0250 | 0.0152 | 0.0440 | 0.0152 | 0.0566 | -0.0229 | 0.0190 | 0.0098 | 0.0316 | 0.0479 |
| Q6-13 | 0.0722 | N/A | 0.1485 | 0.0465 | 0.1506 | -0.0072 | 0.0763 | 0.0257 | 0.0784 | 0.0794 |
| Q7-8 | -0.1716 | -0.1570 | -0.1225 | -0.1136 | -0.1220 | -0.1302 | 0.0491 | 0.0580 | 0.0496 | 0.0414 |
| Q7-9 | 0.0578 | 0.0677 | 0.1298 | 0.1312 | 0.1200 | 0.1134 | 0.0720 | 0.0734 | 0.0622 | 0.0556 |
| Q9-10 | 0.0422 | 0.1216 | 0.0911 | 0.1216 | 0.0872 | 0.0934 | 0.0489 | 0.0794 | 0.0450 | 0.0512 |
| Q9-14 | 0.0361 | N/A | 0.1181 | 0.1019 | 0.1139 | 0.1166 | 0.0820 | 0.0658 | 0.0778 | 0.0805 |
| Q10-11 | -0.0162 | 0.0186 | 0.0488 | 0.0252 | 0.0431 | 0.0863 | 0.0650 | 0.0414 | 0.0593 | 0.1025 |
| Q12-13 | 0.0075 | N/A | -0.0028 | -0.0170 | -0.0055 | -0.0055 | 0.0103 | 0.0245 | 0.0130 | 0.0130 |
| Q13-14 | 0.0175 | 0.0053 | 0.0173 | 0.0053 | 0.0104 | -0.0354 | 0.0002 | 0.0122 | 0.0071 | 0.0529 |
| Q2-1 | 0.2768 | N/A | 0.2221 | 0.2056 | 0.2225 | 0.2177 | 0.0547 | 0.0712 | 0.0543 | 0.0591 |
| Q5-1 | 0.0223 | N/A | -0.0432 | -0.0535 | -0.0445 | -0.0659 | 0.0655 | 0.0758 | 0.0668 | 0.0882 |
| Q3-2 | 0.0160 | N/A | -0.0055 | -0.0158 | -0.0056 | -0.0076 | 0.0215 | 0.0318 | 0.0216 | 0.0236 |
| Q4-2 | 0.0302 | N/A | -0.0228 | -0.0278 | -0.0218 | -0.0343 | 0.0530 | 0.0580 | 0.0520 | 0.0645 |
| Q5-2 | -0.0210 | N/A | -0.0797 | -0.0863 | -0.0813 | -0.1052 | 0.0587 | 0.0653 | 0.0603 | 0.0842 |
| Q4-3 | -0.0484 | N/A | -0.0942 | -0.0829 | -0.0917 | -0.0984 | 0.0458 | 0.0345 | 0.0433 | 0.0500 |
| Q5-4 | 0.1420 | N/A | -0.1197 | -0.1562 | -0.1364 | -0.2093 | 0.2617 | 0.2982 | 0.2784 | 0.3513 |
| Q7-4 | 0.1138 | N/A | 0.1145 | 0.1322 | 0.1233 | 0.1592 | 0.0007 | 0.0184 | 0.0095 | 0.0454 |
| Q9-4 | 0.0173 | N/A | 0.0232 | 0.0280 | 0.0256 | 0.0427 | 0.0059 | 0.0107 | 0.0083 | 0.0254 |
| Q6-5 | -0.0805 | N/A | 0.0677 | 0.0077 | 0.0498 | -0.0699 | 0.1482 | 0.0882 | 0.1303 | 0.0106 |
| Q11-6 | -0.0344 | N/A | -0.0232 | 0.0312 | -0.0347 | 0.1139 | 0.0112 | 0.0656 | 0.0003 | 0.1483 |
| Q12-6 | -0.0235 | N/A | -0.0422 | -0.0150 | -0.0557 | 0.0231 | 0.0187 | 0.0085 | 0.0322 | 0.0466 |
| Q13-6 | -0.0680 | N/A | -0.1455 | -0.0449 | -0.1464 | 0.0085 | 0.0775 | 0.0231 | 0.0784 | 0.0765 |
| Q8-7 | 0.1762 | N/A | 0.1253 | 0.1158 | 0.1246 | 0.1329 | 0.0509 | 0.0604 | 0.0516 | 0.0433 |
| Q9-7 | -0.0498 | N/A | -0.1151 | -0.1212 | -0.1120 | -0.1027 | 0.0653 | 0.0714 | 0.0622 | 0.0529 |
| Q10-9 | -0.0418 | N/A | -0.0799 | -0.1155 | -0.0837 | -0.0863 | 0.0381 | 0.0737 | 0.0419 | 0.0445 |
| Q14-9 | -0.0336 | N/A | -0.1069 | -0.0974 | -0.1086 | -0.1108 | 0.0733 | 0.0638 | 0.0750 | 0.0772 |
| Q11-10 | 0.0164 | N/A | -0.0483 | -0.0248 | -0.0426 | -0.0847 | 0.0647 | 0.0412 | 0.0590 | 0.1011 |
| Q13-12 | -0.0075 | N/A | 0.0046 | 0.0179 | 0.0065 | 0.0063 | 0.0121 | 0.0254 | 0.0140 | 0.0138 |
| Q14-13 | -0.0164 | N/A | -0.0019 | -0.0036 | -0.0085 | 0.0371 | 0.0145 | 0.0128 | 0.0079 | 0.0535 |
| **SUM** | | | | | | | **5.5323** | **3.2120** | **3.7749** | **3.4916** |

For this case, LAV generates best estimation rather than LTAV in other cases. The reason is that LTAV eliminate some correct data.

**Comparison**

Full redundancy and median redundancy estimation with the same condition of error are presented in Table 5.238. Full redundancy cases still generate better estimation.

Table 5.238: Estimation errors of case 2, multi large, 14-bus AC system, comparison

| | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| SUM | 5.1852 | 3.6738 | 2.6666 | 2.5734 | 2.5166 |
| SUM | N/A | 5.5323 | 3.2120 | 3.7749 | 3.4916 |

#### 5.2.2.4 Case 3: Multiple interacting bad data

The three interacting location are selected the same as DC counterpart to ensure consistency. They are: $P_4$, $P_{4-7}$ and $P_{7-9}$. Various values are applied to them considering different kinds of cases for full and median redundancy.

**Two reverse bad data**

The first case is three reverse bad data: $P_4 = 0.4349$, $P_{4-7} = -0.2526$ and $P_{7-9} = -0.2967$.

**Full redundancy**

Full redundancy take all measurements as available value, the system state variables are presented in Table 5.239, Table 5.240 gives the estimation results and error with actual value.

Table 5.239: State Variable of case 3, multi reverse, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0812 | -0.0830 | -0.0816 | -0.0826 | **V1** | 1.0863 | 1.0765 | 1.0893 | 1.0837 |
| $\theta 3$ | -0.2224 | -0.2215 | -0.2175 | -0.2197 | **V2** | 1.0707 | 1.0632 | 1.0736 | 1.0681 |
| $\theta 4$ | -0.1538 | -0.1673 | -0.1713 | -0.1736 | **V3** | 1.0223 | 1.0244 | 1.0283 | 1.0227 |
| $\theta 5$ | -0.1343 | -0.1417 | -0.1423 | -0.1458 | **V4** | 1.0435 | 1.0318 | 1.0407 | 1.0361 |
| $\theta 6$ | -0.2175 | -0.2283 | -0.2086 | -0.2301 | **V5** | 1.0465 | 1.0352 | 1.0465 | 1.0416 |
| $\theta 7$ | -0.2117 | -0.2211 | -0.1950 | -0.2193 | **V6** | 1.0634 | 1.0573 | 1.0628 | 1.0701 |
| $\theta 8$ | -0.2170 | -0.2209 | -0.1941 | -0.2186 | **V7** | 1.0690 | 1.0542 | 1.0673 | 1.0667 |
| $\theta 9$ | -0.2236 | -0.2463 | -0.2047 | -0.2400 | **V8** | 1.0882 | 1.0758 | 1.0865 | 1.0865 |
| $\theta 10$ | -0.2236 | -0.2489 | -0.2039 | -0.2412 | **V9** | 1.0620 | 1.0475 | 1.0604 | 1.0612 |
| $\theta 11$ | -0.2262 | -0.2448 | -0.2105 | -0.2418 | **V10** | 1.0548 | 1.0378 | 1.0540 | 1.0548 |
| $\theta 12$ | -0.2137 | -0.2248 | -0.2030 | -0.2270 | **V11** | 1.0464 | 1.0356 | 1.0463 | 1.0504 |
| $\theta 13$ | -0.2267 | -0.2391 | -0.2152 | -0.2403 | **V12** | 1.0541 | 1.0546 | 1.0545 | 1.0626 |
| $\theta 14$ | -0.2417 | -0.2627 | -0.2223 | -0.2598 | **V13** | 1.0418 | 1.0447 | 1.0421 | 1.0491 |
| | | | | | **V14** | 1.0345 | 1.0309 | 1.0350 | 1.0362 |

Table 5.240: Estimation Results and Errors of case 3, multi reverse, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0863 | 1.0765 | 1.0893 | 1.0837 | 0.0086 | 0.0263 | 0.0165 | 0.0293 | 0.0237 |
| V2 | 1.0450 | 1.0743 | 1.0707 | 1.0632 | 1.0736 | 1.0681 | 0.0293 | 0.0257 | 0.0182 | 0.0286 | 0.0231 |
| V3 | 1.0100 | 0.9739 | 1.0223 | 1.0244 | 1.0283 | 1.0227 | 0.0361 | 0.0123 | 0.0144 | 0.0183 | 0.0127 |
| V4 | 1.0180 | 1.0318 | 1.0435 | 1.0318 | 1.0407 | 1.0361 | 0.0138 | 0.0255 | 0.0138 | 0.0227 | 0.0181 |
| V5 | 1.0200 | 1.0251 | 1.0465 | 1.0352 | 1.0465 | 1.0416 | 0.0051 | 0.0265 | 0.0152 | 0.0265 | 0.0216 |
| V6 | 1.0700 | 1.0491 | 1.0634 | 1.0573 | 1.0628 | 1.0701 | 0.0209 | 0.0066 | 0.0127 | 0.0072 | 0.0001 |
| V7 | 1.0620 | 1.0551 | 1.0690 | 1.0542 | 1.0673 | 1.0667 | 0.0069 | 0.0070 | 0.0078 | 0.0053 | 0.0047 |
| V8 | 1.0900 | 1.0955 | 1.0882 | 1.0758 | 1.0865 | 1.0865 | 0.0055 | 0.0018 | 0.0142 | 0.0035 | 0.0035 |
| V9 | 1.0560 | 1.1131 | 1.0620 | 1.0475 | 1.0604 | 1.0612 | 0.0571 | 0.0060 | 0.0085 | 0.0044 | 0.0052 |
| V10 | 1.0510 | 1.0952 | 1.0548 | 1.0378 | 1.0540 | 1.0548 | 0.0442 | 0.0038 | 0.0132 | 0.0030 | 0.0038 |
| V11 | 1.0570 | 1.0355 | 1.0464 | 1.0356 | 1.0463 | 1.0504 | 0.0215 | 0.0106 | 0.0214 | 0.0107 | 0.0066 |
| V12 | 1.0550 | 1.1034 | 1.0541 | 1.0546 | 1.0545 | 1.0626 | 0.0484 | 0.0009 | 0.0004 | 0.0005 | 0.0076 |
| V13 | 1.0500 | 1.0616 | 1.0418 | 1.0447 | 1.0421 | 1.0491 | 0.0116 | 0.0082 | 0.0053 | 0.0079 | 0.0009 |
| V14 | 1.0360 | 1.0350 | 1.0345 | 1.0309 | 1.0350 | 1.0362 | 0.0010 | 0.0015 | 0.0051 | 0.0010 | 0.0002 |
| P1 | 2.3239 | 2.3508 | 2.2440 | 2.2634 | 2.3050 | 2.3154 | 0.0269 | 0.0799 | 0.0605 | 0.0189 | 0.0085 |
| P2 | 0.1830 | 0.2747 | 0.1273 | 0.2017 | 0.2472 | 0.2567 | 0.0917 | 0.0557 | 0.0187 | 0.0642 | 0.0737 |
| P3 | -0.9420 | -1.0549 | -1.1897 | -1.0549 | -1.0302 | -1.0278 | 0.1129 | 0.2477 | 0.1129 | 0.0882 | 0.0858 |
| **P4** | **-0.4780** | **0.4349** | **-0.0562** | **-0.3782** | **-0.8059** | **-0.5863** | **0.9129** | **0.4218** | **0.0998** | **0.3279** | **0.1083** |
| P5 | -0.0760 | -0.0601 | -0.1517 | -0.0601 | -0.0779 | -0.0467 | 0.0159 | 0.0757 | 0.0159 | 0.0019 | 0.0293 |
| P6 | -0.1120 | -0.1774 | -0.1842 | -0.1774 | -0.1639 | -0.1635 | 0.0654 | 0.0722 | 0.0654 | 0.0519 | 0.0515 |
| P7 | 0.0000 | -0.0217 | -0.1577 | -0.0348 | -0.0345 | -0.0385 | 0.0217 | 0.1577 | 0.0348 | 0.0345 | 0.0385 |
| P8 | 0.0000 | 0.0171 | -0.0351 | 0.0016 | 0.0060 | 0.0047 | 0.0171 | 0.0351 | 0.0016 | 0.0060 | 0.0047 |
| P9 | -0.2950 | -0.1161 | -0.1340 | -0.2652 | -0.0538 | -0.2046 | 0.1789 | 0.1610 | 0.0298 | 0.2412 | 0.0904 |
| P10 | -0.0900 | 0.0485 | -0.0009 | -0.0825 | 0.0303 | -0.0281 | 0.1385 | 0.0891 | 0.0075 | 0.1203 | 0.0619 |
| P11 | -0.0350 | -0.1025 | -0.1028 | -0.1025 | -0.0894 | -0.1061 | 0.0675 | 0.0678 | 0.0675 | 0.0544 | 0.0711 |
| P12 | -0.0610 | 0.0907 | 0.0635 | 0.0692 | 0.0693 | 0.0684 | 0.1517 | 0.1245 | 0.1302 | 0.1303 | 0.1294 |
| P13 | -0.1350 | -0.0987 | -0.1496 | -0.0987 | -0.1474 | -0.1415 | 0.0363 | 0.0146 | 0.0363 | 0.0124 | 0.0065 |
| P14 | -0.1490 | -0.1522 | -0.1462 | -0.1522 | -0.1219 | -0.1666 | 0.0032 | 0.0028 | 0.0032 | 0.0271 | 0.0176 |
| Q1 | -0.1655 | -0.1298 | -0.2033 | -0.2399 | -0.1973 | -0.2020 | 0.0357 | 0.0378 | 0.0744 | 0.0318 | 0.0365 |
| Q2 | 0.3086 | 0.2984 | 0.2785 | 0.2984 | 0.2801 | 0.2724 | 0.0102 | 0.0301 | 0.0102 | 0.0285 | 0.0362 |
| Q3 | 0.0608 | 0.0546 | -0.0210 | 0.0546 | -0.0252 | -0.0288 | 0.0062 | 0.0818 | 0.0062 | 0.0860 | 0.0896 |
| Q4 | 0.0390 | 0.1135 | 0.0522 | 0.0640 | 0.0479 | 0.0321 | 0.0745 | 0.0132 | 0.0250 | 0.0089 | 0.0069 |
| Q5 | -0.0160 | 0.0545 | 0.1163 | 0.0545 | 0.1166 | 0.0844 | 0.0705 | 0.1323 | 0.0705 | 0.1326 | 0.1004 |
| Q6 | 0.0523 | 0.1232 | -0.0326 | -0.0868 | -0.0261 | 0.0099 | 0.0709 | 0.0849 | 0.1391 | 0.0784 | 0.0424 |
| Q7 | 0.0000 | 0.0336 | -0.0283 | -0.0578 | -0.0316 | -0.0213 | 0.0336 | 0.0283 | 0.0578 | 0.0316 | 0.0213 |
| Q8 | 0.1762 | 0.1158 | 0.1187 | 0.1318 | 0.1184 | 0.1219 | 0.0604 | 0.0575 | 0.0444 | 0.0578 | 0.0543 |
| Q9 | -0.1660 | -0.1301 | -0.1656 | -0.1739 | -0.1754 | -0.1596 | 0.0359 | 0.0004 | 0.0079 | 0.0094 | 0.0064 |
| Q10 | -0.0580 | 0.0235 | -0.0447 | -0.0742 | -0.0518 | -0.0464 | 0.0815 | 0.0133 | 0.0162 | 0.0062 | 0.0116 |
| Q11 | -0.0180 | 0.0064 | -0.0868 | -0.0748 | -0.0880 | -0.0778 | 0.0244 | 0.0688 | 0.0568 | 0.0700 | 0.0598 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q12 | -0.0160 | 0.0357 | -0.0439 | -0.0306 | -0.0414 | -0.0353 | 0.0517 | 0.0279 | 0.0146 | 0.0254 | 0.0193 |
| Q13 | -0.0580 | -0.0217 | -0.0991 | -0.0217 | -0.0954 | -0.0861 | 0.0363 | 0.0411 | 0.0363 | 0.0374 | 0.0281 |
| Q14 | -0.0500 | -0.0652 | -0.0558 | -0.0298 | -0.0598 | -0.0532 | 0.0152 | 0.0058 | 0.0202 | 0.0098 | 0.0032 |
| P1-2 | 1.5688 | 1.5835 | 1.5443 | 1.5400 | 1.5606 | 1.5617 | 0.0147 | 0.0245 | 0.0288 | 0.0082 | 0.0071 |
| P1-5 | 0.7551 | 0.7157 | 0.6997 | 0.7233 | 0.7444 | 0.7537 | 0.0394 | 0.0554 | 0.0318 | 0.0107 | 0.0014 |
| P2-3 | 0.7324 | 0.7768 | 0.8079 | 0.7777 | 0.7819 | 0.7808 | 0.0444 | 0.0755 | 0.0453 | 0.0495 | 0.0484 |
| P2-4 | 0.5613 | 0.5039 | 0.4684 | 0.5350 | 0.5789 | 0.5799 | 0.0574 | 0.0929 | 0.0263 | 0.0176 | 0.0186 |
| P2-5 | 0.4152 | 0.3618 | 0.3557 | 0.3888 | 0.4068 | 0.4170 | 0.0534 | 0.0595 | 0.0264 | 0.0084 | 0.0018 |
| P3-4 | -0.2329 | -0.2734 | -0.4092 | -0.3026 | -0.2738 | -0.2726 | 0.0405 | 0.1763 | 0.0697 | 0.0409 | 0.0397 |
| P4-5 | -0.6116 | -0.7588 | -0.4780 | -0.6115 | -0.7194 | -0.6834 | 0.1472 | 0.1336 | 0.0001 | 0.1078 | 0.0718 |
| **P4-7** | **0.2807** | **-0.2526** | **0.3156** | **0.2863** | **0.1285** | **0.2470** | **0.5333** | **0.0349** | **0.0056** | **0.1522** | **0.0337** |
| P4-9 | 0.1608 | 0.1771 | 0.1434 | 0.1584 | 0.0683 | 0.1354 | 0.0163 | 0.0174 | 0.0024 | 0.0925 | 0.0254 |
| P5-6 | 0.4409 | 0.4032 | 0.3938 | 0.4032 | 0.3136 | 0.3996 | 0.0377 | 0.0471 | 0.0377 | 0.1273 | 0.0413 |
| P6-11 | 0.0735 | 0.1420 | 0.0746 | 0.1191 | 0.0429 | 0.0954 | 0.0685 | 0.0011 | 0.0456 | 0.0306 | 0.0219 |
| P6-12 | 0.0779 | -0.0077 | 0.0014 | -0.0077 | -0.0063 | 0.0010 | 0.0856 | 0.0765 | 0.0856 | 0.0842 | 0.0769 |
| P6-13 | 0.1775 | 0.1724 | 0.1335 | 0.1144 | 0.1131 | 0.1396 | 0.0051 | 0.0440 | 0.0631 | 0.0644 | 0.0379 |
| P7-8 | 0.0000 | -0.0121 | 0.0351 | -0.0016 | -0.0060 | -0.0047 | 0.0121 | 0.0351 | 0.0016 | 0.0060 | 0.0047 |
| **P7-9** | **0.2807** | **-0.2967** | **0.1228** | **0.2531** | **0.0999** | **0.2131** | **0.5774** | **0.1579** | **0.0276** | **0.1808** | **0.0676** |
| P9-10 | 0.0523 | 0.0679 | 0.0302 | 0.0679 | 0.0172 | 0.0401 | 0.0156 | 0.0221 | 0.0156 | 0.0351 | 0.0122 |
| P9-14 | 0.0943 | 0.0511 | 0.1021 | 0.0784 | 0.0973 | 0.1039 | 0.0432 | 0.0078 | 0.0159 | 0.0030 | 0.0096 |
| P10-11 | -0.0379 | -0.0394 | 0.0290 | -0.0150 | 0.0473 | 0.0118 | 0.0015 | 0.0669 | 0.0229 | 0.0852 | 0.0497 |
| P12-13 | 0.0161 | 0.0079 | 0.0648 | 0.0615 | 0.0628 | 0.0693 | 0.0082 | 0.0487 | 0.0454 | 0.0467 | 0.0532 |
| P13-14 | 0.0564 | 0.0878 | 0.0460 | 0.0755 | 0.0262 | 0.0648 | 0.0314 | 0.0104 | 0.0191 | 0.0302 | 0.0084 |
| P2-1 | -1.5259 | -1.4712 | -1.5048 | -1.4997 | -1.5204 | -1.5211 | 0.0547 | 0.0211 | 0.0262 | 0.0055 | 0.0048 |
| P5-1 | -0.7275 | -0.6720 | -0.6771 | -0.6987 | -0.7189 | -0.7273 | 0.0555 | 0.0504 | 0.0288 | 0.0086 | 0.0002 |
| P3-2 | -0.7091 | -0.7523 | -0.7805 | -0.7523 | -0.7564 | -0.7552 | 0.0432 | 0.0714 | 0.0432 | 0.0473 | 0.0461 |
| P4-2 | -0.5445 | -0.5406 | -0.4572 | -0.5202 | -0.5619 | -0.5628 | 0.0039 | 0.0873 | 0.0243 | 0.0174 | 0.0183 |
| P5-2 | -0.4061 | -0.4668 | -0.3494 | -0.3810 | -0.3985 | -0.4083 | 0.0607 | 0.0567 | 0.0251 | 0.0076 | 0.0022 |
| P4-3 | 0.2366 | 0.1809 | 0.4201 | 0.3089 | 0.2786 | 0.2774 | 0.0557 | 0.1835 | 0.0723 | 0.0420 | 0.0408 |
| P5-4 | 0.6167 | 0.6164 | 0.4809 | 0.6164 | 0.7259 | 0.6893 | 0.0003 | 0.1358 | 0.0003 | 0.1092 | 0.0726 |
| P7-4 | -0.2807 | -0.2041 | -0.3156 | -0.2863 | -0.1285 | -0.2470 | 0.0766 | 0.0349 | 0.0056 | 0.1522 | 0.0337 |
| P9-4 | -0.1608 | -0.1993 | -0.1434 | -0.1584 | -0.0683 | -0.1354 | 0.0385 | 0.0174 | 0.0024 | 0.0925 | 0.0254 |
| P6-5 | -0.4409 | -0.4223 | -0.3938 | -0.4032 | -0.3136 | -0.3996 | 0.0186 | 0.0471 | 0.0377 | 0.1273 | 0.0413 |
| P11-6 | -0.0730 | -0.0843 | -0.0739 | -0.1176 | -0.0423 | -0.0944 | 0.0113 | 0.0009 | 0.0446 | 0.0307 | 0.0214 |
| P12-6 | -0.0771 | -0.0212 | -0.0013 | 0.0077 | 0.0065 | -0.0009 | 0.0559 | 0.0758 | 0.0848 | 0.0836 | 0.0762 |
| P13-6 | -0.1754 | -0.2299 | -0.1318 | -0.1135 | -0.1116 | -0.1379 | 0.0545 | 0.0436 | 0.0619 | 0.0638 | 0.0375 |
| P8-7 | 0.0000 | 0.0016 | -0.0351 | 0.0016 | 0.0060 | 0.0047 | 0.0016 | 0.0351 | 0.0016 | 0.0060 | 0.0047 |
| P9-7 | -0.2807 | -0.2531 | -0.1228 | -0.2531 | -0.0999 | -0.2131 | 0.0276 | 0.1579 | 0.0276 | 0.1808 | 0.0676 |
| P10-9 | -0.0521 | 0.0029 | -0.0300 | -0.0675 | -0.0171 | -0.0399 | 0.0550 | 0.0221 | 0.0154 | 0.0350 | 0.0122 |
| P14-9 | -0.0931 | -0.0159 | -0.1005 | -0.0776 | -0.0959 | -0.1024 | 0.0772 | 0.0074 | 0.0155 | 0.0028 | 0.0093 |
| P11-10 | 0.0380 | 0.0423 | -0.0289 | 0.0151 | -0.0471 | -0.0118 | 0.0043 | 0.0669 | 0.0229 | 0.0851 | 0.0498 |
| P13-12 | -0.0161 | -0.0907 | -0.0639 | -0.0607 | -0.0620 | -0.0684 | 0.0746 | 0.0478 | 0.0446 | 0.0459 | 0.0523 |
| P14-13 | -0.0559 | -0.0930 | -0.0457 | -0.0746 | -0.0261 | -0.0641 | 0.0371 | 0.0102 | 0.0187 | 0.0298 | 0.0082 |
| Q1-2 | -0.2040 | -0.2571 | -0.2157 | -0.2571 | -0.2189 | -0.2201 | 0.0531 | 0.0117 | 0.0531 | 0.0149 | 0.0161 |
| Q1-5 | 0.0385 | 0.1560 | 0.0125 | 0.0172 | 0.0216 | 0.0181 | 0.1175 | 0.0260 | 0.0213 | 0.0169 | 0.0204 |
| Q2-3 | 0.0356 | 0.0048 | 0.0749 | 0.0268 | 0.0610 | 0.0611 | 0.0308 | 0.0393 | 0.0088 | 0.0254 | 0.0255 |
| Q2-4 | -0.0155 | 0.0219 | -0.0119 | -0.0035 | -0.0046 | -0.0102 | 0.0374 | 0.0036 | 0.0120 | 0.0109 | 0.0053 |
| Q2-5 | 0.0117 | 0.0021 | 0.0019 | 0.0158 | 0.0056 | -0.0005 | 0.0096 | 0.0098 | 0.0041 | 0.0061 | 0.0122 |
| Q3-4 | 0.0447 | 0.0891 | 0.0344 | 0.0699 | 0.0254 | 0.0202 | 0.0444 | 0.0103 | 0.0252 | 0.0193 | 0.0245 |
| Q4-5 | 0.1582 | 0.1200 | 0.0834 | 0.1200 | 0.0954 | 0.0914 | 0.0382 | 0.0748 | 0.0382 | 0.0628 | 0.0668 |
| Q4-7 | -0.0968 | -0.1669 | -0.1207 | -0.1053 | -0.1336 | -0.1494 | 0.0701 | 0.0239 | 0.0085 | 0.0368 | 0.0526 |
| Q4-9 | -0.0043 | -0.0754 | -0.0307 | -0.0237 | -0.0369 | -0.0438 | 0.0711 | 0.0264 | 0.0194 | 0.0326 | 0.0395 |
| Q5-6 | 0.1247 | 0.1491 | -0.0587 | -0.0803 | -0.0621 | -0.1097 | 0.0244 | 0.1834 | 0.2050 | 0.1868 | 0.2344 |
| Q6-11 | 0.0356 | 0.0267 | 0.0551 | 0.0595 | 0.0676 | 0.0611 | 0.0089 | 0.0195 | 0.0239 | 0.0320 | 0.0255 |
| Q6-12 | 0.0250 | 0.0152 | 0.0380 | 0.0152 | 0.0378 | 0.0311 | 0.0098 | 0.0130 | 0.0098 | 0.0128 | 0.0061 |
| Q6-13 | 0.0722 | 0.1432 | 0.1089 | 0.0449 | 0.1115 | 0.1024 | 0.0710 | 0.0367 | 0.0273 | 0.0393 | 0.0302 |
| Q7-8 | -0.1716 | -0.1570 | -0.1164 | -0.1292 | -0.1163 | -0.1196 | 0.0146 | 0.0552 | 0.0424 | 0.0553 | 0.0520 |
| Q7-9 | 0.0578 | 0.0677 | 0.0689 | 0.0677 | 0.0671 | 0.0554 | 0.0099 | 0.0111 | 0.0099 | 0.0093 | 0.0024 |
| Q9-10 | 0.0422 | 0.1216 | 0.0788 | 0.0939 | 0.0742 | 0.0659 | 0.0794 | 0.0366 | 0.0517 | 0.0320 | 0.0237 |
| Q9-14 | 0.0361 | -0.0041 | 0.0605 | 0.0280 | 0.0546 | 0.0501 | 0.0402 | 0.0244 | 0.0081 | 0.0185 | 0.0140 |
| Q10-11 | -0.0162 | 0.0186 | 0.0335 | 0.0186 | 0.0220 | 0.0191 | 0.0348 | 0.0497 | 0.0348 | 0.0382 | 0.0353 |
| Q12-13 | 0.0075 | 0.0493 | -0.0062 | -0.0154 | -0.0039 | -0.0044 | 0.0418 | 0.0137 | 0.0229 | 0.0114 | 0.0119 |
| Q13-14 | 0.0175 | 0.0053 | -0.0006 | 0.0053 | 0.0085 | 0.0076 | 0.0122 | 0.0181 | 0.0122 | 0.0090 | 0.0099 |
| Q2-1 | 0.2768 | 0.2876 | 0.2137 | 0.2593 | 0.2181 | 0.2220 | 0.0108 | 0.0631 | 0.0175 | 0.0587 | 0.0548 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q5-1 | 0.0223 | -0.0360 | -0.0309 | -0.0252 | -0.0285 | -0.0203 | 0.0583 | 0.0532 | 0.0475 | 0.0508 | 0.0426 |
| Q3-2 | 0.0160 | -0.0414 | -0.0555 | -0.0153 | -0.0507 | -0.0490 | 0.0574 | 0.0715 | 0.0313 | 0.0667 | 0.0650 |
| Q4-2 | 0.0302 | 0.0354 | -0.0303 | -0.0263 | -0.0200 | -0.0130 | 0.0052 | 0.0605 | 0.0565 | 0.0502 | 0.0432 |
| Q5-2 | -0.0210 | 0.0151 | -0.0600 | -0.0682 | -0.0581 | -0.0497 | 0.0361 | 0.0390 | 0.0472 | 0.0371 | 0.0287 |
| Q4-3 | -0.0484 | 0.0809 | -0.0340 | -0.0809 | -0.0405 | -0.0350 | 0.1293 | 0.0144 | 0.0325 | 0.0079 | 0.0134 |
| Q5-4 | 0.1420 | 0.1087 | -0.0743 | -0.1046 | -0.0749 | -0.0728 | 0.0333 | 0.2163 | 0.2466 | 0.2169 | 0.2148 |
| Q7-4 | 0.1138 | 0.1232 | 0.1421 | 0.1232 | 0.1401 | 0.1653 | 0.0094 | 0.0283 | 0.0094 | 0.0263 | 0.0515 |
| Q9-4 | 0.0173 | 0.0132 | 0.0413 | 0.0367 | 0.0399 | 0.0540 | 0.0041 | 0.0240 | 0.0194 | 0.0226 | 0.0367 |
| Q6-5 | -0.0805 | -0.1772 | 0.0927 | 0.1173 | 0.0840 | 0.1469 | 0.0967 | 0.1732 | 0.1978 | 0.1645 | 0.2274 |
| Q11-6 | -0.0344 | -0.0563 | -0.0536 | -0.0563 | -0.0665 | -0.0588 | 0.0219 | 0.0192 | 0.0219 | 0.0321 | 0.0244 |
| Q12-6 | -0.0235 | -0.1132 | -0.0377 | -0.0151 | -0.0374 | -0.0309 | 0.0897 | 0.0142 | 0.0084 | 0.0139 | 0.0074 |
| Q13-6 | -0.0680 | -0.0260 | -0.1055 | -0.0431 | -0.1086 | -0.0990 | 0.0420 | 0.0375 | 0.0249 | 0.0406 | 0.0310 |
| Q8-7 | 0.1762 | 0.1318 | 0.1187 | 0.1318 | 0.1184 | 0.1219 | 0.0444 | 0.0575 | 0.0444 | 0.0578 | 0.0543 |
| Q9-7 | -0.0498 | -0.0448 | -0.0670 | -0.0609 | -0.0657 | -0.0508 | 0.0050 | 0.0172 | 0.0111 | 0.0159 | 0.0010 |
| Q10-9 | -0.0418 | -0.0690 | -0.0783 | -0.0928 | -0.0738 | -0.0654 | 0.0272 | 0.0365 | 0.0510 | 0.0320 | 0.0236 |
| Q14-9 | -0.0336 | -0.0184 | -0.0571 | -0.0263 | -0.0516 | -0.0469 | 0.0152 | 0.0235 | 0.0073 | 0.0180 | 0.0133 |
| Q11-10 | 0.0164 | -0.0136 | -0.0332 | -0.0185 | -0.0215 | -0.0190 | 0.0300 | 0.0496 | 0.0349 | 0.0379 | 0.0354 |
| Q13-12 | -0.0075 | 0.0170 | 0.0070 | 0.0161 | 0.0047 | 0.0052 | 0.0245 | 0.0145 | 0.0236 | 0.0122 | 0.0127 |
| Q14-13 | -0.0164 | 0.0206 | 0.0013 | -0.0035 | -0.0083 | -0.0063 | 0.0370 | 0.0177 | 0.0129 | 0.0081 | 0.0101 |
| **SUM** | | | | | | | **4.8640** | **4.0497** | **2.2664** | **3.2721** | **2.1690** |

LAV and LTAV generate much better estimation than other estimators.

### Median redundancy

System state variables, estimation results and error are presented in the following two tables:

Table 5.241: State Variable of case 3, multi reverse, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0851 | -0.0926 | -0.0909 | -0.0858 | **V1** | 1.0523 | 1.0293 | 1.0636 | 1.0705 |
| $\theta 3$ | -0.2345 | -0.2513 | -0.2547 | -0.2313 | **V2** | 1.0358 | 1.0114 | 1.0443 | 1.0537 |
| $\theta 4$ | -0.1482 | -0.1952 | -0.2166 | -0.1806 | **V3** | 0.9918 | 0.9683 | 1.0001 | 1.0143 |
| $\theta 5$ | -0.1356 | -0.1631 | -0.1273 | -0.1483 | **V4** | 1.0086 | 0.9765 | 0.9971 | 1.0169 |
| $\theta 6$ | -0.2251 | -0.2619 | -0.1873 | -0.2300 | **V5** | 1.0063 | 0.9787 | 1.0186 | 1.0171 |
| $\theta 7$ | -0.1881 | -0.2438 | -0.1936 | -0.2246 | **V6** | 1.0124 | 0.9809 | 1.0283 | 0.9947 |
| $\theta 8$ | -0.1921 | -0.2417 | -0.1873 | -0.2234 | **V7** | 1.0268 | 1.0032 | 1.0268 | 1.0463 |
| $\theta 9$ | -0.1962 | -0.2685 | -0.1884 | -0.2450 | **V8** | 1.0458 | 1.0231 | 1.0479 | 1.0677 |
| $\theta 10$ | -0.1957 | -0.2705 | -0.1812 | -0.2463 | **V9** | 1.0146 | 0.9905 | 1.0167 | 1.0352 |
| $\theta 11$ | -0.2128 | -0.2754 | -0.1857 | -0.2488 | **V10** | 1.0070 | 0.9779 | 1.0127 | 1.0264 |
| $\theta 12$ | -0.2235 | -0.2579 | -0.1758 | -0.2309 | **V11** | 0.9972 | 0.9715 | 1.0086 | 1.0083 |
| $\theta 13$ | -0.2305 | -0.2756 | -0.1853 | -0.2442 | **V12** | 0.9981 | 0.9779 | 1.0194 | 1.0007 |
| $\theta 14$ | -0.2248 | -0.3079 | -0.1885 | -0.2677 | **V13** | 0.9860 | 0.9665 | 1.0061 | 0.9876 |
| | | | | | **V14** | 0.9718 | 0.9495 | 0.9904 | 0.9884 |

Table 5.242: Estimation Results and Errors of case 3, multi reverse, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0523 | 1.0293 | 1.0636 | 1.0705 | 0.0077 | 0.0307 | 0.0036 | 0.0105 |
| V2 | 1.0450 | N/A | 1.0358 | 1.0114 | 1.0443 | 1.0537 | 0.0092 | 0.0336 | 0.0007 | 0.0087 |
| V3 | 1.0100 | N/A | 0.9918 | 0.9683 | 1.0001 | 1.0143 | 0.0182 | 0.0417 | 0.0099 | 0.0043 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V4 | 1.0180 | N/A | 1.0086 | 0.9765 | 0.9971 | 1.0169 | 0.0094 | 0.0415 | 0.0209 | 0.0011 |
| V5 | 1.0200 | N/A | 1.0063 | 0.9787 | 1.0186 | 1.0171 | 0.0137 | 0.0413 | 0.0014 | 0.0029 |
| V6 | 1.0700 | N/A | 1.0124 | 0.9809 | 1.0283 | 0.9947 | 0.0576 | 0.0891 | 0.0417 | 0.0753 |
| V7 | 1.0620 | N/A | 1.0268 | 1.0032 | 1.0268 | 1.0463 | 0.0352 | 0.0588 | 0.0352 | 0.0157 |
| V8 | 1.0900 | N/A | 1.0458 | 1.0231 | 1.0479 | 1.0677 | 0.0442 | 0.0669 | 0.0421 | 0.0223 |
| V9 | 1.0560 | N/A | 1.0146 | 0.9905 | 1.0167 | 1.0352 | 0.0414 | 0.0655 | 0.0393 | 0.0208 |
| V10 | 1.0510 | N/A | 1.0070 | 0.9779 | 1.0127 | 1.0264 | 0.0440 | 0.0731 | 0.0383 | 0.0246 |
| V11 | 1.0570 | N/A | 0.9972 | 0.9715 | 1.0086 | 1.0083 | 0.0598 | 0.0855 | 0.0484 | 0.0487 |
| V12 | 1.0550 | N/A | 0.9981 | 0.9779 | 1.0194 | 1.0007 | 0.0569 | 0.0771 | 0.0356 | 0.0543 |
| V13 | 1.0500 | N/A | 0.9860 | 0.9665 | 1.0061 | 0.9876 | 0.0640 | 0.0835 | 0.0439 | 0.0624 |
| V14 | 1.0360 | N/A | 0.9718 | 0.9495 | 0.9904 | 0.9884 | 0.0642 | 0.0865 | 0.0456 | 0.0476 |
| P1 | 2.3239 | 2.3508 | 2.1857 | 2.3434 | 2.3052 | 2.3377 | 0.1382 | 0.0195 | 0.0187 | 0.0138 |
| P2 | 0.1830 | 0.2747 | 0.0332 | 0.2747 | 0.2793 | 0.2683 | 0.1498 | 0.0917 | 0.0963 | 0.0853 |
| P3 | -0.9420 | -1.0549 | -1.2307 | -1.0549 | -1.0343 | -1.0417 | 0.2887 | 0.1129 | 0.0923 | 0.0997 |
| **P4** | **-0.4780** | **0.4349** | **0.1322** | **-0.5866** | **-2.7847** | **-0.6640** | **0.6102** | **0.1086** | **2.3067** | **0.1860** |
| P5 | -0.0760 | -0.0601 | -0.3177 | -0.0601 | 1.5316 | -0.0554 | 0.2417 | 0.0159 | 1.6076 | 0.0206 |
| P6 | -0.1120 | -0.1774 | -0.2762 | -0.2171 | -0.2005 | -0.2003 | 0.1642 | 0.1051 | 0.0885 | 0.0883 |
| P7 | 0.0000 | -0.0217 | -0.0999 | -0.0217 | 0.0267 | -0.0358 | 0.0999 | 0.0217 | 0.0267 | 0.0358 |
| P8 | 0.0000 | 0.0171 | -0.0245 | 0.0121 | 0.0388 | 0.0075 | 0.0245 | 0.0121 | 0.0388 | 0.0075 |
| P9 | -0.2950 | -0.1161 | 0.0030 | -0.1161 | 0.0800 | -0.1373 | 0.2980 | 0.1789 | 0.3750 | 0.1577 |
| P10 | -0.0900 | 0.0485 | 0.0707 | -0.0348 | 0.0890 | -0.0029 | 0.1607 | 0.0552 | 0.1790 | 0.0871 |
| P11 | -0.0350 | -0.1025 | -0.0722 | -0.1025 | -0.0597 | -0.0952 | 0.0372 | 0.0675 | 0.0247 | 0.0602 |
| P12 | -0.0610 | 0.0907 | 0.0288 | 0.0734 | 0.0800 | 0.0688 | 0.0898 | 0.1344 | 0.1410 | 0.1298 |
| P13 | -0.1350 | -0.0987 | -0.1548 | -0.0987 | -0.0860 | -0.1151 | 0.0198 | 0.0363 | 0.0490 | 0.0199 |
| P14 | -0.1490 | -0.1522 | -0.1467 | -0.2528 | -0.0624 | -0.1878 | 0.0023 | 0.1038 | 0.0866 | 0.0388 |
| Q1 | -0.1655 | -0.1298 | -0.1521 | -0.1298 | -0.1372 | -0.1355 | 0.0134 | 0.0357 | 0.0283 | 0.0300 |
| Q2 | 0.3086 | 0.2984 | 0.3037 | 0.2984 | 0.3318 | 0.3205 | 0.0049 | 0.0102 | 0.0232 | 0.0119 |
| Q3 | 0.0608 | 0.0546 | 0.0691 | 0.0546 | 0.0861 | 0.0773 | 0.0083 | 0.0062 | 0.0253 | 0.0165 |
| Q4 | 0.0390 | 0.1135 | 0.0845 | 0.1135 | 0.1227 | 0.1125 | 0.0455 | 0.0745 | 0.0837 | 0.0735 |
| Q5 | -0.0160 | 0.0545 | 0.0209 | 0.0545 | 0.0585 | 0.0493 | 0.0369 | 0.0705 | 0.0745 | 0.0653 |
| Q6 | 0.0523 | 0.1232 | 0.0299 | -0.1751 | 0.0227 | -0.4780 | 0.0224 | 0.2274 | 0.0296 | 0.5303 |
| Q7 | 0.0000 | 0.0336 | -0.0153 | 0.0336 | 0.0094 | 0.0183 | 0.0153 | 0.0336 | 0.0094 | 0.0183 |
| Q8 | 0.1762 | 0.1158 | 0.1131 | 0.1158 | 0.1255 | 0.1298 | 0.0631 | 0.0604 | 0.0507 | 0.0464 |
| Q9 | -0.1660 | -0.1301 | -0.1784 | -0.1301 | -0.1599 | -0.1254 | 0.0124 | 0.0359 | 0.0061 | 0.0406 |
| Q10 | -0.0580 | 0.0235 | -0.0697 | -0.1013 | -0.0612 | -0.0112 | 0.0117 | 0.0433 | 0.0032 | 0.0468 |
| Q11 | -0.0180 | 0.0064 | -0.0965 | -0.0299 | -0.0937 | 0.0183 | 0.0785 | 0.0119 | 0.0757 | 0.0363 |
| Q12 | -0.0160 | 0.0357 | -0.0381 | -0.0313 | -0.0403 | 0.0176 | 0.0221 | 0.0153 | 0.0243 | 0.0336 |
| Q13 | -0.0580 | -0.0217 | -0.1128 | -0.0217 | -0.1148 | -0.0229 | 0.0548 | 0.0363 | 0.0568 | 0.0351 |
| Q14 | -0.0500 | -0.0652 | -0.1229 | -0.0652 | -0.1113 | -0.0777 | 0.0729 | 0.0152 | 0.0613 | 0.0277 |
| P1-2 | 1.5688 | 1.5835 | 1.5197 | 1.5835 | 1.6643 | 1.5854 | 0.0491 | 0.0147 | 0.0955 | 0.0166 |
| P1-5 | 0.7551 | 0.7157 | 0.6659 | 0.7599 | 0.6408 | 0.7522 | 0.0892 | 0.0048 | 0.1143 | 0.0029 |
| P2-3 | 0.7324 | 0.7768 | 0.7961 | 0.8035 | 0.8828 | 0.8013 | 0.0637 | 0.0711 | 0.1504 | 0.0689 |
| P2-4 | 0.5613 | N/A | 0.3882 | 0.5858 | 0.7647 | 0.5925 | 0.1731 | 0.0245 | 0.2034 | 0.0312 |
| P2-5 | 0.4152 | N/A | 0.3278 | 0.4228 | 0.2483 | 0.4171 | 0.0874 | 0.0076 | 0.1669 | 0.0019 |
| P3-4 | -0.2329 | -0.2734 | -0.4628 | -0.2815 | -0.1855 | -0.2679 | 0.2299 | 0.0486 | 0.0474 | 0.0350 |
| P4-5 | -0.6116 | N/A | -0.2591 | -0.6724 | -2.0717 | -0.7180 | 0.3525 | 0.0608 | 1.4601 | 0.1064 |
| **P4-7** | **0.2807** | **-0.3526** | **0.2018** | **0.2329** | **-0.1150** | **0.2290** | **0.0789** | **0.0478** | **0.3957** | **0.0517** |
| P4-9 | 0.1608 | N/A | 0.0912 | 0.1316 | -0.0530 | 0.1258 | 0.0696 | 0.0292 | 0.2138 | 0.0350 |
| P5-6 | 0.4409 | 0.4032 | 0.3878 | 0.4032 | 0.2670 | 0.3517 | 0.0531 | 0.0377 | 0.1739 | 0.0892 |
| P6-11 | 0.0735 | N/A | -0.0208 | 0.0707 | 0.0327 | 0.0507 | 0.0943 | 0.0028 | 0.0408 | 0.0228 |
| P6-12 | 0.0779 | -0.0077 | 0.0169 | -0.0077 | -0.0241 | -0.0063 | 0.0610 | 0.0856 | 0.1020 | 0.0842 |
| P6-13 | 0.1775 | N/A | 0.1154 | 0.1231 | 0.0580 | 0.1069 | 0.0621 | 0.0544 | 0.1195 | 0.0706 |
| P7-8 | 0.0000 | -0.0121 | 0.0245 | -0.0121 | -0.0388 | -0.0075 | 0.0245 | 0.0121 | 0.0388 | 0.0075 |
| **P7-9** | **0.2807** | **-0.2967** | **0.0774** | **0.2233** | **-0.0495** | **0.2007** | **0.2033** | **0.0574** | **0.3302** | **0.0800** |
| P9-10 | 0.0523 | 0.0679 | 0.0240 | 0.0679 | -0.0609 | 0.0494 | 0.0283 | 0.0156 | 0.1132 | 0.0029 |
| P9-14 | 0.0943 | N/A | 0.1475 | 0.1709 | 0.0383 | 0.1397 | 0.0532 | 0.0766 | 0.0560 | 0.0454 |
| P10-11 | -0.0379 | -0.0394 | 0.0945 | 0.0324 | 0.0279 | 0.0462 | 0.1324 | 0.0703 | 0.0658 | 0.0841 |
| P12-13 | 0.0161 | N/A | 0.0454 | 0.0657 | 0.0556 | 0.0624 | 0.0293 | 0.0496 | 0.0395 | 0.0463 |
| P13-14 | 0.0564 | 0.0878 | 0.0032 | 0.0878 | 0.0254 | 0.0526 | 0.0532 | 0.0314 | 0.0310 | 0.0038 |
| P2-1 | -1.5259 | N/A | -1.4790 | -1.5373 | -1.6166 | -1.5426 | 0.0469 | 0.0114 | 0.0907 | 0.0167 |
| P5-1 | -0.7275 | N/A | -0.6438 | -0.7299 | -0.6208 | -0.7248 | 0.0837 | 0.0024 | 0.1067 | 0.0027 |
| P3-2 | -0.7091 | N/A | -0.7679 | -0.7734 | -0.8489 | -0.7739 | 0.0588 | 0.0643 | 0.1398 | 0.0648 |
| P4-2 | -0.5445 | N/A | -0.3799 | -0.5662 | -0.7333 | -0.5740 | 0.1646 | 0.0217 | 0.1888 | 0.0295 |
| P5-2 | -0.4061 | N/A | -0.3218 | -0.4126 | -0.2448 | -0.4077 | 0.0843 | 0.0065 | 0.1613 | 0.0016 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P4-3 | 0.2366 | N/A | 0.4782 | 0.2875 | 0.1884 | 0.2732 | 0.2416 | 0.0509 | 0.0482 | 0.0366 |
| P5-4 | 0.6167 | N/A | 0.2602 | 0.6792 | 2.1301 | 0.7253 | 0.3565 | 0.0625 | 1.5134 | 0.1086 |
| P7-4 | -0.2807 | N/A | -0.2018 | -0.2329 | 0.1150 | -0.2290 | 0.0789 | 0.0478 | 0.3957 | 0.0517 |
| P9-4 | -0.1608 | N/A | -0.0912 | -0.1316 | 0.0530 | -0.1258 | 0.0696 | 0.0292 | 0.2138 | 0.0350 |
| P6-5 | -0.4409 | N/A | -0.3878 | -0.4032 | -0.2670 | -0.3517 | 0.0531 | 0.0377 | 0.1739 | 0.0892 |
| P11-6 | -0.0730 | N/A | 0.0216 | -0.0702 | -0.0319 | -0.0497 | 0.0946 | 0.0028 | 0.0411 | 0.0233 |
| P12-6 | -0.0771 | N/A | -0.0166 | 0.0077 | 0.0244 | 0.0064 | 0.0605 | 0.0848 | 0.1015 | 0.0835 |
| P13-6 | -0.1754 | N/A | -0.1131 | -0.1219 | -0.0564 | -0.1062 | 0.0623 | 0.0535 | 0.1190 | 0.0692 |
| P8-7 | 0.0000 | N/A | -0.0245 | 0.0121 | 0.0388 | 0.0075 | 0.0245 | 0.0121 | 0.0388 | 0.0075 |
| P9-7 | -0.2807 | N/A | -0.0774 | -0.2233 | 0.0495 | -0.2007 | 0.2033 | 0.0574 | 0.3302 | 0.0800 |
| P10-9 | -0.0521 | N/A | -0.0238 | -0.0673 | 0.0611 | -0.0491 | 0.0283 | 0.0152 | 0.1132 | 0.0030 |
| P14-9 | -0.0931 | N/A | -0.1438 | -0.1664 | -0.0373 | -0.1358 | 0.0507 | 0.0733 | 0.0558 | 0.0427 |
| P11-10 | 0.0380 | N/A | -0.0937 | -0.0323 | -0.0278 | -0.0456 | 0.1317 | 0.0703 | 0.0658 | 0.0836 |
| P13-12 | -0.0161 | N/A | -0.0449 | -0.0646 | -0.0549 | -0.0615 | 0.0288 | 0.0485 | 0.0388 | 0.0454 |
| P14-13 | -0.0559 | N/A | -0.0030 | -0.0864 | -0.0251 | -0.0520 | 0.0529 | 0.0305 | 0.0308 | 0.0039 |
| Q1-2 | -0.2040 | -0.2571 | -0.1965 | -0.1874 | -0.1804 | -0.2068 | 0.0075 | 0.0166 | 0.0236 | 0.0028 |
| Q1-5 | 0.0385 | 0.1560 | 0.0445 | 0.0576 | 0.0432 | 0.0713 | 0.0060 | 0.0191 | 0.0047 | 0.0328 |
| Q2-3 | 0.0356 | 0.0048 | 0.0523 | 0.0469 | 0.0467 | 0.0283 | 0.0167 | 0.0113 | 0.0111 | 0.0073 |
| Q2-4 | -0.0155 | N/A | 0.0070 | 0.0022 | 0.0369 | 0.0143 | 0.0225 | 0.0177 | 0.0524 | 0.0298 |
| Q2-5 | 0.0117 | N/A | 0.0386 | 0.0309 | 0.0395 | 0.0592 | 0.0269 | 0.0192 | 0.0278 | 0.0475 |
| Q3-4 | 0.0447 | 0.0891 | 0.0927 | 0.0608 | 0.0813 | 0.0838 | 0.0480 | 0.0161 | 0.0366 | 0.0391 |
| Q4-5 | 0.1582 | N/A | 0.1387 | 0.1740 | 0.2449 | 0.2366 | 0.0195 | 0.0158 | 0.0867 | 0.0784 |
| Q4-7 | -0.0968 | -0.1669 | -0.0854 | -0.1219 | -0.1434 | -0.1411 | 0.0114 | 0.0251 | 0.0466 | 0.0443 |
| Q4-9 | -0.0043 | N/A | -0.0090 | -0.0205 | -0.0354 | -0.0304 | 0.0047 | 0.0162 | 0.0311 | 0.0261 |
| Q5-6 | 0.1247 | 0.1491 | -0.0086 | 0.0107 | -0.0339 | 0.1113 | 0.1333 | 0.1140 | 0.1586 | 0.0134 |
| Q6-11 | 0.0356 | N/A | 0.0879 | 0.0126 | 0.0859 | -0.0915 | 0.0523 | 0.0230 | 0.0503 | 0.1271 |
| Q6-12 | 0.0250 | 0.0152 | 0.0487 | 0.0152 | 0.0474 | -0.0203 | 0.0237 | 0.0098 | 0.0224 | 0.0453 |
| Q6-13 | 0.0722 | N/A | 0.1465 | 0.0465 | 0.1451 | 0.0007 | 0.0743 | 0.0257 | 0.0729 | 0.0715 |
| Q7-8 | -0.1716 | -0.1570 | -0.1109 | -0.1135 | -0.1227 | -0.1272 | 0.0607 | 0.0581 | 0.0489 | 0.0444 |
| Q7-9 | 0.0578 | 0.0677 | 0.1139 | 0.1187 | 0.0951 | 0.1079 | 0.0561 | 0.0609 | 0.0373 | 0.0501 |
| Q9-10 | 0.0422 | 0.1216 | 0.0822 | 0.1216 | 0.0714 | 0.0889 | 0.0400 | 0.0794 | 0.0292 | 0.0467 |
| Q9-14 | 0.0361 | N/A | 0.0927 | 0.0723 | 0.0808 | 0.1145 | 0.0566 | 0.0362 | 0.0447 | 0.0784 |
| Q10-11 | -0.0162 | 0.0186 | 0.0119 | 0.0186 | 0.0095 | 0.0769 | 0.0281 | 0.0348 | 0.0257 | 0.0931 |
| Q12-13 | 0.0075 | N/A | 0.0100 | -0.0162 | 0.0064 | -0.0028 | 0.0025 | 0.0237 | 0.0011 | 0.0103 |
| Q13-14 | 0.0175 | 0.0053 | 0.0388 | 0.0053 | 0.0332 | -0.0273 | 0.0213 | 0.0122 | 0.0157 | 0.0448 |
| Q2-1 | 0.2768 | N/A | 0.2059 | 0.2185 | 0.2087 | 0.2186 | 0.0709 | 0.0583 | 0.0681 | 0.0582 |
| Q5-1 | 0.0223 | N/A | -0.0575 | -0.0328 | -0.0670 | -0.0653 | 0.0798 | 0.0551 | 0.0893 | 0.0876 |
| Q3-2 | 0.0160 | N/A | -0.0236 | -0.0062 | 0.0048 | -0.0065 | 0.0396 | 0.0222 | 0.0112 | 0.0225 |
| Q4-2 | 0.0302 | N/A | -0.0530 | -0.0100 | -0.0123 | -0.0311 | 0.0832 | 0.0402 | 0.0425 | 0.0613 |
| Q5-2 | -0.0210 | N/A | -0.0924 | -0.0683 | -0.1024 | -0.1047 | 0.0714 | 0.0473 | 0.0814 | 0.0837 |
| Q4-3 | -0.0484 | N/A | -0.0791 | -0.0696 | -0.0994 | -0.0968 | 0.0307 | 0.0212 | 0.0510 | 0.0484 |
| Q5-4 | 0.1420 | N/A | -0.1351 | -0.1527 | -0.0605 | -0.2134 | 0.2771 | 0.2947 | 0.2025 | 0.3554 |
| Q7-4 | 0.1138 | N/A | 0.0951 | 0.1367 | 0.1504 | 0.1554 | 0.0187 | 0.0229 | 0.0366 | 0.0416 |
| Q9-4 | 0.0173 | N/A | 0.0134 | 0.0306 | 0.0376 | 0.0391 | 0.0039 | 0.0133 | 0.0203 | 0.0218 |
| Q6-5 | -0.0805 | N/A | 0.0435 | 0.0291 | 0.0503 | -0.0804 | 0.1240 | 0.1096 | 0.1308 | 0.0001 |
| Q11-6 | -0.0344 | N/A | -0.0863 | -0.0115 | -0.0843 | 0.0937 | 0.0519 | 0.0229 | 0.0499 | 0.1281 |
| Q12-6 | -0.0235 | N/A | -0.0480 | -0.0151 | -0.0467 | 0.0204 | 0.0245 | 0.0084 | 0.0232 | 0.0439 |
| Q13-6 | -0.0680 | N/A | -0.1421 | -0.0441 | -0.1421 | 0.0008 | 0.0741 | 0.0239 | 0.0741 | 0.0688 |
| Q8-7 | 0.1762 | N/A | 0.1131 | 0.1158 | 0.1255 | 0.1298 | 0.0631 | 0.0604 | 0.0507 | 0.0464 |
| Q9-7 | -0.0498 | N/A | -0.1119 | -0.1117 | -0.0939 | -0.1027 | 0.0621 | 0.0619 | 0.0441 | 0.0529 |
| Q10-9 | -0.0418 | N/A | -0.0816 | -0.1199 | -0.0706 | -0.0881 | 0.0398 | 0.0781 | 0.0288 | 0.0463 |
| Q14-9 | -0.0336 | N/A | -0.0847 | -0.0628 | -0.0788 | -0.1062 | 0.0511 | 0.0292 | 0.0452 | 0.0726 |
| Q11-10 | 0.0164 | N/A | -0.0102 | -0.0183 | -0.0093 | -0.0754 | 0.0266 | 0.0347 | 0.0257 | 0.0918 |
| Q13-12 | -0.0075 | N/A | -0.0095 | 0.0171 | -0.0058 | 0.0036 | 0.0020 | 0.0246 | 0.0017 | 0.0111 |
| Q14-13 | -0.0164 | N/A | -0.0382 | -0.0024 | -0.0326 | 0.0286 | 0.0218 | 0.0140 | 0.0162 | 0.0450 |
| **SUM** | | | | | | | **5.9810** | **3.5747** | **10.7832** | **3.4798** |

The lack of raw data exaggerates the error of LTS, by detecting wrong bad data and keeping bad data, LTS generate even worse estimation than WLS.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.243.

Table 5.243: Estimation errors of case 3, multi reverse, 14-bus AC system, comparison

| | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| **SUM** | **4.8640** | **4.0497** | **2.2664** | **3.2721** | **2.1690** |
| **SUM** | **N/A** | **5.9810** | **3.5747** | **10.7832** | **3.4798** |

## Two reverse one large bad data

In the non-conforming case, two reverse one large bad data, $P_4 = 0.4608$, $P_{4-7} = -0.2095$ and $P_{7-9} = 0.9871$ are used to replace the original data. As presented, the last one is large error.

### Full redundancy

The five system state variables are presented in Table 5.244, Table 5.245 shows the results and errors between estimation and actual value.

Table 5.244: State Variable of case 2, two reverse, 14-bus AC system, full redundancy

| | WLS | LAV | LTS | LTAV | | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0811 | -0.0849 | -0.0816 | -0.0826 | **V1** | 1.0875 | 1.0768 | 1.0896 | 1.0837 |
| $\theta 3$ | -0.2216 | -0.2235 | -0.2173 | -0.2197 | **V2** | 1.0718 | 1.0630 | 1.0739 | 1.0681 |
| $\theta 4$ | -0.1543 | -0.1693 | -0.1712 | -0.1736 | **V3** | 1.0236 | 1.0245 | 1.0286 | 1.0227 |
| $\theta 5$ | -0.1358 | -0.1437 | -0.1428 | -0.1458 | **V4** | 1.0445 | 1.0320 | 1.0410 | 1.0361 |
| $\theta 6$ | -0.2349 | -0.2322 | -0.2165 | -0.2301 | **V5** | 1.0469 | 1.0354 | 1.0466 | 1.0416 |
| $\theta 7$ | -0.2057 | -0.2277 | -0.1946 | -0.2193 | **V6** | 1.0626 | 1.0590 | 1.0629 | 1.0701 |
| $\theta 8$ | -0.1979 | -0.2258 | -0.1877 | -0.2186 | **V7** | 1.0710 | 1.0542 | 1.0676 | 1.0667 |
| $\theta 9$ | -0.2476 | -0.2577 | -0.2193 | -0.2400 | **V8** | 1.0905 | 1.0758 | 1.0869 | 1.0865 |
| $\theta 10$ | -0.2526 | -0.2596 | -0.2192 | -0.2412 | **V9** | 1.0638 | 1.0476 | 1.0606 | 1.0612 |
| $\theta 11$ | -0.2515 | -0.2521 | -0.2228 | -0.2418 | **V10** | 1.0545 | 1.0362 | 1.0538 | 1.0548 |
| $\theta 12$ | -0.2325 | -0.2264 | -0.2114 | -0.2270 | **V11** | 1.0449 | 1.0355 | 1.0460 | 1.0504 |
| $\theta 13$ | -0.2474 | -0.2427 | -0.2245 | -0.2403 | **V12** | 1.0524 | 1.0575 | 1.0542 | 1.0626 |
| $\theta 14$ | -0.2749 | -0.2700 | -0.2374 | -0.2598 | **V13** | 1.0396 | 1.0455 | 1.0417 | 1.0491 |
| | | | | | **V14** | 1.0307 | 1.0299 | 1.0341 | 1.0362 |

Table 5.245: Estimation Results and Errors of case 3, two reverse, 14-bus AC system, full redundancy

| | Actual | Meas. | WLS | LAV | LTS | LTAV | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0875 | 1.0768 | 1.0896 | 1.0837 | 0.0086 | 0.0275 | 0.0168 | 0.0296 | 0.0237 |
| V2 | 1.0450 | 1.0743 | 1.0718 | 1.0630 | 1.0739 | 1.0681 | 0.0293 | 0.0268 | 0.0180 | 0.0289 | 0.0231 |
| V3 | 1.0100 | 0.9739 | 1.0236 | 1.0245 | 1.0286 | 1.0227 | 0.0361 | 0.0136 | 0.0145 | 0.0186 | 0.0127 |
| V4 | 1.0180 | 1.0318 | 1.0445 | 1.0320 | 1.0410 | 1.0361 | 0.0138 | 0.0265 | 0.0140 | 0.0230 | 0.0181 |
| V5 | 1.0200 | 1.0251 | 1.0469 | 1.0354 | 1.0466 | 1.0416 | 0.0051 | 0.0269 | 0.0154 | 0.0266 | 0.0216 |
| V6 | 1.0700 | 1.0491 | 1.0626 | 1.0590 | 1.0629 | 1.0701 | 0.0209 | 0.0074 | 0.0110 | 0.0071 | 0.0001 |
| V7 | 1.0620 | 1.0551 | 1.0710 | 1.0542 | 1.0676 | 1.0667 | 0.0069 | 0.0090 | 0.0078 | 0.0056 | 0.0047 |
| V8 | 1.0900 | 1.0955 | 1.0905 | 1.0758 | 1.0869 | 1.0865 | 0.0055 | 0.0005 | 0.0142 | 0.0031 | 0.0035 |
| V9 | 1.0560 | 1.1131 | 1.0638 | 1.0476 | 1.0606 | 1.0612 | 0.0571 | 0.0078 | 0.0084 | 0.0046 | 0.0052 |
| V10 | 1.0510 | 1.0952 | 1.0545 | 1.0362 | 1.0538 | 1.0548 | 0.0442 | 0.0035 | 0.0148 | 0.0028 | 0.0038 |
| V11 | 1.0570 | 1.0355 | 1.0449 | 1.0355 | 1.0460 | 1.0504 | 0.0215 | 0.0121 | 0.0215 | 0.0110 | 0.0066 |
| V12 | 1.0550 | 1.1034 | 1.0524 | 1.0575 | 1.0542 | 1.0626 | 0.0484 | 0.0026 | 0.0025 | 0.0008 | 0.0076 |
| V13 | 1.0500 | 1.0616 | 1.0396 | 1.0455 | 1.0417 | 1.0491 | 0.0116 | 0.0104 | 0.0045 | 0.0083 | 0.0009 |
| V14 | 1.0360 | 1.0350 | 1.0307 | 1.0299 | 1.0341 | 1.0362 | 0.0010 | 0.0053 | 0.0061 | 0.0019 | 0.0002 |
| P1 | 2.3239 | 2.3508 | 2.2541 | 2.3096 | 2.3082 | 2.3154 | 0.0269 | 0.0698 | 0.0143 | 0.0157 | 0.0085 |
| P2 | 0.1830 | 0.2747 | 0.1399 | 0.1679 | 0.2502 | 0.2567 | 0.0917 | 0.0431 | 0.0151 | 0.0672 | 0.0737 |
| P3 | -0.9420 | -1.0549 | -1.1803 | -1.0549 | -1.0295 | -1.0278 | 0.1129 | 0.2383 | 0.1129 | 0.0875 | 0.0858 |
| **P4** | **-0.4780** | **0.4349** | **-0.0322** | **-0.3354** | **-0.7623** | **-0.5863** | **0.9129** | **0.4458** | **0.1426** | **0.2843** | **0.1083** |
| P5 | -0.0760 | -0.0601 | -0.1188 | -0.0601 | -0.0646 | -0.0467 | 0.0159 | 0.0428 | 0.0159 | 0.0114 | 0.0293 |
| P6 | -0.1120 | -0.1774 | -0.1889 | -0.1774 | -0.1654 | -0.1635 | 0.0654 | 0.0769 | 0.0654 | 0.0534 | 0.0515 |
| P7 | 0.0000 | -0.0217 | 0.1017 | -0.0217 | 0.0828 | -0.0385 | 0.0217 | 0.1017 | 0.0217 | 0.0828 | 0.0385 |
| P8 | 0.0000 | 0.0171 | 0.0514 | 0.0121 | 0.0452 | 0.0047 | 0.0171 | 0.0514 | 0.0121 | 0.0452 | 0.0047 |
| P9 | -0.2950 | -0.1161 | -0.3867 | -0.3432 | -0.2265 | -0.2046 | 0.1789 | 0.0917 | 0.0482 | 0.0685 | 0.0904 |
| P10 | -0.0900 | 0.0485 | -0.0828 | -0.1012 | 0.0065 | -0.0281 | 0.1385 | 0.0072 | 0.0112 | 0.0965 | 0.0619 |
| P11 | -0.0350 | -0.1025 | -0.1252 | -0.1025 | -0.0961 | -0.1061 | 0.0675 | 0.0902 | 0.0675 | 0.0611 | 0.0711 |
| P12 | -0.0610 | 0.0907 | 0.0629 | 0.0907 | 0.0694 | 0.0684 | 0.1517 | 0.1239 | 0.1517 | 0.1304 | 0.1294 |
| P13 | -0.1350 | -0.0987 | -0.1478 | -0.0987 | -0.1450 | -0.1415 | 0.0363 | 0.0128 | 0.0363 | 0.0100 | 0.0065 |
| P14 | -0.1490 | -0.1522 | -0.2172 | -0.1522 | -0.1398 | -0.1666 | 0.0032 | 0.0682 | 0.0032 | 0.0092 | 0.0176 |
| Q1 | -0.1655 | -0.1298 | -0.2013 | -0.2404 | -0.1967 | -0.2020 | 0.0357 | 0.0358 | 0.0749 | 0.0312 | 0.0365 |
| Q2 | 0.3086 | 0.2984 | 0.2795 | 0.2984 | 0.2806 | 0.2724 | 0.0102 | 0.0291 | 0.0102 | 0.0280 | 0.0362 |
| Q3 | 0.0608 | 0.0546 | -0.0222 | 0.0546 | -0.0254 | -0.0288 | 0.0062 | 0.0830 | 0.0062 | 0.0862 | 0.0896 |
| Q4 | 0.0390 | 0.1135 | 0.0507 | 0.0718 | 0.0481 | 0.0321 | 0.0745 | 0.0117 | 0.0328 | 0.0091 | 0.0069 |
| Q5 | -0.0160 | 0.0545 | 0.1238 | 0.0545 | 0.1190 | 0.0844 | 0.0705 | 0.1398 | 0.0705 | 0.1350 | 0.1004 |
| Q6 | 0.0523 | 0.1232 | -0.0454 | -0.0725 | -0.0319 | 0.0099 | 0.0709 | 0.0977 | 0.1248 | 0.0842 | 0.0424 |
| Q7 | 0.0000 | 0.0336 | -0.0174 | -0.0578 | -0.0284 | -0.0213 | 0.0336 | 0.0174 | 0.0578 | 0.0284 | 0.0213 |
| Q8 | 0.1762 | 0.1158 | 0.1213 | 0.1318 | 0.1194 | 0.1219 | 0.0604 | 0.0549 | 0.0444 | 0.0568 | 0.0543 |
| Q9 | -0.1660 | -0.1301 | -0.1459 | -0.1394 | -0.1679 | -0.1596 | 0.0359 | 0.0201 | 0.0266 | 0.0019 | 0.0064 |
| Q10 | -0.0580 | 0.0235 | -0.0337 | -0.0951 | -0.0469 | -0.0464 | 0.0815 | 0.0243 | 0.0371 | 0.0111 | 0.0116 |
| Q11 | -0.0180 | 0.0064 | -0.0852 | -0.0746 | -0.0872 | -0.0778 | 0.0244 | 0.0672 | 0.0566 | 0.0692 | 0.0598 |
| Q12 | -0.0160 | 0.0357 | -0.0483 | -0.0311 | -0.0436 | -0.0353 | 0.0517 | 0.0323 | 0.0151 | 0.0276 | 0.0193 |
| Q13 | -0.0580 | -0.0217 | -0.1033 | -0.0276 | -0.0979 | -0.0861 | 0.0363 | 0.0453 | 0.0304 | 0.0399 | 0.0281 |
| Q14 | -0.0500 | -0.0652 | -0.0463 | -0.0388 | -0.0565 | -0.0532 | 0.0152 | 0.0037 | 0.0112 | 0.0065 | 0.0032 |
| P1-2 | 1.5688 | 1.5835 | 1.5455 | 1.5761 | 1.5608 | 1.5617 | 0.0147 | 0.0233 | 0.0073 | 0.0080 | 0.0071 |
| P1-5 | 0.7551 | 0.7157 | 0.7085 | 0.7335 | 0.7474 | 0.7537 | 0.0394 | 0.0466 | 0.0216 | 0.0077 | 0.0014 |
| P2-3 | 0.7324 | 0.7768 | 0.8057 | 0.7777 | 0.7815 | 0.7808 | 0.0444 | 0.0733 | 0.0453 | 0.0491 | 0.0484 |
| P2-4 | 0.5613 | 0.5039 | 0.4736 | 0.5351 | 0.5788 | 0.5799 | 0.0574 | 0.0877 | 0.0262 | 0.0175 | 0.0186 |
| P2-5 | 0.4152 | 0.3618 | 0.3666 | 0.3890 | 0.4105 | 0.4170 | 0.0534 | 0.0486 | 0.0262 | 0.0047 | 0.0018 |
| P3-4 | -0.2329 | -0.2734 | -0.4018 | -0.3026 | -0.2734 | -0.2726 | 0.0405 | 0.1689 | 0.0697 | 0.0405 | 0.0397 |
| P4-5 | -0.6116 | -0.7588 | -0.4548 | -0.6115 | -0.7042 | -0.6834 | 0.1472 | 0.1568 | 0.0001 | 0.0926 | 0.0718 |
| **P4-7** | **0.2807** | **-0.2526** | **0.2806** | **0.3106** | **0.1269** | **0.2470** | **0.5333** | **0.0001** | **0.0299** | **0.1538** | **0.0337** |
| P4-9 | 0.1608 | 0.1771 | 0.1919 | 0.1771 | 0.0986 | 0.1354 | 0.0163 | 0.0311 | 0.0163 | 0.0622 | 0.0254 |
| P5-6 | 0.4409 | 0.4032 | 0.4690 | 0.4130 | 0.3488 | 0.3996 | 0.0377 | 0.0281 | 0.0279 | 0.0921 | 0.0413 |
| P6-11 | 0.0735 | 0.1420 | 0.1125 | 0.1383 | 0.0638 | 0.0954 | 0.0685 | 0.0390 | 0.0648 | 0.0097 | 0.0219 |
| P6-12 | 0.0779 | -0.0077 | 0.0078 | -0.0182 | -0.0040 | 0.0010 | 0.0856 | 0.0701 | 0.0961 | 0.0819 | 0.0769 |
| P6-13 | 0.1775 | 0.1724 | 0.1598 | 0.1154 | 0.1235 | 0.1396 | 0.0051 | 0.0177 | 0.0621 | 0.0540 | 0.0379 |
| P7-8 | 0.0000 | -0.0121 | -0.0514 | -0.0121 | -0.0452 | -0.0047 | 0.0121 | 0.0514 | 0.0121 | 0.0452 | 0.0047 |
| **P7-9** | **0.2807** | **0.9967** | **0.4338** | **0.3010** | **0.2549** | **0.2131** | **0.7160** | **0.1531** | **0.0203** | **0.0258** | **0.0676** |
| P9-10 | 0.0523 | 0.0679 | 0.0973 | 0.0679 | 0.0267 | 0.0401 | 0.0156 | 0.0450 | 0.0156 | 0.0256 | 0.0122 |
| P9-14 | 0.0943 | 0.0511 | 0.1416 | 0.0670 | 0.1002 | 0.1039 | 0.0432 | 0.0473 | 0.0273 | 0.0059 | 0.0096 |
| P10-11 | -0.0379 | -0.0394 | 0.0140 | -0.0338 | 0.0330 | 0.0118 | 0.0015 | 0.0519 | 0.0041 | 0.0709 | 0.0497 |
| P12-13 | 0.0161 | 0.0079 | 0.0705 | 0.0725 | 0.0652 | 0.0693 | 0.0082 | 0.0544 | 0.0564 | 0.0491 | 0.0532 |
| P13-14 | 0.0564 | 0.0878 | 0.0793 | 0.0871 | 0.0413 | 0.0648 | 0.0314 | 0.0229 | 0.0307 | 0.0151 | 0.0084 |
| P2-1 | -1.5259 | -1.4712 | -1.5060 | -1.5339 | -1.5206 | -1.5211 | 0.0547 | 0.0199 | 0.0080 | 0.0053 | 0.0048 |
| P5-1 | -0.7275 | -0.6720 | -0.6854 | -0.7082 | -0.7217 | -0.7273 | 0.0555 | 0.0421 | 0.0193 | 0.0058 | 0.0002 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P3-2 | -0.7091 | -0.7523 | -0.7785 | -0.7523 | -0.7561 | -0.7552 | 0.0432 | 0.0694 | 0.0432 | 0.0470 | 0.0461 |
| P4-2 | -0.5445 | -0.5406 | -0.4622 | -0.5204 | -0.5619 | -0.5628 | 0.0039 | 0.0823 | 0.0241 | 0.0174 | 0.0183 |
| P5-2 | -0.4061 | -0.4668 | -0.3599 | -0.3813 | -0.4021 | -0.4083 | 0.0607 | 0.0462 | 0.0248 | 0.0040 | 0.0022 |
| P4-3 | 0.2366 | 0.1809 | 0.4123 | 0.3089 | 0.2782 | 0.2774 | 0.0557 | 0.1757 | 0.0723 | 0.0416 | 0.0408 |
| P5-4 | 0.6167 | 0.6164 | 0.4575 | 0.6164 | 0.7104 | 0.6893 | 0.0003 | 0.1592 | 0.0003 | 0.0937 | 0.0726 |
| P7-4 | -0.2807 | -0.2041 | -0.2806 | -0.3106 | -0.1269 | -0.2470 | 0.0766 | 0.0001 | 0.0299 | 0.1538 | 0.0337 |
| P9-4 | -0.1608 | -0.1993 | -0.1919 | -0.1771 | -0.0986 | -0.1354 | 0.0385 | 0.0311 | 0.0163 | 0.0622 | 0.0254 |
| P6-5 | -0.4409 | -0.4223 | -0.4690 | -0.4130 | -0.3488 | -0.3996 | 0.0186 | 0.0281 | 0.0279 | 0.0921 | 0.0413 |
| P11-6 | -0.0730 | -0.0843 | -0.1113 | -0.1364 | -0.0632 | -0.0944 | 0.0113 | 0.0383 | 0.0634 | 0.0098 | 0.0214 |
| P12-6 | -0.0771 | -0.0212 | -0.0076 | 0.0182 | 0.0042 | -0.0009 | 0.0559 | 0.0695 | 0.0953 | 0.0813 | 0.0762 |
| P13-6 | -0.1754 | -0.2299 | -0.1576 | -0.1144 | -0.1219 | -0.1379 | 0.0545 | 0.0178 | 0.0610 | 0.0535 | 0.0375 |
| P8-7 | 0.0000 | 0.0016 | 0.0514 | 0.0121 | 0.0452 | 0.0047 | 0.0016 | 0.0514 | 0.0121 | 0.0452 | 0.0047 |
| P9-7 | -0.2807 | -0.2531 | -0.4338 | -0.3010 | -0.2549 | -0.2131 | 0.0276 | 0.1531 | 0.0203 | 0.0258 | 0.0676 |
| P10-9 | -0.0521 | 0.0029 | -0.0968 | -0.0674 | -0.0265 | -0.0399 | 0.0550 | 0.0447 | 0.0153 | 0.0256 | 0.0122 |
| P14-9 | -0.0931 | -0.0159 | -0.1389 | -0.0663 | -0.0987 | -0.1024 | 0.0772 | 0.0458 | 0.0268 | 0.0056 | 0.0093 |
| P11-10 | 0.0380 | 0.0423 | -0.0139 | 0.0339 | -0.0329 | -0.0118 | 0.0043 | 0.0519 | 0.0041 | 0.0709 | 0.0498 |
| P13-12 | -0.0161 | -0.0907 | -0.0695 | -0.0714 | -0.0644 | -0.0684 | 0.0746 | 0.0534 | 0.0553 | 0.0483 | 0.0523 |
| P14-13 | -0.0559 | -0.0930 | -0.0783 | -0.0859 | -0.0410 | -0.0641 | 0.0371 | 0.0224 | 0.0300 | 0.0149 | 0.0082 |
| Q1-2 | -0.2040 | -0.2571 | -0.2162 | -0.2571 | -0.2190 | -0.2201 | 0.0531 | 0.0122 | 0.0531 | 0.0150 | 0.0161 |
| Q1-5 | 0.0385 | 0.1560 | 0.0149 | 0.0167 | 0.0224 | 0.0181 | 0.1175 | 0.0236 | 0.0218 | 0.0161 | 0.0204 |
| Q2-3 | 0.0356 | 0.0048 | 0.0741 | 0.0258 | 0.0610 | 0.0611 | 0.0308 | 0.0385 | 0.0098 | 0.0254 | 0.0255 |
| Q2-4 | -0.0155 | 0.0219 | -0.0116 | -0.0057 | -0.0044 | -0.0102 | 0.0374 | 0.0039 | 0.0098 | 0.0111 | 0.0053 |
| Q2-5 | 0.0117 | 0.0021 | 0.0033 | 0.0135 | 0.0060 | -0.0005 | 0.0096 | 0.0084 | 0.0018 | 0.0057 | 0.0122 |
| Q3-4 | 0.0447 | 0.0891 | 0.0335 | 0.0689 | 0.0255 | 0.0202 | 0.0444 | 0.0112 | 0.0242 | 0.0192 | 0.0245 |
| Q4-5 | 0.1582 | 0.1200 | 0.0870 | 0.1200 | 0.0947 | 0.0914 | 0.0382 | 0.0712 | 0.0382 | 0.0635 | 0.0668 |
| Q4-7 | -0.0968 | -0.1669 | -0.1282 | -0.1027 | -0.1339 | -0.1494 | 0.0701 | 0.0314 | 0.0059 | 0.0371 | 0.0526 |
| Q4-9 | -0.0043 | -0.0754 | -0.0286 | -0.0219 | -0.0355 | -0.0438 | 0.0711 | 0.0243 | 0.0176 | 0.0312 | 0.0395 |
| Q5-6 | 0.1247 | 0.1491 | -0.0464 | -0.0859 | -0.0596 | -0.1097 | 0.0244 | 0.1711 | 0.2106 | 0.1843 | 0.2344 |
| Q6-11 | 0.0356 | 0.0267 | 0.0415 | 0.0603 | 0.0602 | 0.0611 | 0.0089 | 0.0059 | 0.0247 | 0.0246 | 0.0255 |
| Q6-12 | 0.0250 | 0.0152 | 0.0387 | 0.0152 | 0.0382 | 0.0311 | 0.0098 | 0.0137 | 0.0098 | 0.0132 | 0.0061 |
| Q6-13 | 0.0722 | 0.1432 | 0.1072 | 0.0518 | 0.1104 | 0.1024 | 0.0710 | 0.0350 | 0.0204 | 0.0382 | 0.0302 |
| Q7-8 | -0.1716 | -0.1570 | -0.1187 | -0.1291 | -0.1169 | -0.1196 | 0.0146 | 0.0529 | 0.0425 | 0.0547 | 0.0520 |
| Q7-9 | 0.0578 | 0.0677 | 0.0786 | 0.0677 | 0.0709 | 0.0554 | 0.0099 | 0.0208 | 0.0099 | 0.0131 | 0.0024 |
| Q9-10 | 0.0422 | 0.1216 | 0.0815 | 0.1151 | 0.0761 | 0.0659 | 0.0794 | 0.0393 | 0.0729 | 0.0339 | 0.0237 |
| Q9-14 | 0.0361 | -0.0041 | 0.0654 | 0.0374 | 0.0576 | 0.0501 | 0.0402 | 0.0293 | 0.0013 | 0.0215 | 0.0140 |
| Q10-11 | -0.0162 | 0.0186 | 0.0466 | 0.0186 | 0.0288 | 0.0191 | 0.0348 | 0.0628 | 0.0348 | 0.0450 | 0.0353 |
| Q12-13 | 0.0075 | 0.0493 | -0.0100 | -0.0160 | -0.0058 | -0.0044 | 0.0418 | 0.0175 | 0.0235 | 0.0133 | 0.0119 |
| Q13-14 | 0.0175 | 0.0053 | -0.0112 | 0.0053 | 0.0027 | 0.0076 | 0.0122 | 0.0287 | 0.0122 | 0.0148 | 0.0099 |
| Q2-1 | 0.2768 | 0.2876 | 0.2138 | 0.2649 | 0.2181 | 0.2220 | 0.0108 | 0.0630 | 0.0119 | 0.0587 | 0.0548 |
| Q5-1 | 0.0223 | -0.0360 | -0.0313 | -0.0220 | -0.0285 | -0.0203 | 0.0583 | 0.0536 | 0.0443 | 0.0508 | 0.0426 |
| Q3-2 | 0.0160 | -0.0414 | -0.0558 | -0.0143 | -0.0509 | -0.0490 | 0.0574 | 0.0718 | 0.0303 | 0.0669 | 0.0650 |
| Q4-2 | 0.0302 | 0.0354 | -0.0300 | -0.0241 | -0.0202 | -0.0130 | 0.0052 | 0.0602 | 0.0543 | 0.0504 | 0.0432 |
| Q5-2 | -0.0210 | 0.0151 | -0.0603 | -0.0659 | -0.0580 | -0.0497 | 0.0361 | 0.0393 | 0.0449 | 0.0370 | 0.0287 |
| Q4-3 | -0.0484 | 0.0809 | -0.0342 | -0.0799 | -0.0406 | -0.0350 | 0.1293 | 0.0142 | 0.0315 | 0.0078 | 0.0134 |
| Q5-4 | 0.1420 | 0.1087 | -0.0787 | -0.1046 | -0.0751 | -0.0728 | 0.0333 | 0.2207 | 0.2466 | 0.2171 | 0.2148 |
| Q7-4 | 0.1138 | 0.1232 | 0.1461 | 0.1232 | 0.1403 | 0.1653 | 0.0094 | 0.0323 | 0.0094 | 0.0265 | 0.0515 |
| Q9-4 | 0.0173 | 0.0132 | 0.0472 | 0.0381 | 0.0410 | 0.0540 | 0.0041 | 0.0299 | 0.0208 | 0.0237 | 0.0367 |
| Q6-5 | -0.0805 | -0.1772 | 0.0940 | 0.1249 | 0.0865 | 0.1469 | 0.0967 | 0.1745 | 0.2054 | 0.1670 | 0.2274 |
| Q11-6 | -0.0344 | -0.0563 | -0.0390 | -0.0563 | -0.0588 | -0.0588 | 0.0219 | 0.0046 | 0.0219 | 0.0244 | 0.0244 |
| Q12-6 | -0.0235 | -0.1132 | -0.0384 | -0.0151 | -0.0378 | -0.0309 | 0.0897 | 0.0149 | 0.0084 | 0.0143 | 0.0074 |
| Q13-6 | -0.0680 | -0.0260 | -0.1030 | -0.0499 | -0.1072 | -0.0990 | 0.0420 | 0.0350 | 0.0181 | 0.0392 | 0.0310 |
| Q8-7 | 0.1762 | 0.1318 | 0.1213 | 0.1318 | 0.1194 | 0.1219 | 0.0444 | 0.0549 | 0.0444 | 0.0568 | 0.0543 |
| Q9-7 | -0.0498 | -0.0448 | -0.0600 | -0.0583 | -0.0641 | -0.0508 | 0.0050 | 0.0102 | 0.0085 | 0.0143 | 0.0010 |
| Q10-9 | -0.0418 | -0.0690 | -0.0803 | -0.1137 | -0.0756 | -0.0654 | 0.0272 | 0.0385 | 0.0719 | 0.0338 | 0.0236 |
| Q14-9 | -0.0336 | -0.0184 | -0.0596 | -0.0359 | -0.0543 | -0.0469 | 0.0152 | 0.0260 | 0.0023 | 0.0207 | 0.0133 |
| Q11-10 | 0.0164 | -0.0136 | -0.0462 | -0.0183 | -0.0284 | -0.0190 | 0.0300 | 0.0626 | 0.0347 | 0.0448 | 0.0354 |
| Q13-12 | -0.0075 | 0.0170 | 0.0109 | 0.0170 | 0.0066 | 0.0052 | 0.0245 | 0.0184 | 0.0245 | 0.0141 | 0.0127 |
| Q14-13 | -0.0164 | 0.0206 | 0.0133 | -0.0029 | -0.0022 | -0.0063 | 0.0370 | 0.0297 | 0.0135 | 0.0142 | 0.0101 |
| **SUM** | | | | | | | **5.0026** | **3.9594** | **2.3378** | **2.8428** | **2.1690** |

## Median redundancy

The same bad data as in full redundancy case, the following two tables show the results and error with actual value of the system.

Table 5.246: State Variable of case 3, two reverse, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0851 | -0.0922 | -0.0851 | -0.0858 | **V1** | 1.0532 | 1.0314 | 1.0527 | 1.0705 |
| $\theta 3$ | -0.2323 | -0.2448 | -0.2335 | -0.2313 | **V2** | 1.0369 | 1.0133 | 1.0363 | 1.0537 |
| $\theta 4$ | -0.1517 | -0.1842 | -0.1498 | -0.1806 | **V3** | 0.9944 | 0.9712 | 0.9929 | 1.0143 |
| $\theta 5$ | -0.1414 | -0.1553 | -0.1383 | -0.1483 | **V4** | 1.0089 | 0.9801 | 1.0087 | 1.0169 |
| $\theta 6$ | -0.2699 | -0.2538 | -0.2458 | -0.2300 | **V5** | 1.0048 | 0.9817 | 1.0056 | 1.0171 |
| $\theta 7$ | -0.1859 | -0.2472 | -0.1870 | -0.2246 | **V6** | 1.0082 | 0.9810 | 1.0104 | 0.9947 |
| $\theta 8$ | -0.1621 | -0.2451 | -0.1781 | -0.2234 | **V7** | 1.0303 | 1.0044 | 1.0282 | 1.0463 |
| $\theta 9$ | -0.2449 | -0.2796 | -0.2188 | -0.2450 | **V8** | 1.0509 | 1.0243 | 1.0477 | 1.0677 |
| $\theta 10$ | -0.2561 | -0.2816 | -0.2236 | -0.2463 | **V9** | 1.0187 | 0.9910 | 1.0161 | 1.0352 |
| $\theta 11$ | -0.2726 | -0.2788 | -0.2404 | -0.2488 | **V10** | 1.0073 | 0.9784 | 1.0066 | 1.0264 |
| $\theta 12$ | -0.2736 | -0.2498 | -0.2466 | -0.2309 | **V11** | 0.9939 | 0.9753 | 0.9953 | 1.0083 |
| $\theta 13$ | -0.2869 | -0.2681 | -0.2565 | -0.2442 | **V12** | 0.9895 | 0.9781 | 0.9941 | 1.0007 |
| $\theta 14$ | -0.3064 | -0.3022 | -0.2621 | -0.2677 | **V13** | 0.9758 | 0.9676 | 0.9812 | 0.9876 |
| | | | | | **V14** | 0.9575 | 0.9547 | 0.9648 | 0.9884 |

Table 5.247: Estimation Results and Errors of case 3, two reverse, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0532 | 1.0314 | 1.0527 | 1.0705 | 0.0068 | 0.0286 | 0.0073 | 0.0105 |
| V2 | 1.0450 | N/A | 1.0369 | 1.0133 | 1.0363 | 1.0537 | 0.0081 | 0.0317 | 0.0087 | 0.0087 |
| V3 | 1.0100 | N/A | 0.9944 | 0.9712 | 0.9929 | 1.0143 | 0.0156 | 0.0388 | 0.0171 | 0.0043 |
| V4 | 1.0180 | N/A | 1.0089 | 0.9801 | 1.0087 | 1.0169 | 0.0091 | 0.0379 | 0.0093 | 0.0011 |
| V5 | 1.0200 | N/A | 1.0048 | 0.9817 | 1.0056 | 1.0171 | 0.0152 | 0.0383 | 0.0144 | 0.0029 |
| V6 | 1.0700 | N/A | 1.0082 | 0.9810 | 1.0104 | 0.9947 | 0.0618 | 0.0890 | 0.0596 | 0.0753 |
| V7 | 1.0620 | N/A | 1.0303 | 1.0044 | 1.0282 | 1.0463 | 0.0317 | 0.0576 | 0.0338 | 0.0157 |
| V8 | 1.0900 | N/A | 1.0509 | 1.0243 | 1.0477 | 1.0677 | 0.0391 | 0.0657 | 0.0423 | 0.0223 |
| V9 | 1.0560 | N/A | 1.0187 | 0.9910 | 1.0161 | 1.0352 | 0.0373 | 0.0650 | 0.0399 | 0.0208 |
| V10 | 1.0510 | N/A | 1.0073 | 0.9784 | 1.0066 | 1.0264 | 0.0437 | 0.0726 | 0.0444 | 0.0246 |
| V11 | 1.0570 | N/A | 0.9939 | 0.9753 | 0.9953 | 1.0083 | 0.0631 | 0.0817 | 0.0617 | 0.0487 |
| V12 | 1.0550 | N/A | 0.9895 | 0.9781 | 0.9941 | 1.0007 | 0.0655 | 0.0769 | 0.0609 | 0.0543 |
| V13 | 1.0500 | N/A | 0.9758 | 0.9676 | 0.9812 | 0.9876 | 0.0742 | 0.0824 | 0.0688 | 0.0624 |
| V14 | 1.0360 | N/A | 0.9575 | 0.9547 | 0.9648 | 0.9884 | 0.0785 | 0.0813 | 0.0712 | 0.0476 |
| P1 | 2.3239 | 2.3508 | 2.2173 | 2.3118 | 2.2003 | 2.3377 | 0.1066 | 0.0121 | 0.1236 | 0.0138 |
| P2 | 0.1830 | 0.2747 | 0.0772 | 0.1536 | 0.0535 | 0.2683 | 0.1058 | 0.0294 | 0.1295 | 0.0853 |
| P3 | -0.9420 | -1.0549 | -1.1902 | -1.0549 | -1.2120 | -1.0417 | 0.2482 | 0.1129 | 0.2700 | 0.0997 |
| **P4** | **-0.4780** | **0.4349** | **0.2020** | **-0.3337** | **0.1643** | **-0.6640** | **0.6800** | **0.1443** | **0.6423** | **0.1860** |
| P5 | -0.0760 | -0.0601 | -0.2792 | -0.0601 | -0.2999 | -0.0554 | 0.2032 | 0.0159 | 0.2239 | 0.0206 |
| P6 | -0.1120 | -0.1774 | -0.2681 | -0.1774 | -0.2724 | -0.2003 | 0.1561 | 0.0654 | 0.1604 | 0.0883 |
| P7 | 0.0000 | -0.0217 | 0.2419 | -0.0217 | 0.0582 | -0.0358 | 0.2419 | 0.0217 | 0.0582 | 0.0358 |
| P8 | 0.0000 | 0.0171 | 0.1462 | 0.0121 | 0.0545 | 0.0075 | 0.1462 | 0.0121 | 0.0545 | 0.0075 |
| P9 | -0.2950 | -0.1161 | -0.3018 | -0.2811 | -0.1390 | -0.1373 | 0.0068 | 0.0139 | 0.1560 | 0.1577 |
| P10 | -0.0900 | 0.0485 | -0.0657 | -0.0731 | 0.0070 | -0.0029 | 0.0243 | 0.0169 | 0.0970 | 0.0871 |
| P11 | -0.0350 | -0.1025 | -0.1365 | -0.1025 | -0.1024 | -0.0952 | 0.1015 | 0.0675 | 0.0674 | 0.0602 |
| P12 | -0.0610 | 0.0907 | 0.0230 | 0.0725 | 0.0262 | 0.0688 | 0.0840 | 0.1335 | 0.0872 | 0.1298 |
| P13 | -0.1350 | -0.0987 | -0.1997 | -0.0987 | -0.1754 | -0.1151 | 0.0647 | 0.0363 | 0.0404 | 0.0199 |
| P14 | -0.1490 | -0.1522 | -0.3241 | -0.2001 | -0.2282 | -0.1878 | 0.1751 | 0.0511 | 0.0792 | 0.0388 |
| Q1 | -0.1655 | -0.1298 | -0.1476 | -0.1298 | -0.1499 | -0.1355 | 0.0179 | 0.0357 | 0.0156 | 0.0300 |
| Q2 | 0.3086 | 0.2984 | 0.3066 | 0.2984 | 0.3051 | 0.3205 | 0.0020 | 0.0102 | 0.0035 | 0.0119 |
| Q3 | 0.0608 | 0.0546 | 0.0704 | 0.0546 | 0.0697 | 0.0773 | 0.0096 | 0.0062 | 0.0089 | 0.0165 |
| Q4 | 0.0390 | 0.1135 | 0.0881 | 0.1135 | 0.0861 | 0.1125 | 0.0491 | 0.0745 | 0.0471 | 0.0735 |
| Q5 | -0.0160 | 0.0545 | 0.0242 | 0.0545 | 0.0227 | 0.0493 | 0.0402 | 0.0705 | 0.0387 | 0.0653 |
| Q6 | 0.0523 | 0.1232 | 0.0123 | -0.2297 | 0.0220 | -0.4780 | 0.0400 | 0.2820 | 0.0303 | 0.5303 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q7 | 0.0000 | 0.0336 | 0.0038 | 0.0336 | -0.0087 | 0.0183 | 0.0038 | 0.0336 | 0.0087 | 0.0183 |
| Q8 | 0.1762 | 0.1158 | 0.1243 | 0.1158 | 0.1166 | 0.1298 | 0.0519 | 0.0604 | 0.0596 | 0.0464 |
| Q9 | -0.1660 | -0.1301 | -0.1364 | -0.1301 | -0.1628 | -0.1254 | 0.0296 | 0.0359 | 0.0032 | 0.0406 |
| Q10 | -0.0580 | 0.0235 | -0.0439 | -0.1013 | -0.0599 | -0.0112 | 0.0141 | 0.0433 | 0.0019 | 0.0468 |
| Q11 | -0.0180 | 0.0064 | -0.0800 | 0.0064 | -0.0899 | 0.0183 | 0.0620 | 0.0244 | 0.0719 | 0.0363 |
| Q12 | -0.0160 | 0.0357 | -0.0544 | -0.0348 | -0.0454 | 0.0176 | 0.0384 | 0.0188 | 0.0294 | 0.0336 |
| Q13 | -0.0580 | -0.0217 | -0.1169 | -0.0217 | -0.1147 | -0.0229 | 0.0589 | 0.0363 | 0.0567 | 0.0351 |
| Q14 | -0.0500 | -0.0652 | -0.1063 | -0.0652 | -0.1175 | -0.0777 | 0.0563 | 0.0152 | 0.0675 | 0.0277 |
| P1-2 | 1.5688 | 1.5835 | 1.5224 | 1.5835 | 1.5209 | 1.5854 | 0.0464 | 0.0147 | 0.0479 | 0.0166 |
| P1-5 | 0.7551 | 0.7157 | 0.6949 | 0.7283 | 0.6793 | 0.7522 | 0.0602 | 0.0268 | 0.0758 | 0.0029 |
| P2-3 | 0.7324 | 0.7768 | 0.7858 | 0.7768 | 0.7913 | 0.8013 | 0.0534 | 0.0444 | 0.0589 | 0.0689 |
| P2-4 | 0.5613 | N/A | 0.4088 | 0.5305 | 0.3977 | 0.5925 | 0.1525 | 0.0308 | 0.1636 | 0.0312 |
| P2-5 | 0.4152 | N/A | 0.3641 | 0.3838 | 0.3446 | 0.4171 | 0.0511 | 0.0314 | 0.0706 | 0.0019 |
| P3-4 | -0.2329 | -0.2734 | -0.4318 | -0.3061 | -0.4485 | -0.2679 | 0.1989 | 0.0732 | 0.2156 | 0.0350 |
| P4-5 | -0.6116 | N/A | -0.1951 | -0.6073 | -0.2293 | -0.7180 | 0.4165 | 0.0043 | 0.3823 | 0.1064 |
| **P4-7** | **0.2807** | **-0.3526** | **0.1741** | **0.3031** | **0.1887** | **0.2290** | **0.1066** | **0.0224** | **0.0920** | **0.0517** |
| P4-9 | 0.1608 | N/A | 0.1775 | 0.1718 | 0.1311 | 0.1258 | 0.0167 | 0.0110 | 0.0297 | 0.0350 |
| P5-6 | 0.4409 | 0.4032 | 0.5525 | 0.4032 | 0.4640 | 0.3517 | 0.1116 | 0.0377 | 0.0231 | 0.0892 |
| P6-11 | 0.0735 | N/A | 0.0395 | 0.1096 | 0.0078 | 0.0507 | 0.0340 | 0.0361 | 0.0657 | 0.0228 |
| P6-12 | 0.0779 | -0.0077 | 0.0406 | -0.0077 | 0.0278 | -0.0063 | 0.0373 | 0.0856 | 0.0501 | 0.0842 |
| P6-13 | 0.1775 | N/A | 0.2043 | 0.1239 | 0.1561 | 0.1069 | 0.0268 | 0.0536 | 0.0214 | 0.0706 |
| P7-8 | 0.0000 | -0.0121 | -0.1462 | -0.0121 | -0.0545 | -0.0075 | 0.1462 | 0.0121 | 0.0545 | 0.0075 |
| **P7-9** | **0.2807** | **0.9967** | **0.5623** | **0.2935** | **0.3014** | **0.2007** | **0.2816** | **0.0128** | **0.0207** | **0.0800** |
| P9-10 | 0.0523 | 0.0679 | 0.1650 | 0.0679 | 0.0893 | 0.0494 | 0.1127 | 0.0156 | 0.0370 | 0.0029 |
| P9-14 | 0.0943 | N/A | 0.2730 | 0.1163 | 0.2042 | 0.1397 | 0.1787 | 0.0220 | 0.1099 | 0.0454 |
| P10-11 | -0.0379 | -0.0394 | 0.0983 | -0.0058 | 0.0958 | 0.0462 | 0.1362 | 0.0321 | 0.1337 | 0.0841 |
| P12-13 | 0.0161 | N/A | 0.0630 | 0.0648 | 0.0536 | 0.0624 | 0.0469 | 0.0487 | 0.0375 | 0.0463 |
| P13-14 | 0.0564 | 0.0878 | 0.0625 | 0.0878 | 0.0307 | 0.0526 | 0.0061 | 0.0314 | 0.0257 | 0.0038 |
| P2-1 | -1.5259 | N/A | -1.4816 | -1.5375 | -1.4801 | -1.5426 | 0.0443 | 0.0116 | 0.0458 | 0.0167 |
| P5-1 | -0.7275 | N/A | -0.6708 | -0.7007 | -0.6563 | -0.7248 | 0.0567 | 0.0268 | 0.0712 | 0.0027 |
| P3-2 | -0.7091 | N/A | -0.7585 | -0.7488 | -0.7635 | -0.7739 | 0.0494 | 0.0397 | 0.0544 | 0.0648 |
| P4-2 | -0.5445 | N/A | -0.3997 | -0.5145 | -0.3890 | -0.5740 | 0.1448 | 0.0300 | 0.1555 | 0.0295 |
| P5-2 | -0.4061 | N/A | -0.3568 | -0.3753 | -0.3380 | -0.4077 | 0.0493 | 0.0308 | 0.0681 | 0.0016 |
| P4-3 | 0.2366 | N/A | 0.4452 | 0.3132 | 0.4629 | 0.2732 | 0.2086 | 0.0766 | 0.2263 | 0.0366 |
| P5-4 | 0.6167 | N/A | 0.1959 | 0.6128 | 0.2303 | 0.7253 | 0.4208 | 0.0039 | 0.3864 | 0.1086 |
| P7-4 | -0.2807 | N/A | -0.1741 | -0.3031 | -0.1887 | -0.2290 | 0.1066 | 0.0224 | 0.0920 | 0.0517 |
| P9-4 | -0.1608 | N/A | -0.1775 | -0.1718 | -0.1311 | -0.1258 | 0.0167 | 0.0110 | 0.0297 | 0.0350 |
| P6-5 | -0.4409 | N/A | -0.5525 | -0.4032 | -0.4640 | -0.3517 | 0.1116 | 0.0377 | 0.0231 | 0.0892 |
| P11-6 | -0.0730 | N/A | -0.0391 | -0.1083 | -0.0073 | -0.0497 | 0.0339 | 0.0353 | 0.0657 | 0.0233 |
| P12-6 | -0.0771 | N/A | -0.0400 | 0.0077 | -0.0273 | 0.0064 | 0.0371 | 0.0848 | 0.0498 | 0.0835 |
| P13-6 | -0.1754 | N/A | -0.2001 | -0.1228 | -0.1531 | -0.1062 | 0.0247 | 0.0526 | 0.0223 | 0.0692 |
| P8-7 | 0.0000 | N/A | 0.1462 | 0.0121 | 0.0545 | 0.0075 | 0.1462 | 0.0121 | 0.0545 | 0.0075 |
| P9-7 | -0.2807 | N/A | -0.5623 | -0.2935 | -0.3014 | -0.2007 | 0.2816 | 0.0128 | 0.0207 | 0.0800 |
| P10-9 | -0.0521 | N/A | -0.1640 | -0.0673 | -0.0888 | -0.0491 | 0.1119 | 0.0152 | 0.0367 | 0.0030 |
| P14-9 | -0.0931 | N/A | -0.2624 | -0.1137 | -0.1979 | -0.1358 | 0.1693 | 0.0206 | 0.1048 | 0.0427 |
| P11-10 | 0.0380 | N/A | -0.0975 | 0.0058 | -0.0951 | -0.0456 | 0.1355 | 0.0322 | 0.1331 | 0.0836 |
| P13-12 | -0.0161 | N/A | -0.0621 | -0.0637 | -0.0529 | -0.0615 | 0.0460 | 0.0476 | 0.0368 | 0.0454 |
| P14-13 | -0.0559 | N/A | -0.0618 | -0.0864 | -0.0303 | -0.0520 | 0.0059 | 0.0305 | 0.0256 | 0.0039 |
| Q1-2 | -0.2040 | -0.2571 | -0.2006 | -0.1853 | -0.1983 | -0.2068 | 0.0034 | 0.0187 | 0.0057 | 0.0028 |
| Q1-5 | 0.0385 | 0.1560 | 0.0530 | 0.0555 | 0.0484 | 0.0713 | 0.0145 | 0.0170 | 0.0099 | 0.0328 |
| Q2-3 | 0.0356 | 0.0048 | 0.0455 | 0.0441 | 0.0492 | 0.0283 | 0.0099 | 0.0085 | 0.0136 | 0.0073 |
| Q2-4 | -0.0155 | N/A | 0.0066 | 0.0046 | 0.0069 | 0.0143 | 0.0221 | 0.0201 | 0.0224 | 0.0298 |
| Q2-5 | 0.0117 | N/A | 0.0445 | 0.0344 | 0.0413 | 0.0592 | 0.0328 | 0.0227 | 0.0296 | 0.0475 |
| Q3-4 | 0.0447 | 0.0891 | 0.0911 | 0.0671 | 0.0920 | 0.0838 | 0.0464 | 0.0224 | 0.0473 | 0.0391 |
| Q4-5 | 0.1582 | N/A | 0.1611 | 0.1658 | 0.1482 | 0.2366 | 0.0029 | 0.0076 | 0.0100 | 0.0784 |
| Q4-7 | -0.0968 | -0.1669 | -0.1027 | -0.1066 | -0.0925 | -0.1411 | 0.0059 | 0.0098 | 0.0043 | 0.0443 |
| Q4-9 | -0.0043 | N/A | -0.0100 | -0.0115 | -0.0093 | -0.0304 | 0.0057 | 0.0072 | 0.0050 | 0.0261 |
| Q5-6 | 0.1247 | 0.1491 | 0.0210 | 0.0226 | 0.0045 | 0.1113 | 0.1037 | 0.1021 | 0.1202 | 0.0134 |
| Q6-11 | 0.0356 | N/A | 0.0538 | -0.0223 | 0.0733 | -0.0915 | 0.0182 | 0.0579 | 0.0377 | 0.1271 |
| Q6-12 | 0.0250 | 0.0152 | 0.0543 | 0.0152 | 0.0512 | -0.0203 | 0.0293 | 0.0098 | 0.0262 | 0.0453 |
| Q6-13 | 0.0722 | N/A | 0.1484 | 0.0390 | 0.1476 | 0.0007 | 0.0762 | 0.0332 | 0.0754 | 0.0715 |
| Q7-8 | -0.1716 | -0.1570 | -0.1184 | -0.1135 | -0.1139 | -0.1272 | 0.0532 | 0.0581 | 0.0577 | 0.0444 |
| Q7-9 | 0.0578 | 0.0677 | 0.1256 | 0.1271 | 0.1175 | 0.1079 | 0.0678 | 0.0693 | 0.0597 | 0.0501 |
| Q9-10 | 0.0422 | 0.1216 | 0.0757 | 0.1216 | 0.0805 | 0.0889 | 0.0335 | 0.0794 | 0.0383 | 0.0467 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q9-14 | 0.0361 | N/A | 0.1092 | 0.0792 | 0.1004 | 0.1145 | 0.0731 | 0.0431 | 0.0643 | 0.0784 |
| Q10-11 | -0.0162 | 0.0186 | 0.0291 | 0.0186 | 0.0195 | 0.0769 | 0.0453 | 0.0348 | 0.0357 | 0.0931 |
| Q12-13 | 0.0075 | N/A | -0.0013 | -0.0197 | 0.0049 | -0.0028 | 0.0088 | 0.0272 | 0.0026 | 0.0103 |
| Q13-14 | 0.0175 | 0.0053 | 0.0212 | -0.0057 | 0.0313 | -0.0273 | 0.0037 | 0.0232 | 0.0138 | 0.0448 |
| Q2-1 | 0.2768 | N/A | 0.2099 | 0.2153 | 0.2077 | 0.2186 | 0.0669 | 0.0615 | 0.0691 | 0.0582 |
| Q5-1 | 0.0223 | N/A | -0.0578 | -0.0416 | -0.0576 | -0.0653 | 0.0801 | 0.0639 | 0.0799 | 0.0876 |
| Q3-2 | 0.0160 | N/A | -0.0207 | -0.0125 | -0.0223 | -0.0065 | 0.0367 | 0.0285 | 0.0383 | 0.0225 |
| Q4-2 | 0.0302 | N/A | -0.0500 | -0.0236 | -0.0518 | -0.0311 | 0.0802 | 0.0538 | 0.0820 | 0.0613 |
| Q5-2 | -0.0210 | N/A | -0.0941 | -0.0775 | -0.0932 | -0.1047 | 0.0731 | 0.0565 | 0.0722 | 0.0837 |
| Q4-3 | -0.0484 | N/A | -0.0827 | -0.0734 | -0.0809 | -0.0968 | 0.0343 | 0.0250 | 0.0325 | 0.0484 |
| Q5-4 | 0.1420 | N/A | -0.1585 | -0.1485 | -0.1451 | -0.2134 | 0.3005 | 0.2905 | 0.2871 | 0.3554 |
| Q7-4 | 0.1138 | N/A | 0.1109 | 0.1285 | 0.1014 | 0.1554 | 0.0029 | 0.0147 | 0.0124 | 0.0416 |
| Q9-4 | 0.0173 | N/A | 0.0268 | 0.0281 | 0.0185 | 0.0391 | 0.0095 | 0.0108 | 0.0012 | 0.0218 |
| Q6-5 | -0.0805 | N/A | 0.0501 | 0.0171 | 0.0455 | -0.0804 | 0.1306 | 0.0976 | 0.1260 | 0.0001 |
| Q11-6 | -0.0344 | N/A | -0.0529 | 0.0249 | -0.0722 | 0.0937 | 0.0185 | 0.0593 | 0.0378 | 0.1281 |
| Q12-6 | -0.0235 | N/A | -0.0531 | -0.0151 | -0.0504 | 0.0204 | 0.0296 | 0.0084 | 0.0269 | 0.0439 |
| Q13-6 | -0.0680 | N/A | -0.1402 | -0.0367 | -0.1417 | 0.0008 | 0.0722 | 0.0313 | 0.0737 | 0.0688 |
| Q8-7 | 0.1762 | N/A | 0.1243 | 0.1158 | 0.1166 | 0.1298 | 0.0519 | 0.0604 | 0.0596 | 0.0464 |
| Q9-7 | -0.0498 | N/A | -0.0912 | -0.1159 | -0.1066 | -0.1027 | 0.0414 | 0.0661 | 0.0568 | 0.0529 |
| Q10-9 | -0.0418 | N/A | -0.0730 | -0.1199 | -0.0793 | -0.0881 | 0.0312 | 0.0781 | 0.0375 | 0.0463 |
| Q14-9 | -0.0336 | N/A | -0.0867 | -0.0738 | -0.0868 | -0.1062 | 0.0531 | 0.0402 | 0.0532 | 0.0726 |
| Q11-10 | 0.0164 | N/A | -0.0271 | -0.0185 | -0.0176 | -0.0754 | 0.0435 | 0.0349 | 0.0340 | 0.0918 |
| Q13-12 | -0.0075 | N/A | 0.0021 | 0.0207 | -0.0044 | 0.0036 | 0.0096 | 0.0282 | 0.0031 | 0.0111 |
| Q14-13 | -0.0164 | N/A | -0.0196 | 0.0086 | -0.0306 | 0.0286 | 0.0032 | 0.0250 | 0.0142 | 0.0450 |
| **SUM** | | | | | | | **6.1414** | **3.1897** | **5.5089** | **3.4798** |

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.248. Full redundancy generate better overall error summation compared with median redundancy case.

Table 5.248: Estimation errors of case 3, two reverse, 14-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **5.0026** | **3.9594** | **2.3378** | **2.8428** | **2.1690** |
| **SUM** | **N/A** | **6.1414** | **3.1897** | **5.5089** | **3.4798** |

## Two large one reverse bad data

Compared with the former case, another reverse bad data are changed to large error, $P_4 = 0.4349$, $P_{4-7} = 0.9526$ and $P_{7-9} = 0.9967$, others keep the same.

### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.229 gives the state variables, and the other tables show the estimation results and error with actual values.

Table 5.249: State variable of case 3, two large, 14-bus AC system, full redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0812 | -0.0813 | -0.0816 | -0.0826 | **V1** | 1.0894 | 1.0772 | 1.0898 | 1.0835 |
| $\theta 3$ | -0.2193 | -0.2197 | -0.2172 | -0.2198 | **V2** | 1.0737 | 1.0643 | 1.0741 | 1.0678 |
| $\theta 4$ | -0.1604 | -0.1679 | -0.1711 | -0.1737 | **V3** | 1.0268 | 1.0255 | 1.0288 | 1.0225 |
| $\theta 5$ | -0.1402 | -0.1423 | -0.1438 | -0.1460 | **V4** | 1.0437 | 1.0318 | 1.0410 | 1.0357 |
| $\theta 6$ | -0.2567 | -0.2330 | -0.2310 | -0.2323 | **V5** | 1.0467 | 1.0352 | 1.0464 | 1.0413 |
| $\theta 7$ | -0.2545 | -0.2327 | -0.2270 | -0.2243 | **V6** | 1.0622 | 1.0572 | 1.0634 | 1.0708 |
| $\theta 8$ | -0.2567 | -0.2308 | -0.2261 | -0.2222 | **V7** | 1.0715 | 1.0551 | 1.0675 | 1.0659 |
| $\theta 9$ | -0.2933 | -0.2660 | -0.2537 | -0.2510 | **V8** | 1.0921 | 1.0767 | 1.0873 | 1.0858 |
| $\theta 10$ | -0.2973 | -0.2629 | -0.2514 | -0.2492 | **V9** | 1.0638 | 1.0486 | 1.0601 | 1.0601 |
| $\theta 11$ | -0.2860 | -0.2556 | -0.2468 | -0.2465 | **V10** | 1.0541 | 1.0408 | 1.0536 | 1.0547 |
| $\theta 12$ | -0.2547 | -0.2279 | -0.2264 | -0.2290 | **V11** | 1.0438 | 1.0401 | 1.0458 | 1.0509 |
| $\theta 13$ | -0.2720 | -0.2444 | -0.2409 | -0.2425 | **V12** | 1.0513 | 1.0553 | 1.0542 | 1.0633 |
| $\theta 14$ | -0.3153 | -0.2742 | -0.2646 | -0.2637 | **V13** | 1.0382 | 1.0432 | 1.0417 | 1.0500 |
| | | | | | **V14** | 1.0294 | 1.0264 | 1.0343 | 1.0377 |

Table 5.250: Estimation Results and Errors of case 2, two large, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0894 | 1.0772 | 1.0898 | 1.0835 | 0.0086 | 0.0294 | 0.0172 | 0.0298 | 0.0235 |
| V2 | 1.0450 | 1.0743 | 1.0737 | 1.0643 | 1.0741 | 1.0678 | 0.0293 | 0.0287 | 0.0193 | 0.0291 | 0.0228 |
| V3 | 1.0100 | 0.9739 | 1.0268 | 1.0255 | 1.0288 | 1.0225 | 0.0361 | 0.0168 | 0.0155 | 0.0188 | 0.0125 |
| V4 | 1.0180 | 1.0318 | 1.0437 | 1.0318 | 1.0410 | 1.0357 | 0.0138 | 0.0257 | 0.0138 | 0.0230 | 0.0177 |
| V5 | 1.0200 | 1.0251 | 1.0467 | 1.0352 | 1.0464 | 1.0413 | 0.0051 | 0.0267 | 0.0152 | 0.0264 | 0.0213 |
| V6 | 1.0700 | 1.0491 | 1.0622 | 1.0572 | 1.0634 | 1.0708 | 0.0209 | 0.0078 | 0.0128 | 0.0066 | 0.0008 |
| V7 | 1.0620 | 1.0551 | 1.0715 | 1.0551 | 1.0675 | 1.0659 | 0.0069 | 0.0095 | 0.0069 | 0.0055 | 0.0039 |
| V8 | 1.0900 | 1.0955 | 1.0921 | 1.0767 | 1.0873 | 1.0858 | 0.0055 | 0.0021 | 0.0133 | 0.0027 | 0.0042 |
| V9 | 1.0560 | 1.1131 | 1.0638 | 1.0486 | 1.0601 | 1.0601 | 0.0571 | 0.0078 | 0.0074 | 0.0041 | 0.0041 |
| V10 | 1.0510 | 1.0952 | 1.0541 | 1.0408 | 1.0536 | 1.0547 | 0.0442 | 0.0031 | 0.0102 | 0.0026 | 0.0037 |
| V11 | 1.0570 | 1.0355 | 1.0438 | 1.0401 | 1.0458 | 1.0509 | 0.0215 | 0.0132 | 0.0169 | 0.0112 | 0.0061 |
| V12 | 1.0550 | 1.1034 | 1.0513 | 1.0553 | 1.0542 | 1.0633 | 0.0484 | 0.0037 | 0.0003 | 0.0008 | 0.0083 |
| V13 | 1.0500 | 1.0616 | 1.0382 | 1.0432 | 1.0417 | 1.0500 | 0.0116 | 0.0118 | 0.0068 | 0.0083 | 0.0000 |
| V14 | 1.0360 | 1.0350 | 1.0294 | 1.0264 | 1.0343 | 1.0377 | 0.0010 | 0.0066 | 0.0096 | 0.0017 | 0.0017 |
| P1 | 2.3239 | 2.3508 | 2.2863 | 2.2374 | 2.3140 | 2.3163 | 0.0269 | 0.0376 | 0.0865 | 0.0099 | 0.0076 |
| P2 | 0.1830 | 0.2747 | 0.1935 | 0.2628 | 0.2558 | 0.2576 | 0.0917 | 0.0105 | 0.0798 | 0.0728 | 0.0746 |
| P3 | -0.9420 | -1.0549 | -1.1187 | -1.0412 | -1.0280 | -1.0275 | 0.1129 | 0.1767 | 0.0992 | 0.0860 | 0.0855 |
| **P4** | **-0.4780** | **0.4349** | **0.1487** | **-0.3117** | **-0.4886** | **-0.5346** | **0.9129** | **0.6267** | **0.1663** | **0.0106** | **0.0566** |
| P5 | -0.0760 | -0.0601 | -0.0501 | -0.0601 | -0.0405 | -0.0430 | 0.0159 | 0.0259 | 0.0159 | 0.0355 | 0.0330 |
| P6 | -0.1120 | -0.1774 | -0.1846 | -0.1774 | -0.1662 | -0.1620 | 0.0654 | 0.0726 | 0.0654 | 0.0542 | 0.0500 |
| P7 | 0.0000 | -0.0217 | -0.0971 | -0.0217 | -0.0350 | -0.0118 | 0.0217 | 0.0971 | 0.0217 | 0.0350 | 0.0118 |
| P8 | 0.0000 | 0.0171 | -0.0149 | 0.0121 | 0.0058 | 0.0136 | 0.0171 | 0.0149 | 0.0121 | 0.0058 | 0.0136 |
| P9 | -0.2950 | -0.1161 | -0.4609 | -0.4743 | -0.3677 | -0.3546 | 0.1789 | 0.1659 | 0.1793 | 0.0727 | 0.0596 |
| P10 | -0.0900 | 0.0485 | -0.1218 | -0.0304 | -0.0073 | -0.0072 | 0.1385 | 0.0318 | 0.0596 | 0.0827 | 0.0828 |
| P11 | -0.0350 | -0.1025 | -0.1351 | -0.1025 | -0.1003 | -0.1006 | 0.0675 | 0.1001 | 0.0675 | 0.0653 | 0.0656 |
| P12 | -0.0610 | 0.0907 | 0.0665 | 0.0880 | 0.0707 | 0.0694 | 0.1517 | 0.1275 | 0.1490 | 0.1317 | 0.1304 |
| P13 | -0.1350 | -0.0987 | -0.1367 | -0.0987 | -0.1371 | -0.1376 | 0.0363 | 0.0017 | 0.0363 | 0.0021 | 0.0026 |
| P14 | -0.1490 | -0.1522 | -0.2396 | -0.1522 | -0.1415 | -0.1428 | 0.0032 | 0.0906 | 0.0032 | 0.0075 | 0.0062 |
| Q1 | -0.1655 | -0.1298 | -0.1951 | -0.2371 | -0.1948 | -0.2017 | 0.0357 | 0.0296 | 0.0716 | 0.0293 | 0.0362 |
| Q2 | 0.3086 | 0.2984 | 0.2847 | 0.2984 | 0.2826 | 0.2729 | 0.0102 | 0.0239 | 0.0102 | 0.0260 | 0.0357 |
| Q3 | 0.0608 | 0.0546 | -0.0205 | 0.0546 | -0.0239 | -0.0279 | 0.0062 | 0.0813 | 0.0062 | 0.0847 | 0.0887 |
| Q4 | 0.0390 | 0.1135 | 0.0613 | 0.0624 | 0.0532 | 0.0346 | 0.0745 | 0.0223 | 0.0234 | 0.0142 | 0.0044 |
| Q5 | -0.0160 | 0.0545 | 0.1343 | 0.0545 | 0.1231 | 0.0836 | 0.0705 | 0.1503 | 0.0705 | 0.1391 | 0.0996 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q6 | 0.0523 | 0.1232 | -0.0625 | -0.1112 | -0.0428 | 0.0090 | 0.0709 | 0.1148 | 0.1635 | 0.0951 | 0.0433 |
| Q7 | 0.0000 | 0.0336 | 0.0044 | -0.0498 | -0.0197 | -0.0192 | 0.0336 | 0.0044 | 0.0498 | 0.0197 | 0.0192 |
| Q8 | 0.1762 | 0.1158 | 0.1275 | 0.1318 | 0.1220 | 0.1229 | 0.0604 | 0.0487 | 0.0444 | 0.0542 | 0.0533 |
| Q9 | -0.1660 | -0.1301 | -0.1217 | -0.1301 | -0.1531 | -0.1561 | 0.0359 | 0.0443 | 0.0359 | 0.0129 | 0.0099 |
| Q10 | -0.0580 | 0.0235 | -0.0173 | -0.0784 | -0.0362 | -0.0437 | 0.0815 | 0.0407 | 0.0204 | 0.0218 | 0.0143 |
| Q11 | -0.0180 | 0.0064 | -0.0835 | -0.0415 | -0.0856 | -0.0772 | 0.0244 | 0.0655 | 0.0235 | 0.0676 | 0.0592 |
| Q12 | -0.0160 | 0.0357 | -0.0550 | -0.0311 | -0.0483 | -0.0361 | 0.0517 | 0.0390 | 0.0151 | 0.0323 | 0.0201 |
| Q13 | -0.0580 | -0.0217 | -0.1117 | -0.0277 | -0.1039 | -0.0876 | 0.0363 | 0.0537 | 0.0303 | 0.0459 | 0.0296 |
| Q14 | -0.0500 | -0.0652 | -0.0379 | -0.0589 | -0.0515 | -0.0531 | 0.0152 | 0.0121 | 0.0089 | 0.0015 | 0.0031 |
| P1-2 | 1.5688 | 1.5835 | 1.5521 | 1.5099 | 1.5612 | 1.5618 | 0.0147 | 0.0167 | 0.0589 | 0.0076 | 0.0070 |
| P1-5 | 0.7551 | 0.7157 | 0.7342 | 0.7275 | 0.7528 | 0.7545 | 0.0394 | 0.0209 | 0.0276 | 0.0023 | 0.0006 |
| P2-3 | 0.7324 | 0.7768 | 0.7948 | 0.7786 | 0.7808 | 0.7808 | 0.0444 | 0.0624 | 0.0462 | 0.0484 | 0.0484 |
| P2-4 | 0.5613 | 0.5039 | 0.5140 | 0.5508 | 0.5787 | 0.5800 | 0.0574 | 0.0473 | 0.0105 | 0.0174 | 0.0187 |
| P2-5 | 0.4152 | 0.3618 | 0.3972 | 0.4046 | 0.4173 | 0.4180 | 0.0534 | 0.0180 | 0.0106 | 0.0021 | 0.0028 |
| P3-4 | -0.2329 | -0.2734 | -0.3503 | -0.2880 | -0.2725 | -0.2723 | 0.0405 | 0.1174 | 0.0551 | 0.0396 | 0.0394 |
| P4-5 | -0.6116 | -0.7588 | -0.4955 | -0.6115 | -0.6767 | -0.6795 | 0.1472 | 0.1161 | 0.0001 | 0.0651 | 0.0679 |
| **P4-7** | **0.2807** | **0.9526** | **0.5137** | **0.3447** | **0.3036** | **0.2731** | **0.6719** | **0.2330** | **0.0640** | **0.0229** | **0.0076** |
| P4-9 | 0.1608 | 0.1771 | 0.2729 | 0.1967 | 0.1689 | 0.1575 | 0.0163 | 0.1121 | 0.0359 | 0.0081 | 0.0033 |
| P5-6 | 0.4409 | 0.4032 | 0.5499 | 0.4223 | 0.4124 | 0.4089 | 0.0377 | 0.1090 | 0.0186 | 0.0285 | 0.0320 |
| P6-11 | 0.0735 | 0.1420 | 0.1723 | 0.1376 | 0.1087 | 0.1076 | 0.0685 | 0.0988 | 0.0641 | 0.0352 | 0.0341 |
| P6-12 | 0.0779 | -0.0077 | 0.0107 | -0.0150 | -0.0014 | 0.0002 | 0.0856 | 0.0672 | 0.0929 | 0.0793 | 0.0777 |
| P6-13 | 0.1775 | 0.1724 | 0.1822 | 0.1223 | 0.1389 | 0.1391 | 0.0051 | 0.0047 | 0.0552 | 0.0386 | 0.0384 |
| P7-8 | 0.0000 | -0.0121 | 0.0149 | -0.0121 | -0.0058 | -0.0136 | 0.0121 | 0.0149 | 0.0121 | 0.0058 | 0.0136 |
| **P7-9** | **0.2807** | **0.9967** | **0.4017** | **0.3351** | **0.2744** | **0.2749** | **0.7160** | **0.1210** | **0.0544** | **0.0063** | **0.0058** |
| P9-10 | 0.0523 | 0.0679 | 0.0880 | -0.0026 | 0.0004 | 0.0017 | 0.0156 | 0.0357 | 0.0549 | 0.0519 | 0.0506 |
| P9-14 | 0.0943 | 0.0511 | 0.1257 | 0.0600 | 0.0752 | 0.0761 | 0.0432 | 0.0314 | 0.0343 | 0.0191 | 0.0182 |
| P10-11 | -0.0379 | -0.0394 | -0.0343 | -0.0333 | -0.0071 | -0.0057 | 0.0015 | 0.0036 | 0.0046 | 0.0308 | 0.0322 |
| P12-13 | 0.0161 | 0.0079 | 0.0771 | 0.0729 | 0.0691 | 0.0696 | 0.0082 | 0.0610 | 0.0568 | 0.0530 | 0.0535 |
| P13-14 | 0.0564 | 0.0878 | 0.1187 | 0.0944 | 0.0681 | 0.0684 | 0.0314 | 0.0623 | 0.0380 | 0.0117 | 0.0120 |
| P2-1 | -1.5259 | -1.4712 | -1.5124 | -1.4712 | -1.5210 | -1.5211 | 0.0547 | 0.0135 | 0.0547 | 0.0049 | 0.0048 |
| P5-1 | -0.7275 | -0.6720 | -0.7093 | -0.7026 | -0.7267 | -0.7280 | 0.0555 | 0.0182 | 0.0249 | 0.0008 | 0.0005 |
| P3-2 | -0.7091 | -0.7523 | -0.7684 | -0.7532 | -0.7555 | -0.7551 | 0.0432 | 0.0593 | 0.0441 | 0.0464 | 0.0460 |
| P4-2 | -0.5445 | -0.5406 | -0.5006 | -0.5351 | -0.5618 | -0.5628 | 0.0039 | 0.0439 | 0.0094 | 0.0173 | 0.0183 |
| P5-2 | -0.4061 | -0.4668 | -0.3893 | -0.3962 | -0.4086 | -0.4092 | 0.0607 | 0.0168 | 0.0099 | 0.0025 | 0.0031 |
| P4-3 | 0.2366 | 0.1809 | 0.3582 | 0.2937 | 0.2773 | 0.2772 | 0.0557 | 0.1216 | 0.0571 | 0.0407 | 0.0406 |
| P5-4 | 0.6167 | 0.6164 | 0.4986 | 0.6164 | 0.6824 | 0.6854 | 0.0003 | 0.1181 | 0.0003 | 0.0657 | 0.0687 |
| P7-4 | -0.2807 | -0.2041 | -0.5137 | -0.3447 | -0.3036 | -0.2731 | 0.0766 | 0.2330 | 0.0640 | 0.0229 | 0.0076 |
| P9-4 | -0.1608 | -0.1993 | -0.2729 | -0.1967 | -0.1689 | -0.1575 | 0.0385 | 0.1121 | 0.0359 | 0.0081 | 0.0033 |
| P6-5 | -0.4409 | -0.4223 | -0.5499 | -0.4223 | -0.4124 | -0.4089 | 0.0186 | 0.1090 | 0.0186 | 0.0285 | 0.0320 |
| P11-6 | -0.0730 | -0.0843 | -0.1698 | -0.1359 | -0.1076 | -0.1063 | 0.0113 | 0.0968 | 0.0629 | 0.0346 | 0.0333 |
| P12-6 | -0.0771 | -0.0212 | -0.0106 | 0.0151 | 0.0016 | -0.0001 | 0.0559 | 0.0665 | 0.0922 | 0.0787 | 0.0770 |
| P13-6 | -0.1754 | -0.2299 | -0.1796 | -0.1213 | -0.1371 | -0.1374 | 0.0545 | 0.0042 | 0.0541 | 0.0383 | 0.0380 |
| P8-7 | 0.0000 | 0.0016 | -0.0149 | 0.0121 | 0.0058 | 0.0136 | 0.0016 | 0.0149 | 0.0121 | 0.0058 | 0.0136 |
| P9-7 | -0.2807 | -0.2531 | -0.4017 | -0.3351 | -0.2744 | -0.2749 | 0.0276 | 0.1210 | 0.0544 | 0.0063 | 0.0058 |
| P10-9 | -0.0521 | 0.0029 | -0.0875 | 0.0029 | -0.0002 | -0.0016 | 0.0550 | 0.0354 | 0.0550 | 0.0519 | 0.0505 |
| P14-9 | -0.0931 | -0.0159 | -0.1233 | -0.0592 | -0.0741 | -0.0751 | 0.0772 | 0.0302 | 0.0339 | 0.0190 | 0.0180 |
| P11-10 | 0.0380 | 0.0423 | 0.0347 | 0.0334 | 0.0072 | 0.0057 | 0.0043 | 0.0033 | 0.0046 | 0.0308 | 0.0323 |
| P13-12 | -0.0161 | -0.0907 | -0.0758 | -0.0718 | -0.0681 | -0.0686 | 0.0746 | 0.0597 | 0.0557 | 0.0520 | 0.0525 |
| P14-13 | -0.0559 | -0.0930 | -0.1164 | -0.0930 | -0.0674 | -0.0677 | 0.0371 | 0.0605 | 0.0371 | 0.0115 | 0.0118 |
| Q1-2 | -0.2040 | -0.2571 | -0.2172 | -0.2571 | -0.2189 | -0.2200 | 0.0531 | 0.0132 | 0.0531 | 0.0149 | 0.0160 |
| Q1-5 | 0.0385 | 0.1560 | 0.0221 | 0.0200 | 0.0240 | 0.0183 | 0.1175 | 0.0164 | 0.0185 | 0.0145 | 0.0202 |
| Q2-3 | 0.0356 | 0.0048 | 0.0681 | 0.0267 | 0.0607 | 0.0610 | 0.0308 | 0.0325 | 0.0089 | 0.0251 | 0.0254 |
| Q2-4 | -0.0155 | 0.0219 | -0.0059 | -0.0007 | -0.0029 | -0.0096 | 0.0374 | 0.0096 | 0.0148 | 0.0126 | 0.0059 |
| Q2-5 | 0.0117 | 0.0021 | 0.0076 | 0.0182 | 0.0069 | -0.0005 | 0.0096 | 0.0041 | 0.0065 | 0.0048 | 0.0122 |
| Q3-4 | 0.0447 | 0.0891 | 0.0334 | 0.0699 | 0.0270 | 0.0209 | 0.0444 | 0.0113 | 0.0252 | 0.0177 | 0.0238 |
| Q4-5 | 0.1582 | 0.1200 | 0.0860 | 0.1200 | 0.0903 | 0.0886 | 0.0382 | 0.0722 | 0.0382 | 0.0679 | 0.0696 |
| Q4-7 | -0.0968 | -0.1669 | -0.1180 | -0.1064 | -0.1267 | -0.1458 | 0.0701 | 0.0212 | 0.0096 | 0.0299 | 0.0490 |
| Q4-9 | -0.0043 | -0.0754 | -0.0209 | -0.0226 | -0.0300 | -0.0408 | 0.0711 | 0.0166 | 0.0183 | 0.0257 | 0.0365 |
| Q5-6 | 0.1247 | 0.1491 | -0.0367 | -0.0779 | -0.0581 | -0.1132 | 0.0244 | 0.1614 | 0.2026 | 0.1828 | 0.2379 |
| Q6-11 | 0.0356 | 0.0267 | 0.0184 | 0.0267 | 0.0429 | 0.0565 | 0.0089 | 0.0172 | 0.0089 | 0.0073 | 0.0209 |
| Q6-12 | 0.0250 | 0.0152 | 0.0400 | 0.0152 | 0.0390 | 0.0311 | 0.0098 | 0.0150 | 0.0098 | 0.0140 | 0.0061 |
| Q6-13 | 0.0722 | 0.1432 | 0.1040 | 0.0521 | 0.1074 | 0.1012 | 0.0710 | 0.0318 | 0.0201 | 0.0352 | 0.0290 |
| Q7-8 | -0.1716 | -0.1570 | -0.1251 | -0.1291 | -0.1197 | -0.1206 | 0.0146 | 0.0465 | 0.0425 | 0.0519 | 0.0510 |
| Q7-9 | 0.0578 | 0.0677 | 0.0828 | 0.0677 | 0.0755 | 0.0595 | 0.0099 | 0.0250 | 0.0099 | 0.0177 | 0.0017 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q9-10 | 0.0422 | 0.1216 | 0.0900 | 0.0977 | 0.0821 | 0.0674 | 0.0794 | 0.0478 | 0.0555 | 0.0399 | 0.0252 |
| Q9-14 | 0.0361 | -0.0041 | 0.0772 | 0.0581 | 0.0660 | 0.0525 | 0.0402 | 0.0411 | 0.0220 | 0.0299 | 0.0164 |
| Q10-11 | -0.0162 | 0.0186 | 0.0715 | 0.0186 | 0.0455 | 0.0234 | 0.0348 | 0.0877 | 0.0348 | 0.0617 | 0.0396 |
| Q12-13 | 0.0075 | 0.0493 | -0.0154 | -0.0160 | -0.0096 | -0.0052 | 0.0418 | 0.0229 | 0.0235 | 0.0171 | 0.0127 |
| Q13-14 | 0.0175 | 0.0053 | -0.0293 | 0.0053 | -0.0105 | 0.0041 | 0.0122 | 0.0468 | 0.0122 | 0.0280 | 0.0134 |
| Q2-1 | 0.2768 | 0.2876 | 0.2149 | 0.2543 | 0.2179 | 0.2220 | 0.0108 | 0.0619 | 0.0225 | 0.0589 | 0.0548 |
| Q5-1 | 0.0223 | -0.0360 | -0.0319 | -0.0270 | -0.0286 | -0.0202 | 0.0583 | 0.0542 | 0.0493 | 0.0509 | 0.0425 |
| Q3-2 | 0.0160 | -0.0414 | -0.0539 | -0.0153 | -0.0509 | -0.0488 | 0.0574 | 0.0699 | 0.0313 | 0.0669 | 0.0648 |
| Q4-2 | 0.0302 | 0.0354 | -0.0297 | -0.0265 | -0.0217 | -0.0135 | 0.0052 | 0.0599 | 0.0567 | 0.0519 | 0.0437 |
| Q5-2 | -0.0210 | 0.0151 | -0.0612 | -0.0689 | -0.0582 | -0.0496 | 0.0361 | 0.0402 | 0.0479 | 0.0372 | 0.0286 |
| Q4-3 | -0.0484 | 0.0809 | -0.0405 | -0.0824 | -0.0421 | -0.0356 | 0.1293 | 0.0079 | 0.0340 | 0.0063 | 0.0128 |
| Q5-4 | 0.1420 | 0.1087 | -0.0762 | -0.1046 | -0.0722 | -0.0702 | 0.0333 | 0.2182 | 0.2466 | 0.2142 | 0.2122 |
| Q7-4 | 0.1138 | 0.1232 | 0.1702 | 0.1314 | 0.1471 | 0.1640 | 0.0094 | 0.0564 | 0.0176 | 0.0333 | 0.0502 |
| Q9-4 | 0.0173 | 0.0132 | 0.0579 | 0.0424 | 0.0447 | 0.0541 | 0.0041 | 0.0406 | 0.0251 | 0.0274 | 0.0368 |
| Q6-5 | -0.0805 | -0.1772 | 0.1018 | 0.1183 | 0.0953 | 0.1522 | 0.0967 | 0.1823 | 0.1988 | 0.1758 | 0.2327 |
| Q11-6 | -0.0344 | -0.0563 | -0.0131 | -0.0232 | -0.0405 | -0.0539 | 0.0219 | 0.0213 | 0.0112 | 0.0061 | 0.0195 |
| Q12-6 | -0.0235 | -0.1132 | -0.0396 | -0.0151 | -0.0386 | -0.0309 | 0.0897 | 0.0161 | 0.0084 | 0.0151 | 0.0074 |
| Q13-6 | -0.0680 | -0.0260 | -0.0989 | -0.0500 | -0.1039 | -0.0978 | 0.0420 | 0.0309 | 0.0180 | 0.0359 | 0.0298 |
| Q8-7 | 0.1762 | 0.1318 | 0.1275 | 0.1318 | 0.1220 | 0.1229 | 0.0444 | 0.0487 | 0.0444 | 0.0542 | 0.0533 |
| Q9-7 | -0.0498 | -0.0448 | -0.0667 | -0.0562 | -0.0677 | -0.0519 | 0.0050 | 0.0169 | 0.0064 | 0.0179 | 0.0021 |
| Q10-9 | -0.0418 | -0.0690 | -0.0888 | -0.0970 | -0.0816 | -0.0671 | 0.0272 | 0.0470 | 0.0552 | 0.0398 | 0.0253 |
| Q14-9 | -0.0336 | -0.0184 | -0.0720 | -0.0564 | -0.0636 | -0.0505 | 0.0152 | 0.0384 | 0.0228 | 0.0300 | 0.0169 |
| Q11-10 | 0.0164 | -0.0136 | -0.0704 | -0.0183 | -0.0451 | -0.0233 | 0.0300 | 0.0868 | 0.0347 | 0.0615 | 0.0397 |
| Q13-12 | -0.0075 | 0.0170 | 0.0165 | 0.0170 | 0.0105 | 0.0060 | 0.0245 | 0.0240 | 0.0245 | 0.0180 | 0.0135 |
| Q14-13 | -0.0164 | 0.0206 | 0.0341 | -0.0024 | 0.0121 | -0.0026 | 0.0370 | 0.0505 | 0.0140 | 0.0285 | 0.0138 |
| **SUM** | | | | | | | **5.1412** | **4.1298** | **2.7754** | **2.1732** | **2.0044** |

### Median redundancy

Table 5.251 presents the state variables, and the other table shows the estimation results and error with actual values.

Table 5.251: State variable of case 3, two large, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0864 | -0.0850 | -0.0860 | -0.0861 | **V1** | 1.0569 | 1.0686 | 1.0585 | 1.0646 |
| $\theta 3$ | -0.2315 | -0.2288 | -0.2320 | -0.2293 | **V2** | 1.0404 | 1.0511 | 1.0418 | 1.0478 |
| $\theta 4$ | -0.1720 | -0.1740 | -0.1691 | -0.1785 | **V3** | 1.0006 | 1.0090 | 1.0005 | 1.0081 |
| $\theta 5$ | -0.1549 | -0.1553 | -0.1507 | -0.1590 | **V4** | 1.0061 | 1.0146 | 1.0072 | 1.0093 |
| $\theta 6$ | -0.3159 | -0.3110 | -0.2835 | -0.3319 | **V5** | 1.0038 | 1.0169 | 1.0076 | 1.0105 |
| $\theta 7$ | -0.2937 | -0.3122 | -0.2865 | -0.3316 | **V6** | 1.0091 | 1.0454 | 1.0220 | 1.0326 |
| $\theta 8$ | -0.2953 | -0.3102 | -0.2862 | -0.3329 | **V7** | 1.0350 | 1.0330 | 1.0284 | 1.0325 |
| $\theta 9$ | -0.3495 | -0.3847 | -0.3442 | -0.4049 | **V8** | 1.0599 | 1.0524 | 1.0512 | 1.0572 |
| $\theta 10$ | -0.3613 | -0.3881 | -0.3398 | -0.4005 | **V9** | 1.0222 | 1.0229 | 1.0153 | 1.0206 |
| $\theta 11$ | -0.3570 | -0.3793 | -0.3250 | -0.3834 | **V10** | 1.0102 | 1.0151 | 1.0091 | 1.0154 |
| $\theta 12$ | -0.3164 | -0.3079 | -0.2777 | -0.3344 | **V11** | 0.9949 | 1.0148 | 1.0015 | 1.0092 |
| $\theta 13$ | -0.3393 | -0.3281 | -0.2959 | -0.3522 | **V12** | 0.9892 | 1.0423 | 1.0087 | 1.0185 |
| $\theta 14$ | -0.3979 | -0.3779 | -0.3393 | -0.4006 | **V13** | 0.9736 | 1.0285 | 0.9941 | 1.0042 |
| | | | | | **V14** | 0.9554 | 1.0036 | 0.9778 | 0.9872 |

Table 5.252: Estimation Results and Errors of case 3, two large, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0569 | 1.0686 | 1.0585 | 1.0646 | 0.0031 | 0.0086 | 0.0015 | 0.0046 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V2 | 1.0450 | N/A | 1.0404 | 1.0511 | 1.0418 | 1.0478 | 0.0046 | 0.0061 | 0.0032 | 0.0028 |
| V3 | 1.0100 | N/A | 1.0006 | 1.0090 | 1.0005 | 1.0081 | 0.0094 | 0.0010 | 0.0095 | 0.0019 |
| V4 | 1.0180 | N/A | 1.0061 | 1.0146 | 1.0072 | 1.0093 | 0.0119 | 0.0034 | 0.0108 | 0.0087 |
| V5 | 1.0200 | N/A | 1.0038 | 1.0169 | 1.0076 | 1.0105 | 0.0162 | 0.0031 | 0.0124 | 0.0095 |
| V6 | 1.0700 | N/A | 1.0091 | 1.0454 | 1.0220 | 1.0326 | 0.0609 | 0.0246 | 0.0480 | 0.0374 |
| V7 | 1.0620 | N/A | 1.0350 | 1.0330 | 1.0284 | 1.0325 | 0.0270 | 0.0290 | 0.0336 | 0.0295 |
| V8 | 1.0900 | N/A | 1.0599 | 1.0524 | 1.0512 | 1.0572 | 0.0301 | 0.0376 | 0.0388 | 0.0328 |
| V9 | 1.0560 | N/A | 1.0222 | 1.0229 | 1.0153 | 1.0206 | 0.0338 | 0.0331 | 0.0407 | 0.0354 |
| V10 | 1.0510 | N/A | 1.0102 | 1.0151 | 1.0091 | 1.0154 | 0.0408 | 0.0359 | 0.0419 | 0.0356 |
| V11 | 1.0570 | N/A | 0.9949 | 1.0148 | 1.0015 | 1.0092 | 0.0621 | 0.0422 | 0.0555 | 0.0478 |
| V12 | 1.0550 | N/A | 0.9892 | 1.0423 | 1.0087 | 1.0185 | 0.0658 | 0.0127 | 0.0463 | 0.0365 |
| V13 | 1.0500 | N/A | 0.9736 | 1.0285 | 0.9941 | 1.0042 | 0.0764 | 0.0215 | 0.0559 | 0.0458 |
| V14 | 1.0360 | N/A | 0.9554 | 1.0036 | 0.9778 | 0.9872 | 0.0806 | 0.0324 | 0.0582 | 0.0488 |
| P1 | 2.3239 | 2.3508 | 2.3201 | 2.3508 | 2.3004 | 2.3676 | 0.0038 | 0.0269 | 0.0235 | 0.0437 |
| P2 | 0.1830 | 0.2747 | 0.2301 | 0.2747 | 0.2024 | 0.2977 | 0.0471 | 0.0917 | 0.0194 | 0.1147 |
| P3 | -0.9420 | -1.0549 | -1.0637 | -1.0549 | -1.0898 | -1.0159 | 0.1217 | 0.1129 | 0.1478 | 0.0739 |
| **P4** | **-0.4780** | **0.4349** | **0.4082** | **0.4349** | **0.3608** | **0.4782** | **0.8862** | **0.9129** | **0.8388** | **0.9562** |
| P5 | -0.0760 | -0.0601 | -0.1160 | -0.0601 | -0.1384 | -0.0202 | 0.0400 | 0.0159 | 0.0624 | 0.0558 |
| P6 | -0.1120 | -0.1774 | -0.2078 | -0.1774 | -0.1974 | -0.2444 | 0.0958 | 0.0654 | 0.0854 | 0.1324 |
| P7 | 0.0000 | -0.0217 | -0.0710 | -0.0217 | -0.0475 | -0.0669 | 0.0710 | 0.0217 | 0.0475 | 0.0669 |
| P8 | 0.0000 | 0.0171 | -0.0100 | 0.0121 | 0.0017 | -0.0080 | 0.0100 | 0.0121 | 0.0017 | 0.0080 |
| P9 | -0.2950 | -0.1161 | -0.4577 | -1.0244 | -0.8610 | -1.1224 | 0.1627 | 0.7294 | 0.5660 | 0.8274 |
| P10 | -0.0900 | 0.0485 | -0.1629 | -0.1070 | -0.0283 | -0.0382 | 0.0729 | 0.0170 | 0.0617 | 0.0518 |
| P11 | -0.0350 | -0.1025 | -0.2047 | -0.3126 | -0.1607 | -0.1971 | 0.1697 | 0.2776 | 0.1257 | 0.1621 |
| P12 | -0.0610 | 0.0907 | 0.0571 | 0.0907 | 0.0767 | 0.0472 | 0.1181 | 0.1517 | 0.1377 | 0.1082 |
| P13 | -0.1350 | -0.0987 | -0.1855 | -0.0987 | -0.1218 | -0.1604 | 0.0505 | 0.0363 | 0.0132 | 0.0254 |
| P14 | -0.1490 | -0.1522 | -0.3766 | -0.1522 | -0.1524 | -0.1625 | 0.2276 | 0.0032 | 0.0034 | 0.0135 |
| Q1 | -0.1655 | -0.1298 | -0.1365 | -0.1298 | -0.1408 | -0.1330 | 0.0290 | 0.0357 | 0.0247 | 0.0325 |
| Q2 | 0.3086 | 0.2984 | 0.3172 | 0.2984 | 0.3148 | 0.3233 | 0.0086 | 0.0102 | 0.0062 | 0.0147 |
| Q3 | 0.0608 | 0.0546 | 0.0754 | 0.0546 | 0.0731 | 0.0768 | 0.0146 | 0.0062 | 0.0123 | 0.0160 |
| Q4 | 0.0390 | 0.1135 | 0.1022 | 0.1135 | 0.0968 | 0.1090 | 0.0632 | 0.0745 | 0.0578 | 0.0700 |
| Q5 | -0.0160 | 0.0545 | 0.0369 | 0.0545 | 0.0376 | 0.0505 | 0.0529 | 0.0705 | 0.0536 | 0.0665 |
| Q6 | 0.0523 | 0.1232 | -0.0220 | -0.0690 | -0.0061 | 0.0115 | 0.0743 | 0.1213 | 0.0584 | 0.0408 |
| Q7 | 0.0000 | 0.0336 | 0.0581 | 0.0336 | 0.0315 | 0.0555 | 0.0581 | 0.0336 | 0.0315 | 0.0555 |
| Q8 | 0.1762 | 0.1158 | 0.1496 | 0.1158 | 0.1364 | 0.1483 | 0.0266 | 0.0604 | 0.0398 | 0.0279 |
| Q9 | -0.1660 | -0.1301 | -0.0782 | -0.1301 | -0.1090 | -0.0904 | 0.0878 | 0.0359 | 0.0570 | 0.0756 |
| Q10 | -0.0580 | 0.0235 | -0.0018 | -0.0500 | -0.0209 | -0.0113 | 0.0562 | 0.0080 | 0.0371 | 0.0467 |
| Q11 | -0.0180 | 0.0064 | -0.0489 | 0.0064 | -0.0587 | -0.0457 | 0.0309 | 0.0244 | 0.0407 | 0.0277 |
| Q12 | -0.0160 | 0.0357 | -0.0812 | -0.0360 | -0.0642 | -0.0536 | 0.0652 | 0.0200 | 0.0482 | 0.0376 |
| Q13 | -0.0580 | -0.0217 | -0.1347 | -0.0217 | -0.1248 | -0.1070 | 0.0767 | 0.0363 | 0.0668 | 0.0490 |
| Q14 | -0.0500 | -0.0652 | -0.0972 | -0.0652 | -0.1048 | -0.0876 | 0.0472 | 0.0152 | 0.0548 | 0.0376 |
| P1-2 | 1.5688 | 1.5835 | 1.5562 | 1.5691 | 1.5548 | 1.5731 | 0.0126 | 0.0003 | 0.0140 | 0.0043 |
| P1-5 | 0.7551 | 0.7157 | 0.7639 | 0.7817 | 0.7457 | 0.7944 | 0.0088 | 0.0266 | 0.0094 | 0.0393 |
| P2-3 | 0.7324 | 0.7768 | 0.7793 | 0.7894 | 0.7863 | 0.7807 | 0.0469 | 0.0570 | 0.0539 | 0.0483 |
| P2-4 | 0.5613 | N/A | 0.5244 | 0.5568 | 0.5121 | 0.5753 | 0.0369 | 0.0045 | 0.0492 | 0.0140 |
| P2-5 | 0.4152 | N/A | 0.4403 | 0.4554 | 0.4166 | 0.4722 | 0.0251 | 0.0402 | 0.0014 | 0.0570 |
| P3-4 | -0.2329 | -0.2734 | -0.3110 | -0.2923 | -0.3305 | -0.2616 | 0.0781 | 0.0594 | 0.0976 | 0.0287 |
| P4-5 | -0.6116 | N/A | -0.3554 | -0.4320 | -0.4039 | -0.4367 | 0.2562 | 0.1796 | 0.2077 | 0.1749 |
| **P4-7** | **0.2807** | **0.9526** | **0.6180** | **0.7059** | **0.5934** | **0.7768** | **0.3373** | **0.4252** | **0.3127** | **0.4961** |
| P4-9 | 0.1608 | N/A | 0.3370 | 0.4028 | 0.3307 | 0.4290 | 0.1762 | 0.2420 | 0.1699 | 0.2682 |
| P5-6 | 0.4409 | 0.4032 | 0.6912 | 0.7021 | 0.5808 | 0.7642 | 0.2503 | 0.2612 | 0.1399 | 0.3233 |
| P6-11 | 0.0735 | N/A | 0.1988 | 0.3637 | 0.2165 | 0.2693 | 0.1253 | 0.2902 | 0.1430 | 0.1958 |
| P6-12 | 0.0779 | -0.0077 | 0.0323 | -0.0058 | 0.0017 | 0.0306 | 0.0456 | 0.0837 | 0.0762 | 0.0473 |
| P6-13 | 0.1775 | N/A | 0.2523 | 0.1668 | 0.1652 | 0.2198 | 0.0748 | 0.0107 | 0.0123 | 0.0423 |
| P7-8 | 0.0000 | -0.0121 | 0.0100 | -0.0121 | -0.0017 | 0.0080 | 0.0100 | 0.0121 | 0.0017 | 0.0080 |
| **P7-9** | **0.2807** | **0.9967** | **0.5369** | **0.6963** | **0.5475** | **0.7019** | **0.2562** | **0.4156** | **0.2668** | **0.4212** |
| P9-10 | 0.0523 | 0.0679 | 0.1741 | 0.0679 | -0.0224 | -0.0268 | 0.1218 | 0.0156 | 0.0747 | 0.0791 |
| P9-14 | 0.0943 | N/A | 0.2421 | 0.0068 | 0.0396 | 0.0353 | 0.1478 | 0.0875 | 0.0547 | 0.0590 |
| P10-11 | -0.0379 | -0.0394 | 0.0101 | -0.0394 | -0.0510 | -0.0652 | 0.0480 | 0.0015 | 0.0131 | 0.0273 |
| P12-13 | 0.0161 | N/A | 0.0888 | 0.0849 | 0.0780 | 0.0775 | 0.0727 | 0.0688 | 0.0619 | 0.0614 |
| P13-14 | 0.0564 | 0.0878 | 0.1482 | 0.1496 | 0.1172 | 0.1317 | 0.0918 | 0.0932 | 0.0608 | 0.0753 |
| P2-1 | -1.5259 | N/A | -1.5139 | -1.5270 | -1.5126 | -1.5305 | 0.0120 | 0.0011 | 0.0133 | 0.0046 |
| P5-1 | -0.7275 | N/A | -0.7349 | -0.7522 | -0.7182 | -0.7636 | 0.0074 | 0.0247 | 0.0093 | 0.0361 |
| P3-2 | -0.7091 | N/A | -0.7527 | -0.7626 | -0.7592 | -0.7543 | 0.0436 | 0.0535 | 0.0501 | 0.0452 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P4-2 | -0.5445 | N/A | -0.5095 | -0.5403 | -0.4979 | -0.5576 | 0.0350 | 0.0042 | 0.0466 | 0.0131 |
| P5-2 | -0.4061 | N/A | -0.4297 | -0.4445 | -0.4072 | -0.4603 | 0.0236 | 0.0384 | 0.0011 | 0.0542 |
| P4-3 | 0.2366 | N/A | 0.3182 | 0.2985 | 0.3385 | 0.2668 | 0.0816 | 0.0619 | 0.1019 | 0.0302 |
| P5-4 | 0.6167 | N/A | 0.3574 | 0.4345 | 0.4063 | 0.4394 | 0.2593 | 0.1822 | 0.2104 | 0.1773 |
| P7-4 | -0.2807 | N/A | -0.6180 | -0.7059 | -0.5934 | -0.7768 | 0.3373 | 0.4252 | 0.3127 | 0.4961 |
| P9-4 | -0.1608 | N/A | -0.3370 | -0.4028 | -0.3307 | -0.4290 | 0.1762 | 0.2420 | 0.1699 | 0.2682 |
| P6-5 | -0.4409 | N/A | -0.6912 | -0.7021 | -0.5808 | -0.7642 | 0.2503 | 0.2612 | 0.1399 | 0.3233 |
| P11-6 | -0.0730 | N/A | -0.1951 | -0.3522 | -0.2123 | -0.2629 | 0.1221 | 0.2792 | 0.1393 | 0.1899 |
| P12-6 | -0.0771 | N/A | -0.0317 | 0.0058 | -0.0013 | -0.0303 | 0.0454 | 0.0829 | 0.0758 | 0.0468 |
| P13-6 | -0.1754 | N/A | -0.2467 | -0.1650 | -0.1623 | -0.2160 | 0.0713 | 0.0104 | 0.0131 | 0.0406 |
| P8-7 | 0.0000 | N/A | -0.0100 | 0.0121 | 0.0017 | -0.0080 | 0.0100 | 0.0121 | 0.0017 | 0.0080 |
| P9-7 | -0.2807 | N/A | -0.5369 | -0.6963 | -0.5475 | -0.7019 | 0.2562 | 0.4156 | 0.2668 | 0.4212 |
| P10-9 | -0.0521 | N/A | -0.1730 | -0.0676 | 0.0227 | 0.0270 | 0.1209 | 0.0155 | 0.0748 | 0.0791 |
| P14-9 | -0.0931 | N/A | -0.2324 | -0.0062 | -0.0376 | -0.0337 | 0.1393 | 0.0869 | 0.0555 | 0.0594 |
| P11-10 | 0.0380 | N/A | -0.0097 | 0.0396 | 0.0515 | 0.0658 | 0.0477 | 0.0016 | 0.0135 | 0.0278 |
| P13-12 | -0.0161 | N/A | -0.0869 | -0.0833 | -0.0767 | -0.0762 | 0.0708 | 0.0672 | 0.0606 | 0.0601 |
| P14-13 | -0.0559 | N/A | -0.1442 | -0.1460 | -0.1148 | -0.1288 | 0.0883 | 0.0901 | 0.0589 | 0.0729 |
| Q1-2 | -0.2040 | -0.2571 | -0.2051 | -0.1904 | -0.2008 | -0.2035 | 0.0011 | 0.0136 | 0.0032 | 0.0005 |
| Q1-5 | 0.0385 | 0.1560 | 0.0686 | 0.0606 | 0.0600 | 0.0705 | 0.0301 | 0.0221 | 0.0215 | 0.0320 |
| Q2-3 | 0.0356 | 0.0048 | 0.0325 | 0.0431 | 0.0390 | 0.0316 | 0.0031 | 0.0075 | 0.0034 | 0.0040 |
| Q2-4 | -0.0155 | N/A | 0.0147 | 0.0205 | 0.0193 | 0.0276 | 0.0302 | 0.0360 | 0.0348 | 0.0431 |
| Q2-5 | 0.0117 | N/A | 0.0515 | 0.0346 | 0.0436 | 0.0480 | 0.0398 | 0.0229 | 0.0319 | 0.0363 |
| Q3-4 | 0.0447 | 0.0891 | 0.0870 | 0.0775 | 0.0893 | 0.0900 | 0.0423 | 0.0328 | 0.0446 | 0.0453 |
| Q4-5 | 0.1582 | N/A | 0.1706 | 0.0857 | 0.1227 | 0.1125 | 0.0124 | 0.0725 | 0.0355 | 0.0457 |
| Q4-7 | -0.0968 | -0.1669 | -0.1045 | -0.0424 | -0.0693 | -0.0552 | 0.0077 | 0.0544 | 0.0275 | 0.0416 |
| Q4-9 | -0.0043 | N/A | 0.0000 | 0.0270 | 0.0138 | 0.0276 | 0.0043 | 0.0313 | 0.0181 | 0.0319 |
| Q5-6 | 0.1247 | 0.1491 | 0.0330 | -0.0682 | -0.0233 | -0.0286 | 0.0917 | 0.1929 | 0.1480 | 0.1533 |
| Q6-11 | 0.0356 | N/A | -0.0183 | -0.0006 | 0.0066 | -0.0004 | 0.0539 | 0.0362 | 0.0290 | 0.0360 |
| Q6-12 | 0.0250 | 0.0152 | 0.0631 | 0.0152 | 0.0527 | 0.0422 | 0.0381 | 0.0098 | 0.0277 | 0.0172 |
| Q6-13 | 0.0722 | N/A | 0.1495 | 0.0515 | 0.1355 | 0.1153 | 0.0773 | 0.0207 | 0.0633 | 0.0431 |
| Q7-8 | -0.1716 | -0.1570 | -0.1461 | -0.1136 | -0.1334 | -0.1448 | 0.0255 | 0.0580 | 0.0382 | 0.0268 |
| Q7-9 | 0.0578 | 0.0677 | 0.1355 | 0.1202 | 0.1375 | 0.1380 | 0.0777 | 0.0624 | 0.0797 | 0.0802 |
| Q9-10 | 0.0422 | 0.1216 | 0.0808 | 0.0694 | 0.0838 | 0.0730 | 0.0386 | 0.0272 | 0.0416 | 0.0308 |
| Q9-14 | 0.0361 | N/A | 0.1431 | 0.0700 | 0.1223 | 0.1095 | 0.1070 | 0.0339 | 0.0862 | 0.0734 |
| Q10-11 | -0.0162 | 0.0186 | 0.0761 | 0.0186 | 0.0623 | 0.0612 | 0.0923 | 0.0348 | 0.0785 | 0.0774 |
| Q12-13 | 0.0075 | N/A | -0.0194 | -0.0208 | -0.0121 | -0.0120 | 0.0269 | 0.0283 | 0.0196 | 0.0195 |
| Q13-14 | 0.0175 | 0.0053 | -0.0173 | 0.0039 | -0.0083 | -0.0124 | 0.0348 | 0.0136 | 0.0258 | 0.0299 |
| Q2-1 | 0.2768 | N/A | 0.2184 | 0.2002 | 0.2130 | 0.2160 | 0.0584 | 0.0766 | 0.0638 | 0.0608 |
| Q5-1 | 0.0223 | N/A | -0.0535 | -0.0456 | -0.0518 | -0.0492 | 0.0758 | 0.0679 | 0.0741 | 0.0715 |
| Q3-2 | 0.0160 | N/A | -0.0116 | -0.0229 | -0.0162 | -0.0132 | 0.0276 | 0.0389 | 0.0322 | 0.0292 |
| Q4-2 | 0.0302 | N/A | -0.0407 | -0.0431 | -0.0475 | -0.0457 | 0.0709 | 0.0733 | 0.0777 | 0.0759 |
| Q5-2 | -0.0210 | N/A | -0.0915 | -0.0751 | -0.0874 | -0.0849 | 0.0705 | 0.0541 | 0.0664 | 0.0639 |
| Q4-3 | -0.0484 | N/A | -0.0945 | -0.0880 | -0.0947 | -0.1027 | 0.0461 | 0.0396 | 0.0463 | 0.0543 |
| Q5-4 | 0.1420 | N/A | -0.1641 | -0.0778 | -0.1153 | -0.1041 | 0.3061 | 0.2198 | 0.2573 | 0.2461 |
| Q7-4 | 0.1138 | N/A | 0.1839 | 0.1418 | 0.1413 | 0.1770 | 0.0701 | 0.0280 | 0.0275 | 0.0632 |
| Q9-4 | 0.0173 | N/A | 0.0605 | 0.0583 | 0.0444 | 0.0702 | 0.0432 | 0.0410 | 0.0271 | 0.0529 |
| Q6-5 | -0.0805 | N/A | 0.0786 | 0.1813 | 0.1014 | 0.1631 | 0.1591 | 0.2618 | 0.1819 | 0.2436 |
| Q11-6 | -0.0344 | N/A | 0.0261 | 0.0246 | 0.0023 | 0.0139 | 0.0605 | 0.0590 | 0.0367 | 0.0483 |
| Q12-6 | -0.0235 | N/A | -0.0618 | -0.0151 | -0.0520 | -0.0416 | 0.0383 | 0.0084 | 0.0285 | 0.0181 |
| Q13-6 | -0.0680 | N/A | -0.1385 | -0.0478 | -0.1298 | -0.1078 | 0.0705 | 0.0202 | 0.0618 | 0.0398 |
| Q8-7 | 0.1762 | N/A | 0.1496 | 0.1158 | 0.1364 | 0.1483 | 0.0266 | 0.0604 | 0.0398 | 0.0279 |
| Q9-7 | -0.0498 | N/A | -0.1040 | -0.0688 | -0.1043 | -0.0852 | 0.0542 | 0.0190 | 0.0545 | 0.0354 |
| Q10-9 | -0.0418 | N/A | -0.0778 | -0.0686 | -0.0832 | -0.0725 | 0.0360 | 0.0268 | 0.0414 | 0.0307 |
| Q14-9 | -0.0336 | N/A | -0.1227 | -0.0687 | -0.1180 | -0.1060 | 0.0891 | 0.0351 | 0.0844 | 0.0724 |
| Q11-10 | 0.0164 | N/A | -0.0750 | -0.0182 | -0.0611 | -0.0597 | 0.0914 | 0.0346 | 0.0775 | 0.0761 |
| Q13-12 | -0.0075 | N/A | 0.0211 | 0.0222 | 0.0134 | 0.0132 | 0.0286 | 0.0297 | 0.0209 | 0.0207 |
| Q14-13 | -0.0164 | N/A | 0.0254 | 0.0035 | 0.0132 | 0.0184 | 0.0418 | 0.0199 | 0.0296 | 0.0348 |
| **SUM** | | | | | | | **5.7168** | **5.8765** | **5.2226** | **6.2692** |

In this case, all estimators generate bad estimation. Though the case is said to be one reverse two large bad data, the reverse one have larger error compared with that of large bad data, plus two more interacting large bad data, these three reduces the local redundancy and ruins the estimation connected with them, it's very hard to estimate especially for median case.

### Comparison

The comparison between full redundancy and median redundancy with the same condition of error are presented in Table 5.253. It's very clear that when local redundancy is lower and errors are large, more correct data have advantage.

Table 5.253: Estimation errors of case 3, two large, 14-bus AC system, comparison

| | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| SUM | 5.1412 | 4.1298 | 2.7754 | 2.1732 | 2.0044 |
| SUM | N/A | 5.7168 | 5.8765 | 5.2226 | 6.2692 |

## Three large bad data

Compared with the base cases, $P_4 = -0.9349$, $P_{4-7} = 0.9526$ and $P_{7-9} = 0.9967$ are introduced to simulate the three large bad data in both full and median redundancy case. The results are presented below.

### Full redundancy

Based on the original raw data and large bad data shown above, Table 5.254 gives the state variables, another table shows the estimation results and error with actual values.

Table 5.254: State variable of case 3, multi large, 14-bus AC system, full redundancy

| | WLS | LAV | LTS | LTAV | | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0817 | -0.0847 | -0.0817 | -0.0827 | **V1** | 1.0923 | 1.0805 | 1.0905 | 1.0845 |
| $\theta 3$ | -0.2141 | -0.2224 | -0.2159 | -0.2185 | **V2** | 1.0766 | 1.0666 | 1.0748 | 1.0688 |
| $\theta 4$ | -0.1798 | -0.1806 | -0.1760 | -0.1782 | **V3** | 1.0330 | 1.0291 | 1.0304 | 1.0243 |
| $\theta 5$ | -0.1491 | -0.1492 | -0.1457 | -0.1477 | **V4** | 1.0405 | 1.0318 | 1.0402 | 1.0354 |
| $\theta 6$ | -0.2469 | -0.2352 | -0.2232 | -0.2261 | **V5** | 1.0468 | 1.0371 | 1.0465 | 1.0417 |
| $\theta 7$ | -0.2363 | -0.2308 | -0.2107 | -0.2118 | **V6** | 1.0622 | 1.0633 | 1.0631 | 1.0699 |
| $\theta 8$ | -0.2317 | -0.2289 | -0.2067 | -0.2089 | **V7** | 1.0693 | 1.0551 | 1.0673 | 1.0662 |
| $\theta 9$ | -0.2727 | -0.2564 | -0.2344 | -0.2320 | **V8** | 1.0895 | 1.0767 | 1.0868 | 1.0859 |
| $\theta 10$ | -0.2758 | -0.2596 | -0.2333 | -0.2329 | **V9** | 1.0620 | 1.0486 | 1.0602 | 1.0608 |
| $\theta 11$ | -0.2688 | -0.2538 | -0.2333 | -0.2352 | **V10** | 1.0531 | 1.0409 | 1.0535 | 1.0546 |
| $\theta 12$ | -0.2431 | -0.2293 | -0.2180 | -0.2224 | **V11** | 1.0440 | 1.0405 | 1.0458 | 1.0504 |
| $\theta 13$ | -0.2594 | -0.2454 | -0.2318 | -0.2354 | **V12** | 1.0524 | 1.0619 | 1.0543 | 1.0627 |
| $\theta 14$ | -0.2940 | -0.2714 | -0.2493 | -0.2518 | **V13** | 1.0393 | 1.0501 | 1.0417 | 1.0492 |
| | | | | | **V14** | 1.0301 | 1.0350 | 1.0341 | 1.0364 |

Table 5.255: Estimation Results and Errors of case 3, multi large, 14-bus AC system, full redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0923 | 1.0805 | 1.0905 | 1.0845 | 0.0086 | 0.0323 | 0.0205 | 0.0305 | 0.0245 |
| V2 | 1.0450 | 1.0743 | 1.0766 | 1.0666 | 1.0748 | 1.0688 | 0.0293 | 0.0316 | 0.0216 | 0.0298 | 0.0238 |
| V3 | 1.0100 | 0.9739 | 1.0330 | 1.0291 | 1.0304 | 1.0243 | 0.0361 | 0.0230 | 0.0191 | 0.0204 | 0.0143 |
| V4 | 1.0180 | 1.0318 | 1.0405 | 1.0318 | 1.0402 | 1.0354 | 0.0138 | 0.0225 | 0.0138 | 0.0222 | 0.0174 |
| V5 | 1.0200 | 1.0251 | 1.0468 | 1.0371 | 1.0465 | 1.0417 | 0.0051 | 0.0268 | 0.0171 | 0.0265 | 0.0217 |
| V6 | 1.0700 | 1.0491 | 1.0622 | 1.0633 | 1.0631 | 1.0699 | 0.0209 | 0.0078 | 0.0067 | 0.0069 | 0.0001 |
| V7 | 1.0620 | 1.0551 | 1.0693 | 1.0551 | 1.0673 | 1.0662 | 0.0069 | 0.0073 | 0.0069 | 0.0053 | 0.0042 |
| V8 | 1.0900 | 1.0955 | 1.0895 | 1.0767 | 1.0868 | 1.0859 | 0.0055 | 0.0005 | 0.0133 | 0.0032 | 0.0041 |
| V9 | 1.0560 | 1.1131 | 1.0620 | 1.0486 | 1.0602 | 1.0608 | 0.0571 | 0.0060 | 0.0074 | 0.0042 | 0.0048 |
| V10 | 1.0510 | 1.0952 | 1.0531 | 1.0409 | 1.0535 | 1.0546 | 0.0442 | 0.0021 | 0.0101 | 0.0025 | 0.0036 |
| V11 | 1.0570 | 1.0355 | 1.0440 | 1.0405 | 1.0458 | 1.0504 | 0.0215 | 0.0130 | 0.0165 | 0.0112 | 0.0066 |
| V12 | 1.0550 | 1.1034 | 1.0524 | 1.0619 | 1.0543 | 1.0627 | 0.0484 | 0.0026 | 0.0069 | 0.0007 | 0.0077 |
| V13 | 1.0500 | 1.0616 | 1.0393 | 1.0501 | 1.0417 | 1.0492 | 0.0116 | 0.0107 | 0.0001 | 0.0083 | 0.0008 |
| V14 | 1.0360 | 1.0350 | 1.0301 | 1.0350 | 1.0341 | 1.0364 | 0.0010 | 0.0059 | 0.0010 | 0.0019 | 0.0004 |
| P1 | 2.3239 | 2.3508 | 2.3538 | 2.3500 | 2.3288 | 2.3305 | 0.0269 | 0.0299 | 0.0261 | 0.0049 | 0.0066 |
| P2 | 0.1830 | 0.2747 | 0.3260 | 0.2747 | 0.2871 | 0.2872 | 0.0917 | 0.1430 | 0.0917 | 0.1041 | 0.1042 |
| P3 | -0.9420 | -1.0549 | -0.9433 | -0.9805 | -0.9844 | -0.9866 | 0.1129 | 0.0013 | 0.0385 | 0.0424 | 0.0446 |
| **P4** | **-0.4780** | **-0.9349** | **-0.6766** | **-0.6993** | **-0.7975** | **-0.8064** | **0.4569** | **0.1986** | **0.2213** | **0.3195** | **0.3284** |
| P5 | -0.0760 | -0.0601 | 0.0328 | 0.0117 | -0.0285 | -0.0300 | 0.0159 | 0.1088 | 0.0877 | 0.0475 | 0.0460 |
| P6 | -0.1120 | -0.1774 | -0.1621 | -0.1774 | -0.1606 | -0.1576 | 0.0654 | 0.0501 | 0.0654 | 0.0486 | 0.0456 |
| P7 | 0.0000 | -0.0217 | 0.0384 | -0.0217 | 0.0278 | 0.0058 | 0.0217 | 0.0384 | 0.0217 | 0.0278 | 0.0058 |
| P8 | 0.0000 | 0.0171 | 0.0303 | 0.0121 | 0.0268 | 0.0195 | 0.0171 | 0.0303 | 0.0121 | 0.0268 | 0.0195 |
| P9 | -0.2950 | -0.1161 | -0.3739 | -0.2717 | -0.2586 | -0.1766 | 0.1789 | 0.0789 | 0.0233 | 0.0364 | 0.1184 |
| P10 | -0.0900 | 0.0485 | -0.0882 | -0.0950 | 0.0006 | -0.0167 | 0.1385 | 0.0018 | 0.0050 | 0.0906 | 0.0733 |
| P11 | -0.0350 | -0.1025 | -0.1203 | -0.1025 | -0.0966 | -0.1017 | 0.0675 | 0.0853 | 0.0675 | 0.0616 | 0.0667 |
| P12 | -0.0610 | 0.0907 | 0.0728 | 0.0907 | 0.0716 | 0.0700 | 0.1517 | 0.1338 | 0.1517 | 0.1326 | 0.1310 |
| P13 | -0.1350 | -0.0987 | -0.1340 | -0.0987 | -0.1401 | -0.1414 | 0.0363 | 0.0010 | 0.0363 | 0.0051 | 0.0064 |
| P14 | -0.1490 | -0.1522 | -0.2129 | -0.1522 | -0.1402 | -0.1581 | 0.0032 | 0.0639 | 0.0032 | 0.0088 | 0.0091 |
| Q1 | -0.1655 | -0.1298 | -0.1891 | -0.2344 | -0.1938 | -0.2003 | 0.0357 | 0.0236 | 0.0689 | 0.0283 | 0.0348 |
| Q2 | 0.3086 | 0.2984 | 0.2859 | 0.2984 | 0.2825 | 0.2732 | 0.0102 | 0.0227 | 0.0102 | 0.0261 | 0.0354 |
| Q3 | 0.0608 | 0.0546 | -0.0253 | 0.0546 | -0.0253 | -0.0293 | 0.0062 | 0.0861 | 0.0062 | 0.0861 | 0.0901 |
| Q4 | 0.0390 | 0.1135 | 0.0516 | 0.0665 | 0.0497 | 0.0313 | 0.0745 | 0.0126 | 0.0275 | 0.0107 | 0.0077 |
| Q5 | -0.0160 | 0.0545 | 0.1326 | 0.0545 | 0.1219 | 0.0845 | 0.0705 | 0.1486 | 0.0705 | 0.1379 | 0.1005 |
| Q6 | 0.0523 | 0.1232 | -0.0549 | -0.0649 | -0.0369 | 0.0122 | 0.0709 | 0.1072 | 0.1172 | 0.0892 | 0.0401 |
| Q7 | 0.0000 | 0.0336 | -0.0082 | -0.0561 | -0.0248 | -0.0232 | 0.0336 | 0.0082 | 0.0561 | 0.0248 | 0.0232 |
| Q8 | 0.1762 | 0.1158 | 0.1247 | 0.1318 | 0.1206 | 0.1215 | 0.0604 | 0.0515 | 0.0444 | 0.0556 | 0.0547 |
| Q9 | -0.1660 | -0.1301 | -0.1373 | -0.2027 | -0.1619 | -0.1634 | 0.0359 | 0.0287 | 0.0367 | 0.0041 | 0.0026 |
| Q10 | -0.0580 | 0.0235 | -0.0272 | -0.0552 | -0.0425 | -0.0491 | 0.0815 | 0.0308 | 0.0028 | 0.0155 | 0.0089 |
| Q11 | -0.0180 | 0.0064 | -0.0848 | -0.0699 | -0.0866 | -0.0783 | 0.0244 | 0.0668 | 0.0519 | 0.0686 | 0.0603 |
| Q12 | -0.0160 | 0.0357 | -0.0524 | -0.0311 | -0.0458 | -0.0343 | 0.0517 | 0.0364 | 0.0151 | 0.0298 | 0.0183 |
| Q13 | -0.0580 | -0.0217 | -0.1074 | -0.0272 | -0.1004 | -0.0847 | 0.0363 | 0.0494 | 0.0308 | 0.0424 | 0.0267 |
| Q14 | -0.0500 | -0.0652 | -0.0434 | -0.0220 | -0.0544 | -0.0548 | 0.0152 | 0.0066 | 0.0280 | 0.0044 | 0.0048 |
| P1-2 | 1.5688 | 1.5835 | 1.5702 | 1.5835 | 1.5656 | 1.5659 | 0.0147 | 0.0014 | 0.0147 | 0.0032 | 0.0029 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P1-5 | 0.7551 | 0.7157 | 0.7835 | 0.7665 | 0.7632 | 0.7646 | 0.0394 | 0.0284 | 0.0114 | 0.0081 | 0.0095 |
| P2-3 | 0.7324 | 0.7768 | 0.7665 | 0.7775 | 0.7740 | 0.7743 | 0.0444 | 0.0341 | 0.0451 | 0.0416 | 0.0419 |
| P2-4 | 0.5613 | 0.5039 | 0.6359 | 0.6105 | 0.6094 | 0.6086 | 0.0574 | 0.0746 | 0.0492 | 0.0481 | 0.0473 |
| P2-5 | 0.4152 | 0.3618 | 0.4534 | 0.4280 | 0.4289 | 0.4294 | 0.0534 | 0.0382 | 0.0128 | 0.0137 | 0.0142 |
| P3-4 | -0.2329 | -0.2734 | -0.2011 | -0.2282 | -0.2352 | -0.2374 | 0.0405 | 0.0318 | 0.0047 | 0.0023 | 0.0045 |
| P4-5 | -0.6116 | -0.7588 | -0.7619 | -0.7588 | -0.7536 | -0.7491 | 0.1472 | 0.1503 | 0.1472 | 0.1420 | 0.1375 |
| **P4-7** | **0.2807** | **0.9526** | **0.3070** | **0.2670** | **0.1886** | **0.1817** | **0.6719** | **0.0263** | **0.0137** | **0.0921** | **0.0990** |
| P4-9 | 0.1608 | 0.1771 | 0.1902 | 0.1520 | 0.1194 | 0.1096 | 0.0163 | 0.0294 | 0.0088 | 0.0414 | 0.0512 |
| P5-6 | 0.4409 | 0.4032 | 0.4621 | 0.4032 | 0.3668 | 0.3715 | 0.0377 | 0.0212 | 0.0377 | 0.0741 | 0.0694 |
| P6-11 | 0.0735 | 0.1420 | 0.1379 | 0.1317 | 0.0821 | 0.0830 | 0.0685 | 0.0644 | 0.0582 | 0.0086 | 0.0095 |
| P6-12 | 0.0779 | -0.0077 | 0.0023 | -0.0188 | -0.0042 | -0.0015 | 0.0856 | 0.0756 | 0.0967 | 0.0821 | 0.0794 |
| P6-13 | 0.1775 | 0.1724 | 0.1598 | 0.1129 | 0.1283 | 0.1324 | 0.0051 | 0.0177 | 0.0646 | 0.0492 | 0.0451 |
| P7-8 | 0.0000 | -0.0121 | -0.0303 | -0.0121 | -0.0268 | -0.0195 | 0.0121 | 0.0303 | 0.0121 | 0.0268 | 0.0195 |
| **P7-9** | **0.2807** | **0.9967** | **0.3758** | **0.2574** | **0.2431** | **0.2070** | **0.7160** | **0.0951** | **0.0233** | **0.0376** | **0.0737** |
| P9-10 | 0.0523 | 0.0679 | 0.0728 | 0.0679 | 0.0150 | 0.0366 | 0.0156 | 0.0205 | 0.0156 | 0.0373 | 0.0157 |
| P9-14 | 0.0943 | 0.0511 | 0.1192 | 0.0698 | 0.0890 | 0.1033 | 0.0432 | 0.0249 | 0.0245 | 0.0053 | 0.0090 |
| P10-11 | -0.0379 | -0.0394 | -0.0157 | -0.0274 | 0.0154 | 0.0197 | 0.0015 | 0.0222 | 0.0105 | 0.0533 | 0.0576 |
| P12-13 | 0.0161 | 0.0079 | 0.0749 | 0.0718 | 0.0672 | 0.0684 | 0.0082 | 0.0588 | 0.0557 | 0.0511 | 0.0523 |
| P13-14 | 0.0564 | 0.0878 | 0.0974 | 0.0841 | 0.0529 | 0.0568 | 0.0314 | 0.0410 | 0.0277 | 0.0035 | 0.0004 |
| P2-1 | -1.5259 | -1.4712 | -1.5298 | -1.5412 | -1.5253 | -1.5251 | 0.0547 | 0.0039 | 0.0153 | 0.0006 | 0.0008 |
| P5-1 | -0.7275 | -0.6720 | -0.7554 | -0.7390 | -0.7364 | -0.7375 | 0.0555 | 0.0279 | 0.0115 | 0.0089 | 0.0100 |
| P3-2 | -0.7091 | -0.7523 | -0.7422 | -0.7523 | -0.7491 | -0.7491 | 0.0432 | 0.0331 | 0.0432 | 0.0400 | 0.0400 |
| P4-2 | -0.5445 | -0.5406 | -0.6156 | -0.5913 | -0.5907 | -0.5897 | 0.0039 | 0.0711 | 0.0468 | 0.0462 | 0.0452 |
| P5-2 | -0.4061 | -0.4668 | -0.4431 | -0.4187 | -0.4197 | -0.4202 | 0.0607 | 0.0370 | 0.0126 | 0.0136 | 0.0141 |
| P4-3 | 0.2366 | 0.1809 | 0.2037 | 0.2319 | 0.2388 | 0.2411 | 0.0557 | 0.0329 | 0.0047 | 0.0022 | 0.0045 |
| P5-4 | 0.6167 | 0.6164 | 0.7692 | 0.7662 | 0.7608 | 0.7562 | 0.0003 | 0.1525 | 0.1495 | 0.1441 | 0.1395 |
| P7-4 | -0.2807 | -0.2041 | -0.3070 | -0.2670 | -0.1886 | -0.1817 | 0.0766 | 0.0263 | 0.0137 | 0.0921 | 0.0990 |
| P9-4 | -0.1608 | -0.1993 | -0.1902 | -0.1520 | -0.1194 | -0.1096 | 0.0385 | 0.0294 | 0.0088 | 0.0414 | 0.0512 |
| P6-5 | -0.4409 | -0.4223 | -0.4621 | -0.4032 | -0.3668 | -0.3715 | 0.0186 | 0.0212 | 0.0377 | 0.0741 | 0.0694 |
| P11-6 | -0.0730 | -0.0843 | -0.1362 | -0.1299 | -0.0813 | -0.0820 | 0.0113 | 0.0632 | 0.0569 | 0.0083 | 0.0090 |
| P12-6 | -0.0771 | -0.0212 | -0.0021 | 0.0189 | 0.0044 | 0.0016 | 0.0559 | 0.0750 | 0.0960 | 0.0815 | 0.0787 |
| P13-6 | -0.1754 | -0.2299 | -0.1577 | -0.1120 | -0.1267 | -0.1307 | 0.0545 | 0.0177 | 0.0634 | 0.0487 | 0.0447 |
| P8-7 | 0.0000 | 0.0016 | 0.0303 | 0.0121 | 0.0268 | 0.0195 | 0.0016 | 0.0303 | 0.0121 | 0.0268 | 0.0195 |
| P9-7 | -0.2807 | -0.2531 | -0.3758 | -0.2574 | -0.2431 | -0.2070 | 0.0276 | 0.0951 | 0.0233 | 0.0376 | 0.0737 |
| P10-9 | -0.0521 | 0.0029 | -0.0725 | -0.0676 | -0.0148 | -0.0364 | 0.0550 | 0.0204 | 0.0155 | 0.0373 | 0.0157 |
| P14-9 | -0.0931 | -0.0159 | -0.1171 | -0.0692 | -0.0877 | -0.1018 | 0.0772 | 0.0240 | 0.0239 | 0.0054 | 0.0087 |
| P11-10 | 0.0380 | 0.0423 | 0.0159 | 0.0274 | -0.0153 | -0.0197 | 0.0043 | 0.0221 | 0.0106 | 0.0533 | 0.0577 |
| P13-12 | -0.0161 | -0.0907 | -0.0738 | -0.0708 | -0.0663 | -0.0675 | 0.0746 | 0.0577 | 0.0547 | 0.0502 | 0.0514 |
| P14-13 | -0.0559 | -0.0930 | -0.0958 | -0.0830 | -0.0525 | -0.0563 | 0.0371 | 0.0399 | 0.0271 | 0.0034 | 0.0004 |
| Q1-2 | -0.2040 | -0.2571 | -0.2206 | -0.2571 | -0.2197 | -0.2208 | 0.0531 | 0.0166 | 0.0531 | 0.0157 | 0.0168 |
| Q1-5 | 0.0385 | 0.1560 | 0.0314 | 0.0227 | 0.0260 | 0.0205 | 0.1175 | 0.0071 | 0.0158 | 0.0125 | 0.0180 |
| Q2-3 | 0.0356 | 0.0048 | 0.0535 | 0.0202 | 0.0571 | 0.0576 | 0.0308 | 0.0179 | 0.0154 | 0.0215 | 0.0220 |
| Q2-4 | -0.0155 | 0.0219 | 0.0018 | -0.0006 | -0.0013 | -0.0079 | 0.0374 | 0.0173 | 0.0149 | 0.0142 | 0.0076 |
| Q2-5 | 0.0117 | 0.0021 | 0.0107 | 0.0145 | 0.0076 | 0.0005 | 0.0096 | 0.0010 | 0.0028 | 0.0041 | 0.0112 |
| Q3-4 | 0.0447 | 0.0891 | 0.0234 | 0.0649 | 0.0242 | 0.0183 | 0.0444 | 0.0213 | 0.0202 | 0.0205 | 0.0264 |
| Q4-5 | 0.1582 | 0.1200 | 0.0992 | 0.1224 | 0.0959 | 0.0950 | 0.0382 | 0.0590 | 0.0358 | 0.0623 | 0.0632 |
| Q4-7 | -0.0968 | -0.1669 | -0.1378 | -0.1108 | -0.1342 | -0.1531 | 0.0701 | 0.0410 | 0.0140 | 0.0374 | 0.0563 |
| Q4-9 | -0.0043 | -0.0754 | -0.0326 | -0.0263 | -0.0350 | -0.0459 | 0.0711 | 0.0283 | 0.0220 | 0.0307 | 0.0416 |
| Q5-6 | 0.1247 | 0.1491 | -0.0463 | -0.0984 | -0.0596 | -0.1109 | 0.0244 | 0.1710 | 0.2231 | 0.1843 | 0.2356 |
| Q6-11 | 0.0356 | 0.0267 | 0.0326 | 0.0600 | 0.0532 | 0.0657 | 0.0089 | 0.0030 | 0.0244 | 0.0176 | 0.0301 |
| Q6-12 | 0.0250 | 0.0152 | 0.0398 | 0.0152 | 0.0386 | 0.0310 | 0.0098 | 0.0148 | 0.0098 | 0.0136 | 0.0060 |
| Q6-13 | 0.0722 | 0.1432 | 0.1068 | 0.0513 | 0.1093 | 0.1033 | 0.0710 | 0.0346 | 0.0209 | 0.0371 | 0.0311 |
| Q7-8 | -0.1716 | -0.1570 | -0.1223 | -0.1291 | -0.1183 | -0.1192 | 0.0146 | 0.0493 | 0.0425 | 0.0533 | 0.0524 |
| Q7-9 | 0.0578 | 0.0677 | 0.0779 | 0.0659 | 0.0717 | 0.0545 | 0.0099 | 0.0201 | 0.0081 | 0.0139 | 0.0033 |
| Q9-10 | 0.0422 | 0.1216 | 0.0845 | 0.0697 | 0.0783 | 0.0642 | 0.0794 | 0.0423 | 0.0275 | 0.0361 | 0.0220 |
| Q9-14 | 0.0361 | -0.0041 | 0.0700 | 0.0203 | 0.0608 | 0.0479 | 0.0402 | 0.0339 | 0.0158 | 0.0247 | 0.0118 |
| Q10-11 | -0.0162 | 0.0186 | 0.0564 | 0.0138 | 0.0353 | 0.0147 | 0.0348 | 0.0726 | 0.0300 | 0.0515 | 0.0309 |
| Q12-13 | 0.0075 | 0.0493 | -0.0129 | -0.0160 | -0.0075 | -0.0035 | 0.0418 | 0.0204 | 0.0235 | 0.0150 | 0.0110 |
| Q13-14 | 0.0175 | 0.0053 | -0.0188 | 0.0053 | -0.0027 | 0.0111 | 0.0122 | 0.0363 | 0.0122 | 0.0202 | 0.0064 |
| Q2-1 | 0.2768 | 0.2876 | 0.2199 | 0.2644 | 0.2191 | 0.2230 | 0.0108 | 0.0569 | 0.0124 | 0.0577 | 0.0538 |
| Q5-1 | 0.0223 | -0.0360 | -0.0278 | -0.0196 | -0.0278 | -0.0197 | 0.0583 | 0.0501 | 0.0419 | 0.0501 | 0.0420 |
| Q3-2 | 0.0160 | -0.0414 | -0.0488 | -0.0103 | -0.0495 | -0.0477 | 0.0574 | 0.0648 | 0.0263 | 0.0655 | 0.0637 |
| Q4-2 | 0.0302 | 0.0354 | -0.0162 | -0.0162 | -0.0179 | -0.0100 | 0.0052 | 0.0464 | 0.0464 | 0.0481 | 0.0402 |
| Q5-2 | -0.0210 | 0.0151 | -0.0575 | -0.0626 | -0.0575 | -0.0493 | 0.0361 | 0.0365 | 0.0416 | 0.0365 | 0.0283 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Q4-3 | -0.0484 | 0.0809 | -0.0443 | -0.0827 | -0.0425 | -0.0361 | 0.1293 | 0.0041 | 0.0343 | 0.0059 | 0.0123 |
| Q5-4 | 0.1420 | 0.1087 | -0.0762 | -0.0990 | -0.0735 | -0.0726 | 0.0333 | 0.2182 | 0.2410 | 0.2155 | 0.2146 |
| Q7-4 | 0.1138 | 0.1232 | 0.1592 | 0.1269 | 0.1443 | 0.1639 | 0.0094 | 0.0454 | 0.0131 | 0.0305 | 0.0501 |
| Q9-4 | 0.0173 | 0.0132 | 0.0512 | 0.0384 | 0.0427 | 0.0530 | 0.0041 | 0.0339 | 0.0211 | 0.0254 | 0.0357 |
| Q6-5 | -0.0805 | -0.1772 | 0.0926 | 0.1360 | 0.0892 | 0.1435 | 0.0967 | 0.1731 | 0.2165 | 0.1697 | 0.2240 |
| Q11-6 | -0.0344 | -0.0563 | -0.0291 | -0.0563 | -0.0515 | -0.0638 | 0.0219 | 0.0053 | 0.0219 | 0.0171 | 0.0294 |
| Q12-6 | -0.0235 | -0.1132 | -0.0394 | -0.0151 | -0.0382 | -0.0308 | 0.0897 | 0.0159 | 0.0084 | 0.0147 | 0.0073 |
| Q13-6 | -0.0680 | -0.0260 | -0.1026 | -0.0495 | -0.1061 | -0.1001 | 0.0420 | 0.0346 | 0.0185 | 0.0381 | 0.0321 |
| Q8-7 | 0.1762 | 0.1318 | 0.1247 | 0.1318 | 0.1206 | 0.1215 | 0.0444 | 0.0515 | 0.0444 | 0.0556 | 0.0547 |
| Q9-7 | -0.0498 | -0.0448 | -0.0637 | -0.0589 | -0.0655 | -0.0500 | 0.0050 | 0.0139 | 0.0091 | 0.0157 | 0.0002 |
| Q10-9 | -0.0418 | -0.0690 | -0.0836 | -0.0690 | -0.0778 | -0.0638 | 0.0272 | 0.0418 | 0.0272 | 0.0360 | 0.0220 |
| Q14-9 | -0.0336 | -0.0184 | -0.0654 | -0.0190 | -0.0580 | -0.0448 | 0.0152 | 0.0318 | 0.0146 | 0.0244 | 0.0112 |
| Q11-10 | 0.0164 | -0.0136 | -0.0558 | -0.0136 | -0.0351 | -0.0146 | 0.0300 | 0.0722 | 0.0300 | 0.0515 | 0.0310 |
| Q13-12 | -0.0075 | 0.0170 | 0.0140 | 0.0170 | 0.0084 | 0.0043 | 0.0245 | 0.0215 | 0.0245 | 0.0159 | 0.0118 |
| Q14-13 | -0.0164 | 0.0206 | 0.0219 | -0.0031 | 0.0036 | -0.0100 | 0.0370 | 0.0383 | 0.0133 | 0.0200 | 0.0064 |
| **SUM** | | | | | | | **4.6852** | **2.9283** | **2.4469** | **2.6866** | **2.6020** |

### Median redundancy

The following two tables show the results and errors in detail.

Table 5.256: State Variable of case 3, multi large, 14-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0886 | -0.0866 | -0.0880 | -0.0862 | **V1** | 1.0636 | 1.0686 | 1.0594 | 1.0713 |
| $\theta 3$ | -0.2348 | -0.2338 | -0.2371 | -0.2321 | **V2** | 1.0465 | 1.0522 | 1.0423 | 1.0545 |
| $\theta 4$ | -0.2034 | -0.1832 | -0.1886 | -0.1857 | **V3** | 1.0082 | 1.0112 | 1.0017 | 1.0152 |
| $\theta 5$ | -0.1660 | -0.1504 | -0.1543 | -0.1498 | **V4** | 1.0040 | 1.0174 | 1.0045 | 1.0163 |
| $\theta 6$ | -0.2896 | -0.2237 | -0.2328 | -0.2223 | **V5** | 1.0079 | 1.0198 | 1.0082 | 1.0176 |
| $\theta 7$ | -0.2818 | -0.2185 | -0.2139 | -0.2172 | **V6** | 1.0117 | 1.0176 | 1.0132 | 0.9953 |
| $\theta 8$ | -0.2756 | -0.2158 | -0.2108 | -0.2137 | **V7** | 1.0335 | 1.0448 | 1.0278 | 1.0459 |
| $\theta 9$ | -0.3298 | -0.2368 | -0.2281 | -0.2354 | **V8** | 1.0572 | 1.0640 | 1.0481 | 1.0672 |
| $\theta 10$ | -0.3374 | -0.2385 | -0.2289 | -0.2357 | **V9** | 1.0205 | 1.0323 | 1.0158 | 1.0348 |
| $\theta 11$ | -0.3287 | -0.2413 | -0.2358 | -0.2384 | **V10** | 1.0095 | 1.0202 | 1.0076 | 1.0265 |
| $\theta 12$ | -0.2810 | -0.2158 | -0.2237 | -0.2215 | **V11** | 0.9962 | 1.0149 | 0.9979 | 1.0089 |
| $\theta 13$ | -0.3042 | -0.2323 | -0.2366 | -0.2343 | **V12** | 0.9951 | 1.0168 | 1.0003 | 1.0020 |
| $\theta 14$ | -0.3606 | -0.2508 | -0.2510 | -0.2553 | **V13** | 0.9789 | 1.0059 | 0.9866 | 0.9890 |
| | | | | | **V14** | 0.9603 | 0.9947 | 0.9707 | 0.9900 |

Table 5.257: Estimation Results and Errors of case 3, multi large, 14-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0686 | 1.0636 | 1.0686 | 1.0594 | 1.0713 | 0.0036 | 0.0086 | 0.0006 | 0.0113 |
| V2 | 1.0450 | N/A | 1.0465 | 1.0522 | 1.0423 | 1.0545 | 0.0015 | 0.0072 | 0.0027 | 0.0095 |
| V3 | 1.0100 | N/A | 1.0082 | 1.0112 | 1.0017 | 1.0152 | 0.0018 | 0.0012 | 0.0083 | 0.0052 |
| V4 | 1.0180 | N/A | 1.0040 | 1.0174 | 1.0045 | 1.0163 | 0.0140 | 0.0006 | 0.0135 | 0.0017 |
| V5 | 1.0200 | N/A | 1.0079 | 1.0198 | 1.0082 | 1.0176 | 0.0121 | 0.0002 | 0.0118 | 0.0024 |
| V6 | 1.0700 | N/A | 1.0117 | 1.0176 | 1.0132 | 0.9953 | 0.0583 | 0.0524 | 0.0568 | 0.0747 |
| V7 | 1.0620 | N/A | 1.0335 | 1.0448 | 1.0278 | 1.0459 | 0.0285 | 0.0172 | 0.0342 | 0.0161 |
| V8 | 1.0900 | N/A | 1.0572 | 1.0640 | 1.0481 | 1.0672 | 0.0328 | 0.0260 | 0.0419 | 0.0228 |
| V9 | 1.0560 | N/A | 1.0205 | 1.0323 | 1.0158 | 1.0348 | 0.0355 | 0.0237 | 0.0402 | 0.0212 |
| V10 | 1.0510 | N/A | 1.0095 | 1.0202 | 1.0076 | 1.0265 | 0.0415 | 0.0308 | 0.0434 | 0.0245 |
| V11 | 1.0570 | N/A | 0.9962 | 1.0149 | 0.9979 | 1.0089 | 0.0608 | 0.0421 | 0.0591 | 0.0481 |
| V12 | 1.0550 | N/A | 0.9951 | 1.0168 | 1.0003 | 1.0020 | 0.0599 | 0.0382 | 0.0547 | 0.0530 |
| V13 | 1.0500 | N/A | 0.9789 | 1.0059 | 0.9866 | 0.9890 | 0.0711 | 0.0441 | 0.0634 | 0.0610 |
| V14 | 1.0360 | N/A | 0.9603 | 0.9947 | 0.9707 | 0.9900 | 0.0757 | 0.0413 | 0.0653 | 0.0460 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P1 | 2.3239 | 2.3508 | 2.4429 | 2.3508 | 2.3567 | 2.3564 | 0.1190 | 0.0269 | 0.0328 | 0.0325 |
| P2 | 0.1830 | 0.2747 | 0.4208 | 0.2747 | 0.2935 | 0.2979 | 0.2378 | 0.0917 | 0.1105 | 0.1149 |
| P3 | -0.9420 | -1.0549 | -0.9174 | -1.0537 | -1.0249 | -1.0194 | 0.0246 | 0.1117 | 0.0829 | 0.0774 |
| **P4** | **-0.4780** | **-0.9349** | **-0.7230** | **-0.7604** | **-0.9092** | **-0.8993** | **0.2450** | **0.2824** | **0.4312** | **0.4213** |
| P5 | -0.0760 | -0.0601 | 0.1063 | -0.0601 | -0.0284 | -0.0204 | 0.1823 | 0.0159 | 0.0476 | 0.0556 |
| P6 | -0.1120 | -0.1774 | -0.1531 | -0.1774 | -0.2010 | -0.1931 | 0.0411 | 0.0654 | 0.0890 | 0.0811 |
| P7 | 0.0000 | -0.0217 | 0.0253 | -0.0217 | -0.0127 | -0.0064 | 0.0253 | 0.0217 | 0.0127 | 0.0064 |
| P8 | 0.0000 | 0.0171 | 0.0379 | 0.0171 | 0.0191 | 0.0222 | 0.0379 | 0.0171 | 0.0191 | 0.0222 |
| P9 | -0.2950 | -0.1161 | -0.3953 | -0.1161 | -0.0332 | -0.1112 | 0.1003 | 0.1789 | 0.2618 | 0.1838 |
| P10 | -0.0900 | 0.0485 | -0.1377 | -0.0444 | 0.0075 | 0.0097 | 0.0477 | 0.0456 | 0.0975 | 0.0997 |
| P11 | -0.0350 | -0.1025 | -0.1763 | -0.1025 | -0.0908 | -0.0851 | 0.1413 | 0.0675 | 0.0558 | 0.0501 |
| P12 | -0.0610 | 0.0907 | 0.0944 | 0.0907 | 0.0727 | 0.0741 | 0.1554 | 0.1517 | 0.1337 | 0.1351 |
| P13 | -0.1350 | -0.0987 | -0.1316 | -0.0987 | -0.1170 | -0.1051 | 0.0034 | 0.0363 | 0.0180 | 0.0299 |
| P14 | -0.1490 | -0.1522 | -0.3145 | -0.1522 | -0.1796 | -0.1706 | 0.1655 | 0.0032 | 0.0306 | 0.0216 |
| Q1 | -0.1655 | -0.1298 | -0.1313 | -0.1647 | -0.1430 | -0.1347 | 0.0342 | 0.0008 | 0.0225 | 0.0308 |
| Q2 | 0.3086 | 0.2984 | 0.3222 | 0.2984 | 0.3107 | 0.3214 | 0.0136 | 0.0102 | 0.0021 | 0.0128 |
| Q3 | 0.0608 | 0.0546 | 0.0735 | 0.0546 | 0.0688 | 0.0773 | 0.0127 | 0.0062 | 0.0080 | 0.0165 |
| Q4 | 0.0390 | 0.1135 | 0.1075 | 0.1135 | 0.0940 | 0.1138 | 0.0685 | 0.0745 | 0.0550 | 0.0748 |
| Q5 | -0.0160 | 0.0545 | 0.0475 | 0.0545 | 0.0339 | 0.0510 | 0.0635 | 0.0705 | 0.0499 | 0.0670 |
| Q6 | 0.0523 | 0.1232 | -0.0312 | -0.2598 | 0.0044 | -0.4740 | 0.0835 | 0.3121 | 0.0479 | 0.5263 |
| Q7 | 0.0000 | 0.0336 | 0.0436 | 0.0336 | 0.0004 | 0.0165 | 0.0436 | 0.0336 | 0.0004 | 0.0165 |
| Q8 | 0.1762 | 0.1158 | 0.1425 | 0.1158 | 0.1209 | 0.1289 | 0.0337 | 0.0604 | 0.0553 | 0.0473 |
| Q9 | -0.1660 | -0.1301 | -0.0904 | -0.1301 | -0.1528 | -0.1285 | 0.0756 | 0.0359 | 0.0132 | 0.0375 |
| Q10 | -0.0580 | 0.0235 | -0.0088 | -0.1015 | -0.0520 | -0.0136 | 0.0492 | 0.0435 | 0.0060 | 0.0444 |
| Q11 | -0.0180 | 0.0064 | -0.0577 | 0.0064 | -0.0865 | 0.0155 | 0.0397 | 0.0244 | 0.0685 | 0.0335 |
| Q12 | -0.0160 | 0.0357 | -0.0850 | -0.0316 | -0.0556 | 0.0182 | 0.0690 | 0.0156 | 0.0396 | 0.0342 |
| Q13 | -0.0580 | -0.0217 | -0.1441 | -0.0217 | -0.1261 | -0.0237 | 0.0861 | 0.0363 | 0.0681 | 0.0343 |
| Q14 | -0.0500 | -0.0652 | -0.1081 | -0.0964 | -0.1192 | -0.0788 | 0.0581 | 0.0464 | 0.0692 | 0.0288 |
| P1-2 | 1.5688 | 1.5835 | 1.6168 | 1.5930 | 1.5930 | 1.5957 | 0.0480 | 0.0242 | 0.0242 | 0.0269 |
| P1-5 | 0.7551 | 0.7157 | 0.8261 | 0.7578 | 0.7637 | 0.7608 | 0.0710 | 0.0027 | 0.0086 | 0.0057 |
| P2-3 | 0.7324 | 0.7768 | 0.7930 | 0.8083 | 0.8029 | 0.8044 | 0.0606 | 0.0759 | 0.0705 | 0.0720 |
| P2-4 | 0.5613 | N/A | 0.7020 | 0.5986 | 0.6137 | 0.6216 | 0.1407 | 0.0373 | 0.0524 | 0.0603 |
| P2-5 | 0.4152 | N/A | 0.4974 | 0.4173 | 0.4258 | 0.4242 | 0.0822 | 0.0021 | 0.0106 | 0.0090 |
| P3-4 | -0.2329 | -0.2734 | -0.1517 | -0.2734 | -0.2502 | -0.2426 | 0.0812 | 0.0405 | 0.0173 | 0.0097 |
| P4-5 | -0.6116 | N/A | -0.8380 | -0.7470 | -0.7730 | -0.8055 | 0.2264 | 0.1354 | 0.1614 | 0.1939 |
| **P4-7** | **0.2807** | **0.9526** | **0.3972** | **0.1834** | **0.1277** | **0.1635** | **0.1165** | **0.0973** | **0.1530** | **0.1172** |
| P4-9 | 0.1608 | N/A | 0.2397 | 0.1043 | 0.0746 | 0.0969 | 0.0789 | 0.0565 | 0.0862 | 0.0639 |
| P5-6 | 0.4409 | 0.4032 | 0.5354 | 0.3234 | 0.3413 | 0.3123 | 0.0945 | 0.1175 | 0.0996 | 0.1286 |
| P6-11 | 0.0735 | N/A | 0.1934 | 0.0803 | 0.0427 | 0.0401 | 0.1199 | 0.0068 | 0.0308 | 0.0334 |
| P6-12 | 0.0779 | -0.0077 | -0.0021 | -0.0247 | -0.0093 | -0.0127 | 0.0800 | 0.1026 | 0.0872 | 0.0906 |
| P6-13 | 0.1775 | N/A | 0.1910 | 0.0905 | 0.1068 | 0.0918 | 0.0135 | 0.0870 | 0.0707 | 0.0857 |
| P7-8 | 0.0000 | -0.0121 | -0.0379 | -0.0171 | -0.0191 | -0.0222 | 0.0379 | 0.0171 | 0.0191 | 0.0222 |
| **P7-9** | **0.2807** | **0.9967** | **0.4604** | **0.1788** | **0.1341** | **0.1793** | **0.1797** | **0.1019** | **0.1466** | **0.1014** |
| P9-10 | 0.0523 | 0.0679 | 0.1252 | 0.0679 | 0.0416 | 0.0370 | 0.0729 | 0.0156 | 0.0107 | 0.0153 |
| P9-14 | 0.0943 | N/A | 0.1796 | 0.0990 | 0.1340 | 0.1279 | 0.0853 | 0.0047 | 0.0397 | 0.0336 |
| P10-11 | -0.0379 | -0.0394 | -0.0131 | 0.0230 | 0.0488 | 0.0464 | 0.0248 | 0.0609 | 0.0867 | 0.0843 |
| P12-13 | 0.0161 | N/A | 0.0917 | 0.0659 | 0.0631 | 0.0613 | 0.0756 | 0.0498 | 0.0470 | 0.0452 |
| P13-14 | 0.0564 | 0.0878 | 0.1452 | 0.0560 | 0.0498 | 0.0466 | 0.0888 | 0.0004 | 0.0066 | 0.0098 |
| P2-1 | -1.5259 | N/A | -1.5716 | -1.5495 | -1.5489 | -1.5523 | 0.0457 | 0.0236 | 0.0230 | 0.0264 |
| P5-1 | -0.7275 | N/A | -0.7927 | -0.7301 | -0.7349 | -0.7327 | 0.0652 | 0.0026 | 0.0074 | 0.0052 |
| P3-2 | -0.7091 | N/A | -0.7658 | -0.7803 | -0.7747 | -0.7768 | 0.0567 | 0.0712 | 0.0656 | 0.0677 |
| P4-2 | -0.5445 | N/A | -0.6757 | -0.5797 | -0.5934 | -0.6013 | 0.1312 | 0.0352 | 0.0489 | 0.0568 |
| P5-2 | -0.4061 | N/A | -0.4842 | -0.4081 | -0.4160 | -0.4145 | 0.0781 | 0.0020 | 0.0099 | 0.0084 |
| P4-3 | 0.2366 | N/A | 0.1537 | 0.2787 | 0.2549 | 0.2470 | 0.0829 | 0.0421 | 0.0183 | 0.0104 |
| P5-4 | 0.6167 | N/A | 0.8477 | 0.7547 | 0.7813 | 0.8146 | 0.2310 | 0.1380 | 0.1646 | 0.1979 |
| P7-4 | -0.2807 | N/A | -0.3972 | -0.1834 | -0.1277 | -0.1635 | 0.1165 | 0.0973 | 0.1530 | 0.1172 |
| P9-4 | -0.1608 | N/A | -0.2397 | -0.1043 | -0.0746 | -0.0969 | 0.0789 | 0.0565 | 0.0862 | 0.0639 |
| P6-5 | -0.4409 | N/A | -0.5354 | -0.3234 | -0.3413 | -0.3123 | 0.0945 | 0.1175 | 0.0996 | 0.1286 |
| P11-6 | -0.0730 | N/A | -0.1900 | -0.0796 | -0.0423 | -0.0393 | 0.1170 | 0.0066 | 0.0307 | 0.0337 |
| P12-6 | -0.0771 | N/A | 0.0026 | 0.0248 | 0.0097 | 0.0128 | 0.0797 | 0.1019 | 0.0868 | 0.0899 |
| P13-6 | -0.1754 | N/A | -0.1870 | -0.0898 | -0.1046 | -0.0912 | 0.0116 | 0.0856 | 0.0708 | 0.0842 |
| P8-7 | 0.0000 | N/A | 0.0379 | 0.0171 | 0.0191 | 0.0222 | 0.0379 | 0.0171 | 0.0191 | 0.0222 |
| P9-7 | -0.2807 | N/A | -0.4604 | -0.1788 | -0.1341 | -0.1793 | 0.1797 | 0.1019 | 0.1466 | 0.1014 |
| P10-9 | -0.0521 | N/A | -0.1245 | -0.0673 | -0.0413 | -0.0368 | 0.0724 | 0.0152 | 0.0108 | 0.0153 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P14-9 | -0.0931 | N/A | -0.1731 | -0.0967 | -0.1303 | -0.1245 | 0.0800 | 0.0036 | 0.0372 | 0.0314 |
| P11-10 | 0.0380 | N/A | 0.0136 | -0.0229 | -0.0485 | -0.0458 | 0.0244 | 0.0609 | 0.0865 | 0.0838 |
| P13-12 | -0.0161 | N/A | -0.0898 | -0.0649 | -0.0622 | -0.0605 | 0.0737 | 0.0488 | 0.0461 | 0.0444 |
| P14-13 | -0.0559 | N/A | -0.1414 | -0.0555 | -0.0493 | -0.0461 | 0.0855 | 0.0004 | 0.0066 | 0.0098 |
| Q1-2 | -0.2040 | -0.2571 | -0.2073 | -0.2141 | -0.2028 | -0.2069 | 0.0033 | 0.0101 | 0.0012 | 0.0029 |
| Q1-5 | 0.0385 | 0.1560 | 0.0760 | 0.0494 | 0.0599 | 0.0722 | 0.0375 | 0.0109 | 0.0214 | 0.0337 |
| Q2-3 | 0.0356 | 0.0048 | 0.0227 | 0.0355 | 0.0342 | 0.0270 | 0.0129 | 0.0001 | 0.0014 | 0.0086 |
| Q2-4 | -0.0155 | N/A | 0.0225 | 0.0012 | 0.0147 | 0.0157 | 0.0380 | 0.0167 | 0.0302 | 0.0312 |
| Q2-5 | 0.0117 | N/A | 0.0494 | 0.0337 | 0.0407 | 0.0587 | 0.0377 | 0.0220 | 0.0290 | 0.0470 |
| Q3-4 | 0.0447 | 0.0891 | 0.0741 | 0.0653 | 0.0759 | 0.0819 | 0.0294 | 0.0206 | 0.0312 | 0.0372 |
| Q4-5 | 0.1582 | N/A | 0.1902 | 0.1918 | 0.1698 | 0.2399 | 0.0320 | 0.0336 | 0.0116 | 0.0817 |
| Q4-7 | -0.0968 | -0.1669 | -0.1290 | -0.1334 | -0.1131 | -0.1445 | 0.0322 | 0.0366 | 0.0163 | 0.0477 |
| Q4-9 | -0.0043 | N/A | -0.0155 | -0.0253 | -0.0196 | -0.0326 | 0.0112 | 0.0210 | 0.0153 | 0.0283 |
| Q5-6 | 0.1247 | 0.1491 | 0.0168 | 0.0212 | -0.0082 | 0.1078 | 0.1079 | 0.1035 | 0.1329 | 0.0169 |
| Q6-11 | 0.0356 | N/A | -0.0095 | -0.0235 | 0.0580 | -0.0865 | 0.0451 | 0.0591 | 0.0224 | 0.1221 |
| Q6-12 | 0.0250 | 0.0152 | 0.0667 | 0.0152 | 0.0558 | -0.0200 | 0.0417 | 0.0098 | 0.0308 | 0.0450 |
| Q6-13 | 0.0722 | N/A | 0.1584 | 0.0458 | 0.1528 | 0.0023 | 0.0862 | 0.0264 | 0.0806 | 0.0699 |
| Q7-8 | -0.1716 | -0.1570 | -0.1391 | -0.1137 | -0.1185 | -0.1263 | 0.0325 | 0.0579 | 0.0531 | 0.0453 |
| Q7-9 | 0.0578 | 0.0677 | 0.1332 | 0.1211 | 0.1135 | 0.1066 | 0.0754 | 0.0633 | 0.0557 | 0.0488 |
| Q9-10 | 0.0422 | 0.1216 | 0.0863 | 0.1216 | 0.0829 | 0.0885 | 0.0441 | 0.0794 | 0.0407 | 0.0463 |
| Q9-14 | 0.0361 | N/A | 0.1443 | 0.0971 | 0.1071 | 0.1122 | 0.1082 | 0.0610 | 0.0710 | 0.0761 |
| Q10-11 | -0.0162 | 0.0186 | 0.0756 | 0.0186 | 0.0302 | 0.0742 | 0.0918 | 0.0348 | 0.0464 | 0.0904 |
| Q12-13 | 0.0075 | N/A | -0.0194 | -0.0166 | -0.0007 | -0.0019 | 0.0269 | 0.0241 | 0.0082 | 0.0094 |
| Q13-14 | 0.0175 | 0.0053 | -0.0147 | 0.0053 | 0.0208 | -0.0252 | 0.0322 | 0.0122 | 0.0033 | 0.0427 |
| Q2-1 | 0.2768 | N/A | 0.2276 | 0.2280 | 0.2211 | 0.2199 | 0.0492 | 0.0488 | 0.0557 | 0.0569 |
| Q5-1 | 0.0223 | N/A | -0.0436 | -0.0424 | -0.0466 | -0.0639 | 0.0659 | 0.0647 | 0.0689 | 0.0862 |
| Q3-2 | 0.0160 | N/A | -0.0006 | -0.0107 | -0.0071 | -0.0046 | 0.0166 | 0.0267 | 0.0231 | 0.0206 |
| Q4-2 | 0.0302 | N/A | -0.0141 | -0.0167 | -0.0244 | -0.0269 | 0.0443 | 0.0469 | 0.0546 | 0.0571 |
| Q5-2 | -0.0210 | N/A | -0.0820 | -0.0798 | -0.0835 | -0.1034 | 0.0610 | 0.0588 | 0.0625 | 0.0824 |
| Q4-3 | -0.0484 | N/A | -0.0949 | -0.0781 | -0.0897 | -0.0970 | 0.0465 | 0.0297 | 0.0413 | 0.0486 |
| Q5-4 | 0.1420 | N/A | -0.1593 | -0.1676 | -0.1437 | -0.2111 | 0.3013 | 0.3096 | 0.2857 | 0.3531 |
| Q7-4 | 0.1138 | N/A | 0.1643 | 0.1436 | 0.1190 | 0.1539 | 0.0505 | 0.0298 | 0.0052 | 0.0401 |
| Q9-4 | 0.0173 | N/A | 0.0463 | 0.0313 | 0.0228 | 0.0380 | 0.0290 | 0.0140 | 0.0055 | 0.0207 |
| Q6-5 | -0.0805 | N/A | 0.0495 | 0.0025 | 0.0351 | -0.0830 | 0.1300 | 0.0830 | 0.1156 | 0.0025 |
| Q11-6 | -0.0344 | N/A | 0.0168 | 0.0248 | -0.0569 | 0.0883 | 0.0512 | 0.0592 | 0.0225 | 0.1227 |
| Q12-6 | -0.0235 | N/A | -0.0656 | -0.0150 | -0.0550 | 0.0202 | 0.0421 | 0.0085 | 0.0315 | 0.0437 |
| Q13-6 | -0.0680 | N/A | -0.1506 | -0.0445 | -0.1484 | -0.0012 | 0.0826 | 0.0235 | 0.0804 | 0.0668 |
| Q8-7 | 0.1762 | N/A | 0.1425 | 0.1158 | 0.1209 | 0.1289 | 0.0337 | 0.0604 | 0.0553 | 0.0473 |
| Q9-7 | -0.0498 | N/A | -0.1095 | -0.1164 | -0.1103 | -0.1022 | 0.0597 | 0.0666 | 0.0605 | 0.0524 |
| Q10-9 | -0.0418 | N/A | -0.0844 | -0.1201 | -0.0822 | -0.0878 | 0.0426 | 0.0783 | 0.0404 | 0.0460 |
| Q14-9 | -0.0336 | N/A | -0.1305 | -0.0922 | -0.0994 | -0.1049 | 0.0969 | 0.0586 | 0.0658 | 0.0713 |
| Q11-10 | 0.0164 | N/A | -0.0745 | -0.0184 | -0.0296 | -0.0728 | 0.0909 | 0.0348 | 0.0460 | 0.0892 |
| Q13-12 | -0.0075 | N/A | 0.0212 | 0.0175 | 0.0015 | 0.0027 | 0.0287 | 0.0250 | 0.0090 | 0.0102 |
| Q14-13 | -0.0164 | N/A | 0.0224 | -0.0042 | -0.0198 | 0.0261 | 0.0388 | 0.0122 | 0.0034 | 0.0425 |
| **SUM** | | | | | | | **4.9928** | **3.4331** | **3.8266** | **4.1176** |

### Comparison

Full redundancy and median redundancy estimation with the same condition of error are presented in Table 5.258.

Table 5.258: Estimation errors of case 3, multi large, 14-bus AC system, comparison

| | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|
| **SUM** | **4.6852** | **2.9283** | **2.4469** | **2.6866** | **2.6020** |
| **SUM** | **N/A** | **4.9928** | **3.4331** | **3.8266** | **4.1176** |

#### 5.2.2.5 Summary

Table 5.259 and Table 5.260 presents the comparison between each estimator for full and median redundancy cases respectively.

Table 5.259: Comparison of full redundancy 14-bus estimation between estimators

| N.o | Type | Meas. | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|---|
| **Case 0** | **Random** | 2.9714 | 2.1408 | 2.0361 | 2.1408 | **1.9526** |
| **Case 1** | **Reverse** | 3.8412 | 3.6465 | 2.6437 | 2.2273 | **2.0061** |
| | **Large** | 3.3852 | 2.6711 | **2.4469** | 2.6711 | 2.5112 |
| **Case 2** | **Reverse** | 4.0586 | 3.6727 | 2.7602 | 2.2691 | **2.1637** |
| | **2Re-La** | 4.8458 | 4.2585 | 2.5858 | 2.2761 | **2.1360** |
| | **2La-Re** | 5.6412 | 4.7040 | 2.5548 | 2.4080 | **2.1046** |
| | **Large** | 5.1852 | 3.6738 | 2.6666 | 2.5734 | **2.5166** |
| **Case 3** | **Reverse** | 4.8640 | 4.0497 | 2.2664 | 3.2721 | **2.1690** |
| | **2Re-La** | 5.0026 | 3.9594 | 2.3378 | 2.8428 | **2.1690** |
| | **2La-Re** | 5.1412 | 4.1298 | 2.7754 | 2.1732 | **2.0044** |
| | **Large** | 4.6852 | 2.9283 | **2.4469** | 2.6866 | 2.6020 |

Table 5.260: Comparison of median redundancy 14-bus estimation between estimators

| N.o | Type | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| **Case 0** | **Random** | 2.5633 | 2.9987 | **2.5633** | 3.0435 |
| **Case 1** | **Reverse** | 4.7656 | 3.3894 | 4.7656 | **3.1157** |
| | **Large** | 3.6031 | **3.4331** | 3.6031 | 4.0409 |
| **Case 2** | **Reverse** | 4.7477 | 3.6854 | 4.7477 | **3.2348** |
| | **2Re-La** | 5.4858 | 4.1392 | 6.3330 | **4.1127** |
| | **2La-Re** | 6.4459 | 3.8904 | 5.8919 | **3.4916** |
| | **Large** | 5.5323 | **3.2120** | 3.7749 | 3.4916 |
| **Case 3** | **Reverse** | 5.9810 | 3.5747 | 10.7832 | **3.4798** |
| | **2Re-La** | 6.1414 | **3.1897** | 5.5089 | 3.4798 |
| | **2La-Re** | 5.7168 | 5.8765 | **5.2226** | 6.2692 |
| | **Large** | 4.9928 | **3.4331** | 3.8266 | 4.1176 |

WLS generates the worst results for almost all cases, but the overall estimation are still better than measurements for full redundancy case. LAV's performance is very good and stable by detecting most large bad data, which also leads to the excellent performance of LTAV, LTAV generate best results in almost all cases for full redundancy and half cases in median redundancy cases. As for LTS, though it generate best estimation in some cases, but its performance is not stable since it based on bad data detection of WLS.

Comparing between full and median redundancy cases, the former one has huge advantage among all cases, no matter in bad data detection , overall estimation and the stability of estimators. In full redundancy case, estimator's performance are very stable, but for median case, fail to detect bad data or eliminate more data many induce huge error. Full redundancy cases have almost 2.5 times of raw data of median counterparts, when their are bad data, full redundancy cases' local redundancy are far more larger, making it has huge advantage in both bad data estimation and elimination, which further induce estimators' stability issue.

## 5.3 30-Bus System

For 30 bus system, only median redundancy AC state estimation are conducted. All the data used in this section are presented in Appendix 6.2. WLS, LAV, LTS and LTAV are four estimators that are used in this part as previous AC state estimation.

The parameters used in the estimation are:

1. Tolerance of iterative loop for all estimators: 0.0001;

2. Fixed trimmed number for LTS in the sample precess: $nt = 0$; for LTAV: $nt = 0$;

3. Sample set for LTS: $nsample = 1$; for LTAV: $nsample = 1$;

The median redundancy rate are selected to be 1.9 for 30 bus state estimation, i.e, the number of measurement equal to $1.9 \times Ns$, where $Ns$ is the number of state

variable, $Ns = 59$ in this section. The number of measurements is $Nm = 113$.

The next step is topology planning, it's the key point to guarantee the observability of the system. Following the detail steps in section 3.2, Chapter 3, the location to put meters are determined. Since real and reactive power always come in pair, only real power injection and flow are considered, reactive measurements are added at the same location.

Power injection at each bus are selected at first, the involvement of power injection ensure a good coverage of power flow connected with each bus. Then other 26 meters are placed according to topology algorithm step by step. The placement of meters are shown in Figure 5.6. The raw data are presented in Table. 5.261.

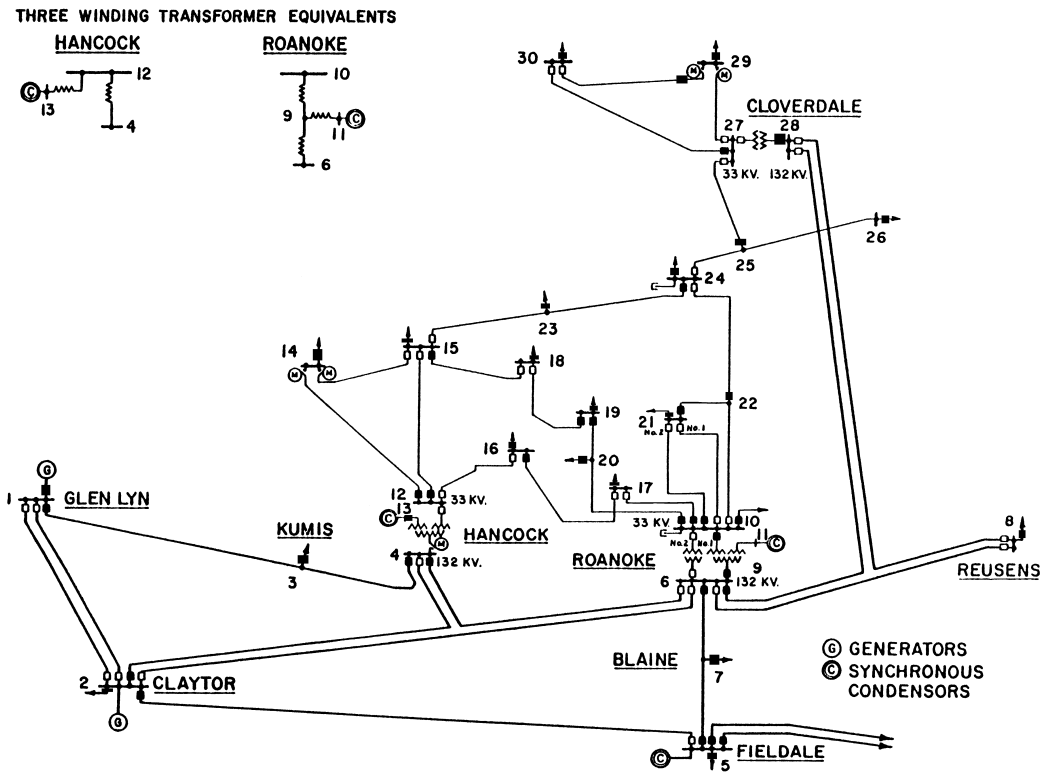


Figure 5.6: 30-bus system with meters for median redundancy.

### 5.3.1 Case 0: basic case with only random noise

The base case only have random small noise rather than large error. Table 5.261 shows the actual values of real power injection and flow from load flow. Random noise with normal distribution $\sigma = 0.01665$ for voltage measurements and $\sigma = 0.05$ for power measurements are added to the actual value to simulate the raw data for estimation in the same table.

Table 5.261: Raw data for 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | | **Actual** | **Meas.** | | **Actual** | **Meas.** |
|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | Q8 | 0.0611 | 0.0240 | P10-20 | 0.0903 | 0.1383 |
| P1 | 2.6096 | 2.6540 | Q9 | 0.0000 | -0.0531 | P10-17 | 0.0533 | 0.0595 |
| P2 | 0.1830 | 0.1256 | Q10 | -0.0200 | 0.0975 | P10-21 | 0.1579 | 0.2297 |
| P3 | -0.0240 | -0.0774 | Q11 | 0.1606 | 0.1298 | P21-22 | -0.0183 | -0.0282 |
| P4 | -0.0760 | -0.1165 | Q12 | -0.0750 | -0.0376 | P22-24 | 0.0574 | 0.2028 |
| P5 | -0.9420 | -1.0892 | Q13 | 0.1045 | 0.0949 | P23-24 | 0.0180 | 0.0593 |
| P6 | 0.0000 | 0.0719 | Q14 | -0.0160 | 0.0284 | P25-27 | -0.0476 | -0.0710 |
| P7 | -0.2280 | -0.2117 | Q15 | -0.0250 | -0.0632 | P28-27 | 0.1807 | 0.1671 |
| P8 | -0.3000 | -0.3377 | Q16 | -0.0180 | -0.0881 | P27-30 | 0.0709 | 0.0570 |
| P9 | 0.0000 | 0.0685 | Q17 | -0.0580 | -0.1291 | P29-30 | 0.0370 | 0.0721 |
| P10 | -0.0580 | -0.1436 | Q18 | -0.0090 | 0.0154 | P6-28 | 0.1867 | 0.1690 |
| P11 | 0.0000 | -0.0051 | Q19 | -0.0340 | -0.0429 | Q1-3 | 0.0428 | 0.0595 |
| P12 | -0.1120 | -0.1241 | Q20 | -0.0070 | -0.0168 | Q2-4 | 0.0475 | 0.0671 |
| P13 | 0.0000 | 0.0160 | Q21 | -0.1120 | -0.0410 | Q3-4 | -0.0385 | -0.0159 |
| P14 | -0.0620 | -0.0464 | Q22 | 0.0000 | 0.0146 | Q2-5 | 0.0278 | 0.0213 |
| P15 | -0.0820 | -0.1252 | Q23 | -0.0160 | -0.0061 | Q4-6 | -0.1591 | -0.1829 |
| P16 | -0.0350 | -0.0365 | Q24 | -0.0670 | 0.0124 | Q5-7 | 0.1149 | 0.1580 |
| P17 | -0.0900 | -0.0982 | Q25 | 0.0000 | -0.0402 | Q6-7 | -0.0278 | -0.0959 |
| P18 | -0.0320 | -0.0006 | Q26 | -0.0230 | 0.0118 | Q6-9 | -0.0809 | -0.1233 |
| P19 | -0.0950 | -0.0403 | Q27 | 0.0000 | 0.0418 | Q9-10 | 0.0588 | 0.1108 |
| P20 | -0.0220 | 0.0335 | Q28 | 0.0000 | -0.0122 | Q12-14 | 0.0240 | 0.0570 |
| P21 | -0.1750 | -0.2182 | Q29 | -0.0090 | 0.0018 | Q12-15 | 0.0679 | 0.0645 |
| P22 | 0.0000 | 0.0039 | Q30 | -0.0190 | -0.0773 | Q16-17 | 0.0144 | -0.0008 |
| P23 | -0.0320 | -0.0927 | P1-3 | 0.8765 | 0.8817 | Q15-18 | 0.0160 | 0.0172 |
| P24 | -0.0870 | -0.1427 | P2-4 | 0.4365 | 0.4726 | Q18-19 | 0.0062 | 0.0088 |
| P25 | 0.0000 | -0.0003 | P3-4 | 0.8214 | 0.9507 | Q19-20 | -0.0279 | 0.0134 |
| P26 | -0.0350 | 0.0416 | P2-5 | 0.8236 | 0.7903 | Q10-20 | 0.0371 | 0.1134 |
| P27 | 0.0000 | -0.0385 | P4-6 | 0.7213 | 0.7172 | Q10-17 | 0.0443 | 0.0676 |
| P28 | 0.0000 | 0.0186 | P5-7 | -0.1478 | -0.2445 | Q10-21 | 0.1001 | 0.0896 |
| P29 | -0.0240 | -0.0353 | P6-7 | 0.3813 | 0.3594 | Q21-22 | -0.0143 | -0.0051 |
| P30 | -0.1060 | -0.0501 | P6-9 | 0.2772 | 0.3192 | Q22-24 | 0.0306 | 0.0781 |
| Q1 | -0.2042 | -0.2587 | P9-10 | 0.2772 | 0.2500 | Q23-24 | 0.0124 | 0.0278 |
| Q2 | 0.4337 | 0.4353 | P12-14 | 0.0786 | 0.1031 | Q25-27 | -0.0037 | 0.0094 |
| Q3 | -0.0120 | 0.0156 | P12-15 | 0.1789 | 0.2159 | Q28-27 | 0.0504 | 0.0033 |
| Q4 | -0.0160 | 0.0390 | P16-17 | 0.0369 | -0.0700 | Q27-30 | 0.0166 | 0.0093 |
| Q5 | 0.1666 | 0.2438 | P15-18 | 0.0602 | 0.0182 | Q29-30 | 0.0061 | -0.0205 |
| Q6 | 0.0000 | 0.0043 | P18-19 | 0.0278 | 0.0955 | Q6-28 | 0.0011 | -0.0427 |
| Q7 | -0.1090 | -0.1836 | P19-20 | -0.0673 | -0.1209 | | | |

Table 5.262 shows the state variables, Table 5.263 presents the estimation results of each estimator and original actual value, measured value and errors between each estimator and actual value.

Table 5.262: State Variable of case 0, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0965 | -0.0969 | -0.0965 | -0.0965 | **V1** | 1.0676 | 1.0690 | 1.0676 | 1.0676 |
| $\theta 3$ | -0.1319 | -0.1306 | -0.1319 | -0.1319 | **V2** | 1.0534 | 1.0541 | 1.0534 | 1.0534 |
| $\theta 4$ | -0.1636 | -0.1629 | -0.1636 | -0.1636 | **V3** | 1.0319 | 1.0354 | 1.0319 | 1.0319 |
| $\theta 5$ | -0.2550 | -0.2560 | -0.2550 | -0.2550 | **V4** | 1.0232 | 1.0260 | 1.0232 | 1.0232 |
| $\theta 6$ | -0.1928 | -0.1919 | -0.1928 | -0.1928 | **V5** | 1.0225 | 1.0202 | 1.0225 | 1.0225 |
| $\theta 7$ | -0.2237 | -0.2204 | -0.2237 | -0.2237 | **V6** | 1.0199 | 1.0203 | 1.0199 | 1.0199 |
| $\theta 8$ | -0.2062 | -0.2054 | -0.2062 | -0.2062 | **V7** | 1.0127 | 1.0111 | 1.0127 | 1.0127 |
| $\theta 9$ | -0.2531 | -0.2548 | -0.2531 | -0.2531 | **V8** | 1.0177 | 1.0184 | 1.0177 | 1.0177 |
| $\theta 10$ | -0.2874 | -0.2949 | -0.2874 | -0.2874 | **V9** | 1.0317 | 1.0232 | 1.0317 | 1.0317 |
| $\theta 11$ | -0.2578 | -0.2558 | -0.2578 | -0.2578 | **V10** | 1.0242 | 1.0187 | 1.0242 | 1.0242 |
| $\theta 12$ | -0.2764 | -0.2772 | -0.2764 | -0.2764 | **V11** | 1.0639 | 1.0490 | 1.0639 | 1.0639 |
| $\theta 13$ | -0.2750 | -0.2751 | -0.2750 | -0.2750 | **V12** | 1.0135 | 1.0141 | 1.0135 | 1.0135 |
| $\theta 14$ | -0.2971 | -0.2983 | -0.2971 | -0.2971 | **V13** | 1.0272 | 1.0271 | 1.0272 | 1.0272 |
| $\theta 15$ | -0.2985 | -0.3005 | -0.2985 | -0.2985 | **V14** | 1.0010 | 1.0060 | 1.0010 | 1.0010 |
| $\theta 16$ | -0.2852 | -0.2878 | -0.2852 | -0.2852 | **V15** | 0.9938 | 0.9944 | 0.9938 | 0.9938 |
| $\theta 17$ | -0.2875 | -0.2949 | -0.2875 | -0.2875 | **V16** | 1.0057 | 1.0017 | 1.0057 | 1.0057 |
| $\theta 18$ | -0.2998 | -0.3065 | -0.2998 | -0.2998 | **V17** | 1.0120 | 1.0042 | 1.0120 | 1.0120 |
| $\theta 19$ | -0.3008 | -0.3097 | -0.3008 | -0.3008 | **V18** | 0.9929 | 0.9868 | 0.9929 | 0.9929 |
| $\theta 20$ | -0.2957 | -0.3023 | -0.2957 | -0.2957 | **V19** | 0.9910 | 0.9819 | 0.9910 | 0.9910 |
| $\theta 21$ | -0.2987 | -0.3075 | -0.2987 | -0.2987 | **V20** | 0.9965 | 0.9876 | 0.9965 | 0.9965 |
| $\theta 22$ | -0.2984 | -0.3069 | -0.2984 | -0.2984 | **V21** | 1.0127 | 1.0094 | 1.0127 | 1.0127 |
| $\theta 23$ | -0.3155 | -0.3176 | -0.3155 | -0.3155 | **V22** | 1.0127 | 1.0099 | 1.0127 | 1.0127 |
| $\theta 24$ | -0.3155 | -0.3157 | -0.3155 | -0.3155 | **V23** | 0.9874 | 0.9898 | 0.9874 | 0.9874 |
| $\theta 25$ | -0.2839 | -0.2776 | -0.2839 | -0.2839 | **V24** | 0.9928 | 0.9980 | 0.9928 | 0.9928 |
| $\theta 26$ | -0.2750 | -0.2652 | -0.2750 | -0.2750 | **V25** | 0.9969 | 1.0104 | 0.9969 | 0.9969 |
| $\theta 27$ | -0.2714 | -0.2655 | -0.2714 | -0.2714 | **V26** | 1.0064 | 1.0251 | 1.0064 | 1.0064 |
| $\theta 28$ | -0.2028 | -0.2017 | -0.2028 | -0.2028 | **V27** | 1.0059 | 1.0194 | 1.0059 | 1.0059 |
| $\theta 29$ | -0.2825 | -0.2756 | -0.2825 | -0.2825 | **V28** | 1.0172 | 1.0186 | 1.0172 | 1.0172 |
| $\theta 30$ | -0.2888 | -0.2807 | -0.2888 | -0.2888 | **V29** | 0.9877 | 1.0090 | 0.9877 | 0.9877 |
| | | | | | **V30** | 0.9732 | 1.0001 | 0.9732 | 0.9732 |

Table 5.263: Estimation Results and Errors of case 0, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0676 | 1.0690 | 1.0676 | 1.0676 | 0.0076 | 0.0090 | 0.0076 | 0.0076 |
| V2 | 1.0450 | N/A | 1.0534 | 1.0541 | 1.0534 | 1.0534 | 0.0084 | 0.0091 | 0.0084 | 0.0084 |
| V3 | 1.0210 | N/A | 1.0319 | 1.0354 | 1.0319 | 1.0319 | 0.0109 | 0.0144 | 0.0109 | 0.0109 |
| V4 | 1.0120 | N/A | 1.0232 | 1.0260 | 1.0232 | 1.0232 | 0.0112 | 0.0140 | 0.0112 | 0.0112 |
| V5 | 1.0100 | N/A | 1.0225 | 1.0202 | 1.0225 | 1.0225 | 0.0125 | 0.0102 | 0.0125 | 0.0125 |
| V6 | 1.0110 | N/A | 1.0199 | 1.0203 | 1.0199 | 1.0199 | 0.0089 | 0.0093 | 0.0089 | 0.0089 |
| V7 | 1.0030 | N/A | 1.0127 | 1.0111 | 1.0127 | 1.0127 | 0.0097 | 0.0081 | 0.0097 | 0.0097 |
| V8 | 1.0100 | N/A | 1.0177 | 1.0184 | 1.0177 | 1.0177 | 0.0077 | 0.0084 | 0.0077 | 0.0077 |
| V9 | 1.0510 | N/A | 1.0317 | 1.0232 | 1.0317 | 1.0317 | 0.0193 | 0.0278 | 0.0193 | 0.0193 |
| V10 | 1.0450 | N/A | 1.0242 | 1.0187 | 1.0242 | 1.0242 | 0.0208 | 0.0263 | 0.0208 | 0.0208 |
| V11 | 1.0820 | N/A | 1.0639 | 1.0490 | 1.0639 | 1.0639 | 0.0181 | 0.0330 | 0.0181 | 0.0181 |
| V12 | 1.0570 | N/A | 1.0135 | 1.0141 | 1.0135 | 1.0135 | 0.0435 | 0.0429 | 0.0435 | 0.0435 |
| V13 | 1.0710 | N/A | 1.0272 | 1.0271 | 1.0272 | 1.0272 | 0.0438 | 0.0439 | 0.0438 | 0.0438 |
| V14 | 1.0430 | N/A | 1.0010 | 1.0060 | 1.0010 | 1.0010 | 0.0420 | 0.0370 | 0.0420 | 0.0420 |
| V15 | 1.0380 | N/A | 0.9938 | 0.9944 | 0.9938 | 0.9938 | 0.0442 | 0.0436 | 0.0442 | 0.0442 |
| V16 | 1.0450 | N/A | 1.0057 | 1.0017 | 1.0057 | 1.0057 | 0.0393 | 0.0433 | 0.0393 | 0.0393 |
| V17 | 1.0400 | N/A | 1.0120 | 1.0042 | 1.0120 | 1.0120 | 0.0280 | 0.0358 | 0.0280 | 0.0280 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V18 | 1.0280 | N/A | 0.9929 | 0.9868 | 0.9929 | 0.9929 | 0.0351 | 0.0412 | 0.0351 | 0.0351 |
| V19 | 1.0260 | N/A | 0.9910 | 0.9819 | 0.9910 | 0.9910 | 0.0350 | 0.0441 | 0.0350 | 0.0350 |
| V20 | 1.0300 | N/A | 0.9965 | 0.9876 | 0.9965 | 0.9965 | 0.0335 | 0.0424 | 0.0335 | 0.0335 |
| V21 | 1.0330 | N/A | 1.0127 | 1.0094 | 1.0127 | 1.0127 | 0.0203 | 0.0236 | 0.0203 | 0.0203 |
| V22 | 1.0340 | N/A | 1.0127 | 1.0099 | 1.0127 | 1.0127 | 0.0213 | 0.0241 | 0.0213 | 0.0213 |
| V23 | 1.0270 | N/A | 0.9874 | 0.9898 | 0.9874 | 0.9874 | 0.0396 | 0.0372 | 0.0396 | 0.0396 |
| V24 | 1.0220 | N/A | 0.9928 | 0.9980 | 0.9928 | 0.9928 | 0.0292 | 0.0240 | 0.0292 | 0.0292 |
| V25 | 1.0180 | N/A | 0.9969 | 1.0104 | 0.9969 | 0.9969 | 0.0211 | 0.0076 | 0.0211 | 0.0211 |
| V26 | 1.0000 | N/A | 1.0064 | 1.0251 | 1.0064 | 1.0064 | 0.0064 | 0.0251 | 0.0064 | 0.0064 |
| V27 | 1.0240 | N/A | 1.0059 | 1.0194 | 1.0059 | 1.0059 | 0.0181 | 0.0046 | 0.0181 | 0.0181 |
| V28 | 1.0070 | N/A | 1.0172 | 1.0186 | 1.0172 | 1.0172 | 0.0102 | 0.0116 | 0.0102 | 0.0102 |
| V29 | 1.0040 | N/A | 0.9877 | 1.0090 | 0.9877 | 0.9877 | 0.0163 | 0.0050 | 0.0163 | 0.0163 |
| V30 | 0.9920 | N/A | 0.9732 | 1.0001 | 0.9732 | 0.9732 | 0.0188 | 0.0081 | 0.0188 | 0.0188 |
| P1 | 2.6096 | 2.6540 | 2.6906 | 2.6977 | 2.6906 | 2.6906 | 0.0810 | 0.0881 | 0.0810 | 0.0810 |
| P2 | 0.1830 | 0.1256 | 0.1493 | 0.1256 | 0.1493 | 0.1493 | 0.0337 | 0.0574 | 0.0337 | 0.0337 |
| P3 | -0.0240 | -0.0774 | 0.0052 | 0.0403 | 0.0052 | 0.0052 | 0.0292 | 0.0643 | 0.0292 | 0.0292 |
| P4 | -0.0760 | -0.1165 | -0.1154 | -0.1165 | -0.1154 | -0.1154 | 0.0394 | 0.0405 | 0.0394 | 0.0394 |
| P5 | -0.9420 | -1.0892 | -1.0424 | -1.0818 | -1.0424 | -1.0424 | 0.1004 | 0.1398 | 0.1004 | 0.1004 |
| P6 | 0.0000 | 0.0719 | 0.0752 | 0.0719 | 0.0752 | 0.0752 | 0.0752 | 0.0719 | 0.0752 | 0.0752 |
| P7 | -0.2280 | -0.2117 | -0.1626 | -0.1075 | -0.1626 | -0.1626 | 0.0654 | 0.1205 | 0.0654 | 0.0654 |
| P8 | -0.3000 | -0.3377 | -0.3346 | -0.3377 | -0.3346 | -0.3346 | 0.0346 | 0.0377 | 0.0346 | 0.0346 |
| P9 | 0.0000 | 0.0685 | 0.0489 | 0.0685 | 0.0489 | 0.0489 | 0.0489 | 0.0685 | 0.0489 | 0.0489 |
| P10 | -0.0580 | -0.1436 | -0.0920 | -0.1436 | -0.0920 | -0.0920 | 0.0340 | 0.0856 | 0.0340 | 0.0340 |
| P11 | 0.0000 | -0.0051 | -0.0244 | -0.0051 | -0.0244 | -0.0244 | 0.0244 | 0.0051 | 0.0244 | 0.0244 |
| P12 | -0.1120 | -0.1241 | -0.1296 | -0.1241 | -0.1296 | -0.1296 | 0.0176 | 0.0121 | 0.0176 | 0.0176 |
| P13 | 0.0000 | 0.0160 | 0.0105 | 0.0160 | 0.0105 | 0.0105 | 0.0105 | 0.0160 | 0.0105 | 0.0105 |
| P14 | -0.0620 | -0.0464 | -0.0648 | -0.0464 | -0.0648 | -0.0648 | 0.0028 | 0.0156 | 0.0028 | 0.0028 |
| P15 | -0.0820 | -0.1252 | -0.1304 | -0.1252 | -0.1304 | -0.1304 | 0.0484 | 0.0432 | 0.0484 | 0.0484 |
| P16 | -0.0350 | -0.0365 | -0.0498 | -0.0365 | -0.0498 | -0.0498 | 0.0148 | 0.0015 | 0.0148 | 0.0148 |
| P17 | -0.0900 | -0.0982 | -0.0516 | -0.0902 | -0.0516 | -0.0516 | 0.0384 | 0.0002 | 0.0384 | 0.0384 |
| P18 | -0.0320 | -0.0006 | 0.0059 | -0.0006 | 0.0059 | 0.0059 | 0.0379 | 0.0314 | 0.0379 | 0.0379 |
| P19 | -0.0950 | -0.0403 | -0.1033 | -0.1520 | -0.1033 | -0.1033 | 0.0083 | 0.0570 | 0.0083 | 0.0083 |
| P20 | -0.0220 | 0.0335 | 0.0085 | 0.0335 | 0.0085 | 0.0085 | 0.0305 | 0.0555 | 0.0305 | 0.0305 |
| P21 | -0.1750 | -0.2182 | -0.1997 | -0.2182 | -0.1997 | -0.1997 | 0.0247 | 0.0432 | 0.0247 | 0.0247 |
| P22 | 0.0000 | 0.0039 | 0.0409 | 0.0039 | 0.0409 | 0.0409 | 0.0409 | 0.0039 | 0.0409 | 0.0409 |
| P23 | -0.0320 | -0.0927 | -0.0861 | -0.0927 | -0.0861 | -0.0861 | 0.0541 | 0.0607 | 0.0541 | 0.0541 |
| P24 | -0.0870 | -0.1427 | -0.1869 | -0.1510 | -0.1869 | -0.1869 | 0.0999 | 0.0640 | 0.0999 | 0.0999 |
| P25 | 0.0000 | -0.0003 | -0.0143 | -0.0003 | -0.0143 | -0.0143 | 0.0143 | 0.0003 | 0.0143 | 0.0143 |
| P26 | -0.0350 | 0.0416 | 0.0279 | 0.0416 | 0.0279 | 0.0279 | 0.0629 | 0.0766 | 0.0629 | 0.0629 |
| P27 | 0.0000 | -0.0385 | -0.0288 | -0.0385 | -0.0288 | -0.0288 | 0.0288 | 0.0385 | 0.0288 | 0.0288 |
| P28 | 0.0000 | 0.0186 | 0.0218 | 0.0186 | 0.0218 | 0.0218 | 0.0218 | 0.0186 | 0.0218 | 0.0218 |
| P29 | -0.0240 | -0.0353 | -0.0152 | -0.0129 | -0.0152 | -0.0152 | 0.0088 | 0.0111 | 0.0088 | 0.0088 |
| P30 | -0.1060 | -0.0501 | -0.0668 | -0.0501 | -0.0668 | -0.0668 | 0.0392 | 0.0559 | 0.0392 | 0.0392 |
| Q1 | -0.2042 | -0.2587 | -0.2557 | -0.2587 | -0.2557 | -0.2557 | 0.0515 | 0.0545 | 0.0515 | 0.0515 |
| Q2 | 0.4337 | 0.4353 | 0.4332 | 0.4353 | 0.4332 | 0.4332 | 0.0005 | 0.0016 | 0.0005 | 0.0005 |
| Q3 | -0.0120 | 0.0156 | -0.0067 | 0.0156 | -0.0067 | -0.0067 | 0.0053 | 0.0276 | 0.0053 | 0.0053 |
| Q4 | -0.0160 | 0.0390 | -0.0211 | 0.0390 | -0.0211 | -0.0211 | 0.0051 | 0.0550 | 0.0051 | 0.0051 |
| Q5 | 0.1666 | 0.2438 | 0.2490 | 0.2438 | 0.2490 | 0.2490 | 0.0824 | 0.0772 | 0.0824 | 0.0824 |
| Q6 | 0.0000 | 0.0043 | -0.0030 | 0.0043 | -0.0030 | -0.0030 | 0.0030 | 0.0043 | 0.0030 | 0.0030 |
| Q7 | -0.1090 | -0.1836 | -0.1460 | -0.1836 | -0.1460 | -0.1460 | 0.0370 | 0.0746 | 0.0370 | 0.0370 |
| Q8 | 0.0611 | 0.0240 | 0.0205 | 0.0240 | 0.0205 | 0.0205 | 0.0406 | 0.0371 | 0.0406 | 0.0406 |
| Q9 | 0.0000 | -0.0531 | -0.0160 | -0.0531 | -0.0160 | -0.0160 | 0.0160 | 0.0531 | 0.0160 | 0.0160 |
| Q10 | -0.0200 | 0.0975 | 0.0850 | 0.0975 | 0.0850 | 0.0850 | 0.1050 | 0.1175 | 0.1050 | 0.1050 |
| Q11 | 0.1606 | 0.1298 | 0.1647 | 0.1298 | 0.1647 | 0.1647 | 0.0041 | 0.0308 | 0.0041 | 0.0041 |
| Q12 | -0.0750 | -0.0376 | -0.0324 | -0.0376 | -0.0324 | -0.0324 | 0.0426 | 0.0374 | 0.0426 | 0.0426 |
| Q13 | 0.1045 | 0.0949 | 0.0999 | 0.0949 | 0.0999 | 0.0999 | 0.0046 | 0.0096 | 0.0046 | 0.0046 |
| Q14 | -0.0160 | 0.0284 | 0.0058 | 0.0284 | 0.0058 | 0.0058 | 0.0218 | 0.0444 | 0.0218 | 0.0218 |
| Q15 | -0.0250 | -0.0632 | -0.0681 | -0.0632 | -0.0681 | -0.0681 | 0.0431 | 0.0382 | 0.0431 | 0.0431 |
| Q16 | -0.0180 | -0.0881 | -0.0484 | -0.0513 | -0.0484 | -0.0484 | 0.0304 | 0.0333 | 0.0304 | 0.0304 |
| Q17 | -0.0580 | -0.1291 | -0.0932 | -0.1291 | -0.0932 | -0.0932 | 0.0352 | 0.0711 | 0.0352 | 0.0352 |
| Q18 | -0.0090 | 0.0154 | 0.0077 | 0.0037 | 0.0077 | 0.0077 | 0.0167 | 0.0127 | 0.0167 | 0.0167 |
| Q19 | -0.0340 | -0.0429 | -0.0429 | -0.0429 | -0.0429 | -0.0429 | 0.0089 | 0.0089 | 0.0089 | 0.0089 |
| Q20 | -0.0070 | -0.0168 | -0.0600 | -0.0857 | -0.0600 | -0.0600 | 0.0530 | 0.0787 | 0.0530 | 0.0530 |
| Q21 | -0.1120 | -0.0410 | -0.0651 | -0.0410 | -0.0651 | -0.0651 | 0.0469 | 0.0710 | 0.0469 | 0.0469 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q22 | 0.0000 | 0.0146 | 0.0008 | 0.0146 | 0.0008 | 0.0008 | 0.0008 | 0.0146 | 0.0008 | 0.0008 |
| Q23 | -0.0160 | -0.0061 | -0.0075 | -0.0061 | -0.0075 | -0.0075 | 0.0085 | 0.0099 | 0.0085 | 0.0085 |
| Q24 | -0.0670 | 0.0124 | -0.0275 | -0.0213 | -0.0275 | -0.0275 | 0.0395 | 0.0457 | 0.0395 | 0.0395 |
| Q25 | 0.0000 | -0.0402 | -0.0455 | -0.0402 | -0.0455 | -0.0455 | 0.0455 | 0.0402 | 0.0455 | 0.0455 |
| Q26 | -0.0230 | 0.0118 | 0.0066 | 0.0118 | 0.0066 | 0.0066 | 0.0296 | 0.0348 | 0.0296 | 0.0296 |
| Q27 | 0.0000 | 0.0418 | 0.0413 | 0.0418 | 0.0413 | 0.0413 | 0.0413 | 0.0418 | 0.0413 | 0.0413 |
| Q28 | 0.0000 | -0.0122 | 0.0024 | -0.0122 | 0.0024 | 0.0024 | 0.0024 | 0.0122 | 0.0024 | 0.0024 |
| Q29 | -0.0090 | 0.0018 | -0.0034 | 0.0018 | -0.0034 | -0.0034 | 0.0056 | 0.0108 | 0.0056 | 0.0056 |
| Q30 | -0.0190 | -0.0773 | -0.0480 | -0.0249 | -0.0480 | -0.0480 | 0.0290 | 0.0059 | 0.0290 | 0.0290 |
| P1-2 | 1.7331 | N/A | 1.8011 | 1.8160 | 1.8011 | 1.8011 | 0.0680 | 0.0829 | 0.0680 | 0.0680 |
| P1-3 | 0.8765 | 0.8817 | 0.8895 | 0.8817 | 0.8895 | 0.8895 | 0.0130 | 0.0052 | 0.0130 | 0.0130 |
| P2-4 | 0.4365 | 0.4726 | 0.4339 | 0.4254 | 0.4339 | 0.4339 | 0.0026 | 0.0111 | 0.0026 | 0.0026 |
| P3-4 | 0.8214 | 0.9507 | 0.8633 | 0.8912 | 0.8633 | 0.8633 | 0.0419 | 0.0698 | 0.0419 | 0.0419 |
| P2-5 | 0.8236 | 0.7903 | 0.8640 | 0.8695 | 0.8640 | 0.8640 | 0.0404 | 0.0459 | 0.0404 | 0.0404 |
| P2-6 | 0.6038 | N/A | 0.5968 | 0.5903 | 0.5968 | 0.5968 | 0.0070 | 0.0135 | 0.0070 | 0.0070 |
| P4-6 | 0.7213 | 0.7172 | 0.7064 | 0.7172 | 0.7064 | 0.7064 | 0.0149 | 0.0041 | 0.0149 | 0.0149 |
| P5-7 | -0.1478 | -0.2445 | -0.2102 | -0.2445 | -0.2102 | -0.2102 | 0.0624 | 0.0967 | 0.0624 | 0.0624 |
| P6-7 | 0.3813 | 0.3594 | 0.3798 | 0.3594 | 0.3798 | 0.3798 | 0.0015 | 0.0219 | 0.0015 | 0.0015 |
| P6-8 | 0.2956 | N/A | 0.3203 | 0.3215 | 0.3203 | 0.3203 | 0.0247 | 0.0259 | 0.0247 | 0.0247 |
| P6-9 | 0.2772 | 0.3192 | 0.3049 | 0.3158 | 0.3049 | 0.3049 | 0.0277 | 0.0386 | 0.0277 | 0.0277 |
| P6-10 | 0.1584 | N/A | 0.1775 | 0.1922 | 0.1775 | 0.1775 | 0.0191 | 0.0338 | 0.0191 | 0.0191 |
| P9-11 | 0.0000 | N/A | 0.0244 | 0.0051 | 0.0244 | 0.0244 | 0.0244 | 0.0051 | 0.0244 | 0.0244 |
| P9-10 | 0.2772 | 0.2500 | 0.3294 | 0.3792 | 0.3294 | 0.3294 | 0.0522 | 0.1020 | 0.0522 | 0.0522 |
| P4-12 | 0.4419 | N/A | 0.4562 | 0.4638 | 0.4562 | 0.4562 | 0.0143 | 0.0219 | 0.0143 | 0.0143 |
| P12-13 | 0.0000 | N/A | -0.0105 | -0.0160 | -0.0105 | -0.0105 | 0.0105 | 0.0160 | 0.0105 | 0.0105 |
| P12-14 | 0.0786 | 0.1031 | 0.0865 | 0.0812 | 0.0865 | 0.0865 | 0.0079 | 0.0026 | 0.0079 | 0.0079 |
| P12-15 | 0.1789 | 0.2159 | 0.1980 | 0.2058 | 0.1980 | 0.1980 | 0.0191 | 0.0269 | 0.0191 | 0.0191 |
| P12-16 | 0.0724 | N/A | 0.0526 | 0.0687 | 0.0526 | 0.0526 | 0.0198 | 0.0037 | 0.0198 | 0.0198 |
| P14-15 | 0.0158 | N/A | 0.0208 | 0.0340 | 0.0208 | 0.0208 | 0.0050 | 0.0182 | 0.0050 | 0.0050 |
| P16-17 | 0.0369 | -0.0700 | 0.0026 | 0.0317 | 0.0026 | 0.0026 | 0.0343 | 0.0052 | 0.0343 | 0.0343 |
| P15-18 | 0.0602 | 0.0182 | 0.0064 | 0.0353 | 0.0064 | 0.0064 | 0.0538 | 0.0249 | 0.0538 | 0.0538 |
| P18-19 | 0.0278 | 0.0955 | 0.0122 | 0.0345 | 0.0122 | 0.0122 | 0.0156 | 0.0067 | 0.0156 | 0.0156 |
| P19-20 | -0.0673 | -0.1209 | -0.0911 | -0.1175 | -0.0911 | -0.0911 | 0.0238 | 0.0502 | 0.0238 | 0.0238 |
| P10-20 | 0.0903 | 0.1383 | 0.0844 | 0.0864 | 0.0844 | 0.0844 | 0.0059 | 0.0039 | 0.0059 | 0.0059 |
| P10-17 | 0.0533 | 0.0595 | 0.0497 | 0.0595 | 0.0497 | 0.0497 | 0.0036 | 0.0062 | 0.0036 | 0.0036 |
| P10-21 | 0.1579 | 0.2297 | 0.1887 | 0.1913 | 0.1887 | 0.1887 | 0.0308 | 0.0334 | 0.0308 | 0.0308 |
| P10-22 | 0.0762 | N/A | 0.0921 | 0.0907 | 0.0921 | 0.0921 | 0.0159 | 0.0145 | 0.0159 | 0.0159 |
| P21-22 | -0.0183 | -0.0282 | -0.0124 | -0.0282 | -0.0124 | -0.0124 | 0.0059 | 0.0099 | 0.0059 | 0.0059 |
| P15-23 | 0.0504 | N/A | 0.0792 | 0.0760 | 0.0792 | 0.0792 | 0.0288 | 0.0256 | 0.0288 | 0.0288 |
| P22-24 | 0.0574 | 0.2028 | 0.1199 | 0.0657 | 0.1199 | 0.1199 | 0.0625 | 0.0083 | 0.0625 | 0.0625 |
| P23-24 | 0.0180 | 0.0593 | -0.0076 | -0.0173 | -0.0076 | -0.0076 | 0.0256 | 0.0353 | 0.0256 | 0.0256 |
| P24-25 | -0.0121 | N/A | -0.0764 | -0.1032 | -0.0764 | -0.0764 | 0.0643 | 0.0911 | 0.0643 | 0.0643 |
| P25-26 | 0.0354 | N/A | -0.0277 | -0.0411 | -0.0277 | -0.0277 | 0.0631 | 0.0765 | 0.0631 | 0.0631 |
| P25-27 | -0.0476 | -0.0710 | -0.0643 | -0.0645 | -0.0643 | -0.0643 | 0.0167 | 0.0169 | 0.0167 | 0.0167 |
| P28-27 | 0.1807 | 0.1671 | 0.1773 | 0.1671 | 0.1773 | 0.1773 | 0.0034 | 0.0136 | 0.0034 | 0.0034 |
| P27-29 | 0.0619 | N/A | 0.0390 | 0.0301 | 0.0390 | 0.0390 | 0.0229 | 0.0318 | 0.0229 | 0.0229 |
| P27-30 | 0.0709 | 0.0570 | 0.0446 | 0.0336 | 0.0446 | 0.0446 | 0.0263 | 0.0373 | 0.0263 | 0.0263 |
| P29-30 | 0.0370 | 0.0721 | 0.0234 | 0.0170 | 0.0234 | 0.0234 | 0.0136 | 0.0200 | 0.0136 | 0.0136 |
| P8-28 | -0.0054 | N/A | -0.0155 | -0.0174 | -0.0155 | -0.0155 | 0.0101 | 0.0120 | 0.0101 | 0.0101 |
| P6-28 | 0.1867 | 0.1690 | 0.1714 | 0.1664 | 0.1714 | 0.1714 | 0.0153 | 0.0203 | 0.0153 | 0.0153 |
| P2-1 | -1.6809 | N/A | -1.7454 | -1.7597 | -1.7454 | -1.7454 | 0.0645 | 0.0788 | 0.0645 | 0.0645 |
| P3-1 | -0.8454 | N/A | -0.8581 | -0.8509 | -0.8581 | -0.8581 | 0.0127 | 0.0055 | 0.0127 | 0.0127 |
| P4-2 | -0.4263 | N/A | -0.4241 | -0.4160 | -0.4241 | -0.4241 | 0.0022 | 0.0103 | 0.0022 | 0.0022 |
| P4-3 | -0.8129 | N/A | -0.8540 | -0.8814 | -0.8540 | -0.8540 | 0.0411 | 0.0685 | 0.0411 | 0.0411 |
| P5-2 | -0.7942 | N/A | -0.8322 | -0.8373 | -0.8322 | -0.8322 | 0.0380 | 0.0431 | 0.0380 | 0.0380 |
| P6-2 | -0.5843 | N/A | -0.5781 | -0.5720 | -0.5781 | -0.5781 | 0.0062 | 0.0123 | 0.0062 | 0.0062 |
| P6-4 | -0.7150 | N/A | -0.7006 | -0.7113 | -0.7006 | -0.7006 | 0.0144 | 0.0037 | 0.0144 | 0.0144 |
| P7-5 | 0.1495 | N/A | 0.2135 | 0.2486 | 0.2135 | 0.2135 | 0.0640 | 0.0991 | 0.0640 | 0.0640 |
| P7-6 | -0.3775 | N/A | -0.3761 | -0.3561 | -0.3761 | -0.3761 | 0.0014 | 0.0214 | 0.0014 | 0.0014 |
| P8-6 | -0.2946 | N/A | -0.3191 | -0.3203 | -0.3191 | -0.3191 | 0.0245 | 0.0257 | 0.0245 | 0.0245 |
| P9-6 | -0.2772 | N/A | -0.3049 | -0.3158 | -0.3049 | -0.3049 | 0.0277 | 0.0386 | 0.0277 | 0.0277 |
| P10-6 | -0.1584 | N/A | -0.1775 | -0.1922 | -0.1775 | -0.1775 | 0.0191 | 0.0338 | 0.0191 | 0.0191 |
| P11-9 | 0.0000 | N/A | -0.0244 | -0.0051 | -0.0244 | -0.0244 | 0.0244 | 0.0051 | 0.0244 | 0.0244 |
| P10-9 | -0.2772 | N/A | -0.3294 | -0.3792 | -0.3294 | -0.3294 | 0.0522 | 0.1020 | 0.0522 | 0.0522 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P12-4 | -0.4419 | N/A | -0.4562 | -0.4638 | -0.4562 | -0.4562 | 0.0143 | 0.0219 | 0.0143 | 0.0143 |
| P13-12 | 0.0000 | N/A | 0.0105 | 0.0160 | 0.0105 | 0.0105 | 0.0105 | 0.0160 | 0.0105 | 0.0105 |
| P14-12 | -0.0778 | N/A | -0.0856 | -0.0804 | -0.0856 | -0.0856 | 0.0078 | 0.0026 | 0.0078 | 0.0078 |
| P15-12 | -0.1767 | N/A | -0.1953 | -0.2029 | -0.1953 | -0.1953 | 0.0186 | 0.0262 | 0.0186 | 0.0186 |
| P16-12 | -0.0719 | N/A | -0.0523 | -0.0682 | -0.0523 | -0.0523 | 0.0196 | 0.0037 | 0.0196 | 0.0196 |
| P15-14 | -0.0158 | N/A | -0.0207 | -0.0336 | -0.0207 | -0.0207 | 0.0049 | 0.0178 | 0.0049 | 0.0049 |
| P17-16 | -0.0368 | N/A | -0.0025 | -0.0316 | -0.0025 | -0.0025 | 0.0343 | 0.0052 | 0.0343 | 0.0343 |
| P18-15 | -0.0598 | N/A | -0.0064 | -0.0351 | -0.0064 | -0.0064 | 0.0534 | 0.0247 | 0.0534 | 0.0534 |
| P19-18 | -0.0277 | N/A | -0.0122 | -0.0344 | -0.0122 | -0.0122 | 0.0155 | 0.0067 | 0.0155 | 0.0155 |
| P20-19 | 0.0674 | N/A | 0.0914 | 0.1180 | 0.0914 | 0.0914 | 0.0240 | 0.0506 | 0.0240 | 0.0240 |
| P20-10 | -0.0894 | N/A | -0.0829 | -0.0845 | -0.0829 | -0.0829 | 0.0065 | 0.0049 | 0.0065 | 0.0065 |
| P17-10 | -0.0532 | N/A | -0.0491 | -0.0587 | -0.0491 | -0.0491 | 0.0041 | 0.0055 | 0.0041 | 0.0041 |
| P21-10 | -0.1567 | N/A | -0.1873 | -0.1900 | -0.1873 | -0.1873 | 0.0306 | 0.0333 | 0.0306 | 0.0306 |
| P22-10 | -0.0757 | N/A | -0.0914 | -0.0901 | -0.0914 | -0.0914 | 0.0157 | 0.0144 | 0.0157 | 0.0157 |
| P22-21 | 0.0183 | N/A | 0.0124 | 0.0282 | 0.0124 | 0.0124 | 0.0059 | 0.0099 | 0.0059 | 0.0059 |
| P23-15 | -0.0500 | N/A | -0.0785 | -0.0754 | -0.0785 | -0.0785 | 0.0285 | 0.0254 | 0.0285 | 0.0285 |
| P24-22 | -0.0569 | N/A | -0.1182 | -0.0652 | -0.1182 | -0.1182 | 0.0613 | 0.0083 | 0.0613 | 0.0613 |
| P24-23 | -0.0180 | N/A | 0.0076 | 0.0174 | 0.0076 | 0.0076 | 0.0256 | 0.0354 | 0.0256 | 0.0256 |
| P25-24 | 0.0122 | N/A | 0.0777 | 0.1053 | 0.0777 | 0.0777 | 0.0655 | 0.0931 | 0.0655 | 0.0655 |
| P26-25 | -0.0350 | N/A | 0.0279 | 0.0416 | 0.0279 | 0.0279 | 0.0629 | 0.0766 | 0.0629 | 0.0629 |
| P27-25 | 0.0479 | N/A | 0.0648 | 0.0649 | 0.0648 | 0.0648 | 0.0169 | 0.0170 | 0.0169 | 0.0169 |
| P27-28 | -0.1807 | N/A | -0.1773 | -0.1671 | -0.1773 | -0.1773 | 0.0034 | 0.0136 | 0.0034 | 0.0034 |
| P29-27 | -0.0610 | N/A | -0.0386 | -0.0299 | -0.0386 | -0.0386 | 0.0224 | 0.0311 | 0.0224 | 0.0224 |
| P30-27 | -0.0693 | N/A | -0.0437 | -0.0332 | -0.0437 | -0.0437 | 0.0256 | 0.0361 | 0.0256 | 0.0256 |
| P30-29 | -0.0367 | N/A | -0.0231 | -0.0169 | -0.0231 | -0.0231 | 0.0136 | 0.0198 | 0.0136 | 0.0136 |
| P28-8 | 0.0055 | N/A | 0.0155 | 0.0174 | 0.0155 | 0.0155 | 0.0100 | 0.0119 | 0.0100 | 0.0100 |
| P28-6 | -0.1862 | N/A | -0.1710 | -0.1659 | -0.1710 | -0.1710 | 0.0152 | 0.0203 | 0.0152 | 0.0152 |
| Q1-2 | -0.2470 | N/A | -0.2774 | -0.2685 | -0.2774 | -0.2774 | 0.0304 | 0.0215 | 0.0304 | 0.0304 |
| Q1-3 | 0.0428 | 0.0595 | 0.0218 | 0.0098 | 0.0218 | 0.0218 | 0.0210 | 0.0330 | 0.0210 | 0.0210 |
| Q2-4 | 0.0475 | 0.0671 | 0.0341 | 0.0245 | 0.0341 | 0.0341 | 0.0134 | 0.0230 | 0.0134 | 0.0134 |
| Q3-4 | -0.0385 | -0.0159 | -0.0549 | -0.0420 | -0.0549 | -0.0549 | 0.0164 | 0.0035 | 0.0164 | 0.0164 |
| Q2-5 | 0.0278 | 0.0213 | 0.0033 | 0.0186 | 0.0033 | 0.0033 | 0.0245 | 0.0092 | 0.0245 | 0.0245 |
| Q2-6 | 0.0137 | N/A | 0.0110 | 0.0144 | 0.0110 | 0.0110 | 0.0027 | 0.0007 | 0.0027 | 0.0027 |
| Q4-6 | -0.1591 | -0.1829 | -0.1145 | -0.0604 | -0.1145 | -0.1145 | 0.0446 | 0.0987 | 0.0446 | 0.0446 |
| Q5-7 | 0.1149 | 0.1580 | 0.1639 | 0.1721 | 0.1639 | 0.1639 | 0.0490 | 0.0572 | 0.0490 | 0.0490 |
| Q6-7 | -0.0278 | -0.0959 | -0.0368 | -0.0065 | -0.0368 | -0.0368 | 0.0090 | 0.0213 | 0.0090 | 0.0090 |
| Q6-8 | -0.0720 | N/A | -0.0404 | -0.0468 | -0.0404 | -0.0404 | 0.0316 | 0.0252 | 0.0316 | 0.0316 |
| Q6-9 | -0.0809 | -0.1233 | -0.0487 | -0.0043 | -0.0487 | -0.0487 | 0.0322 | 0.0766 | 0.0322 | 0.0322 |
| Q6-10 | 0.0019 | N/A | 0.0005 | 0.0128 | 0.0005 | 0.0005 | 0.0014 | 0.0109 | 0.0014 | 0.0014 |
| Q9-11 | -0.1560 | N/A | -0.1597 | -0.1266 | -0.1597 | -0.1597 | 0.0037 | 0.0294 | 0.0037 | 0.0037 |
| Q9-10 | 0.0588 | 0.1108 | 0.0759 | 0.0493 | 0.0759 | 0.0759 | 0.0171 | 0.0095 | 0.0171 | 0.0171 |
| Q4-12 | 0.1441 | N/A | 0.0645 | 0.0740 | 0.0645 | 0.0645 | 0.0796 | 0.0701 | 0.0796 | 0.0796 |
| Q12-13 | -0.1032 | N/A | -0.0986 | -0.0937 | -0.0986 | -0.0986 | 0.0046 | 0.0095 | 0.0046 | 0.0046 |
| Q12-14 | 0.0240 | 0.0570 | 0.0090 | -0.0059 | 0.0090 | 0.0090 | 0.0150 | 0.0299 | 0.0150 | 0.0150 |
| Q12-15 | 0.0679 | 0.0645 | 0.0546 | 0.0512 | 0.0546 | 0.0546 | 0.0133 | 0.0167 | 0.0133 | 0.0133 |
| Q12-16 | 0.0335 | N/A | 0.0152 | 0.0311 | 0.0152 | 0.0152 | 0.0183 | 0.0024 | 0.0183 | 0.0183 |
| Q14-15 | 0.0065 | N/A | 0.0129 | 0.0209 | 0.0129 | 0.0129 | 0.0064 | 0.0144 | 0.0064 | 0.0064 |
| Q16-17 | 0.0144 | -0.0008 | -0.0337 | -0.0213 | -0.0337 | -0.0337 | 0.0481 | 0.0357 | 0.0481 | 0.0481 |
| Q15-18 | 0.0160 | 0.0172 | 0.0010 | 0.0172 | 0.0010 | 0.0010 | 0.0150 | 0.0012 | 0.0150 | 0.0150 |
| Q18-19 | 0.0062 | 0.0088 | 0.0087 | 0.0205 | 0.0087 | 0.0087 | 0.0025 | 0.0143 | 0.0025 | 0.0025 |
| Q19-20 | -0.0279 | 0.0134 | -0.0342 | -0.0226 | -0.0342 | -0.0342 | 0.0063 | 0.0053 | 0.0063 | 0.0063 |
| Q10-20 | 0.0371 | 0.1134 | 0.0982 | 0.1134 | 0.0982 | 0.0982 | 0.0611 | 0.0763 | 0.0611 | 0.0611 |
| Q10-17 | 0.0443 | 0.0676 | 0.1287 | 0.1529 | 0.1287 | 0.1287 | 0.0844 | 0.1086 | 0.0844 | 0.0844 |
| Q10-21 | 0.1001 | 0.0896 | 0.0710 | 0.0386 | 0.0710 | 0.0710 | 0.0291 | 0.0615 | 0.0291 | 0.0291 |
| Q10-22 | 0.0460 | N/A | 0.0341 | 0.0167 | 0.0341 | 0.0341 | 0.0119 | 0.0293 | 0.0119 | 0.0119 |
| Q21-22 | -0.0143 | -0.0051 | 0.0030 | -0.0051 | 0.0030 | 0.0030 | 0.0173 | 0.0092 | 0.0173 | 0.0173 |
| Q15-23 | 0.0291 | N/A | -0.0071 | -0.0144 | -0.0071 | -0.0071 | 0.0362 | 0.0435 | 0.0362 | 0.0362 |
| Q22-24 | 0.0306 | 0.0781 | 0.0366 | 0.0249 | 0.0366 | 0.0366 | 0.0060 | 0.0057 | 0.0060 | 0.0060 |
| Q23-24 | 0.0124 | 0.0278 | -0.0158 | -0.0217 | -0.0158 | -0.0158 | 0.0282 | 0.0341 | 0.0282 | 0.0282 |
| Q24-25 | 0.0201 | N/A | 0.0328 | 0.0237 | 0.0328 | 0.0328 | 0.0127 | 0.0036 | 0.0127 | 0.0127 |
| Q25-26 | 0.0237 | N/A | -0.0063 | -0.0111 | -0.0063 | -0.0063 | 0.0300 | 0.0348 | 0.0300 | 0.0300 |
| Q25-27 | -0.0037 | 0.0094 | -0.0087 | -0.0091 | -0.0087 | -0.0087 | 0.0050 | 0.0054 | 0.0050 | 0.0050 |
| Q28-27 | 0.0504 | 0.0033 | 0.0352 | 0.0033 | 0.0352 | 0.0352 | 0.0152 | 0.0471 | 0.0152 | 0.0152 |
| Q27-29 | 0.0167 | N/A | 0.0235 | 0.0095 | 0.0235 | 0.0235 | 0.0068 | 0.0072 | 0.0068 | 0.0068 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q27-30 | 0.0166 | 0.0093 | 0.0310 | 0.0149 | 0.0310 | 0.0310 | 0.0144 | 0.0017 | 0.0144 | 0.0144 |
| Q29-30 | 0.0061 | -0.0205 | 0.0192 | 0.0109 | 0.0192 | 0.0192 | 0.0131 | 0.0048 | 0.0131 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0148 | -0.0177 | -0.0148 | -0.0148 | 0.0094 | 0.0123 | 0.0094 | 0.0094 |
| Q6-28 | 0.0011 | -0.0427 | -0.0085 | -0.0231 | -0.0085 | -0.0085 | 0.0096 | 0.0242 | 0.0096 | 0.0096 |
| Q2-1 | 0.3447 | N/A | 0.3848 | 0.3778 | 0.3848 | 0.3848 | 0.0401 | 0.0331 | 0.0401 | 0.0401 |
| Q3-1 | 0.0265 | N/A | 0.0483 | 0.0576 | 0.0483 | 0.0483 | 0.0218 | 0.0311 | 0.0218 | 0.0218 |
| Q4-2 | -0.0554 | N/A | -0.0438 | -0.0357 | -0.0438 | -0.0438 | 0.0116 | 0.0197 | 0.0116 | 0.0116 |
| Q4-3 | 0.0544 | N/A | 0.0727 | 0.0612 | 0.0727 | 0.0727 | 0.0183 | 0.0068 | 0.0183 | 0.0183 |
| Q5-2 | 0.0517 | N/A | 0.0852 | 0.0717 | 0.0852 | 0.0852 | 0.0335 | 0.0200 | 0.0335 | 0.0335 |
| Q6-2 | 0.0058 | N/A | 0.0055 | 0.0008 | 0.0055 | 0.0055 | 0.0003 | 0.0050 | 0.0003 | 0.0003 |
| Q6-4 | 0.1719 | N/A | 0.1253 | 0.0713 | 0.1253 | 0.1253 | 0.0466 | 0.1006 | 0.0466 | 0.0466 |
| Q7-5 | -0.1313 | N/A | -0.1767 | -0.1828 | -0.1767 | -0.1767 | 0.0454 | 0.0515 | 0.0454 | 0.0454 |
| Q7-6 | 0.0223 | N/A | 0.0306 | -0.0008 | 0.0306 | 0.0306 | 0.0083 | 0.0231 | 0.0083 | 0.0083 |
| Q8-6 | 0.0666 | N/A | 0.0352 | 0.0417 | 0.0352 | 0.0352 | 0.0314 | 0.0249 | 0.0314 | 0.0314 |
| Q9-6 | 0.0972 | N/A | 0.0677 | 0.0242 | 0.0677 | 0.0677 | 0.0295 | 0.0730 | 0.0295 | 0.0295 |
| Q10-6 | 0.0110 | N/A | 0.0163 | 0.0070 | 0.0163 | 0.0163 | 0.0053 | 0.0040 | 0.0053 | 0.0053 |
| Q11-9 | 0.1606 | N/A | 0.1647 | 0.1298 | 0.1647 | 0.1647 | 0.0041 | 0.0308 | 0.0041 | 0.0041 |
| Q10-9 | -0.0508 | N/A | -0.0641 | -0.0339 | -0.0641 | -0.0641 | 0.0133 | 0.0169 | 0.0133 | 0.0133 |
| Q12-4 | -0.0972 | N/A | -0.0126 | -0.0203 | -0.0126 | -0.0126 | 0.0846 | 0.0769 | 0.0846 | 0.0846 |
| Q13-12 | 0.1045 | N/A | 0.0999 | 0.0949 | 0.0999 | 0.0999 | 0.0046 | 0.0096 | 0.0046 | 0.0046 |
| Q14-12 | -0.0225 | N/A | -0.0071 | 0.0075 | -0.0071 | -0.0071 | 0.0154 | 0.0300 | 0.0154 | 0.0154 |
| Q15-12 | -0.0636 | N/A | -0.0492 | -0.0455 | -0.0492 | -0.0492 | 0.0144 | 0.0181 | 0.0144 | 0.0144 |
| Q16-12 | -0.0324 | N/A | -0.0147 | -0.0300 | -0.0147 | -0.0147 | 0.0177 | 0.0024 | 0.0177 | 0.0177 |
| Q15-14 | -0.0064 | N/A | -0.0128 | -0.0205 | -0.0128 | -0.0128 | 0.0064 | 0.0141 | 0.0064 | 0.0064 |
| Q17-16 | -0.0141 | N/A | 0.0340 | 0.0216 | 0.0340 | 0.0340 | 0.0481 | 0.0357 | 0.0481 | 0.0481 |
| Q18-15 | -0.0152 | N/A | -0.0010 | -0.0169 | -0.0010 | -0.0010 | 0.0142 | 0.0017 | 0.0142 | 0.0142 |
| Q19-18 | -0.0061 | N/A | -0.0087 | -0.0203 | -0.0087 | -0.0087 | 0.0026 | 0.0142 | 0.0026 | 0.0026 |
| Q20-19 | 0.0283 | N/A | 0.0349 | 0.0236 | 0.0349 | 0.0349 | 0.0066 | 0.0047 | 0.0066 | 0.0066 |
| Q20-10 | -0.0353 | N/A | -0.0949 | -0.1093 | -0.0949 | -0.0949 | 0.0596 | 0.0740 | 0.0596 | 0.0596 |
| Q17-10 | -0.0439 | N/A | -0.1272 | -0.1507 | -0.1272 | -0.1272 | 0.0833 | 0.1068 | 0.0833 | 0.0833 |
| Q21-10 | -0.0977 | N/A | -0.0681 | -0.0359 | -0.0681 | -0.0681 | 0.0296 | 0.0618 | 0.0296 | 0.0296 |
| Q22-10 | -0.0449 | N/A | -0.0327 | -0.0155 | -0.0327 | -0.0327 | 0.0122 | 0.0294 | 0.0122 | 0.0122 |
| Q22-21 | 0.0143 | N/A | -0.0030 | 0.0051 | -0.0030 | -0.0030 | 0.0173 | 0.0092 | 0.0173 | 0.0173 |
| Q23-15 | -0.0284 | N/A | 0.0084 | 0.0156 | 0.0084 | 0.0084 | 0.0368 | 0.0440 | 0.0368 | 0.0368 |
| Q24-22 | -0.0299 | N/A | -0.0339 | -0.0241 | -0.0339 | -0.0339 | 0.0040 | 0.0058 | 0.0040 | 0.0040 |
| Q24-23 | -0.0123 | N/A | 0.0159 | 0.0219 | 0.0159 | 0.0159 | 0.0282 | 0.0342 | 0.0282 | 0.0282 |
| Q25-24 | -0.0200 | N/A | -0.0305 | -0.0200 | -0.0305 | -0.0305 | 0.0105 | 0.0000 | 0.0105 | 0.0105 |
| Q26-25 | -0.0230 | N/A | 0.0066 | 0.0118 | 0.0066 | 0.0066 | 0.0296 | 0.0348 | 0.0296 | 0.0296 |
| Q27-25 | 0.0042 | N/A | 0.0096 | 0.0100 | 0.0096 | 0.0096 | 0.0054 | 0.0058 | 0.0054 | 0.0054 |
| Q27-28 | -0.0375 | N/A | -0.0227 | 0.0074 | -0.0227 | -0.0227 | 0.0148 | 0.0449 | 0.0148 | 0.0148 |
| Q29-27 | -0.0151 | N/A | -0.0226 | -0.0091 | -0.0226 | -0.0226 | 0.0075 | 0.0060 | 0.0075 | 0.0075 |
| Q30-27 | -0.0136 | N/A | -0.0292 | -0.0142 | -0.0292 | -0.0292 | 0.0156 | 0.0006 | 0.0156 | 0.0156 |
| Q30-29 | -0.0054 | N/A | -0.0188 | -0.0107 | -0.0188 | -0.0188 | 0.0134 | 0.0053 | 0.0134 | 0.0134 |
| Q28-8 | -0.0380 | N/A | -0.0295 | -0.0267 | -0.0295 | -0.0295 | 0.0085 | 0.0113 | 0.0085 | 0.0085 |
| Q28-6 | -0.0123 | N/A | -0.0033 | 0.0112 | -0.0033 | -0.0033 | 0.0090 | 0.0235 | 0.0090 | 0.0090 |
| **SUM** | | | | | | | **6.5405** | **7.8971** | **6.5405** | **6.5405** |

Since this is the base case, no large error exists, LTS and LTAV don't trim any data before final re-estimation, the results are the same as WLS.

### 5.3.2 Case 1: Single bad data

For the single bad data cases, one reverse and one large bad data are used to simulate two variants. The location is selected in the power injection of Bus 2.

#### 5.3.2.1 Reverse bad data

Based on the raw data in Table 5.261, a reverse bad data is used to replace the original one $P_2 = -0.1256$, others remain the same. With the reverse bad data presented above, the following two tables show the state variables, estimation and error between estimation and actual value.

Table 5.264: State Variable of case 1, reverse, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1000 | -0.1041 | -0.1000 | -0.1000 | **V1** | 1.0681 | 1.0690 | 1.0681 | 1.0681 |
| $\theta 3$ | -0.1278 | -0.1307 | -0.1278 | -0.1278 | **V2** | 1.0528 | 1.0522 | 1.0528 | 1.0528 |
| $\theta 4$ | -0.1592 | -0.1637 | -0.1592 | -0.1592 | **V3** | 1.0336 | 1.0361 | 1.0336 | 1.0336 |
| $\theta 5$ | -0.2519 | -0.2568 | -0.2519 | -0.2519 | **V4** | 1.0250 | 1.0266 | 1.0250 | 1.0250 |
| $\theta 6$ | -0.1879 | -0.1927 | -0.1879 | -0.1879 | **V5** | 1.0237 | 1.0210 | 1.0237 | 1.0237 |
| $\theta 7$ | -0.2189 | -0.2212 | -0.2189 | -0.2189 | **V6** | 1.0217 | 1.0211 | 1.0217 | 1.0217 |
| $\theta 8$ | -0.2007 | -0.2062 | -0.2007 | -0.2007 | **V7** | 1.0143 | 1.0119 | 1.0143 | 1.0143 |
| $\theta 9$ | -0.2445 | -0.2514 | -0.2445 | -0.2445 | **V8** | 1.0196 | 1.0191 | 1.0196 | 1.0196 |
| $\theta 10$ | -0.2779 | -0.2891 | -0.2779 | -0.2779 | **V9** | 1.0329 | 1.0241 | 1.0329 | 1.0329 |
| $\theta 11$ | -0.2481 | -0.2523 | -0.2481 | -0.2481 | **V10** | 1.0253 | 1.0195 | 1.0253 | 1.0253 |
| $\theta 12$ | -0.2662 | -0.2729 | -0.2662 | -0.2662 | **V11** | 1.0649 | 1.0498 | 1.0649 | 1.0649 |
| $\theta 13$ | -0.2637 | -0.2708 | -0.2637 | -0.2637 | **V12** | 1.0143 | 1.0142 | 1.0143 | 1.0143 |
| $\theta 14$ | -0.2860 | -0.2931 | -0.2860 | -0.2860 | **V13** | 1.0275 | 1.0272 | 1.0275 | 1.0275 |
| $\theta 15$ | -0.2877 | -0.2945 | -0.2877 | -0.2877 | **V14** | 1.0019 | 1.0064 | 1.0019 | 1.0019 |
| $\theta 16$ | -0.2749 | -0.2824 | -0.2749 | -0.2749 | **V15** | 0.9948 | 0.9954 | 0.9948 | 0.9948 |
| $\theta 17$ | -0.2776 | -0.2891 | -0.2776 | -0.2776 | **V16** | 1.0067 | 1.0012 | 1.0067 | 1.0067 |
| $\theta 18$ | -0.2890 | -0.2966 | -0.2890 | -0.2890 | **V17** | 1.0131 | 1.0046 | 1.0131 | 1.0131 |
| $\theta 19$ | -0.2903 | -0.2975 | -0.2903 | -0.2903 | **V18** | 0.9940 | 0.9897 | 0.9940 | 0.9940 |
| $\theta 20$ | -0.2853 | -0.2916 | -0.2853 | -0.2853 | **V19** | 0.9921 | 0.9857 | 0.9921 | 0.9921 |
| $\theta 21$ | -0.2889 | -0.3017 | -0.2889 | -0.2889 | **V20** | 0.9976 | 0.9904 | 0.9976 | 0.9976 |
| $\theta 22$ | -0.2886 | -0.3011 | -0.2886 | -0.2886 | **V21** | 1.0138 | 1.0102 | 1.0138 | 1.0138 |
| $\theta 23$ | -0.3048 | -0.3116 | -0.3048 | -0.3048 | **V22** | 1.0139 | 1.0106 | 1.0139 | 1.0139 |
| $\theta 24$ | -0.3056 | -0.3099 | -0.3056 | -0.3056 | **V23** | 0.9885 | 0.9907 | 0.9885 | 0.9885 |
| $\theta 25$ | -0.2752 | -0.2759 | -0.2752 | -0.2752 | **V24** | 0.9941 | 0.9988 | 0.9941 | 0.9941 |
| $\theta 26$ | -0.2665 | -0.2635 | -0.2665 | -0.2665 | **V25** | 0.9984 | 1.0112 | 0.9984 | 0.9984 |
| $\theta 27$ | -0.2634 | -0.2662 | -0.2634 | -0.2634 | **V26** | 1.0080 | 1.0258 | 1.0080 | 1.0080 |
| $\theta 28$ | -0.1972 | -0.2025 | -0.1972 | -0.1972 | **V27** | 1.0073 | 1.0201 | 1.0073 | 1.0073 |
| $\theta 29$ | -0.2740 | -0.2795 | -0.2740 | -0.2740 | **V28** | 1.0191 | 1.0193 | 1.0191 | 1.0191 |
| $\theta 30$ | -0.2802 | -0.2838 | -0.2802 | -0.2802 | **V29** | 0.9893 | 1.0092 | 0.9893 | 0.9893 |
| | | | | | **V30** | 0.9749 | 1.0018 | 0.9749 | 0.9749 |

Table 5.265: Estimation Results and Errors of case 1, reverse, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0681 | 1.0690 | 1.0681 | 1.0681 | 0.0081 | 0.0090 | 0.0081 | 0.0081 |
| V2 | 1.0450 | N/A | 1.0528 | 1.0522 | 1.0528 | 1.0528 | 0.0078 | 0.0072 | 0.0078 | 0.0078 |
| V3 | 1.0210 | N/A | 1.0336 | 1.0361 | 1.0336 | 1.0336 | 0.0126 | 0.0151 | 0.0126 | 0.0126 |
| V4 | 1.0120 | N/A | 1.0250 | 1.0266 | 1.0250 | 1.0250 | 0.0130 | 0.0146 | 0.0130 | 0.0130 |
| V5 | 1.0100 | N/A | 1.0237 | 1.0210 | 1.0237 | 1.0237 | 0.0137 | 0.0110 | 0.0137 | 0.0137 |
| V6 | 1.0110 | N/A | 1.0217 | 1.0211 | 1.0217 | 1.0217 | 0.0107 | 0.0101 | 0.0107 | 0.0107 |
| V7 | 1.0030 | N/A | 1.0143 | 1.0119 | 1.0143 | 1.0143 | 0.0113 | 0.0089 | 0.0113 | 0.0113 |
| V8 | 1.0100 | N/A | 1.0196 | 1.0191 | 1.0196 | 1.0196 | 0.0096 | 0.0091 | 0.0096 | 0.0096 |
| V9 | 1.0510 | N/A | 1.0329 | 1.0241 | 1.0329 | 1.0329 | 0.0181 | 0.0269 | 0.0181 | 0.0181 |
| V10 | 1.0450 | N/A | 1.0253 | 1.0195 | 1.0253 | 1.0253 | 0.0197 | 0.0255 | 0.0197 | 0.0197 |
| V11 | 1.0820 | N/A | 1.0649 | 1.0498 | 1.0649 | 1.0649 | 0.0171 | 0.0322 | 0.0171 | 0.0171 |
| V12 | 1.0570 | N/A | 1.0143 | 1.0142 | 1.0143 | 1.0143 | 0.0427 | 0.0428 | 0.0427 | 0.0427 |
| V13 | 1.0710 | N/A | 1.0275 | 1.0272 | 1.0275 | 1.0275 | 0.0435 | 0.0438 | 0.0435 | 0.0435 |
| V14 | 1.0430 | N/A | 1.0019 | 1.0064 | 1.0019 | 1.0019 | 0.0411 | 0.0366 | 0.0411 | 0.0411 |
| V15 | 1.0380 | N/A | 0.9948 | 0.9954 | 0.9948 | 0.9948 | 0.0432 | 0.0426 | 0.0432 | 0.0432 |
| V16 | 1.0450 | N/A | 1.0067 | 1.0012 | 1.0067 | 1.0067 | 0.0383 | 0.0438 | 0.0383 | 0.0383 |
| V17 | 1.0400 | N/A | 1.0131 | 1.0046 | 1.0131 | 1.0131 | 0.0269 | 0.0354 | 0.0269 | 0.0269 |
| V18 | 1.0280 | N/A | 0.9940 | 0.9897 | 0.9940 | 0.9940 | 0.0340 | 0.0383 | 0.0340 | 0.0340 |
| V19 | 1.0260 | N/A | 0.9921 | 0.9857 | 0.9921 | 0.9921 | 0.0339 | 0.0403 | 0.0339 | 0.0339 |
| V20 | 1.0300 | N/A | 0.9976 | 0.9904 | 0.9976 | 0.9976 | 0.0324 | 0.0396 | 0.0324 | 0.0324 |
| V21 | 1.0330 | N/A | 1.0138 | 1.0102 | 1.0138 | 1.0138 | 0.0192 | 0.0228 | 0.0192 | 0.0192 |
| V22 | 1.0340 | N/A | 1.0139 | 1.0106 | 1.0139 | 1.0139 | 0.0201 | 0.0234 | 0.0201 | 0.0201 |
| V23 | 1.0270 | N/A | 0.9885 | 0.9907 | 0.9885 | 0.9885 | 0.0385 | 0.0363 | 0.0385 | 0.0385 |
| V24 | 1.0220 | N/A | 0.9941 | 0.9988 | 0.9941 | 0.9941 | 0.0279 | 0.0232 | 0.0279 | 0.0279 |
| V25 | 1.0180 | N/A | 0.9984 | 1.0112 | 0.9984 | 0.9984 | 0.0196 | 0.0068 | 0.0196 | 0.0196 |
| V26 | 1.0000 | N/A | 1.0080 | 1.0258 | 1.0080 | 1.0080 | 0.0080 | 0.0258 | 0.0080 | 0.0080 |
| V27 | 1.0240 | N/A | 1.0073 | 1.0201 | 1.0073 | 1.0073 | 0.0167 | 0.0039 | 0.0167 | 0.0167 |
| V28 | 1.0070 | N/A | 1.0191 | 1.0193 | 1.0191 | 1.0191 | 0.0121 | 0.0123 | 0.0121 | 0.0121 |
| V29 | 1.0040 | N/A | 0.9893 | 1.0092 | 0.9893 | 0.9893 | 0.0147 | 0.0052 | 0.0147 | 0.0147 |
| V30 | 0.9920 | N/A | 0.9749 | 1.0018 | 0.9749 | 0.9749 | 0.0171 | 0.0098 | 0.0171 | 0.0171 |
| P1 | 2.6096 | 2.6540 | 2.7348 | 2.8361 | 2.7348 | 2.7348 | 0.1252 | 0.2265 | 0.1252 | 0.1252 |
| **P2** | **0.1830** | **-0.1256** | **-0.0558** | **-0.1256** | **-0.0558** | **-0.0558** | **0.2388** | **0.3086** | **0.2388** | **0.2388** |
| P3 | -0.0240 | -0.0774 | 0.0259 | 0.0604 | 0.0259 | 0.0259 | 0.0499 | 0.0844 | 0.0499 | 0.0499 |
| P4 | -0.0760 | -0.1165 | -0.0979 | -0.1165 | -0.0979 | -0.0979 | 0.0219 | 0.0405 | 0.0219 | 0.0219 |
| P5 | -0.9420 | -1.0892 | -1.0226 | -1.0462 | -1.0226 | -1.0226 | 0.0806 | 0.1042 | 0.0806 | 0.0806 |
| P6 | 0.0000 | 0.0719 | 0.0887 | 0.0779 | 0.0887 | 0.0887 | 0.0887 | 0.0779 | 0.0887 | 0.0887 |
| P7 | -0.2280 | -0.2117 | -0.1506 | -0.1075 | -0.1506 | -0.1506 | 0.0774 | 0.1205 | 0.0774 | 0.0774 |
| P8 | -0.3000 | -0.3377 | -0.3225 | -0.3377 | -0.3225 | -0.3225 | 0.0225 | 0.0377 | 0.0225 | 0.0225 |
| P9 | 0.0000 | 0.0685 | 0.0543 | 0.0685 | 0.0543 | 0.0543 | 0.0543 | 0.0685 | 0.0543 | 0.0543 |
| P10 | -0.0580 | -0.1436 | -0.0884 | -0.1338 | -0.0884 | -0.0884 | 0.0304 | 0.0758 | 0.0304 | 0.0304 |
| P11 | 0.0000 | -0.0051 | -0.0190 | -0.0051 | -0.0190 | -0.0190 | 0.0190 | 0.0051 | 0.0190 | 0.0190 |
| P12 | -0.1120 | -0.1241 | -0.1220 | -0.1241 | -0.1220 | -0.1220 | 0.0100 | 0.0121 | 0.0100 | 0.0100 |
| P13 | 0.0000 | 0.0160 | 0.0181 | 0.0160 | 0.0181 | 0.0181 | 0.0181 | 0.0160 | 0.0181 | 0.0181 |
| P14 | -0.0620 | -0.0464 | -0.0611 | -0.0464 | -0.0611 | -0.0611 | 0.0009 | 0.0156 | 0.0009 | 0.0009 |
| P15 | -0.0820 | -0.1252 | -0.1276 | -0.1252 | -0.1276 | -0.1276 | 0.0456 | 0.0432 | 0.0456 | 0.0456 |
| P16 | -0.0350 | -0.0365 | -0.0464 | -0.0365 | -0.0464 | -0.0464 | 0.0114 | 0.0015 | 0.0114 | 0.0114 |
| P17 | -0.0900 | -0.0982 | -0.0501 | -0.0867 | -0.0501 | -0.0501 | 0.0399 | 0.0033 | 0.0399 | 0.0399 |
| P18 | -0.0320 | -0.0006 | 0.0068 | -0.0006 | 0.0068 | 0.0068 | 0.0388 | 0.0314 | 0.0388 | 0.0388 |
| P19 | -0.0950 | -0.0403 | -0.1025 | -0.1122 | -0.1025 | -0.1025 | 0.0075 | 0.0172 | 0.0075 | 0.0075 |
| P20 | -0.0220 | 0.0335 | 0.0103 | 0.0335 | 0.0103 | 0.0103 | 0.0323 | 0.0555 | 0.0323 | 0.0323 |
| P21 | -0.1750 | -0.2182 | -0.1977 | -0.2182 | -0.1977 | -0.1977 | 0.0227 | 0.0432 | 0.0227 | 0.0227 |
| P22 | 0.0000 | 0.0039 | 0.0425 | 0.0039 | 0.0425 | 0.0425 | 0.0425 | 0.0039 | 0.0425 | 0.0425 |
| P23 | -0.0320 | -0.0927 | -0.0846 | -0.0927 | -0.0846 | -0.0846 | 0.0526 | 0.0607 | 0.0526 | 0.0526 |
| P24 | -0.0870 | -0.1427 | -0.1856 | -0.1427 | -0.1856 | -0.1856 | 0.0986 | 0.0557 | 0.0986 | 0.0986 |
| P25 | 0.0000 | -0.0003 | -0.0144 | -0.0003 | -0.0144 | -0.0144 | 0.0144 | 0.0003 | 0.0144 | 0.0144 |
| P26 | -0.0350 | 0.0416 | 0.0278 | 0.0416 | 0.0278 | 0.0278 | 0.0628 | 0.0766 | 0.0628 | 0.0628 |
| P27 | 0.0000 | -0.0385 | -0.0275 | -0.0385 | -0.0275 | -0.0275 | 0.0275 | 0.0385 | 0.0275 | 0.0275 |
| P28 | 0.0000 | 0.0186 | 0.0271 | 0.0186 | 0.0271 | 0.0271 | 0.0271 | 0.0186 | 0.0271 | 0.0271 |
| P29 | -0.0240 | -0.0353 | -0.0141 | -0.0223 | -0.0141 | -0.0141 | 0.0099 | 0.0017 | 0.0099 | 0.0099 |
| P30 | -0.1060 | -0.0501 | -0.0660 | -0.0501 | -0.0660 | -0.0660 | 0.0400 | 0.0559 | 0.0400 | 0.0400 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q1 | -0.2042 | -0.2587 | -0.2558 | -0.2587 | -0.2558 | -0.2558 | 0.0516 | 0.0545 | 0.0516 | 0.0516 |
| Q2 | 0.4337 | 0.4353 | 0.4325 | 0.4353 | 0.4325 | 0.4325 | 0.0012 | 0.0016 | 0.0012 | 0.0012 |
| Q3 | -0.0120 | 0.0156 | -0.0097 | 0.0156 | -0.0097 | -0.0097 | 0.0023 | 0.0276 | 0.0023 | 0.0023 |
| Q4 | -0.0160 | 0.0390 | -0.0240 | 0.0390 | -0.0240 | -0.0240 | 0.0080 | 0.0550 | 0.0080 | 0.0080 |
| Q5 | 0.1666 | 0.2438 | 0.2456 | 0.2438 | 0.2456 | 0.2456 | 0.0790 | 0.0772 | 0.0790 | 0.0790 |
| Q6 | 0.0000 | 0.0043 | -0.0054 | 0.0043 | -0.0054 | -0.0054 | 0.0054 | 0.0043 | 0.0054 | 0.0054 |
| Q7 | -0.1090 | -0.1836 | -0.1479 | -0.1836 | -0.1479 | -0.1479 | 0.0389 | 0.0746 | 0.0389 | 0.0389 |
| Q8 | 0.0611 | 0.0240 | 0.0183 | 0.0240 | 0.0183 | 0.0183 | 0.0428 | 0.0371 | 0.0428 | 0.0428 |
| Q9 | 0.0000 | -0.0531 | -0.0173 | -0.0531 | -0.0173 | -0.0173 | 0.0173 | 0.0531 | 0.0173 | 0.0173 |
| Q10 | -0.0200 | 0.0975 | 0.0842 | 0.0975 | 0.0842 | 0.0842 | 0.1042 | 0.1175 | 0.1042 | 0.1042 |
| Q11 | 0.1606 | 0.1298 | 0.1635 | 0.1298 | 0.1635 | 0.1635 | 0.0029 | 0.0308 | 0.0029 | 0.0029 |
| Q12 | -0.0750 | -0.0376 | -0.0354 | -0.0376 | -0.0354 | -0.0354 | 0.0396 | 0.0374 | 0.0396 | 0.0396 |
| Q13 | 0.1045 | 0.0949 | 0.0970 | 0.0949 | 0.0970 | 0.0970 | 0.0075 | 0.0096 | 0.0075 | 0.0075 |
| Q14 | -0.0160 | 0.0284 | 0.0039 | 0.0284 | 0.0039 | 0.0039 | 0.0199 | 0.0444 | 0.0199 | 0.0199 |
| Q15 | -0.0250 | -0.0632 | -0.0693 | -0.0632 | -0.0693 | -0.0693 | 0.0443 | 0.0382 | 0.0443 | 0.0443 |
| Q16 | -0.0180 | -0.0881 | -0.0489 | -0.0599 | -0.0489 | -0.0489 | 0.0309 | 0.0419 | 0.0309 | 0.0309 |
| Q17 | -0.0580 | -0.1291 | -0.0934 | -0.1291 | -0.0934 | -0.0934 | 0.0354 | 0.0711 | 0.0354 | 0.0354 |
| Q18 | -0.0090 | 0.0154 | 0.0075 | 0.0050 | 0.0075 | 0.0075 | 0.0165 | 0.0140 | 0.0165 | 0.0165 |
| Q19 | -0.0340 | -0.0429 | -0.0430 | -0.0429 | -0.0430 | -0.0430 | 0.0090 | 0.0089 | 0.0090 | 0.0090 |
| Q20 | -0.0070 | -0.0168 | -0.0606 | -0.0884 | -0.0606 | -0.0606 | 0.0536 | 0.0814 | 0.0536 | 0.0536 |
| Q21 | -0.1120 | -0.0410 | -0.0656 | -0.0410 | -0.0656 | -0.0656 | 0.0464 | 0.0710 | 0.0464 | 0.0464 |
| Q22 | 0.0000 | 0.0146 | 0.0004 | 0.0146 | 0.0004 | 0.0004 | 0.0004 | 0.0146 | 0.0004 | 0.0004 |
| Q23 | -0.0160 | -0.0061 | -0.0079 | -0.0061 | -0.0079 | -0.0079 | 0.0081 | 0.0099 | 0.0081 | 0.0081 |
| Q24 | -0.0670 | 0.0124 | -0.0283 | -0.0275 | -0.0283 | -0.0283 | 0.0387 | 0.0395 | 0.0387 | 0.0387 |
| Q25 | 0.0000 | -0.0402 | -0.0451 | -0.0402 | -0.0451 | -0.0451 | 0.0451 | 0.0402 | 0.0451 | 0.0451 |
| Q26 | -0.0230 | 0.0118 | 0.0070 | 0.0118 | 0.0070 | 0.0070 | 0.0300 | 0.0348 | 0.0300 | 0.0300 |
| Q27 | 0.0000 | 0.0418 | 0.0411 | 0.0418 | 0.0411 | 0.0411 | 0.0411 | 0.0418 | 0.0411 | 0.0411 |
| Q28 | 0.0000 | -0.0122 | 0.0013 | -0.0122 | 0.0013 | 0.0013 | 0.0013 | 0.0122 | 0.0013 | 0.0013 |
| Q29 | -0.0090 | 0.0018 | -0.0036 | 0.0018 | -0.0036 | -0.0036 | 0.0054 | 0.0108 | 0.0054 | 0.0054 |
| Q30 | -0.0190 | -0.0773 | -0.0482 | -0.0199 | -0.0482 | -0.0482 | 0.0292 | 0.0009 | 0.0292 | 0.0292 |
| P1-2 | 1.7331 | N/A | 1.8719 | 1.9544 | 1.8719 | 1.8719 | 0.1388 | 0.2213 | 0.1388 | 0.1388 |
| P1-3 | 0.8765 | 0.8817 | 0.8629 | 0.8817 | 0.8629 | 0.8629 | 0.0136 | 0.0052 | 0.0136 | 0.0136 |
| P2-4 | 0.4365 | 0.4726 | 0.3845 | 0.3837 | 0.3845 | 0.3845 | 0.0520 | 0.0528 | 0.0520 | 0.0520 |
| P3-4 | 0.8214 | 0.9507 | 0.8593 | 0.9113 | 0.8593 | 0.8593 | 0.0379 | 0.0899 | 0.0379 | 0.0379 |
| P2-5 | 0.8236 | 0.7903 | 0.8267 | 0.8312 | 0.8267 | 0.8267 | 0.0031 | 0.0076 | 0.0031 | 0.0031 |
| P2-6 | 0.6038 | N/A | 0.5450 | 0.5488 | 0.5450 | 0.5450 | 0.0588 | 0.0550 | 0.0588 | 0.0588 |
| P4-6 | 0.7213 | 0.7172 | 0.6954 | 0.7172 | 0.6954 | 0.6954 | 0.0259 | 0.0041 | 0.0259 | 0.0259 |
| P5-7 | -0.1478 | -0.2445 | -0.2250 | -0.2445 | -0.2250 | -0.2250 | 0.0772 | 0.0967 | 0.0772 | 0.0772 |
| P6-7 | 0.3813 | 0.3594 | 0.3830 | 0.3594 | 0.3830 | 0.3830 | 0.0017 | 0.0219 | 0.0017 | 0.0017 |
| P6-8 | 0.2956 | N/A | 0.3076 | 0.3215 | 0.3076 | 0.3076 | 0.0120 | 0.0259 | 0.0120 | 0.0120 |
| P6-9 | 0.2772 | 0.3192 | 0.2868 | 0.2947 | 0.2868 | 0.2868 | 0.0096 | 0.0175 | 0.0096 | 0.0096 |
| P6-10 | 0.1584 | N/A | 0.1694 | 0.1802 | 0.1694 | 0.1694 | 0.0110 | 0.0218 | 0.0110 | 0.0110 |
| P9-11 | 0.0000 | N/A | 0.0190 | 0.0051 | 0.0190 | 0.0190 | 0.0190 | 0.0051 | 0.0190 | 0.0190 |
| P9-10 | 0.2772 | 0.2500 | 0.3221 | 0.3581 | 0.3221 | 0.3221 | 0.0449 | 0.0809 | 0.0449 | 0.0449 |
| P4-12 | 0.4419 | N/A | 0.4337 | 0.4434 | 0.4337 | 0.4337 | 0.0082 | 0.0015 | 0.0082 | 0.0082 |
| P12-13 | 0.0000 | N/A | -0.0181 | -0.0160 | -0.0181 | -0.0181 | 0.0181 | 0.0160 | 0.0181 | 0.0181 |
| P12-14 | 0.0786 | 0.1031 | 0.0836 | 0.0777 | 0.0836 | 0.0836 | 0.0050 | 0.0009 | 0.0050 | 0.0050 |
| P12-15 | 0.1789 | 0.2159 | 0.1946 | 0.1924 | 0.1946 | 0.1946 | 0.0157 | 0.0135 | 0.0157 | 0.0157 |
| P12-16 | 0.0724 | N/A | 0.0516 | 0.0652 | 0.0516 | 0.0516 | 0.0208 | 0.0072 | 0.0208 | 0.0208 |
| P14-15 | 0.0158 | N/A | 0.0216 | 0.0306 | 0.0216 | 0.0216 | 0.0058 | 0.0148 | 0.0058 | 0.0058 |
| P16-17 | 0.0369 | -0.0700 | 0.0049 | 0.0282 | 0.0049 | 0.0049 | 0.0320 | 0.0087 | 0.0320 | 0.0320 |
| P15-18 | 0.0602 | 0.0182 | 0.0063 | 0.0182 | 0.0063 | 0.0063 | 0.0539 | 0.0420 | 0.0539 | 0.0539 |
| P18-19 | 0.0278 | 0.0955 | 0.0131 | 0.0175 | 0.0131 | 0.0131 | 0.0147 | 0.0103 | 0.0147 | 0.0147 |
| P19-20 | -0.0673 | -0.1209 | -0.0894 | -0.0947 | -0.0894 | -0.0894 | 0.0221 | 0.0274 | 0.0221 | 0.0221 |
| P10-20 | 0.0903 | 0.1383 | 0.0809 | 0.0631 | 0.0809 | 0.0809 | 0.0094 | 0.0272 | 0.0094 | 0.0094 |
| P10-17 | 0.0533 | 0.0595 | 0.0458 | 0.0595 | 0.0458 | 0.0458 | 0.0075 | 0.0062 | 0.0075 | 0.0075 |
| P10-21 | 0.1579 | 0.2297 | 0.1858 | 0.1913 | 0.1858 | 0.1858 | 0.0279 | 0.0334 | 0.0279 | 0.0279 |
| P10-22 | 0.0762 | N/A | 0.0906 | 0.0907 | 0.0906 | 0.0906 | 0.0144 | 0.0145 | 0.0144 | 0.0144 |
| P21-22 | -0.0183 | -0.0282 | -0.0132 | -0.0282 | -0.0132 | -0.0132 | 0.0051 | 0.0099 | 0.0051 | 0.0051 |
| P15-23 | 0.0504 | N/A | 0.0795 | 0.0767 | 0.0795 | 0.0795 | 0.0291 | 0.0263 | 0.0291 | 0.0291 |
| P22-24 | 0.0574 | 0.2028 | 0.1192 | 0.0657 | 0.1192 | 0.1192 | 0.0618 | 0.0083 | 0.0618 | 0.0618 |
| P23-24 | 0.0180 | 0.0593 | -0.0058 | -0.0166 | -0.0058 | -0.0058 | 0.0238 | 0.0346 | 0.0238 | 0.0238 |
| P24-25 | -0.0121 | N/A | -0.0739 | -0.0942 | -0.0739 | -0.0739 | 0.0618 | 0.0821 | 0.0618 | 0.0618 |
| P25-26 | 0.0354 | N/A | -0.0276 | -0.0411 | -0.0276 | -0.0276 | 0.0630 | 0.0765 | 0.0630 | 0.0630 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P25-27 | -0.0476 | -0.0710 | -0.0620 | -0.0551 | -0.0620 | -0.0620 | 0.0144 | 0.0075 | 0.0144 | 0.0144 |
| P28-27 | 0.1807 | 0.1671 | 0.1717 | 0.1671 | 0.1717 | 0.1717 | 0.0090 | 0.0136 | 0.0090 | 0.0090 |
| P27-29 | 0.0619 | N/A | 0.0379 | 0.0369 | 0.0379 | 0.0379 | 0.0240 | 0.0250 | 0.0240 | 0.0240 |
| P27-30 | 0.0709 | 0.0570 | 0.0438 | 0.0363 | 0.0438 | 0.0438 | 0.0271 | 0.0346 | 0.0271 | 0.0271 |
| P29-30 | 0.0370 | 0.0721 | 0.0233 | 0.0144 | 0.0233 | 0.0233 | 0.0137 | 0.0226 | 0.0137 | 0.0137 |
| P8-28 | -0.0054 | N/A | -0.0160 | -0.0174 | -0.0160 | -0.0160 | 0.0106 | 0.0120 | 0.0106 | 0.0106 |
| P6-28 | 0.1867 | 0.1690 | 0.1609 | 0.1664 | 0.1609 | 0.1609 | 0.0258 | 0.0203 | 0.0258 | 0.0258 |
| P2-1 | -1.6809 | N/A | -1.8119 | -1.8893 | -1.8119 | -1.8119 | 0.1310 | 0.2084 | 0.1310 | 0.1310 |
| P3-1 | -0.8454 | N/A | -0.8333 | -0.8509 | -0.8333 | -0.8333 | 0.0121 | 0.0055 | 0.0121 | 0.0121 |
| P4-2 | -0.4263 | N/A | -0.3768 | -0.3760 | -0.3768 | -0.3768 | 0.0495 | 0.0503 | 0.0495 | 0.0495 |
| P4-3 | -0.8129 | N/A | -0.8501 | -0.9011 | -0.8501 | -0.8501 | 0.0372 | 0.0882 | 0.0372 | 0.0372 |
| P5-2 | -0.7942 | N/A | -0.7975 | -0.8017 | -0.7975 | -0.7975 | 0.0033 | 0.0075 | 0.0033 | 0.0033 |
| P6-2 | -0.5843 | N/A | -0.5294 | -0.5329 | -0.5294 | -0.5294 | 0.0549 | 0.0514 | 0.0549 | 0.0549 |
| P6-4 | -0.7150 | N/A | -0.6898 | -0.7114 | -0.6898 | -0.6898 | 0.0252 | 0.0036 | 0.0252 | 0.0252 |
| P7-5 | 0.1495 | N/A | 0.2286 | 0.2486 | 0.2286 | 0.2286 | 0.0791 | 0.0991 | 0.0791 | 0.0791 |
| P7-6 | -0.3775 | N/A | -0.3793 | -0.3561 | -0.3793 | -0.3793 | 0.0018 | 0.0214 | 0.0018 | 0.0018 |
| P8-6 | -0.2946 | N/A | -0.3065 | -0.3203 | -0.3065 | -0.3065 | 0.0119 | 0.0257 | 0.0119 | 0.0119 |
| P9-6 | -0.2772 | N/A | -0.2868 | -0.2947 | -0.2868 | -0.2868 | 0.0096 | 0.0175 | 0.0096 | 0.0096 |
| P10-6 | -0.1584 | N/A | -0.1694 | -0.1802 | -0.1694 | -0.1694 | 0.0110 | 0.0218 | 0.0110 | 0.0110 |
| P11-9 | 0.0000 | N/A | -0.0190 | -0.0051 | -0.0190 | -0.0190 | 0.0190 | 0.0051 | 0.0190 | 0.0190 |
| P10-9 | -0.2772 | N/A | -0.3221 | -0.3581 | -0.3221 | -0.3221 | 0.0449 | 0.0809 | 0.0449 | 0.0449 |
| P12-4 | -0.4419 | N/A | -0.4337 | -0.4434 | -0.4337 | -0.4337 | 0.0082 | 0.0015 | 0.0082 | 0.0082 |
| P13-12 | 0.0000 | N/A | 0.0181 | 0.0160 | 0.0181 | 0.0181 | 0.0181 | 0.0160 | 0.0181 | 0.0181 |
| P14-12 | -0.0778 | N/A | -0.0827 | -0.0770 | -0.0827 | -0.0827 | 0.0049 | 0.0008 | 0.0049 | 0.0049 |
| P15-12 | -0.1767 | N/A | -0.1919 | -0.1898 | -0.1919 | -0.1919 | 0.0152 | 0.0131 | 0.0152 | 0.0152 |
| P16-12 | -0.0719 | N/A | -0.0514 | -0.0647 | -0.0514 | -0.0514 | 0.0205 | 0.0072 | 0.0205 | 0.0205 |
| P15-14 | -0.0158 | N/A | -0.0214 | -0.0303 | -0.0214 | -0.0214 | 0.0056 | 0.0145 | 0.0056 | 0.0056 |
| P17-16 | -0.0368 | N/A | -0.0049 | -0.0281 | -0.0049 | -0.0049 | 0.0319 | 0.0087 | 0.0319 | 0.0319 |
| P18-15 | -0.0598 | N/A | -0.0063 | -0.0181 | -0.0063 | -0.0063 | 0.0535 | 0.0417 | 0.0535 | 0.0535 |
| P19-18 | -0.0277 | N/A | -0.0131 | -0.0175 | -0.0131 | -0.0131 | 0.0146 | 0.0102 | 0.0146 | 0.0146 |
| P20-19 | 0.0674 | N/A | 0.0897 | 0.0951 | 0.0897 | 0.0897 | 0.0223 | 0.0277 | 0.0223 | 0.0223 |
| P20-10 | -0.0894 | N/A | -0.0794 | -0.0616 | -0.0794 | -0.0794 | 0.0100 | 0.0278 | 0.0100 | 0.0100 |
| P17-10 | -0.0532 | N/A | -0.0453 | -0.0586 | -0.0453 | -0.0453 | 0.0079 | 0.0054 | 0.0079 | 0.0079 |
| P21-10 | -0.1567 | N/A | -0.1845 | -0.1900 | -0.1845 | -0.1845 | 0.0278 | 0.0333 | 0.0278 | 0.0278 |
| P22-10 | -0.0757 | N/A | -0.0899 | -0.0901 | -0.0899 | -0.0899 | 0.0142 | 0.0144 | 0.0142 | 0.0142 |
| P22-21 | 0.0183 | N/A | 0.0132 | 0.0282 | 0.0132 | 0.0132 | 0.0051 | 0.0099 | 0.0051 | 0.0051 |
| P23-15 | -0.0500 | N/A | -0.0788 | -0.0761 | -0.0788 | -0.0788 | 0.0288 | 0.0261 | 0.0288 | 0.0288 |
| P24-22 | -0.0569 | N/A | -0.1175 | -0.0652 | -0.1175 | -0.1175 | 0.0606 | 0.0083 | 0.0606 | 0.0606 |
| P24-23 | -0.0180 | N/A | 0.0058 | 0.0167 | 0.0058 | 0.0058 | 0.0238 | 0.0347 | 0.0238 | 0.0238 |
| P25-24 | 0.0122 | N/A | 0.0751 | 0.0959 | 0.0751 | 0.0751 | 0.0629 | 0.0837 | 0.0629 | 0.0629 |
| P26-25 | -0.0350 | N/A | 0.0278 | 0.0416 | 0.0278 | 0.0278 | 0.0628 | 0.0766 | 0.0628 | 0.0628 |
| P27-25 | 0.0479 | N/A | 0.0624 | 0.0554 | 0.0624 | 0.0624 | 0.0145 | 0.0075 | 0.0145 | 0.0145 |
| P27-28 | -0.1807 | N/A | -0.1717 | -0.1671 | -0.1717 | -0.1717 | 0.0090 | 0.0136 | 0.0090 | 0.0090 |
| P29-27 | -0.0610 | N/A | -0.0374 | -0.0366 | -0.0374 | -0.0374 | 0.0236 | 0.0244 | 0.0236 | 0.0236 |
| P30-27 | -0.0693 | N/A | -0.0429 | -0.0358 | -0.0429 | -0.0429 | 0.0264 | 0.0335 | 0.0264 | 0.0264 |
| P30-29 | -0.0367 | N/A | -0.0231 | -0.0143 | -0.0231 | -0.0231 | 0.0136 | 0.0224 | 0.0136 | 0.0136 |
| P28-8 | 0.0055 | N/A | 0.0160 | 0.0174 | 0.0160 | 0.0160 | 0.0105 | 0.0119 | 0.0105 | 0.0105 |
| P28-6 | -0.1862 | N/A | -0.1605 | -0.1659 | -0.1605 | -0.1605 | 0.0257 | 0.0203 | 0.0257 | 0.0257 |
| Q1-2 | -0.2470 | N/A | -0.2736 | -0.2643 | -0.2736 | -0.2736 | 0.0266 | 0.0173 | 0.0266 | 0.0266 |
| Q1-3 | 0.0428 | 0.0595 | 0.0179 | 0.0056 | 0.0179 | 0.0179 | 0.0249 | 0.0372 | 0.0249 | 0.0249 |
| Q2-4 | 0.0475 | 0.0671 | 0.0328 | 0.0201 | 0.0328 | 0.0328 | 0.0147 | 0.0274 | 0.0147 | 0.0147 |
| Q3-4 | -0.0385 | -0.0159 | -0.0549 | -0.0461 | -0.0549 | -0.0549 | 0.0164 | 0.0076 | 0.0164 | 0.0164 |
| Q2-5 | 0.0278 | 0.0213 | -0.0028 | 0.0074 | -0.0028 | -0.0028 | 0.0306 | 0.0204 | 0.0306 | 0.0306 |
| Q2-6 | 0.0137 | N/A | 0.0087 | 0.0080 | 0.0087 | 0.0087 | 0.0050 | 0.0057 | 0.0050 | 0.0050 |
| Q4-6 | -0.1591 | -0.1829 | -0.1134 | -0.0646 | -0.1134 | -0.1134 | 0.0457 | 0.0945 | 0.0457 | 0.0457 |
| Q5-7 | 0.1149 | 0.1580 | 0.1655 | 0.1722 | 0.1655 | 0.1655 | 0.0506 | 0.0573 | 0.0506 | 0.0506 |
| Q6-7 | -0.0278 | -0.0959 | -0.0358 | -0.0067 | -0.0358 | -0.0358 | 0.0080 | 0.0211 | 0.0080 | 0.0080 |
| Q6-8 | -0.0720 | N/A | -0.0385 | -0.0468 | -0.0385 | -0.0385 | 0.0335 | 0.0252 | 0.0335 | 0.0335 |
| Q6-9 | -0.0809 | -0.1233 | -0.0471 | -0.0062 | -0.0471 | -0.0471 | 0.0338 | 0.0747 | 0.0338 | 0.0338 |
| Q6-10 | 0.0019 | N/A | 0.0011 | 0.0117 | 0.0011 | 0.0011 | 0.0008 | 0.0098 | 0.0008 | 0.0008 |
| Q9-11 | -0.1560 | N/A | -0.1586 | -0.1266 | -0.1586 | -0.1586 | 0.0026 | 0.0294 | 0.0026 | 0.0026 |
| Q9-10 | 0.0588 | 0.1108 | 0.0774 | 0.0500 | 0.0774 | 0.0774 | 0.0186 | 0.0088 | 0.0186 | 0.0186 |
| Q4-12 | 0.1441 | N/A | 0.0660 | 0.0736 | 0.0660 | 0.0660 | 0.0781 | 0.0705 | 0.0781 | 0.0781 |
| Q12-13 | -0.1032 | N/A | -0.0957 | -0.0937 | -0.0957 | -0.0957 | 0.0075 | 0.0095 | 0.0075 | 0.0075 |

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|---|---|---|---|---|---|---|---|---|---|---|
| Q12-14 | 0.0240 | 0.0570 | 0.0098 | -0.0055 | 0.0098 | 0.0098 | 0.0142 | 0.0295 | 0.0142 | 0.0142 |
| Q12-15 | 0.0679 | 0.0645 | 0.0550 | 0.0503 | 0.0550 | 0.0550 | 0.0129 | 0.0176 | 0.0129 | 0.0129 |
| Q12-16 | 0.0335 | N/A | 0.0145 | 0.0358 | 0.0145 | 0.0145 | 0.0190 | 0.0023 | 0.0190 | 0.0190 |
| Q14-15 | 0.0065 | N/A | 0.0119 | 0.0214 | 0.0119 | 0.0119 | 0.0054 | 0.0149 | 0.0054 | 0.0054 |
| Q16-17 | 0.0144 | -0.0008 | -0.0350 | -0.0251 | -0.0350 | -0.0350 | 0.0494 | 0.0395 | 0.0494 | 0.0494 |
| Q15-18 | 0.0160 | 0.0172 | 0.0004 | 0.0172 | 0.0004 | 0.0004 | 0.0156 | 0.0012 | 0.0156 | 0.0156 |
| Q18-19 | 0.0062 | 0.0088 | 0.0079 | 0.0221 | 0.0079 | 0.0079 | 0.0017 | 0.0159 | 0.0017 | 0.0017 |
| Q19-20 | -0.0279 | 0.0134 | -0.0352 | -0.0209 | -0.0352 | -0.0352 | 0.0073 | 0.0070 | 0.0073 | 0.0073 |
| Q10-20 | 0.0371 | 0.1134 | 0.0997 | 0.1134 | 0.0997 | 0.0997 | 0.0626 | 0.0763 | 0.0626 | 0.0626 |
| Q10-17 | 0.0443 | 0.0676 | 0.1301 | 0.1568 | 0.1301 | 0.1301 | 0.0858 | 0.1125 | 0.0858 | 0.0858 |
| Q10-21 | 0.1001 | 0.0896 | 0.0715 | 0.0386 | 0.0715 | 0.0715 | 0.0286 | 0.0615 | 0.0286 | 0.0286 |
| Q10-22 | 0.0460 | N/A | 0.0344 | 0.0167 | 0.0344 | 0.0344 | 0.0116 | 0.0293 | 0.0116 | 0.0116 |
| Q21-22 | -0.0143 | -0.0051 | 0.0031 | -0.0051 | 0.0031 | 0.0031 | 0.0174 | 0.0092 | 0.0174 | 0.0174 |
| Q15-23 | 0.0291 | N/A | -0.0081 | -0.0140 | -0.0081 | -0.0081 | 0.0372 | 0.0431 | 0.0372 | 0.0372 |
| Q22-24 | 0.0306 | 0.0781 | 0.0366 | 0.0249 | 0.0366 | 0.0366 | 0.0060 | 0.0057 | 0.0060 | 0.0060 |
| Q23-24 | 0.0124 | 0.0278 | -0.0173 | -0.0213 | -0.0173 | -0.0173 | 0.0297 | 0.0337 | 0.0297 | 0.0297 |
| Q24-25 | 0.0201 | N/A | 0.0307 | 0.0180 | 0.0307 | 0.0307 | 0.0106 | 0.0021 | 0.0106 | 0.0106 |
| Q25-26 | 0.0237 | N/A | -0.0067 | -0.0111 | -0.0067 | -0.0067 | 0.0304 | 0.0348 | 0.0304 | 0.0304 |
| Q25-27 | -0.0037 | 0.0094 | -0.0099 | -0.0142 | -0.0099 | -0.0099 | 0.0062 | 0.0105 | 0.0062 | 0.0062 |
| Q28-27 | 0.0504 | 0.0033 | 0.0361 | 0.0033 | 0.0361 | 0.0361 | 0.0143 | 0.0471 | 0.0143 | 0.0143 |
| Q27-29 | 0.0167 | N/A | 0.0236 | 0.0076 | 0.0236 | 0.0236 | 0.0069 | 0.0091 | 0.0069 | 0.0069 |
| Q27-30 | 0.0166 | 0.0093 | 0.0311 | 0.0120 | 0.0311 | 0.0311 | 0.0145 | 0.0046 | 0.0145 | 0.0145 |
| Q29-30 | 0.0061 | -0.0205 | 0.0192 | 0.0088 | 0.0192 | 0.0192 | 0.0131 | 0.0027 | 0.0131 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0147 | -0.0177 | -0.0147 | -0.0147 | 0.0093 | 0.0123 | 0.0093 | 0.0093 |
| Q6-28 | 0.0011 | -0.0427 | -0.0070 | -0.0231 | -0.0070 | -0.0070 | 0.0081 | 0.0242 | 0.0081 | 0.0081 |
| Q2-1 | 0.3447 | N/A | 0.3939 | 0.3999 | 0.3939 | 0.3939 | 0.0492 | 0.0552 | 0.0492 | 0.0492 |
| Q3-1 | 0.0265 | N/A | 0.0451 | 0.0617 | 0.0451 | 0.0451 | 0.0186 | 0.0352 | 0.0186 | 0.0186 |
| Q4-2 | -0.0554 | N/A | -0.0489 | -0.0365 | -0.0489 | -0.0489 | 0.0065 | 0.0189 | 0.0065 | 0.0065 |
| Q4-3 | 0.0544 | N/A | 0.0723 | 0.0665 | 0.0723 | 0.0723 | 0.0179 | 0.0121 | 0.0179 | 0.0179 |
| Q5-2 | 0.0517 | N/A | 0.0801 | 0.0716 | 0.0801 | 0.0801 | 0.0284 | 0.0199 | 0.0284 | 0.0284 |
| Q6-2 | 0.0058 | N/A | -0.0016 | -0.0001 | -0.0016 | -0.0016 | 0.0074 | 0.0059 | 0.0074 | 0.0074 |
| Q6-4 | 0.1719 | N/A | 0.1234 | 0.0756 | 0.1234 | 0.1234 | 0.0485 | 0.0963 | 0.0485 | 0.0485 |
| Q7-5 | -0.1313 | N/A | -0.1776 | -0.1829 | -0.1776 | -0.1776 | 0.0463 | 0.0516 | 0.0463 | 0.0463 |
| Q7-6 | 0.0223 | N/A | 0.0297 | -0.0007 | 0.0297 | 0.0297 | 0.0074 | 0.0230 | 0.0074 | 0.0074 |
| Q8-6 | 0.0666 | N/A | 0.0330 | 0.0417 | 0.0330 | 0.0330 | 0.0336 | 0.0249 | 0.0336 | 0.0336 |
| Q9-6 | 0.0972 | N/A | 0.0639 | 0.0235 | 0.0639 | 0.0639 | 0.0333 | 0.0737 | 0.0333 | 0.0333 |
| Q10-6 | 0.0110 | N/A | 0.0142 | 0.0057 | 0.0142 | 0.0142 | 0.0032 | 0.0053 | 0.0032 | 0.0032 |
| Q11-9 | 0.1606 | N/A | 0.1635 | 0.1298 | 0.1635 | 0.1635 | 0.0029 | 0.0308 | 0.0029 | 0.0029 |
| Q10-9 | -0.0508 | N/A | -0.0661 | -0.0363 | -0.0661 | -0.0661 | 0.0153 | 0.0145 | 0.0153 | 0.0153 |
| Q12-4 | -0.0972 | N/A | -0.0191 | -0.0245 | -0.0191 | -0.0191 | 0.0781 | 0.0727 | 0.0781 | 0.0781 |
| Q13-12 | 0.1045 | N/A | 0.0970 | 0.0949 | 0.0970 | 0.0970 | 0.0075 | 0.0096 | 0.0075 | 0.0075 |
| Q14-12 | -0.0225 | N/A | -0.0080 | 0.0070 | -0.0080 | -0.0080 | 0.0145 | 0.0295 | 0.0145 | 0.0145 |
| Q15-12 | -0.0636 | N/A | -0.0499 | -0.0453 | -0.0499 | -0.0499 | 0.0137 | 0.0183 | 0.0137 | 0.0137 |
| Q16-12 | -0.0324 | N/A | -0.0140 | -0.0347 | -0.0140 | -0.0140 | 0.0184 | 0.0023 | 0.0184 | 0.0184 |
| Q15-14 | -0.0064 | N/A | -0.0118 | -0.0211 | -0.0118 | -0.0118 | 0.0054 | 0.0147 | 0.0054 | 0.0054 |
| Q17-16 | -0.0141 | N/A | 0.0352 | 0.0254 | 0.0352 | 0.0352 | 0.0493 | 0.0395 | 0.0493 | 0.0493 |
| Q18-15 | -0.0152 | N/A | -0.0004 | -0.0171 | -0.0004 | -0.0004 | 0.0148 | 0.0019 | 0.0148 | 0.0148 |
| Q19-18 | -0.0061 | N/A | -0.0078 | -0.0220 | -0.0078 | -0.0078 | 0.0017 | 0.0159 | 0.0017 | 0.0017 |
| Q20-19 | 0.0283 | N/A | 0.0358 | 0.0216 | 0.0358 | 0.0358 | 0.0075 | 0.0067 | 0.0075 | 0.0075 |
| Q20-10 | -0.0353 | N/A | -0.0964 | -0.1100 | -0.0964 | -0.0964 | 0.0611 | 0.0747 | 0.0611 | 0.0611 |
| Q17-10 | -0.0439 | N/A | -0.1286 | -0.1545 | -0.1286 | -0.1286 | 0.0847 | 0.1106 | 0.0847 | 0.0847 |
| Q21-10 | -0.0977 | N/A | -0.0687 | -0.0359 | -0.0687 | -0.0687 | 0.0290 | 0.0618 | 0.0290 | 0.0290 |
| Q22-10 | -0.0449 | N/A | -0.0331 | -0.0155 | -0.0331 | -0.0331 | 0.0118 | 0.0294 | 0.0118 | 0.0118 |
| Q22-21 | 0.0143 | N/A | -0.0031 | 0.0051 | -0.0031 | -0.0031 | 0.0174 | 0.0092 | 0.0174 | 0.0174 |
| Q23-15 | -0.0284 | N/A | 0.0094 | 0.0152 | 0.0094 | 0.0094 | 0.0378 | 0.0436 | 0.0378 | 0.0378 |
| Q24-22 | -0.0299 | N/A | -0.0339 | -0.0241 | -0.0339 | -0.0339 | 0.0040 | 0.0058 | 0.0040 | 0.0040 |
| Q24-23 | -0.0123 | N/A | 0.0174 | 0.0215 | 0.0174 | 0.0174 | 0.0297 | 0.0338 | 0.0297 | 0.0297 |
| Q25-24 | -0.0200 | N/A | -0.0286 | -0.0149 | -0.0286 | -0.0286 | 0.0086 | 0.0051 | 0.0086 | 0.0086 |
| Q26-25 | -0.0230 | N/A | 0.0070 | 0.0118 | 0.0070 | 0.0070 | 0.0300 | 0.0348 | 0.0300 | 0.0300 |
| Q27-25 | 0.0042 | N/A | 0.0107 | 0.0148 | 0.0107 | 0.0107 | 0.0065 | 0.0106 | 0.0065 | 0.0065 |
| Q27-28 | -0.0375 | N/A | -0.0244 | 0.0073 | -0.0244 | -0.0244 | 0.0131 | 0.0448 | 0.0131 | 0.0131 |
| Q29-27 | -0.0151 | N/A | -0.0228 | -0.0070 | -0.0228 | -0.0228 | 0.0077 | 0.0081 | 0.0077 | 0.0077 |
| Q30-27 | -0.0136 | N/A | -0.0294 | -0.0112 | -0.0294 | -0.0294 | 0.0158 | 0.0024 | 0.0158 | 0.0158 |
| Q30-29 | -0.0054 | N/A | -0.0188 | -0.0087 | -0.0188 | -0.0188 | 0.0134 | 0.0033 | 0.0134 | 0.0134 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q28-8 | -0.0380 | N/A | -0.0297 | -0.0267 | -0.0297 | -0.0297 | 0.0083 | 0.0113 | 0.0083 | 0.0083 |
| Q28-6 | -0.0123 | N/A | -0.0051 | 0.0112 | -0.0051 | -0.0051 | 0.0072 | 0.0235 | 0.0072 | 0.0072 |
| **SUM** | | | | | | | **7.0127** | **8.4977** | **7.0127** | **7.0127** |

The same situation as the base case, since the bad data are small, noth LTS and LTAV don't detect it, and generate the same estimation as WLS.

#### 5.3.2.2 Large bad data

Change the bad data in section 5.3.2.1 from reverse to large bad data by letting $P_2 = 0.9256$, others remain the same. State variables are presented in Table 5.266, Table 5.267 shows the estimated value and errors between each estimator and actual value.

Table 5.266: State Variable of case 1, large, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0849 | -0.0915 | -0.0849 | -0.0946 | **V1** | 1.0667 | 1.0690 | 1.0667 | 1.0674 |
| $\theta 3$ | -0.1450 | -0.1399 | -0.1450 | -0.1341 | **V2** | 1.0560 | 1.0561 | 1.0560 | 1.0537 |
| $\theta 4$ | -0.1774 | -0.1702 | -0.1774 | -0.1658 | **V3** | 1.0275 | 1.0325 | 1.0275 | 1.0311 |
| $\theta 5$ | -0.2647 | -0.2621 | -0.2647 | -0.2566 | **V4** | 1.0186 | 1.0240 | 1.0186 | 1.0224 |
| $\theta 6$ | -0.2084 | -0.1995 | -0.2084 | -0.1954 | **V5** | 1.0198 | 1.0208 | 1.0198 | 1.0220 |
| $\theta 7$ | -0.2388 | -0.2329 | -0.2388 | -0.2262 | **V6** | 1.0151 | 1.0189 | 1.0151 | 1.0190 |
| $\theta 8$ | -0.2236 | -0.2131 | -0.2236 | -0.2090 | **V7** | 1.0084 | 1.0107 | 1.0084 | 1.0119 |
| $\theta 9$ | -0.2804 | -0.2633 | -0.2804 | -0.2576 | **V8** | 1.0127 | 1.0169 | 1.0127 | 1.0168 |
| $\theta 10$ | -0.3172 | -0.3038 | -0.3172 | -0.2923 | **V9** | 1.0291 | 1.0219 | 1.0291 | 1.0311 |
| $\theta 11$ | -0.2882 | -0.2643 | -0.2882 | -0.2627 | **V10** | 1.0223 | 1.0176 | 1.0223 | 1.0237 |
| $\theta 12$ | -0.3084 | -0.2894 | -0.3084 | -0.2817 | **V11** | 1.0622 | 1.0477 | 1.0622 | 1.0635 |
| $\theta 13$ | -0.3102 | -0.2872 | -0.3102 | -0.2808 | **V12** | 1.0127 | 1.0118 | 1.0127 | 1.0132 |
| $\theta 14$ | -0.3319 | -0.3157 | -0.3319 | -0.3028 | **V13** | 1.0278 | 1.0247 | 1.0278 | 1.0271 |
| $\theta 15$ | -0.3321 | -0.3134 | -0.3321 | -0.3040 | **V14** | 0.9999 | 1.0000 | 0.9999 | 1.0006 |
| $\theta 16$ | -0.3178 | -0.2985 | -0.3178 | -0.2906 | **V15** | 0.9926 | 0.9920 | 0.9926 | 0.9934 |
| $\theta 17$ | -0.3184 | -0.3039 | -0.3184 | -0.2925 | **V16** | 1.0042 | 1.0007 | 1.0042 | 1.0053 |
| $\theta 18$ | -0.3333 | -0.3177 | -0.3333 | -0.3052 | **V17** | 1.0101 | 1.0032 | 1.0101 | 1.0115 |
| $\theta 19$ | -0.3338 | -0.3197 | -0.3338 | -0.3062 | **V18** | 0.9913 | 0.9852 | 0.9913 | 0.9925 |
| $\theta 20$ | -0.3283 | -0.3119 | -0.3283 | -0.3010 | **V19** | 0.9892 | 0.9804 | 0.9892 | 0.9905 |
| $\theta 21$ | -0.3292 | -0.3165 | -0.3292 | -0.3037 | **V20** | 0.9946 | 0.9860 | 0.9946 | 0.9960 |
| $\theta 22$ | -0.3290 | -0.3159 | -0.3290 | -0.3034 | **V21** | 1.0106 | 1.0082 | 1.0106 | 1.0121 |
| $\theta 23$ | -0.3487 | -0.3290 | -0.3487 | -0.3209 | **V22** | 1.0106 | 1.0087 | 1.0106 | 1.0122 |
| $\theta 24$ | -0.3466 | -0.3251 | -0.3466 | -0.3206 | **V23** | 0.9856 | 0.9878 | 0.9856 | 0.9870 |
| $\theta 25$ | -0.3110 | -0.2884 | -0.3110 | -0.2883 | **V24** | 0.9904 | 0.9966 | 0.9904 | 0.9922 |
| $\theta 26$ | -0.3017 | -0.2760 | -0.3017 | -0.2793 | **V25** | 0.9937 | 1.0091 | 0.9937 | 0.9962 |
| $\theta 27$ | -0.2965 | -0.2772 | -0.2965 | -0.2755 | **V26** | 1.0028 | 1.0237 | 1.0028 | 1.0057 |
| $\theta 28$ | -0.2204 | -0.2098 | -0.2204 | -0.2056 | **V27** | 1.0026 | 1.0180 | 1.0026 | 1.0052 |
| $\theta 29$ | -0.3094 | -0.2912 | -0.3094 | -0.2869 | **V28** | 1.0122 | 1.0170 | 1.0122 | 1.0163 |
| $\theta 30$ | -0.3157 | -0.2951 | -0.3157 | -0.2932 | **V29** | 0.9839 | 1.0063 | 0.9839 | 0.9869 |
| | | | | | **V30** | 0.9694 | 0.9988 | 0.9694 | 0.9724 |

Table 5.267: Estimation Results and Errors of case 1, large, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0667 | 1.0690 | 1.0667 | 1.0674 | 0.0067 | 0.0090 | 0.0067 | 0.0074 |
| V2 | 1.0450 | N/A | 1.0560 | 1.0561 | 1.0560 | 1.0537 | 0.0110 | 0.0111 | 0.0110 | 0.0087 |
| V3 | 1.0210 | N/A | 1.0275 | 1.0325 | 1.0275 | 1.0311 | 0.0065 | 0.0115 | 0.0065 | 0.0101 |
| V4 | 1.0120 | N/A | 1.0186 | 1.0240 | 1.0186 | 1.0224 | 0.0066 | 0.0120 | 0.0066 | 0.0104 |
| V5 | 1.0100 | N/A | 1.0198 | 1.0208 | 1.0198 | 1.0220 | 0.0098 | 0.0108 | 0.0098 | 0.0120 |
| V6 | 1.0110 | N/A | 1.0151 | 1.0189 | 1.0151 | 1.0190 | 0.0041 | 0.0079 | 0.0041 | 0.0080 |
| V7 | 1.0030 | N/A | 1.0084 | 1.0107 | 1.0084 | 1.0119 | 0.0054 | 0.0077 | 0.0054 | 0.0089 |
| V8 | 1.0100 | N/A | 1.0127 | 1.0169 | 1.0127 | 1.0168 | 0.0027 | 0.0069 | 0.0027 | 0.0068 |
| V9 | 1.0510 | N/A | 1.0291 | 1.0219 | 1.0291 | 1.0311 | 0.0219 | 0.0291 | 0.0219 | 0.0199 |
| V10 | 1.0450 | N/A | 1.0223 | 1.0176 | 1.0223 | 1.0237 | 0.0227 | 0.0274 | 0.0227 | 0.0213 |
| V11 | 1.0820 | N/A | 1.0622 | 1.0477 | 1.0622 | 1.0635 | 0.0198 | 0.0343 | 0.0198 | 0.0185 |
| V12 | 1.0570 | N/A | 1.0127 | 1.0118 | 1.0127 | 1.0132 | 0.0443 | 0.0452 | 0.0443 | 0.0438 |
| V13 | 1.0710 | N/A | 1.0278 | 1.0247 | 1.0278 | 1.0271 | 0.0432 | 0.0463 | 0.0432 | 0.0439 |
| V14 | 1.0430 | N/A | 0.9999 | 1.0000 | 0.9999 | 1.0006 | 0.0431 | 0.0430 | 0.0431 | 0.0424 |
| V15 | 1.0380 | N/A | 0.9926 | 0.9920 | 0.9926 | 0.9934 | 0.0454 | 0.0460 | 0.0454 | 0.0446 |
| V16 | 1.0450 | N/A | 1.0042 | 1.0007 | 1.0042 | 1.0053 | 0.0408 | 0.0443 | 0.0408 | 0.0397 |
| V17 | 1.0400 | N/A | 1.0101 | 1.0032 | 1.0101 | 1.0115 | 0.0299 | 0.0368 | 0.0299 | 0.0285 |
| V18 | 1.0280 | N/A | 0.9913 | 0.9852 | 0.9913 | 0.9925 | 0.0367 | 0.0428 | 0.0367 | 0.0355 |
| V19 | 1.0260 | N/A | 0.9892 | 0.9804 | 0.9892 | 0.9905 | 0.0368 | 0.0456 | 0.0368 | 0.0355 |
| V20 | 1.0300 | N/A | 0.9946 | 0.9860 | 0.9946 | 0.9960 | 0.0354 | 0.0440 | 0.0354 | 0.0340 |
| V21 | 1.0330 | N/A | 1.0106 | 1.0082 | 1.0106 | 1.0121 | 0.0224 | 0.0248 | 0.0224 | 0.0209 |
| V22 | 1.0340 | N/A | 1.0106 | 1.0087 | 1.0106 | 1.0122 | 0.0234 | 0.0253 | 0.0234 | 0.0218 |
| V23 | 1.0270 | N/A | 0.9856 | 0.9878 | 0.9856 | 0.9870 | 0.0414 | 0.0392 | 0.0414 | 0.0400 |
| V24 | 1.0220 | N/A | 0.9904 | 0.9966 | 0.9904 | 0.9922 | 0.0316 | 0.0254 | 0.0316 | 0.0298 |
| V25 | 1.0180 | N/A | 0.9937 | 1.0091 | 0.9937 | 0.9962 | 0.0243 | 0.0089 | 0.0243 | 0.0218 |
| V26 | 1.0000 | N/A | 1.0028 | 1.0237 | 1.0028 | 1.0057 | 0.0028 | 0.0237 | 0.0028 | 0.0057 |
| V27 | 1.0240 | N/A | 1.0026 | 1.0180 | 1.0026 | 1.0052 | 0.0214 | 0.0060 | 0.0214 | 0.0188 |
| V28 | 1.0070 | N/A | 1.0122 | 1.0170 | 1.0122 | 1.0163 | 0.0052 | 0.0100 | 0.0052 | 0.0093 |
| V29 | 1.0040 | N/A | 0.9839 | 1.0063 | 0.9839 | 0.9869 | 0.0201 | 0.0023 | 0.0201 | 0.0171 |
| V30 | 0.9920 | N/A | 0.9694 | 0.9988 | 0.9694 | 0.9724 | 0.0226 | 0.0068 | 0.0226 | 0.0196 |
| P1 | 2.6096 | 2.6540 | 2.5494 | 2.6540 | 2.5494 | 2.6678 | 0.0602 | 0.0444 | 0.0602 | 0.0582 |
| **P2** | **0.1830** | **0.9256** | **0.8033** | **0.4494** | **0.8033** | **0.2549** | **0.6203** | **0.2664** | **0.6203** | **0.0719** |
| P3 | -0.0240 | -0.0774 | -0.0605 | -0.0774 | -0.0605 | -0.0055 | 0.0365 | 0.0534 | 0.0365 | 0.0185 |
| P4 | -0.0760 | -0.1165 | -0.1711 | -0.1165 | -0.1711 | -0.1245 | 0.0951 | 0.0405 | 0.0951 | 0.0485 |
| P5 | -0.9420 | -1.0892 | -1.1047 | -1.0892 | -1.1047 | -1.0526 | 0.1627 | 0.1472 | 0.1627 | 0.1106 |
| P6 | 0.0000 | 0.0719 | 0.0323 | 0.0636 | 0.0323 | 0.0682 | 0.0323 | 0.0636 | 0.0323 | 0.0682 |
| P7 | -0.2280 | -0.2117 | -0.2002 | -0.2117 | -0.2002 | -0.1687 | 0.0278 | 0.0163 | 0.0278 | 0.0593 |
| P8 | -0.3000 | -0.3377 | -0.3731 | -0.3377 | -0.3731 | -0.3409 | 0.0731 | 0.0377 | 0.0731 | 0.0409 |
| P9 | 0.0000 | 0.0685 | 0.0318 | 0.0685 | 0.0318 | 0.0461 | 0.0318 | 0.0685 | 0.0318 | 0.0461 |
| P10 | -0.0580 | -0.1436 | -0.1034 | -0.1436 | -0.1034 | -0.0938 | 0.0454 | 0.0856 | 0.0454 | 0.0358 |
| P11 | 0.0000 | -0.0051 | -0.0412 | -0.0051 | -0.0412 | -0.0271 | 0.0412 | 0.0051 | 0.0412 | 0.0271 |
| P12 | -0.1120 | -0.1241 | -0.1532 | -0.1241 | -0.1532 | -0.1335 | 0.0412 | 0.0121 | 0.0412 | 0.0215 |
| P13 | 0.0000 | 0.0160 | -0.0130 | 0.0160 | -0.0130 | 0.0066 | 0.0130 | 0.0160 | 0.0130 | 0.0066 |
| P14 | -0.0620 | -0.0464 | -0.0762 | -0.0868 | -0.0762 | -0.0667 | 0.0142 | 0.0248 | 0.0142 | 0.0047 |
| P15 | -0.0820 | -0.1252 | -0.1390 | -0.1252 | -0.1390 | -0.1318 | 0.0570 | 0.0432 | 0.0570 | 0.0498 |
| P16 | -0.0350 | -0.0365 | -0.0602 | -0.0365 | -0.0602 | -0.0515 | 0.0252 | 0.0015 | 0.0252 | 0.0165 |
| P17 | -0.0900 | -0.0982 | -0.0564 | -0.0817 | -0.0564 | -0.0524 | 0.0336 | 0.0083 | 0.0336 | 0.0376 |
| P18 | -0.0320 | -0.0006 | 0.0029 | -0.0006 | 0.0029 | 0.0054 | 0.0349 | 0.0314 | 0.0349 | 0.0374 |
| P19 | -0.0950 | -0.0403 | -0.1057 | -0.1476 | -0.1057 | -0.1037 | 0.0107 | 0.0526 | 0.0107 | 0.0087 |
| P20 | -0.0220 | 0.0335 | 0.0027 | 0.0335 | 0.0027 | 0.0075 | 0.0247 | 0.0555 | 0.0247 | 0.0295 |
| P21 | -0.1750 | -0.2182 | -0.2060 | -0.2182 | -0.2060 | -0.2008 | 0.0310 | 0.0432 | 0.0310 | 0.0258 |
| P22 | 0.0000 | 0.0039 | 0.0359 | 0.0039 | 0.0359 | 0.0401 | 0.0359 | 0.0039 | 0.0359 | 0.0401 |
| P23 | -0.0320 | -0.0927 | -0.0906 | -0.0927 | -0.0906 | -0.0868 | 0.0586 | 0.0607 | 0.0586 | 0.0548 |
| P24 | -0.0870 | -0.1427 | -0.1911 | -0.1427 | -0.1911 | -0.1876 | 0.1041 | 0.0557 | 0.1041 | 0.1006 |
| P25 | 0.0000 | -0.0003 | -0.0141 | -0.0003 | -0.0141 | -0.0143 | 0.0141 | 0.0003 | 0.0141 | 0.0143 |
| P26 | -0.0350 | 0.0416 | 0.0281 | 0.0416 | 0.0281 | 0.0279 | 0.0631 | 0.0766 | 0.0631 | 0.0629 |
| P27 | 0.0000 | -0.0385 | -0.0330 | -0.0385 | -0.0330 | -0.0295 | 0.0330 | 0.0385 | 0.0330 | 0.0295 |
| P28 | 0.0000 | 0.0186 | 0.0050 | 0.0186 | 0.0050 | 0.0191 | 0.0050 | 0.0186 | 0.0050 | 0.0191 |
| P29 | -0.0240 | -0.0353 | -0.0187 | -0.0251 | -0.0187 | -0.0158 | 0.0053 | 0.0011 | 0.0053 | 0.0082 |
| P30 | -0.1060 | -0.0501 | -0.0694 | -0.0501 | -0.0694 | -0.0672 | 0.0366 | 0.0559 | 0.0366 | 0.0388 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q1 | -0.2042 | -0.2587 | -0.2536 | -0.2587 | -0.2536 | -0.2555 | 0.0494 | 0.0545 | 0.0494 | 0.0513 |
| Q2 | 0.4337 | 0.4353 | 0.4370 | 0.4353 | 0.4370 | 0.4337 | 0.0033 | 0.0016 | 0.0033 | 0.0000 |
| Q3 | -0.0120 | 0.0156 | 0.0048 | 0.0156 | 0.0048 | -0.0050 | 0.0168 | 0.0276 | 0.0168 | 0.0070 |
| Q4 | -0.0160 | 0.0390 | -0.0102 | 0.0390 | -0.0102 | -0.0195 | 0.0058 | 0.0550 | 0.0058 | 0.0035 |
| Q5 | 0.1666 | 0.2438 | 0.2616 | 0.2470 | 0.2616 | 0.2509 | 0.0950 | 0.0804 | 0.0950 | 0.0843 |
| Q6 | 0.0000 | 0.0043 | 0.0059 | 0.0043 | 0.0059 | -0.0017 | 0.0059 | 0.0043 | 0.0059 | 0.0017 |
| Q7 | -0.1090 | -0.1836 | -0.1391 | -0.1415 | -0.1391 | -0.1450 | 0.0301 | 0.0325 | 0.0301 | 0.0360 |
| Q8 | 0.0611 | 0.0240 | 0.0287 | 0.0240 | 0.0287 | 0.0217 | 0.0324 | 0.0371 | 0.0324 | 0.0394 |
| Q9 | 0.0000 | -0.0531 | -0.0111 | -0.0531 | -0.0111 | -0.0153 | 0.0111 | 0.0531 | 0.0111 | 0.0153 |
| Q10 | -0.0200 | 0.0975 | 0.0878 | 0.0975 | 0.0878 | 0.0854 | 0.1078 | 0.1175 | 0.1078 | 0.1054 |
| Q11 | 0.1606 | 0.1298 | 0.1693 | 0.1298 | 0.1693 | 0.1654 | 0.0087 | 0.0308 | 0.0087 | 0.0048 |
| Q12 | -0.0750 | -0.0376 | -0.0218 | -0.0376 | -0.0218 | -0.0308 | 0.0532 | 0.0374 | 0.0532 | 0.0442 |
| Q13 | 0.1045 | 0.0949 | 0.1102 | 0.0949 | 0.1102 | 0.1015 | 0.0057 | 0.0096 | 0.0057 | 0.0030 |
| Q14 | -0.0160 | 0.0284 | 0.0125 | 0.0284 | 0.0125 | 0.0069 | 0.0285 | 0.0444 | 0.0285 | 0.0229 |
| Q15 | -0.0250 | -0.0632 | -0.0637 | -0.0632 | -0.0637 | -0.0674 | 0.0387 | 0.0382 | 0.0387 | 0.0424 |
| Q16 | -0.0180 | -0.0881 | -0.0462 | -0.0461 | -0.0462 | -0.0481 | 0.0282 | 0.0281 | 0.0282 | 0.0301 |
| Q17 | -0.0580 | -0.1291 | -0.0924 | -0.1291 | -0.0924 | -0.0931 | 0.0344 | 0.0711 | 0.0344 | 0.0351 |
| Q18 | -0.0090 | 0.0154 | 0.0087 | 0.0061 | 0.0087 | 0.0079 | 0.0177 | 0.0151 | 0.0177 | 0.0169 |
| Q19 | -0.0340 | -0.0429 | -0.0424 | -0.0429 | -0.0424 | -0.0428 | 0.0084 | 0.0089 | 0.0084 | 0.0088 |
| Q20 | -0.0070 | -0.0168 | -0.0578 | -0.0881 | -0.0578 | -0.0597 | 0.0508 | 0.0811 | 0.0508 | 0.0527 |
| Q21 | -0.1120 | -0.0410 | -0.0634 | -0.0410 | -0.0634 | -0.0649 | 0.0486 | 0.0710 | 0.0486 | 0.0471 |
| Q22 | 0.0000 | 0.0146 | 0.0024 | 0.0146 | 0.0024 | 0.0011 | 0.0024 | 0.0146 | 0.0024 | 0.0011 |
| Q23 | -0.0160 | -0.0061 | -0.0059 | -0.0061 | -0.0059 | -0.0072 | 0.0101 | 0.0099 | 0.0101 | 0.0088 |
| Q24 | -0.0670 | 0.0124 | -0.0250 | -0.0247 | -0.0250 | -0.0271 | 0.0420 | 0.0423 | 0.0420 | 0.0399 |
| Q25 | 0.0000 | -0.0402 | -0.0467 | -0.0402 | -0.0467 | -0.0457 | 0.0467 | 0.0402 | 0.0467 | 0.0457 |
| Q26 | -0.0230 | 0.0118 | 0.0054 | 0.0118 | 0.0054 | 0.0064 | 0.0284 | 0.0348 | 0.0284 | 0.0294 |
| Q27 | 0.0000 | 0.0418 | 0.0422 | 0.0418 | 0.0422 | 0.0415 | 0.0422 | 0.0418 | 0.0422 | 0.0415 |
| Q28 | 0.0000 | -0.0122 | 0.0065 | -0.0122 | 0.0065 | 0.0030 | 0.0065 | 0.0122 | 0.0065 | 0.0030 |
| Q29 | -0.0090 | 0.0018 | -0.0026 | 0.0018 | -0.0026 | -0.0033 | 0.0064 | 0.0108 | 0.0064 | 0.0057 |
| Q30 | -0.0190 | -0.0773 | -0.0475 | -0.0215 | -0.0475 | -0.0480 | 0.0285 | 0.0025 | 0.0285 | 0.0290 |
| P1-2 | 1.7331 | N/A | 1.5753 | 1.7105 | 1.5753 | 1.7646 | 0.1578 | 0.0226 | 0.1578 | 0.0315 |
| P1-3 | 0.8765 | 0.8817 | 0.9740 | 0.9435 | 0.9740 | 0.9032 | 0.0975 | 0.0670 | 0.0975 | 0.0267 |
| P2-4 | 0.4365 | 0.4726 | 0.5913 | 0.5057 | 0.5913 | 0.4593 | 0.1548 | 0.0692 | 0.1548 | 0.0228 |
| P3-4 | 0.8214 | 0.9507 | 0.8757 | 0.8308 | 0.8757 | 0.8653 | 0.0543 | 0.0094 | 0.0543 | 0.0439 |
| P2-5 | 0.8236 | 0.7903 | 0.9823 | 0.9337 | 0.9823 | 0.8831 | 0.1587 | 0.1101 | 0.1587 | 0.0595 |
| P2-6 | 0.6038 | N/A | 0.7620 | 0.6703 | 0.7620 | 0.6235 | 0.1582 | 0.0665 | 0.1582 | 0.0197 |
| P4-6 | 0.7213 | 0.7172 | 0.7415 | 0.7172 | 0.7415 | 0.7121 | 0.0202 | 0.0041 | 0.0202 | 0.0092 |
| P5-7 | -0.1478 | -0.2445 | -0.1632 | -0.1925 | -0.1632 | -0.2026 | 0.0154 | 0.0447 | 0.0154 | 0.0548 |
| P6-7 | 0.3813 | 0.3594 | 0.3694 | 0.4115 | 0.3694 | 0.3781 | 0.0119 | 0.0302 | 0.0119 | 0.0032 |
| P6-8 | 0.2956 | N/A | 0.3606 | 0.3233 | 0.3606 | 0.3269 | 0.0650 | 0.0277 | 0.0650 | 0.0313 |
| P6-9 | 0.2772 | 0.3192 | 0.3615 | 0.3192 | 0.3615 | 0.3141 | 0.0843 | 0.0420 | 0.0843 | 0.0369 |
| P6-10 | 0.1584 | N/A | 0.2028 | 0.1941 | 0.2028 | 0.1816 | 0.0444 | 0.0357 | 0.0444 | 0.0232 |
| P9-11 | 0.0000 | N/A | 0.0412 | 0.0051 | 0.0412 | 0.0271 | 0.0412 | 0.0051 | 0.0412 | 0.0271 |
| P9-10 | 0.2772 | 0.2500 | 0.3521 | 0.3826 | 0.3521 | 0.3331 | 0.0749 | 0.1054 | 0.0749 | 0.0559 |
| P4-12 | 0.4419 | N/A | 0.5267 | 0.4811 | 0.5267 | 0.4677 | 0.0848 | 0.0392 | 0.0848 | 0.0258 |
| P12-13 | 0.0000 | N/A | 0.0130 | -0.0160 | 0.0130 | -0.0066 | 0.0130 | 0.0160 | 0.0130 | 0.0066 |
| P12-14 | 0.0786 | 0.1031 | 0.0958 | 0.1031 | 0.0958 | 0.0880 | 0.0172 | 0.0245 | 0.0172 | 0.0094 |
| P12-15 | 0.1789 | 0.2159 | 0.2089 | 0.2099 | 0.2089 | 0.1998 | 0.0300 | 0.0310 | 0.0300 | 0.0209 |
| P12-16 | 0.0724 | N/A | 0.0559 | 0.0599 | 0.0559 | 0.0531 | 0.0165 | 0.0125 | 0.0165 | 0.0193 |
| P14-15 | 0.0158 | N/A | 0.0185 | 0.0151 | 0.0185 | 0.0204 | 0.0027 | 0.0007 | 0.0027 | 0.0046 |
| P16-17 | 0.0369 | -0.0700 | -0.0047 | 0.0230 | -0.0047 | 0.0014 | 0.0416 | 0.0139 | 0.0416 | 0.0355 |
| P15-18 | 0.0602 | 0.0182 | 0.0068 | 0.0275 | 0.0068 | 0.0064 | 0.0534 | 0.0327 | 0.0534 | 0.0538 |
| P18-19 | 0.0278 | 0.0955 | 0.0097 | 0.0268 | 0.0097 | 0.0118 | 0.0181 | 0.0010 | 0.0181 | 0.0160 |
| P19-20 | -0.0673 | -0.1209 | -0.0961 | -0.1209 | -0.0961 | -0.0919 | 0.0288 | 0.0536 | 0.0288 | 0.0246 |
| P10-20 | 0.0903 | 0.1383 | 0.0953 | 0.0898 | 0.0953 | 0.0862 | 0.0050 | 0.0005 | 0.0050 | 0.0041 |
| P10-17 | 0.0533 | 0.0595 | 0.0617 | 0.0595 | 0.0617 | 0.0517 | 0.0084 | 0.0062 | 0.0084 | 0.0016 |
| P10-21 | 0.1579 | 0.2297 | 0.1976 | 0.1924 | 0.1976 | 0.1901 | 0.0397 | 0.0345 | 0.0397 | 0.0322 |
| P10-22 | 0.0762 | N/A | 0.0969 | 0.0914 | 0.0969 | 0.0929 | 0.0207 | 0.0152 | 0.0207 | 0.0167 |
| P21-22 | -0.0183 | -0.0282 | -0.0099 | -0.0271 | -0.0099 | -0.0120 | 0.0084 | 0.0088 | 0.0084 | 0.0063 |
| P15-23 | 0.0504 | N/A | 0.0784 | 0.0691 | 0.0784 | 0.0790 | 0.0280 | 0.0187 | 0.0280 | 0.0286 |
| P22-24 | 0.0574 | 0.2028 | 0.1222 | 0.0676 | 0.1222 | 0.1203 | 0.0648 | 0.0102 | 0.0648 | 0.0629 |
| P23-24 | 0.0180 | 0.0593 | -0.0128 | -0.0241 | -0.0128 | -0.0085 | 0.0308 | 0.0421 | 0.0308 | 0.0265 |
| P24-25 | -0.0121 | N/A | -0.0836 | -0.0999 | -0.0836 | -0.0776 | 0.0715 | 0.0878 | 0.0715 | 0.0655 |
| P25-26 | 0.0354 | N/A | -0.0279 | -0.0411 | -0.0279 | -0.0277 | 0.0633 | 0.0765 | 0.0633 | 0.0631 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P25-27 | -0.0476 | -0.0710 | -0.0715 | -0.0610 | -0.0715 | -0.0655 | 0.0239 | 0.0134 | 0.0239 | 0.0179 |
| P28-27 | 0.1807 | 0.1671 | 0.1949 | 0.1761 | 0.1949 | 0.1801 | 0.0142 | 0.0046 | 0.0142 | 0.0006 |
| P27-29 | 0.0619 | N/A | 0.0427 | 0.0390 | 0.0427 | 0.0396 | 0.0192 | 0.0229 | 0.0192 | 0.0223 |
| P27-30 | 0.0709 | 0.0570 | 0.0472 | 0.0371 | 0.0472 | 0.0450 | 0.0237 | 0.0338 | 0.0237 | 0.0259 |
| P29-30 | 0.0370 | 0.0721 | 0.0235 | 0.0135 | 0.0235 | 0.0234 | 0.0135 | 0.0235 | 0.0135 | 0.0136 |
| P8-28 | -0.0054 | N/A | -0.0140 | -0.0156 | -0.0140 | -0.0153 | 0.0086 | 0.0102 | 0.0086 | 0.0099 |
| P6-28 | 0.1867 | 0.1690 | 0.2047 | 0.1736 | 0.2047 | 0.1768 | 0.0180 | 0.0131 | 0.0180 | 0.0099 |
| P2-1 | -1.6809 | N/A | -1.5323 | -1.6603 | -1.5323 | -1.7111 | 0.1486 | 0.0206 | 0.1486 | 0.0302 |
| P3-1 | -0.8454 | N/A | -0.9362 | -0.9082 | -0.9362 | -0.8708 | 0.0908 | 0.0628 | 0.0908 | 0.0254 |
| P4-2 | -0.4263 | N/A | -0.5732 | -0.4925 | -0.5732 | -0.4483 | 0.1469 | 0.0662 | 0.1469 | 0.0220 |
| P4-3 | -0.8129 | N/A | -0.8661 | -0.8222 | -0.8661 | -0.8560 | 0.0532 | 0.0093 | 0.0532 | 0.0431 |
| P5-2 | -0.7942 | N/A | -0.9414 | -0.8967 | -0.9414 | -0.8500 | 0.1472 | 0.1025 | 0.1472 | 0.0558 |
| P6-2 | -0.5843 | N/A | -0.7317 | -0.6468 | -0.7317 | -0.6031 | 0.1474 | 0.0625 | 0.1474 | 0.0188 |
| P6-4 | -0.7150 | N/A | -0.7350 | -0.7113 | -0.7350 | -0.7062 | 0.0200 | 0.0037 | 0.0200 | 0.0088 |
| P7-5 | 0.1495 | N/A | 0.1657 | 0.1954 | 0.1657 | 0.2057 | 0.0162 | 0.0459 | 0.0162 | 0.0562 |
| P7-6 | -0.3775 | N/A | -0.3658 | -0.4071 | -0.3658 | -0.3744 | 0.0117 | 0.0296 | 0.0117 | 0.0031 |
| P8-6 | -0.2946 | N/A | -0.3591 | -0.3221 | -0.3591 | -0.3256 | 0.0645 | 0.0275 | 0.0645 | 0.0310 |
| P9-6 | -0.2772 | N/A | -0.3615 | -0.3192 | -0.3615 | -0.3141 | 0.0843 | 0.0420 | 0.0843 | 0.0369 |
| P10-6 | -0.1584 | N/A | -0.2028 | -0.1941 | -0.2028 | -0.1816 | 0.0444 | 0.0357 | 0.0444 | 0.0232 |
| P11-9 | 0.0000 | N/A | -0.0412 | -0.0051 | -0.0412 | -0.0271 | 0.0412 | 0.0051 | 0.0412 | 0.0271 |
| P10-9 | -0.2772 | N/A | -0.3521 | -0.3826 | -0.3521 | -0.3331 | 0.0749 | 0.1054 | 0.0749 | 0.0559 |
| P12-4 | -0.4419 | N/A | -0.5267 | -0.4811 | -0.5267 | -0.4677 | 0.0848 | 0.0392 | 0.0848 | 0.0258 |
| P13-12 | 0.0000 | N/A | -0.0130 | 0.0160 | -0.0130 | 0.0066 | 0.0130 | 0.0160 | 0.0130 | 0.0066 |
| P14-12 | -0.0778 | N/A | -0.0947 | -0.1018 | -0.0947 | -0.0871 | 0.0169 | 0.0240 | 0.0169 | 0.0093 |
| P15-12 | -0.1767 | N/A | -0.2059 | -0.2069 | -0.2059 | -0.1970 | 0.0292 | 0.0302 | 0.0292 | 0.0203 |
| P16-12 | -0.0719 | N/A | -0.0556 | -0.0595 | -0.0556 | -0.0528 | 0.0163 | 0.0124 | 0.0163 | 0.0191 |
| P15-14 | -0.0158 | N/A | -0.0184 | -0.0149 | -0.0184 | -0.0203 | 0.0026 | 0.0009 | 0.0026 | 0.0045 |
| P17-16 | -0.0368 | N/A | 0.0047 | -0.0230 | 0.0047 | -0.0013 | 0.0415 | 0.0138 | 0.0415 | 0.0355 |
| P18-15 | -0.0598 | N/A | -0.0068 | -0.0274 | -0.0068 | -0.0064 | 0.0530 | 0.0324 | 0.0530 | 0.0534 |
| P19-18 | -0.0277 | N/A | -0.0097 | -0.0267 | -0.0097 | -0.0118 | 0.0180 | 0.0010 | 0.0180 | 0.0159 |
| P20-19 | 0.0674 | N/A | 0.0964 | 0.1214 | 0.0964 | 0.0922 | 0.0290 | 0.0540 | 0.0290 | 0.0248 |
| P20-10 | -0.0894 | N/A | -0.0937 | -0.0879 | -0.0937 | -0.0847 | 0.0043 | 0.0015 | 0.0043 | 0.0047 |
| P17-10 | -0.0532 | N/A | -0.0611 | -0.0587 | -0.0611 | -0.0511 | 0.0079 | 0.0055 | 0.0079 | 0.0021 |
| P21-10 | -0.1567 | N/A | -0.1961 | -0.1911 | -0.1961 | -0.1888 | 0.0394 | 0.0344 | 0.0394 | 0.0321 |
| P22-10 | -0.0757 | N/A | -0.0962 | -0.0908 | -0.0962 | -0.0922 | 0.0205 | 0.0151 | 0.0205 | 0.0165 |
| P22-21 | 0.0183 | N/A | 0.0099 | 0.0271 | 0.0099 | 0.0120 | 0.0084 | 0.0088 | 0.0084 | 0.0063 |
| P23-15 | -0.0500 | N/A | -0.0778 | -0.0686 | -0.0778 | -0.0784 | 0.0278 | 0.0186 | 0.0278 | 0.0284 |
| P24-22 | -0.0569 | N/A | -0.1203 | -0.0670 | -0.1203 | -0.1185 | 0.0634 | 0.0101 | 0.0634 | 0.0616 |
| P24-23 | -0.0180 | N/A | 0.0129 | 0.0242 | 0.0129 | 0.0085 | 0.0309 | 0.0422 | 0.0309 | 0.0265 |
| P25-24 | 0.0122 | N/A | 0.0853 | 0.1019 | 0.0853 | 0.0790 | 0.0731 | 0.0897 | 0.0731 | 0.0668 |
| P26-25 | -0.0350 | N/A | 0.0281 | 0.0416 | 0.0281 | 0.0279 | 0.0631 | 0.0766 | 0.0631 | 0.0629 |
| P27-25 | 0.0479 | N/A | 0.0720 | 0.0614 | 0.0720 | 0.0660 | 0.0241 | 0.0135 | 0.0241 | 0.0181 |
| P27-28 | -0.1807 | N/A | -0.1949 | -0.1761 | -0.1949 | -0.1801 | 0.0142 | 0.0046 | 0.0142 | 0.0006 |
| P29-27 | -0.0610 | N/A | -0.0421 | -0.0387 | -0.0421 | -0.0391 | 0.0189 | 0.0223 | 0.0189 | 0.0219 |
| P30-27 | -0.0693 | N/A | -0.0462 | -0.0366 | -0.0462 | -0.0441 | 0.0231 | 0.0327 | 0.0231 | 0.0252 |
| P30-29 | -0.0367 | N/A | -0.0232 | -0.0135 | -0.0232 | -0.0232 | 0.0135 | 0.0232 | 0.0135 | 0.0135 |
| P28-8 | 0.0055 | N/A | 0.0140 | 0.0156 | 0.0140 | 0.0153 | 0.0085 | 0.0101 | 0.0085 | 0.0098 |
| P28-6 | -0.1862 | N/A | -0.2040 | -0.1731 | -0.2040 | -0.1763 | 0.0178 | 0.0131 | 0.0178 | 0.0099 |
| Q1-2 | -0.2470 | N/A | -0.2868 | -0.2787 | -0.2868 | -0.2792 | 0.0398 | 0.0317 | 0.0398 | 0.0322 |
| Q1-3 | 0.0428 | 0.0595 | 0.0331 | 0.0200 | 0.0331 | 0.0237 | 0.0097 | 0.0228 | 0.0097 | 0.0191 |
| Q2-4 | 0.0475 | 0.0671 | 0.0389 | 0.0280 | 0.0389 | 0.0348 | 0.0086 | 0.0195 | 0.0086 | 0.0127 |
| Q3-4 | -0.0385 | -0.0159 | -0.0555 | -0.0483 | -0.0555 | -0.0550 | 0.0170 | 0.0098 | 0.0170 | 0.0165 |
| Q2-5 | 0.0278 | 0.0213 | 0.0230 | 0.0213 | 0.0230 | 0.0065 | 0.0048 | 0.0065 | 0.0048 | 0.0213 |
| Q2-6 | 0.0137 | N/A | 0.0191 | 0.0165 | 0.0191 | 0.0123 | 0.0054 | 0.0028 | 0.0054 | 0.0014 |
| Q4-6 | -0.1591 | -0.1829 | -0.1190 | -0.0747 | -0.1190 | -0.1151 | 0.0401 | 0.0844 | 0.0401 | 0.0440 |
| Q5-7 | 0.1149 | 0.1580 | 0.1577 | 0.1580 | 0.1577 | 0.1630 | 0.0428 | 0.0431 | 0.0428 | 0.0481 |
| Q6-7 | -0.0278 | -0.0959 | -0.0399 | -0.0344 | -0.0399 | -0.0373 | 0.0121 | 0.0066 | 0.0121 | 0.0095 |
| Q6-8 | -0.0720 | N/A | -0.0475 | -0.0467 | -0.0475 | -0.0414 | 0.0245 | 0.0253 | 0.0245 | 0.0306 |
| Q6-9 | -0.0809 | -0.1233 | -0.0553 | -0.0048 | -0.0553 | -0.0496 | 0.0256 | 0.0761 | 0.0256 | 0.0313 |
| Q6-10 | 0.0019 | N/A | -0.0020 | 0.0125 | -0.0020 | 0.0002 | 0.0039 | 0.0106 | 0.0039 | 0.0017 |
| Q9-11 | -0.1560 | N/A | -0.1637 | -0.1266 | -0.1637 | -0.1602 | 0.0077 | 0.0294 | 0.0077 | 0.0042 |
| Q9-10 | 0.0588 | 0.1108 | 0.0703 | 0.0483 | 0.0703 | 0.0751 | 0.0115 | 0.0105 | 0.0115 | 0.0163 |
| Q4-12 | 0.1441 | N/A | 0.0580 | 0.0774 | 0.0580 | 0.0636 | 0.0861 | 0.0667 | 0.0861 | 0.0805 |
| Q12-13 | -0.1032 | N/A | -0.1086 | -0.0937 | -0.1086 | -0.1001 | 0.0054 | 0.0095 | 0.0054 | 0.0031 |

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|---|---|---|---|---|---|---|---|---|---|---|
| Q12-14 | 0.0240 | 0.0570 | 0.0058 | -0.0019 | 0.0058 | 0.0085 | 0.0182 | 0.0259 | 0.0182 | 0.0155 |
| Q12-15 | 0.0679 | 0.0645 | 0.0525 | 0.0494 | 0.0525 | 0.0543 | 0.0154 | 0.0185 | 0.0154 | 0.0136 |
| Q12-16 | 0.0335 | N/A | 0.0172 | 0.0280 | 0.0172 | 0.0156 | 0.0163 | 0.0055 | 0.0163 | 0.0179 |
| Q14-15 | 0.0065 | N/A | 0.0161 | 0.0239 | 0.0161 | 0.0134 | 0.0096 | 0.0174 | 0.0096 | 0.0069 |
| Q16-17 | 0.0144 | -0.0008 | -0.0296 | -0.0190 | -0.0296 | -0.0331 | 0.0440 | 0.0334 | 0.0440 | 0.0475 |
| Q15-18 | 0.0160 | 0.0172 | 0.0028 | 0.0172 | 0.0028 | 0.0013 | 0.0132 | 0.0012 | 0.0132 | 0.0147 |
| Q18-19 | 0.0062 | 0.0088 | 0.0114 | 0.0231 | 0.0114 | 0.0091 | 0.0052 | 0.0169 | 0.0052 | 0.0029 |
| Q19-20 | -0.0279 | 0.0134 | -0.0310 | -0.0200 | -0.0310 | -0.0337 | 0.0031 | 0.0079 | 0.0031 | 0.0058 |
| Q10-20 | 0.0371 | 0.1134 | 0.0931 | 0.1134 | 0.0931 | 0.0974 | 0.0560 | 0.0763 | 0.0560 | 0.0603 |
| Q10-17 | 0.0443 | 0.0676 | 0.1237 | 0.1504 | 0.1237 | 0.1279 | 0.0794 | 0.1061 | 0.0794 | 0.0836 |
| Q10-21 | 0.1001 | 0.0896 | 0.0692 | 0.0387 | 0.0692 | 0.0708 | 0.0309 | 0.0614 | 0.0309 | 0.0293 |
| Q10-22 | 0.0460 | N/A | 0.0331 | 0.0167 | 0.0331 | 0.0340 | 0.0129 | 0.0293 | 0.0129 | 0.0120 |
| Q21-22 | -0.0143 | -0.0051 | 0.0026 | -0.0051 | 0.0026 | 0.0030 | 0.0169 | 0.0092 | 0.0169 | 0.0173 |
| Q15-23 | 0.0291 | N/A | -0.0039 | -0.0132 | -0.0039 | -0.0065 | 0.0330 | 0.0423 | 0.0330 | 0.0356 |
| Q22-24 | 0.0306 | 0.0781 | 0.0366 | 0.0249 | 0.0366 | 0.0366 | 0.0060 | 0.0057 | 0.0060 | 0.0060 |
| Q23-24 | 0.0124 | 0.0278 | -0.0110 | -0.0203 | -0.0110 | -0.0151 | 0.0234 | 0.0327 | 0.0234 | 0.0275 |
| Q24-25 | 0.0201 | N/A | 0.0398 | 0.0214 | 0.0398 | 0.0339 | 0.0197 | 0.0013 | 0.0197 | 0.0138 |
| Q25-26 | 0.0237 | N/A | -0.0051 | -0.0111 | -0.0051 | -0.0061 | 0.0288 | 0.0348 | 0.0288 | 0.0298 |
| Q25-27 | -0.0037 | 0.0094 | -0.0047 | -0.0111 | -0.0047 | -0.0081 | 0.0010 | 0.0074 | 0.0010 | 0.0044 |
| Q28-27 | 0.0504 | 0.0033 | 0.0319 | 0.0033 | 0.0319 | 0.0347 | 0.0185 | 0.0471 | 0.0185 | 0.0157 |
| Q27-29 | 0.0167 | N/A | 0.0228 | 0.0083 | 0.0228 | 0.0234 | 0.0061 | 0.0084 | 0.0061 | 0.0067 |
| Q27-30 | 0.0166 | 0.0093 | 0.0306 | 0.0130 | 0.0306 | 0.0309 | 0.0140 | 0.0036 | 0.0140 | 0.0143 |
| Q29-30 | 0.0061 | -0.0205 | 0.0192 | 0.0095 | 0.0192 | 0.0192 | 0.0131 | 0.0034 | 0.0131 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0149 | -0.0176 | -0.0149 | -0.0148 | 0.0095 | 0.0122 | 0.0095 | 0.0094 |
| Q6-28 | 0.0011 | -0.0427 | -0.0144 | -0.0228 | -0.0144 | -0.0093 | 0.0155 | 0.0239 | 0.0155 | 0.0104 |
| Q2-1 | 0.3447 | N/A | 0.3560 | 0.3694 | 0.3560 | 0.3801 | 0.0113 | 0.0247 | 0.0113 | 0.0354 |
| Q3-1 | 0.0265 | N/A | 0.0603 | 0.0639 | 0.0603 | 0.0500 | 0.0338 | 0.0374 | 0.0338 | 0.0235 |
| Q4-2 | -0.0554 | N/A | -0.0235 | -0.0277 | -0.0235 | -0.0410 | 0.0319 | 0.0277 | 0.0319 | 0.0144 |
| Q4-3 | 0.0544 | N/A | 0.0743 | 0.0640 | 0.0743 | 0.0729 | 0.0199 | 0.0096 | 0.0199 | 0.0185 |
| Q5-2 | 0.0517 | N/A | 0.1039 | 0.0890 | 0.1039 | 0.0879 | 0.0522 | 0.0373 | 0.0522 | 0.0362 |
| Q6-2 | 0.0058 | N/A | 0.0328 | 0.0145 | 0.0328 | 0.0095 | 0.0270 | 0.0087 | 0.0270 | 0.0037 |
| Q6-4 | 0.1719 | N/A | 0.1322 | 0.0859 | 0.1322 | 0.1263 | 0.0397 | 0.0860 | 0.0397 | 0.0456 |
| Q7-5 | -0.1313 | N/A | -0.1725 | -0.1718 | -0.1725 | -0.1761 | 0.0412 | 0.0405 | 0.0412 | 0.0448 |
| Q7-6 | 0.0223 | N/A | 0.0334 | 0.0303 | 0.0334 | 0.0311 | 0.0111 | 0.0080 | 0.0111 | 0.0088 |
| Q8-6 | 0.0666 | N/A | 0.0436 | 0.0416 | 0.0436 | 0.0364 | 0.0230 | 0.0250 | 0.0230 | 0.0302 |
| Q9-6 | 0.0972 | N/A | 0.0823 | 0.0252 | 0.0823 | 0.0699 | 0.0149 | 0.0720 | 0.0149 | 0.0273 |
| Q10-6 | 0.0110 | N/A | 0.0242 | 0.0077 | 0.0242 | 0.0175 | 0.0132 | 0.0033 | 0.0132 | 0.0065 |
| Q11-9 | 0.1606 | N/A | 0.1693 | 0.1298 | 0.1693 | 0.1654 | 0.0087 | 0.0308 | 0.0087 | 0.0048 |
| Q10-9 | -0.0508 | N/A | -0.0570 | -0.0327 | -0.0570 | -0.0631 | 0.0062 | 0.0181 | 0.0062 | 0.0123 |
| Q12-4 | -0.0972 | N/A | 0.0112 | -0.0194 | 0.0112 | -0.0091 | 0.1084 | 0.0778 | 0.1084 | 0.0881 |
| Q13-12 | 0.1045 | N/A | 0.1102 | 0.0949 | 0.1102 | 0.1015 | 0.0057 | 0.0096 | 0.0057 | 0.0030 |
| Q14-12 | -0.0225 | N/A | -0.0035 | 0.0045 | -0.0035 | -0.0065 | 0.0190 | 0.0270 | 0.0190 | 0.0160 |
| Q15-12 | -0.0636 | N/A | -0.0466 | -0.0435 | -0.0466 | -0.0489 | 0.0170 | 0.0201 | 0.0170 | 0.0147 |
| Q16-12 | -0.0324 | N/A | -0.0166 | -0.0271 | -0.0166 | -0.0150 | 0.0158 | 0.0053 | 0.0158 | 0.0174 |
| Q15-14 | -0.0064 | N/A | -0.0159 | -0.0237 | -0.0159 | -0.0133 | 0.0095 | 0.0173 | 0.0095 | 0.0069 |
| Q17-16 | -0.0141 | N/A | 0.0298 | 0.0192 | 0.0298 | 0.0333 | 0.0439 | 0.0333 | 0.0439 | 0.0474 |
| Q18-15 | -0.0152 | N/A | -0.0028 | -0.0170 | -0.0028 | -0.0013 | 0.0124 | 0.0018 | 0.0124 | 0.0139 |
| Q19-18 | -0.0061 | N/A | -0.0114 | -0.0229 | -0.0114 | -0.0091 | 0.0053 | 0.0168 | 0.0053 | 0.0030 |
| Q20-19 | 0.0283 | N/A | 0.0317 | 0.0210 | 0.0317 | 0.0344 | 0.0034 | 0.0073 | 0.0034 | 0.0061 |
| Q20-10 | -0.0353 | N/A | -0.0895 | -0.1092 | -0.0895 | -0.0940 | 0.0542 | 0.0739 | 0.0542 | 0.0587 |
| Q17-10 | -0.0439 | N/A | -0.1222 | -0.1483 | -0.1222 | -0.1264 | 0.0783 | 0.1044 | 0.0783 | 0.0825 |
| Q21-10 | -0.0977 | N/A | -0.0661 | -0.0359 | -0.0661 | -0.0678 | 0.0316 | 0.0618 | 0.0316 | 0.0299 |
| Q22-10 | -0.0449 | N/A | -0.0316 | -0.0155 | -0.0316 | -0.0326 | 0.0133 | 0.0294 | 0.0133 | 0.0123 |
| Q22-21 | 0.0143 | N/A | -0.0026 | 0.0051 | -0.0026 | -0.0030 | 0.0169 | 0.0092 | 0.0169 | 0.0173 |
| Q23-15 | -0.0284 | N/A | 0.0052 | 0.0142 | 0.0052 | 0.0078 | 0.0336 | 0.0426 | 0.0336 | 0.0362 |
| Q24-22 | -0.0299 | N/A | -0.0338 | -0.0240 | -0.0338 | -0.0339 | 0.0039 | 0.0059 | 0.0039 | 0.0040 |
| Q24-23 | -0.0123 | N/A | 0.0111 | 0.0206 | 0.0111 | 0.0152 | 0.0234 | 0.0329 | 0.0234 | 0.0275 |
| Q25-24 | -0.0200 | N/A | -0.0369 | -0.0180 | -0.0369 | -0.0315 | 0.0169 | 0.0020 | 0.0169 | 0.0115 |
| Q26-25 | -0.0230 | N/A | 0.0054 | 0.0118 | 0.0054 | 0.0064 | 0.0284 | 0.0348 | 0.0284 | 0.0294 |
| Q27-25 | 0.0042 | N/A | 0.0058 | 0.0119 | 0.0058 | 0.0090 | 0.0016 | 0.0077 | 0.0016 | 0.0048 |
| Q27-28 | -0.0375 | N/A | -0.0169 | 0.0086 | -0.0169 | -0.0218 | 0.0206 | 0.0461 | 0.0206 | 0.0157 |
| Q29-27 | -0.0151 | N/A | -0.0218 | -0.0077 | -0.0218 | -0.0225 | 0.0067 | 0.0074 | 0.0067 | 0.0074 |
| Q30-27 | -0.0136 | N/A | -0.0287 | -0.0121 | -0.0287 | -0.0292 | 0.0151 | 0.0015 | 0.0151 | 0.0156 |
| Q30-29 | -0.0054 | N/A | -0.0188 | -0.0094 | -0.0188 | -0.0188 | 0.0134 | 0.0040 | 0.0134 | 0.0134 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q28-8 | -0.0380 | N/A | -0.0289 | -0.0266 | -0.0289 | -0.0294 | 0.0091 | 0.0114 | 0.0091 | 0.0086 |
| Q28-6 | -0.0123 | N/A | 0.0035 | 0.0111 | 0.0035 | -0.0023 | 0.0158 | 0.0234 | 0.0158 | 0.0100 |
| **SUM** | | | | | | | **9.0223** | **8.0533** | **9.0223** | **6.6611** |

In this case, LTAV generate best estimation by successfully eliminate bad data, LAV performs well. LTS is not able to successfully detect the bad data.

### 5.3.3 Case 2: Multiple non-interacting bad data

For multiple non-interacting bad data case, Four power injection/flow that are not connected directly with each other are selected. Four reverse bad data, four large bad data and two reverse two large bad data are conducted. Four non-interacting location $P_2$, $P_{12-14}$, $P_{10-21}$ and $P_{25-27}$ are selected to replace the original one.

#### 5.3.3.1 Four reverse bad data

The two reverse bad data are $P_2 = -0.1256$, $P_{12-14} = -0.1031$, $P_{10-21} = -0.2297$ and $P_{25-27} = 0.0710$. Table 5.268 presents the state variables generated by the estimation of each estimator; The estimated system results and the error between them and the actual value are illustrated in Table 5.269.

Table 5.268: State Variable of case 2, multi reverse, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1000 | -0.1041 | -0.1000 | -0.1000 | **V1** | 1.0676 | 1.0690 | 1.0676 | 1.0678 |
| $\theta 3$ | -0.1272 | -0.1307 | -0.1272 | -0.1275 | **V2** | 1.0523 | 1.0522 | 1.0523 | 1.0525 |
| $\theta 4$ | -0.1582 | -0.1638 | -0.1582 | -0.1586 | **V3** | 1.0333 | 1.0361 | 1.0333 | 1.0335 |
| $\theta 5$ | -0.2522 | -0.2568 | -0.2522 | -0.2520 | **V4** | 1.0248 | 1.0266 | 1.0248 | 1.0250 |
| $\theta 6$ | -0.1870 | -0.1927 | -0.1870 | -0.1875 | **V5** | 1.0230 | 1.0210 | 1.0230 | 1.0233 |
| $\theta 7$ | -0.2188 | -0.2212 | -0.2188 | -0.2189 | **V6** | 1.0216 | 1.0210 | 1.0216 | 1.0217 |
| $\theta 8$ | -0.2003 | -0.2062 | -0.2003 | -0.2006 | **V7** | 1.0139 | 1.0118 | 1.0139 | 1.0141 |
| $\theta 9$ | -0.2387 | -0.2502 | -0.2387 | -0.2413 | **V8** | 1.0192 | 1.0189 | 1.0192 | 1.0194 |
| $\theta 10$ | -0.2652 | -0.2872 | -0.2652 | -0.2727 | **V9** | 1.0329 | 1.0251 | 1.0329 | 1.0337 |
| $\theta 11$ | -0.2463 | -0.2512 | -0.2463 | -0.2455 | **V10** | 1.0252 | 1.0209 | 1.0252 | 1.0262 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\theta$12 | -0.2565 | -0.2733 | -0.2565 | -0.2591 | **V11** | 1.0647 | 1.0508 | 1.0647 | 1.0657 |
| $\theta$13 | -0.2563 | -0.2712 | -0.2563 | -0.2587 | **V12** | 1.0130 | 1.0152 | 1.0130 | 1.0149 |
| $\theta$14 | -0.2645 | -0.2938 | -0.2645 | -0.2675 | **V13** | 1.0260 | 1.0282 | 1.0260 | 1.0279 |
| $\theta$15 | -0.2737 | -0.2955 | -0.2737 | -0.2779 | **V14** | 1.0056 | 1.0072 | 1.0056 | 1.0077 |
| $\theta$16 | -0.2673 | -0.2821 | -0.2673 | -0.2701 | **V15** | 0.9953 | 0.9959 | 0.9953 | 0.9966 |
| $\theta$17 | -0.2683 | -0.2873 | -0.2683 | -0.2729 | **V16** | 1.0044 | 1.0039 | 1.0044 | 1.0068 |
| $\theta$18 | -0.2805 | -0.3005 | -0.2805 | -0.2821 | **V17** | 1.0115 | 1.0065 | 1.0115 | 1.0137 |
| $\theta$19 | -0.2835 | -0.3030 | -0.2835 | -0.2844 | **V18** | 0.9921 | 0.9888 | 0.9921 | 0.9954 |
| $\theta$20 | -0.2783 | -0.2953 | -0.2783 | -0.2798 | **V19** | 0.9894 | 0.9839 | 0.9894 | 0.9933 |
| $\theta$21 | -0.2650 | -0.2976 | -0.2650 | -0.2814 | **V20** | 0.9949 | 0.9895 | 0.9949 | 0.9987 |
| $\theta$22 | -0.2642 | -0.2970 | -0.2642 | -0.2809 | **V21** | 1.0190 | 1.0128 | 1.0190 | 1.0158 |
| $\theta$23 | -0.2828 | -0.3115 | -0.2828 | -0.2946 | **V22** | 1.0194 | 1.0132 | 1.0194 | 1.0160 |
| $\theta$24 | -0.2785 | -0.3083 | -0.2785 | -0.2953 | **V23** | 0.9938 | 0.9914 | 0.9938 | 0.9916 |
| $\theta$25 | -0.2517 | -0.2763 | -0.2517 | -0.2631 | **V24** | 1.0020 | 0.9997 | 1.0020 | 0.9974 |
| $\theta$26 | -0.2299 | -0.2639 | -0.2299 | -0.2413 | **V25** | 1.0061 | 1.0083 | 1.0061 | 1.0016 |
| $\theta$27 | -0.2568 | -0.2679 | -0.2568 | -0.2646 | **V26** | 1.0249 | 1.0229 | 1.0249 | 1.0190 |
| $\theta$28 | -0.1963 | -0.2026 | -0.1963 | -0.1971 | **V27** | 1.0066 | 1.0148 | 1.0066 | 1.0036 |
| $\theta$29 | -0.2771 | -0.2837 | -0.2771 | -0.2847 | **V28** | 1.0189 | 1.0186 | 1.0189 | 1.0186 |
| $\theta$30 | -0.2836 | -0.2848 | -0.2836 | -0.2913 | **V29** | 0.9834 | 0.9986 | 0.9834 | 0.9792 |
| | | | | | **V30** | 0.9687 | 0.9888 | 0.9687 | 0.9644 |

Table 5.269: Estimation Results and Errors of case 2, multi reverse, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0676 | 1.0690 | 1.0676 | 1.0678 | 0.0076 | 0.0090 | 0.0076 | 0.0078 |
| V2 | 1.0450 | N/A | 1.0523 | 1.0522 | 1.0523 | 1.0525 | 0.0073 | 0.0072 | 0.0073 | 0.0075 |
| V3 | 1.0210 | N/A | 1.0333 | 1.0361 | 1.0333 | 1.0335 | 0.0123 | 0.0151 | 0.0123 | 0.0125 |
| V4 | 1.0120 | N/A | 1.0248 | 1.0266 | 1.0248 | 1.0250 | 0.0128 | 0.0146 | 0.0128 | 0.0130 |
| V5 | 1.0100 | N/A | 1.0230 | 1.0210 | 1.0230 | 1.0233 | 0.0130 | 0.0110 | 0.0130 | 0.0133 |
| V6 | 1.0110 | N/A | 1.0216 | 1.0210 | 1.0216 | 1.0217 | 0.0106 | 0.0100 | 0.0106 | 0.0107 |
| V7 | 1.0030 | N/A | 1.0139 | 1.0118 | 1.0139 | 1.0141 | 0.0109 | 0.0088 | 0.0109 | 0.0111 |
| V8 | 1.0100 | N/A | 1.0192 | 1.0189 | 1.0192 | 1.0194 | 0.0092 | 0.0089 | 0.0092 | 0.0094 |
| V9 | 1.0510 | N/A | 1.0329 | 1.0251 | 1.0329 | 1.0337 | 0.0181 | 0.0259 | 0.0181 | 0.0173 |
| V10 | 1.0450 | N/A | 1.0252 | 1.0209 | 1.0252 | 1.0262 | 0.0198 | 0.0241 | 0.0198 | 0.0188 |
| V11 | 1.0820 | N/A | 1.0647 | 1.0508 | 1.0647 | 1.0657 | 0.0173 | 0.0312 | 0.0173 | 0.0163 |
| V12 | 1.0570 | N/A | 1.0130 | 1.0152 | 1.0130 | 1.0149 | 0.0440 | 0.0418 | 0.0440 | 0.0421 |
| V13 | 1.0710 | N/A | 1.0260 | 1.0282 | 1.0260 | 1.0279 | 0.0450 | 0.0428 | 0.0450 | 0.0431 |
| V14 | 1.0430 | N/A | 1.0056 | 1.0072 | 1.0056 | 1.0077 | 0.0374 | 0.0358 | 0.0374 | 0.0353 |
| V15 | 1.0380 | N/A | 0.9953 | 0.9959 | 0.9953 | 0.9966 | 0.0427 | 0.0421 | 0.0427 | 0.0414 |
| V16 | 1.0450 | N/A | 1.0044 | 1.0039 | 1.0044 | 1.0068 | 0.0406 | 0.0411 | 0.0406 | 0.0382 |
| V17 | 1.0400 | N/A | 1.0115 | 1.0065 | 1.0115 | 1.0137 | 0.0285 | 0.0335 | 0.0285 | 0.0263 |
| V18 | 1.0280 | N/A | 0.9921 | 0.9888 | 0.9921 | 0.9954 | 0.0359 | 0.0392 | 0.0359 | 0.0326 |
| V19 | 1.0260 | N/A | 0.9894 | 0.9839 | 0.9894 | 0.9933 | 0.0366 | 0.0421 | 0.0366 | 0.0327 |
| V20 | 1.0300 | N/A | 0.9949 | 0.9895 | 0.9949 | 0.9987 | 0.0351 | 0.0405 | 0.0351 | 0.0313 |
| V21 | 1.0330 | N/A | 1.0190 | 1.0128 | 1.0190 | 1.0158 | 0.0140 | 0.0202 | 0.0140 | 0.0172 |
| V22 | 1.0340 | N/A | 1.0194 | 1.0132 | 1.0194 | 1.0160 | 0.0146 | 0.0208 | 0.0146 | 0.0180 |
| V23 | 1.0270 | N/A | 0.9938 | 0.9914 | 0.9938 | 0.9916 | 0.0332 | 0.0356 | 0.0332 | 0.0354 |
| V24 | 1.0220 | N/A | 1.0020 | 0.9997 | 1.0020 | 0.9974 | 0.0200 | 0.0223 | 0.0200 | 0.0246 |
| V25 | 1.0180 | N/A | 1.0061 | 1.0083 | 1.0061 | 1.0016 | 0.0119 | 0.0097 | 0.0119 | 0.0164 |
| V26 | 1.0000 | N/A | 1.0249 | 1.0229 | 1.0249 | 1.0190 | 0.0249 | 0.0229 | 0.0249 | 0.0190 |
| V27 | 1.0240 | N/A | 1.0066 | 1.0148 | 1.0066 | 1.0036 | 0.0174 | 0.0092 | 0.0174 | 0.0204 |
| V28 | 1.0070 | N/A | 1.0189 | 1.0186 | 1.0189 | 1.0186 | 0.0119 | 0.0116 | 0.0119 | 0.0116 |
| V29 | 1.0040 | N/A | 0.9834 | 0.9986 | 0.9834 | 0.9792 | 0.0206 | 0.0054 | 0.0206 | 0.0248 |
| V30 | 0.9920 | N/A | 0.9687 | 0.9888 | 0.9687 | 0.9644 | 0.0233 | 0.0032 | 0.0233 | 0.0276 |
| P1 | 2.6096 | 2.6540 | 2.7283 | 2.8365 | 2.7283 | 2.7314 | 0.1187 | 0.2269 | 0.1187 | 0.1218 |
| **P2** | **0.1830** | **-0.1256** | **-0.0647** | **-0.1256** | **-0.0647** | **-0.0604** | **0.2477** | **0.3086** | **0.2477** | **0.2434** |
| P3 | -0.0240 | -0.0774 | 0.0189 | 0.0621 | 0.0189 | 0.0214 | 0.0429 | 0.0861 | 0.0429 | 0.0454 |
| P4 | -0.0760 | -0.1165 | -0.1154 | -0.1165 | -0.1154 | -0.1092 | 0.0394 | 0.0405 | 0.0394 | 0.0332 |
| P5 | -0.9420 | -1.0892 | -1.0275 | -1.0464 | -1.0275 | -1.0249 | 0.0855 | 0.1044 | 0.0855 | 0.0829 |
| P6 | 0.0000 | 0.0719 | 0.0726 | 0.0719 | 0.0726 | 0.0821 | 0.0726 | 0.0719 | 0.0726 | 0.0821 |
| P7 | -0.2280 | -0.2117 | -0.1574 | -0.1075 | -0.1574 | -0.1534 | 0.0706 | 0.1205 | 0.0706 | 0.0746 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P8 | -0.3000 | -0.3377 | -0.3384 | -0.3377 | -0.3384 | -0.3288 | 0.0384 | 0.0377 | 0.0384 | 0.0288 |
| P9 | 0.0000 | 0.0685 | 0.0331 | 0.0685 | 0.0331 | 0.0514 | 0.0331 | 0.0685 | 0.0331 | 0.0514 |
| P10 | -0.0580 | -0.1436 | -0.1659 | -0.1436 | -0.1659 | -0.1016 | 0.1079 | 0.0856 | 0.1079 | 0.0436 |
| P11 | 0.0000 | -0.0051 | -0.0400 | -0.0051 | -0.0400 | -0.0220 | 0.0400 | 0.0051 | 0.0400 | 0.0220 |
| P12 | -0.1120 | -0.1241 | -0.1383 | -0.1241 | -0.1383 | -0.1367 | 0.0263 | 0.0121 | 0.0263 | 0.0247 |
| P13 | 0.0000 | 0.0160 | 0.0018 | 0.0160 | 0.0018 | 0.0034 | 0.0018 | 0.0160 | 0.0018 | 0.0034 |
| P14 | -0.0620 | -0.0464 | 0.0091 | -0.0464 | 0.0091 | 0.0133 | 0.0711 | 0.0156 | 0.0711 | 0.0753 |
| P15 | -0.0820 | -0.1252 | -0.1358 | -0.1252 | -0.1358 | -0.1295 | 0.0538 | 0.0432 | 0.0538 | 0.0475 |
| P16 | -0.0350 | -0.0365 | -0.0665 | -0.0365 | -0.0665 | -0.0570 | 0.0315 | 0.0015 | 0.0315 | 0.0220 |
| P17 | -0.0900 | -0.0982 | -0.0832 | -0.0809 | -0.0832 | -0.0574 | 0.0068 | 0.0091 | 0.0068 | 0.0326 |
| P18 | -0.0320 | -0.0006 | -0.0046 | -0.0006 | -0.0046 | 0.0034 | 0.0274 | 0.0314 | 0.0274 | 0.0354 |
| P19 | -0.0950 | -0.0403 | -0.1175 | -0.1508 | -0.1175 | -0.1058 | 0.0225 | 0.0558 | 0.0225 | 0.0108 |
| P20 | -0.0220 | 0.0335 | -0.0151 | 0.0335 | -0.0151 | 0.0073 | 0.0069 | 0.0555 | 0.0069 | 0.0293 |
| P21 | -0.1750 | -0.2182 | -0.0630 | -0.1879 | -0.0630 | -0.1751 | 0.1120 | 0.0129 | 0.1120 | 0.0001 |
| P22 | 0.0000 | 0.0039 | 0.1272 | 0.0337 | 0.1272 | 0.0541 | 0.1272 | 0.0337 | 0.1272 | 0.0541 |
| P23 | -0.0320 | -0.0927 | -0.0634 | -0.0927 | -0.0634 | -0.0815 | 0.0314 | 0.0607 | 0.0314 | 0.0495 |
| P24 | -0.0870 | -0.1427 | -0.1438 | -0.1427 | -0.1438 | -0.1765 | 0.0568 | 0.0557 | 0.0568 | 0.0895 |
| P25 | 0.0000 | -0.0003 | 0.0231 | -0.0003 | 0.0231 | 0.0206 | 0.0231 | 0.0003 | 0.0231 | 0.0206 |
| P26 | -0.0350 | 0.0416 | 0.0644 | 0.0416 | 0.0644 | 0.0622 | 0.0994 | 0.0766 | 0.0994 | 0.0972 |
| P27 | 0.0000 | -0.0385 | -0.0537 | -0.0385 | -0.0537 | -0.0541 | 0.0537 | 0.0385 | 0.0537 | 0.0541 |
| P28 | 0.0000 | 0.0186 | 0.0127 | 0.0186 | 0.0127 | 0.0225 | 0.0127 | 0.0186 | 0.0127 | 0.0225 |
| P29 | -0.0240 | -0.0353 | -0.0362 | -0.0353 | -0.0362 | -0.0366 | 0.0122 | 0.0113 | 0.0122 | 0.0126 |
| P30 | -0.1060 | -0.0501 | -0.0826 | -0.0501 | -0.0826 | -0.0828 | 0.0234 | 0.0559 | 0.0234 | 0.0232 |
| Q1 | -0.2042 | -0.2587 | -0.2562 | -0.2587 | -0.2562 | -0.2565 | 0.0520 | 0.0545 | 0.0520 | 0.0523 |
| Q2 | 0.4337 | 0.4353 | 0.4319 | 0.4353 | 0.4319 | 0.4316 | 0.0018 | 0.0016 | 0.0018 | 0.0021 |
| Q3 | -0.0120 | 0.0156 | -0.0102 | 0.0156 | -0.0102 | -0.0108 | 0.0018 | 0.0276 | 0.0018 | 0.0012 |
| Q4 | -0.0160 | 0.0390 | -0.0247 | 0.0390 | -0.0247 | -0.0262 | 0.0087 | 0.0550 | 0.0087 | 0.0102 |
| Q5 | 0.1666 | 0.2438 | 0.2457 | 0.2438 | 0.2457 | 0.2454 | 0.0791 | 0.0772 | 0.0791 | 0.0788 |
| Q6 | 0.0000 | 0.0043 | -0.0063 | 0.0043 | -0.0063 | -0.0063 | 0.0063 | 0.0043 | 0.0063 | 0.0063 |
| Q7 | -0.1090 | -0.1836 | -0.1480 | -0.1836 | -0.1480 | -0.1482 | 0.0390 | 0.0746 | 0.0390 | 0.0392 |
| Q8 | 0.0611 | 0.0240 | 0.0174 | 0.0240 | 0.0174 | 0.0179 | 0.0437 | 0.0371 | 0.0437 | 0.0432 |
| Q9 | 0.0000 | -0.0531 | -0.0183 | -0.0531 | -0.0183 | -0.0166 | 0.0183 | 0.0531 | 0.0183 | 0.0166 |
| Q10 | -0.0200 | 0.0975 | 0.0820 | 0.0975 | 0.0820 | 0.0854 | 0.1020 | 0.1175 | 0.1020 | 0.1054 |
| Q11 | 0.1606 | 0.1298 | 0.1626 | 0.1298 | 0.1626 | 0.1642 | 0.0020 | 0.0308 | 0.0020 | 0.0036 |
| Q12 | -0.0750 | -0.0376 | -0.0376 | -0.0376 | -0.0376 | -0.0369 | 0.0374 | 0.0374 | 0.0374 | 0.0381 |
| Q13 | 0.1045 | 0.0949 | 0.0949 | 0.0949 | 0.0949 | 0.0956 | 0.0096 | 0.0096 | 0.0096 | 0.0089 |
| Q14 | -0.0160 | 0.0284 | -0.0104 | 0.0284 | -0.0104 | -0.0103 | 0.0056 | 0.0444 | 0.0056 | 0.0057 |
| Q15 | -0.0250 | -0.0632 | -0.0657 | -0.0632 | -0.0657 | -0.0651 | 0.0407 | 0.0382 | 0.0407 | 0.0401 |
| Q16 | -0.0180 | -0.0881 | -0.0497 | -0.0493 | -0.0497 | -0.0482 | 0.0317 | 0.0313 | 0.0317 | 0.0302 |
| Q17 | -0.0580 | -0.1291 | -0.0942 | -0.1291 | -0.0942 | -0.0922 | 0.0362 | 0.0711 | 0.0362 | 0.0342 |
| Q18 | -0.0090 | 0.0154 | 0.0086 | 0.0057 | 0.0086 | 0.0093 | 0.0176 | 0.0147 | 0.0176 | 0.0183 |
| Q19 | -0.0340 | -0.0429 | -0.0427 | -0.0429 | -0.0427 | -0.0421 | 0.0087 | 0.0089 | 0.0087 | 0.0081 |
| Q20 | -0.0070 | -0.0168 | -0.0602 | -0.0877 | -0.0602 | -0.0599 | 0.0532 | 0.0807 | 0.0532 | 0.0529 |
| Q21 | -0.1120 | -0.0410 | -0.0683 | -0.0410 | -0.0683 | -0.0650 | 0.0437 | 0.0710 | 0.0437 | 0.0470 |
| Q22 | 0.0000 | 0.0146 | -0.0031 | 0.0146 | -0.0031 | 0.0014 | 0.0031 | 0.0146 | 0.0031 | 0.0014 |
| Q23 | -0.0160 | -0.0061 | -0.0061 | -0.0061 | -0.0061 | -0.0046 | 0.0099 | 0.0099 | 0.0099 | 0.0114 |
| Q24 | -0.0670 | 0.0124 | -0.0294 | -0.0229 | -0.0294 | -0.0268 | 0.0376 | 0.0441 | 0.0376 | 0.0402 |
| Q25 | 0.0000 | -0.0402 | -0.0438 | -0.0402 | -0.0438 | -0.0463 | 0.0438 | 0.0402 | 0.0438 | 0.0463 |
| Q26 | -0.0230 | 0.0118 | 0.0082 | 0.0118 | 0.0082 | 0.0057 | 0.0312 | 0.0348 | 0.0312 | 0.0287 |
| Q27 | 0.0000 | 0.0418 | 0.0414 | 0.0418 | 0.0414 | 0.0386 | 0.0414 | 0.0418 | 0.0414 | 0.0386 |
| Q28 | 0.0000 | -0.0122 | 0.0009 | -0.0122 | 0.0009 | 0.0036 | 0.0009 | 0.0122 | 0.0009 | 0.0036 |
| Q29 | -0.0090 | 0.0018 | -0.0032 | 0.0018 | -0.0032 | -0.0055 | 0.0058 | 0.0108 | 0.0058 | 0.0035 |
| Q30 | -0.0190 | -0.0773 | -0.0478 | -0.0373 | -0.0478 | -0.0496 | 0.0288 | 0.0183 | 0.0288 | 0.0306 |
| P1-2 | 1.7331 | N/A | 1.8700 | 1.9548 | 1.8700 | 1.8709 | 0.1369 | 0.2217 | 0.1369 | 0.1378 |
| P1-3 | 0.8765 | 0.8817 | 0.8583 | 0.8817 | 0.8583 | 0.8604 | 0.0182 | 0.0052 | 0.0182 | 0.0161 |
| P2-4 | 0.4365 | 0.4726 | 0.3783 | 0.3838 | 0.3783 | 0.3809 | 0.0582 | 0.0527 | 0.0582 | 0.0556 |
| P3-4 | 0.8214 | 0.9507 | 0.8480 | 0.9130 | 0.8480 | 0.8524 | 0.0266 | 0.0916 | 0.0266 | 0.0310 |
| P2-5 | 0.8236 | 0.7903 | 0.8281 | 0.8314 | 0.8281 | 0.8273 | 0.0045 | 0.0078 | 0.0045 | 0.0037 |
| P2-6 | 0.6038 | N/A | 0.5390 | 0.5489 | 0.5390 | 0.5424 | 0.0648 | 0.0549 | 0.0648 | 0.0614 |
| P4-6 | 0.7213 | 0.7172 | 0.6964 | 0.7172 | 0.6964 | 0.6998 | 0.0249 | 0.0041 | 0.0249 | 0.0215 |
| P5-7 | -0.1478 | -0.2445 | -0.2287 | -0.2445 | -0.2287 | -0.2267 | 0.0809 | 0.0967 | 0.0809 | 0.0789 |
| P6-7 | 0.3813 | 0.3594 | 0.3937 | 0.3594 | 0.3937 | 0.3877 | 0.0124 | 0.0219 | 0.0124 | 0.0064 |
| P6-8 | 0.2956 | N/A | 0.3213 | 0.3223 | 0.3213 | 0.3146 | 0.0257 | 0.0267 | 0.0257 | 0.0190 |
| P6-9 | 0.2772 | 0.3192 | 0.2622 | 0.2888 | 0.2622 | 0.2730 | 0.0150 | 0.0116 | 0.0150 | 0.0042 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P6-10 | 0.1584 | N/A | 0.1472 | 0.1768 | 0.1472 | 0.1604 | 0.0112 | 0.0184 | 0.0112 | 0.0020 |
| P9-11 | 0.0000 | N/A | 0.0400 | 0.0051 | 0.0400 | 0.0220 | 0.0400 | 0.0051 | 0.0400 | 0.0220 |
| P9-10 | 0.2772 | 0.2500 | 0.2553 | 0.3522 | 0.2553 | 0.3024 | 0.0219 | 0.0750 | 0.0219 | 0.0252 |
| P4-12 | 0.4419 | N/A | 0.3980 | 0.4452 | 0.3980 | 0.4077 | 0.0439 | 0.0033 | 0.0439 | 0.0342 |
| P12-13 | 0.0000 | N/A | -0.0018 | -0.0160 | -0.0018 | -0.0034 | 0.0018 | 0.0160 | 0.0018 | 0.0034 |
| **P12-14** | **0.0786** | **-0.1031** | **0.0375** | **0.0794** | **0.0375** | **0.0384** | **0.0411** | **0.0008** | **0.0411** | **0.0402** |
| P12-15 | 0.1789 | 0.2159 | 0.1615 | 0.1986 | 0.1615 | 0.1738 | 0.0174 | 0.0197 | 0.0174 | 0.0051 |
| P12-16 | 0.0724 | N/A | 0.0624 | 0.0591 | 0.0624 | 0.0622 | 0.0100 | 0.0133 | 0.0100 | 0.0102 |
| P14-15 | 0.0158 | N/A | 0.0464 | 0.0322 | 0.0464 | 0.0515 | 0.0306 | 0.0164 | 0.0306 | 0.0357 |
| P16-17 | 0.0369 | -0.0700 | -0.0045 | 0.0222 | -0.0045 | 0.0048 | 0.0414 | 0.0147 | 0.0414 | 0.0321 |
| P15-18 | 0.0602 | 0.0182 | 0.0309 | 0.0308 | 0.0309 | 0.0174 | 0.0293 | 0.0294 | 0.0293 | 0.0428 |
| P18-19 | 0.0278 | 0.0955 | 0.0261 | 0.0300 | 0.0261 | 0.0208 | 0.0017 | 0.0022 | 0.0017 | 0.0070 |
| P19-20 | -0.0673 | -0.1209 | -0.0914 | -0.1209 | -0.0914 | -0.0851 | 0.0241 | 0.0536 | 0.0241 | 0.0178 |
| P10-20 | 0.0903 | 0.1383 | 0.1088 | 0.0898 | 0.1088 | 0.0795 | 0.0185 | 0.0005 | 0.0185 | 0.0108 |
| P10-17 | 0.0533 | 0.0595 | 0.0886 | 0.0595 | 0.0886 | 0.0533 | 0.0353 | 0.0062 | 0.0353 | 0.0000 |
| **P10-21** | **0.1579** | **-0.2297** | **0.0294** | **0.1607** | **0.0294** | **0.1546** | **0.1285** | **0.0028** | **0.1285** | **0.0033** |
| P10-22 | 0.0762 | N/A | 0.0098 | 0.0755 | 0.0098 | 0.0738 | 0.0664 | 0.0007 | 0.0664 | 0.0024 |
| P21-22 | -0.0183 | -0.0282 | -0.0338 | -0.0282 | -0.0338 | -0.0215 | 0.0155 | 0.0099 | 0.0155 | 0.0032 |
| P15-23 | 0.0504 | N/A | 0.0389 | 0.0718 | 0.0389 | 0.0757 | 0.0115 | 0.0214 | 0.0115 | 0.0253 |
| P22-24 | 0.0574 | 0.2028 | 0.1031 | 0.0806 | 0.1031 | 0.1059 | 0.0457 | 0.0232 | 0.0457 | 0.0485 |
| P23-24 | 0.0180 | 0.0593 | -0.0246 | -0.0214 | -0.0246 | -0.0064 | 0.0426 | 0.0394 | 0.0426 | 0.0244 |
| P24-25 | -0.0121 | N/A | -0.0668 | -0.0845 | -0.0668 | -0.0784 | 0.0547 | 0.0724 | 0.0547 | 0.0663 |
| P25-26 | 0.0354 | N/A | -0.0634 | -0.0411 | -0.0634 | -0.0612 | 0.0988 | 0.0765 | 0.0988 | 0.0966 |
| **P25-27** | **-0.0476** | **0.0710** | **0.0187** | **-0.0451** | **0.0187** | **0.0020** | **0.0663** | **0.0025** | **0.0663** | **0.0496** |
| P28-27 | 0.1807 | 0.1671 | 0.1565 | 0.1705 | 0.1565 | 0.1742 | 0.0242 | 0.0102 | 0.0242 | 0.0065 |
| P27-29 | 0.0619 | N/A | 0.0611 | 0.0466 | 0.0611 | 0.0616 | 0.0008 | 0.0153 | 0.0008 | 0.0003 |
| P27-30 | 0.0709 | 0.0570 | 0.0603 | 0.0401 | 0.0603 | 0.0606 | 0.0106 | 0.0308 | 0.0106 | 0.0103 |
| P29-30 | 0.0370 | 0.0721 | 0.0240 | 0.0107 | 0.0240 | 0.0240 | 0.0130 | 0.0263 | 0.0130 | 0.0130 |
| P8-28 | -0.0054 | N/A | -0.0183 | -0.0166 | -0.0183 | -0.0154 | 0.0129 | 0.0112 | 0.0129 | 0.0100 |
| P6-28 | 0.1867 | 0.1690 | 0.1626 | 0.1690 | 0.1626 | 0.1676 | 0.0241 | 0.0177 | 0.0241 | 0.0191 |
| P2-1 | -1.6809 | N/A | -1.8101 | -1.8897 | -1.8101 | -1.8110 | 0.1292 | 0.2088 | 0.1292 | 0.1301 |
| P3-1 | -0.8454 | N/A | -0.8290 | -0.8509 | -0.8290 | -0.8310 | 0.0164 | 0.0055 | 0.0164 | 0.0144 |
| P4-2 | -0.4263 | N/A | -0.3708 | -0.3762 | -0.3708 | -0.3733 | 0.0555 | 0.0501 | 0.0555 | 0.0530 |
| P4-3 | -0.8129 | N/A | -0.8390 | -0.9027 | -0.8390 | -0.8434 | 0.0261 | 0.0898 | 0.0261 | 0.0305 |
| P5-2 | -0.7942 | N/A | -0.7988 | -0.8019 | -0.7988 | -0.7982 | 0.0046 | 0.0077 | 0.0046 | 0.0040 |
| P6-2 | -0.5843 | N/A | -0.5237 | -0.5330 | -0.5237 | -0.5269 | 0.0606 | 0.0513 | 0.0606 | 0.0574 |
| P6-4 | -0.7150 | N/A | -0.6908 | -0.7114 | -0.6908 | -0.6941 | 0.0242 | 0.0036 | 0.0242 | 0.0209 |
| P7-5 | 0.1495 | N/A | 0.2324 | 0.2486 | 0.2324 | 0.2304 | 0.0829 | 0.0991 | 0.0829 | 0.0809 |
| P7-6 | -0.3775 | N/A | -0.3897 | -0.3561 | -0.3897 | -0.3838 | 0.0122 | 0.0214 | 0.0122 | 0.0063 |
| P8-6 | -0.2946 | N/A | -0.3201 | -0.3211 | -0.3201 | -0.3134 | 0.0255 | 0.0265 | 0.0255 | 0.0188 |
| P9-6 | -0.2772 | N/A | -0.2622 | -0.2888 | -0.2622 | -0.2730 | 0.0150 | 0.0116 | 0.0150 | 0.0042 |
| P10-6 | -0.1584 | N/A | -0.1472 | -0.1768 | -0.1472 | -0.1604 | 0.0112 | 0.0184 | 0.0112 | 0.0020 |
| P11-9 | 0.0000 | N/A | -0.0400 | -0.0051 | -0.0400 | -0.0220 | 0.0400 | 0.0051 | 0.0400 | 0.0220 |
| P10-9 | -0.2772 | N/A | -0.2553 | -0.3522 | -0.2553 | -0.3024 | 0.0219 | 0.0750 | 0.0219 | 0.0252 |
| P12-4 | -0.4419 | N/A | -0.3980 | -0.4452 | -0.3980 | -0.4077 | 0.0439 | 0.0033 | 0.0439 | 0.0342 |
| P13-12 | 0.0000 | N/A | 0.0018 | 0.0160 | 0.0018 | 0.0034 | 0.0018 | 0.0160 | 0.0018 | 0.0034 |
| P14-12 | -0.0778 | N/A | -0.0374 | -0.0786 | -0.0374 | -0.0382 | 0.0404 | 0.0008 | 0.0404 | 0.0396 |
| P15-12 | -0.1767 | N/A | -0.1596 | -0.1959 | -0.1596 | -0.1717 | 0.0171 | 0.0192 | 0.0171 | 0.0050 |
| P16-12 | -0.0719 | N/A | -0.0620 | -0.0587 | -0.0620 | -0.0618 | 0.0099 | 0.0132 | 0.0099 | 0.0101 |
| P15-14 | -0.0158 | N/A | -0.0460 | -0.0319 | -0.0460 | -0.0509 | 0.0302 | 0.0161 | 0.0302 | 0.0351 |
| P17-16 | -0.0368 | N/A | 0.0046 | -0.0222 | 0.0046 | -0.0047 | 0.0414 | 0.0146 | 0.0414 | 0.0321 |
| P18-15 | -0.0598 | N/A | -0.0308 | -0.0306 | -0.0308 | -0.0174 | 0.0290 | 0.0292 | 0.0290 | 0.0424 |
| P19-18 | -0.0277 | N/A | -0.0261 | -0.0299 | -0.0261 | -0.0207 | 0.0016 | 0.0022 | 0.0016 | 0.0070 |
| P20-19 | 0.0674 | N/A | 0.0918 | 0.1214 | 0.0918 | 0.0854 | 0.0244 | 0.0540 | 0.0244 | 0.0180 |
| P20-10 | -0.0894 | N/A | -0.1069 | -0.0879 | -0.1069 | -0.0781 | 0.0175 | 0.0015 | 0.0175 | 0.0113 |
| P17-10 | -0.0532 | N/A | -0.0878 | -0.0587 | -0.0878 | -0.0527 | 0.0346 | 0.0055 | 0.0346 | 0.0005 |
| P21-10 | -0.1567 | N/A | -0.0292 | -0.1597 | -0.0292 | -0.1536 | 0.1275 | 0.0030 | 0.1275 | 0.0031 |
| P22-10 | -0.0757 | N/A | -0.0097 | -0.0751 | -0.0097 | -0.0733 | 0.0660 | 0.0006 | 0.0660 | 0.0024 |
| P22-21 | 0.0183 | N/A | 0.0338 | 0.0282 | 0.0338 | 0.0215 | 0.0155 | 0.0099 | 0.0155 | 0.0032 |
| P23-15 | -0.0500 | N/A | -0.0388 | -0.0713 | -0.0388 | -0.0751 | 0.0112 | 0.0213 | 0.0112 | 0.0251 |
| P24-22 | -0.0569 | N/A | -0.1018 | -0.0798 | -0.1018 | -0.1045 | 0.0449 | 0.0229 | 0.0449 | 0.0476 |
| P24-23 | -0.0180 | N/A | 0.0248 | 0.0215 | 0.0248 | 0.0064 | 0.0428 | 0.0395 | 0.0428 | 0.0244 |
| P25-24 | 0.0122 | N/A | 0.0678 | 0.0859 | 0.0678 | 0.0798 | 0.0556 | 0.0737 | 0.0556 | 0.0676 |
| P26-25 | -0.0350 | N/A | 0.0644 | 0.0416 | 0.0644 | 0.0622 | 0.0994 | 0.0766 | 0.0994 | 0.0972 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P27-25 | 0.0479 | N/A | -0.0187 | 0.0453 | -0.0187 | -0.0020 | 0.0666 | 0.0026 | 0.0666 | 0.0499 |
| P27-28 | -0.1807 | N/A | -0.1565 | -0.1705 | -0.1565 | -0.1742 | 0.0242 | 0.0102 | 0.0242 | 0.0065 |
| P29-27 | -0.0610 | N/A | -0.0602 | -0.0460 | -0.0602 | -0.0606 | 0.0008 | 0.0150 | 0.0008 | 0.0004 |
| P30-27 | -0.0693 | N/A | -0.0588 | -0.0394 | -0.0588 | -0.0591 | 0.0105 | 0.0299 | 0.0105 | 0.0102 |
| P30-29 | -0.0367 | N/A | -0.0237 | -0.0107 | -0.0237 | -0.0237 | 0.0130 | 0.0260 | 0.0130 | 0.0130 |
| P28-8 | 0.0055 | N/A | 0.0183 | 0.0167 | 0.0183 | 0.0154 | 0.0128 | 0.0112 | 0.0128 | 0.0099 |
| P28-6 | -0.1862 | N/A | -0.1622 | -0.1685 | -0.1622 | -0.1672 | 0.0240 | 0.0177 | 0.0240 | 0.0190 |
| Q1-2 | -0.2470 | N/A | -0.2734 | -0.2641 | -0.2734 | -0.2736 | 0.0264 | 0.0171 | 0.0264 | 0.0266 |
| Q1-3 | 0.0428 | 0.0595 | 0.0172 | 0.0054 | 0.0172 | 0.0171 | 0.0256 | 0.0374 | 0.0256 | 0.0257 |
| Q2-4 | 0.0475 | 0.0671 | 0.0325 | 0.0197 | 0.0325 | 0.0320 | 0.0150 | 0.0278 | 0.0150 | 0.0155 |
| Q3-4 | -0.0385 | -0.0159 | -0.0550 | -0.0463 | -0.0550 | -0.0561 | 0.0165 | 0.0078 | 0.0165 | 0.0176 |
| Q2-5 | 0.0278 | 0.0213 | -0.0022 | 0.0076 | -0.0022 | -0.0024 | 0.0300 | 0.0202 | 0.0300 | 0.0302 |
| Q2-6 | 0.0137 | N/A | 0.0081 | 0.0084 | 0.0081 | 0.0083 | 0.0056 | 0.0053 | 0.0056 | 0.0054 |
| Q4-6 | -0.1591 | -0.1829 | -0.1139 | -0.0617 | -0.1139 | -0.1118 | 0.0452 | 0.0974 | 0.0452 | 0.0473 |
| Q5-7 | 0.1149 | 0.1580 | 0.1657 | 0.1723 | 0.1657 | 0.1654 | 0.0508 | 0.0574 | 0.0508 | 0.0505 |
| Q6-7 | -0.0278 | -0.0959 | -0.0350 | -0.0068 | -0.0350 | -0.0351 | 0.0072 | 0.0210 | 0.0072 | 0.0073 |
| Q6-8 | -0.0720 | N/A | -0.0373 | -0.0444 | -0.0373 | -0.0368 | 0.0347 | 0.0276 | 0.0347 | 0.0352 |
| Q6-9 | -0.0809 | -0.1233 | -0.0490 | -0.0118 | -0.0490 | -0.0518 | 0.0319 | 0.0691 | 0.0319 | 0.0291 |
| Q6-10 | 0.0019 | N/A | -0.0009 | 0.0084 | -0.0009 | -0.0015 | 0.0028 | 0.0065 | 0.0028 | 0.0034 |
| Q9-11 | -0.1560 | N/A | -0.1575 | -0.1266 | -0.1575 | -0.1591 | 0.0015 | 0.0294 | 0.0015 | 0.0031 |
| Q9-10 | 0.0588 | 0.1108 | 0.0760 | 0.0451 | 0.0760 | 0.0753 | 0.0172 | 0.0137 | 0.0172 | 0.0165 |
| Q4-12 | 0.1441 | N/A | 0.0668 | 0.0699 | 0.0668 | 0.0611 | 0.0773 | 0.0742 | 0.0773 | 0.0830 |
| Q12-13 | -0.1032 | N/A | -0.0937 | -0.0937 | -0.0937 | -0.0943 | 0.0095 | 0.0095 | 0.0095 | 0.0089 |
| Q12-14 | 0.0240 | 0.0570 | 0.0115 | -0.0054 | 0.0115 | 0.0100 | 0.0125 | 0.0294 | 0.0125 | 0.0140 |
| Q12-15 | 0.0679 | 0.0645 | 0.0570 | 0.0519 | 0.0570 | 0.0553 | 0.0109 | 0.0160 | 0.0109 | 0.0126 |
| Q12-16 | 0.0335 | N/A | 0.0147 | 0.0302 | 0.0147 | 0.0118 | 0.0188 | 0.0033 | 0.0188 | 0.0217 |
| Q14-15 | 0.0065 | N/A | 0.0007 | 0.0214 | 0.0007 | -0.0006 | 0.0058 | 0.0149 | 0.0058 | 0.0071 |
| Q16-17 | 0.0144 | -0.0008 | -0.0358 | -0.0199 | -0.0358 | -0.0372 | 0.0502 | 0.0343 | 0.0502 | 0.0516 |
| Q15-18 | 0.0160 | 0.0172 | -0.0005 | 0.0172 | -0.0005 | -0.0031 | 0.0165 | 0.0012 | 0.0165 | 0.0191 |
| Q18-19 | 0.0062 | 0.0088 | 0.0079 | 0.0226 | 0.0079 | 0.0062 | 0.0017 | 0.0164 | 0.0017 | 0.0000 |
| Q19-20 | -0.0279 | 0.0134 | -0.0349 | -0.0205 | -0.0349 | -0.0359 | 0.0070 | 0.0074 | 0.0070 | 0.0080 |
| Q10-20 | 0.0371 | 0.1134 | 0.1001 | 0.1134 | 0.1001 | 0.0996 | 0.0630 | 0.0763 | 0.0630 | 0.0625 |
| Q10-17 | 0.0443 | 0.0676 | 0.1323 | 0.1513 | 0.1323 | 0.1312 | 0.0880 | 0.1070 | 0.0880 | 0.0869 |
| Q10-21 | 0.1001 | 0.0896 | 0.0705 | 0.0379 | 0.0705 | 0.0706 | 0.0296 | 0.0622 | 0.0296 | 0.0295 |
| Q10-22 | 0.0460 | N/A | 0.0350 | 0.0166 | 0.0350 | 0.0342 | 0.0110 | 0.0294 | 0.0110 | 0.0118 |
| Q21-22 | -0.0143 | -0.0051 | 0.0018 | -0.0051 | 0.0018 | 0.0035 | 0.0161 | 0.0092 | 0.0161 | 0.0178 |
| Q15-23 | 0.0291 | N/A | -0.0117 | -0.0127 | -0.0117 | -0.0122 | 0.0408 | 0.0418 | 0.0408 | 0.0413 |
| Q22-24 | 0.0306 | 0.0781 | 0.0334 | 0.0252 | 0.0334 | 0.0381 | 0.0028 | 0.0054 | 0.0028 | 0.0075 |
| Q23-24 | 0.0124 | 0.0278 | -0.0181 | -0.0199 | -0.0181 | -0.0180 | 0.0305 | 0.0323 | 0.0305 | 0.0304 |
| Q24-25 | 0.0201 | N/A | 0.0269 | 0.0238 | 0.0269 | 0.0338 | 0.0068 | 0.0037 | 0.0068 | 0.0137 |
| Q25-26 | 0.0237 | N/A | -0.0066 | -0.0111 | -0.0066 | -0.0043 | 0.0303 | 0.0348 | 0.0303 | 0.0280 |
| Q25-27 | -0.0037 | 0.0094 | -0.0120 | -0.0078 | -0.0120 | -0.0106 | 0.0083 | 0.0041 | 0.0083 | 0.0069 |
| Q28-27 | 0.0504 | 0.0033 | 0.0365 | 0.0154 | 0.0365 | 0.0446 | 0.0139 | 0.0350 | 0.0139 | 0.0058 |
| Q27-29 | 0.0167 | N/A | 0.0242 | 0.0151 | 0.0242 | 0.0267 | 0.0075 | 0.0016 | 0.0075 | 0.0100 |
| Q27-30 | 0.0166 | 0.0093 | 0.0318 | 0.0227 | 0.0318 | 0.0336 | 0.0152 | 0.0061 | 0.0152 | 0.0170 |
| Q29-30 | 0.0061 | -0.0205 | 0.0193 | 0.0160 | 0.0193 | 0.0194 | 0.0132 | 0.0099 | 0.0132 | 0.0133 |
| Q8-28 | -0.0054 | N/A | -0.0147 | -0.0153 | -0.0147 | -0.0136 | 0.0093 | 0.0099 | 0.0093 | 0.0082 |
| Q6-28 | 0.0011 | -0.0427 | -0.0062 | -0.0133 | -0.0062 | -0.0017 | 0.0073 | 0.0144 | 0.0073 | 0.0028 |
| Q2-1 | 0.3447 | N/A | 0.3935 | 0.3997 | 0.3935 | 0.3938 | 0.0488 | 0.0550 | 0.0488 | 0.0491 |
| Q3-1 | 0.0265 | N/A | 0.0447 | 0.0619 | 0.0447 | 0.0453 | 0.0182 | 0.0354 | 0.0182 | 0.0188 |
| Q4-2 | -0.0554 | N/A | -0.0493 | -0.0361 | -0.0493 | -0.0486 | 0.0061 | 0.0193 | 0.0061 | 0.0068 |
| Q4-3 | 0.0544 | N/A | 0.0717 | 0.0669 | 0.0717 | 0.0731 | 0.0173 | 0.0125 | 0.0173 | 0.0187 |
| Q5-2 | 0.0517 | N/A | 0.0801 | 0.0715 | 0.0801 | 0.0800 | 0.0284 | 0.0198 | 0.0284 | 0.0283 |
| Q6-2 | 0.0058 | N/A | -0.0020 | -0.0004 | -0.0020 | -0.0015 | 0.0078 | 0.0062 | 0.0078 | 0.0073 |
| Q6-4 | 0.1719 | N/A | 0.1240 | 0.0726 | 0.1240 | 0.1221 | 0.0479 | 0.0993 | 0.0479 | 0.0498 |
| Q7-5 | -0.1313 | N/A | -0.1776 | -0.1830 | -0.1776 | -0.1775 | 0.0463 | 0.0517 | 0.0463 | 0.0462 |
| Q7-6 | 0.0223 | N/A | 0.0296 | -0.0006 | 0.0296 | 0.0293 | 0.0073 | 0.0229 | 0.0073 | 0.0070 |
| Q8-6 | 0.0666 | N/A | 0.0321 | 0.0393 | 0.0321 | 0.0315 | 0.0345 | 0.0273 | 0.0345 | 0.0351 |
| Q9-6 | 0.0972 | N/A | 0.0632 | 0.0284 | 0.0632 | 0.0672 | 0.0340 | 0.0688 | 0.0340 | 0.0300 |
| Q10-6 | 0.0110 | N/A | 0.0125 | 0.0083 | 0.0125 | 0.0152 | 0.0015 | 0.0027 | 0.0015 | 0.0042 |
| Q11-9 | 0.1606 | N/A | 0.1626 | 0.1298 | 0.1626 | 0.1642 | 0.0020 | 0.0308 | 0.0020 | 0.0036 |
| Q10-9 | -0.0508 | N/A | -0.0687 | -0.0319 | -0.0687 | -0.0653 | 0.0179 | 0.0189 | 0.0179 | 0.0145 |
| Q12-4 | -0.0972 | N/A | -0.0271 | -0.0206 | -0.0271 | -0.0197 | 0.0701 | 0.0766 | 0.0701 | 0.0775 |
| Q13-12 | 0.1045 | N/A | 0.0949 | 0.0949 | 0.0949 | 0.0956 | 0.0096 | 0.0096 | 0.0096 | 0.0089 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q14-12 | -0.0225 | N/A | -0.0111 | 0.0070 | -0.0111 | -0.0096 | 0.0114 | 0.0295 | 0.0114 | 0.0129 |
| Q15-12 | -0.0636 | N/A | -0.0533 | -0.0465 | -0.0533 | -0.0511 | 0.0103 | 0.0171 | 0.0103 | 0.0125 |
| Q16-12 | -0.0324 | N/A | -0.0139 | -0.0293 | -0.0139 | -0.0111 | 0.0185 | 0.0031 | 0.0185 | 0.0213 |
| Q15-14 | -0.0064 | N/A | -0.0003 | -0.0211 | -0.0003 | 0.0012 | 0.0061 | 0.0147 | 0.0061 | 0.0076 |
| Q17-16 | -0.0141 | N/A | 0.0361 | 0.0201 | 0.0361 | 0.0374 | 0.0502 | 0.0342 | 0.0502 | 0.0515 |
| Q18-15 | -0.0152 | N/A | 0.0007 | -0.0169 | 0.0007 | 0.0031 | 0.0159 | 0.0017 | 0.0159 | 0.0183 |
| Q19-18 | -0.0061 | N/A | -0.0078 | -0.0224 | -0.0078 | -0.0062 | 0.0017 | 0.0163 | 0.0017 | 0.0001 |
| Q20-19 | 0.0283 | N/A | 0.0355 | 0.0215 | 0.0355 | 0.0365 | 0.0072 | 0.0068 | 0.0072 | 0.0082 |
| Q20-10 | -0.0353 | N/A | -0.0957 | -0.1092 | -0.0957 | -0.0964 | 0.0604 | 0.0739 | 0.0604 | 0.0611 |
| Q17-10 | -0.0439 | N/A | -0.1303 | -0.1492 | -0.1303 | -0.1296 | 0.0864 | 0.1053 | 0.0864 | 0.0857 |
| Q21-10 | -0.0977 | N/A | -0.0701 | -0.0359 | -0.0701 | -0.0685 | 0.0276 | 0.0618 | 0.0276 | 0.0292 |
| Q22-10 | -0.0449 | N/A | -0.0348 | -0.0157 | -0.0348 | -0.0333 | 0.0101 | 0.0292 | 0.0101 | 0.0116 |
| Q22-21 | 0.0143 | N/A | -0.0018 | 0.0051 | -0.0018 | -0.0035 | 0.0161 | 0.0092 | 0.0161 | 0.0178 |
| Q23-15 | -0.0284 | N/A | 0.0120 | 0.0138 | 0.0120 | 0.0134 | 0.0404 | 0.0422 | 0.0404 | 0.0418 |
| Q24-22 | -0.0299 | N/A | -0.0314 | -0.0239 | -0.0314 | -0.0359 | 0.0015 | 0.0060 | 0.0015 | 0.0060 |
| Q24-23 | -0.0123 | N/A | 0.0184 | 0.0202 | 0.0184 | 0.0181 | 0.0307 | 0.0325 | 0.0307 | 0.0304 |
| Q25-24 | -0.0200 | N/A | -0.0252 | -0.0213 | -0.0252 | -0.0314 | 0.0052 | 0.0013 | 0.0052 | 0.0114 |
| Q26-25 | -0.0230 | N/A | 0.0082 | 0.0118 | 0.0082 | 0.0057 | 0.0312 | 0.0348 | 0.0312 | 0.0287 |
| Q27-25 | 0.0042 | N/A | 0.0121 | 0.0082 | 0.0121 | 0.0106 | 0.0079 | 0.0040 | 0.0079 | 0.0064 |
| Q27-28 | -0.0375 | N/A | -0.0266 | -0.0043 | -0.0266 | -0.0323 | 0.0109 | 0.0332 | 0.0109 | 0.0052 |
| Q29-27 | -0.0151 | N/A | -0.0224 | -0.0142 | -0.0224 | -0.0249 | 0.0073 | 0.0009 | 0.0073 | 0.0098 |
| Q30-27 | -0.0136 | N/A | -0.0290 | -0.0214 | -0.0290 | -0.0307 | 0.0154 | 0.0078 | 0.0154 | 0.0171 |
| Q30-29 | -0.0054 | N/A | -0.0188 | -0.0158 | -0.0188 | -0.0189 | 0.0134 | 0.0104 | 0.0134 | 0.0135 |
| Q28-8 | -0.0380 | N/A | -0.0297 | -0.0290 | -0.0297 | -0.0308 | 0.0083 | 0.0090 | 0.0083 | 0.0072 |
| Q28-6 | -0.0123 | N/A | -0.0058 | 0.0014 | -0.0058 | -0.0102 | 0.0065 | 0.0137 | 0.0065 | 0.0021 |
| **SUM** | | | | | | | **7.9108** | **8.3359** | **7.9108** | **7.1024** |

Since all bad data are reverse and sufficient small, it's very hard to detect them, LTS shares the same estimation with WLS by not eliminating any bad data, LTAV successfully detecte two bad data and generate best results in this case.

#### 5.3.3.2 Two non-conforming bad data

$P_2 = -0.1256$, $P_{12-14} = -0.1031$, $P_{10-21} = 0.9297$ and $P_{25-27} = -0.9710$ instead of the original correct value are used to simulate the non-conforming bad data. The following two tables show the results and error with actual value of the system.

Table 5.270: State Variable of case 2, non-conforming, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0999 | -0.1041 | -0.0983 | -0.1000 | **V1** | 1.0697 | 1.0690 | 1.0777 | 1.0679 |
| $\theta 3$ | -0.1283 | -0.1307 | -0.1258 | -0.1276 | **V2** | 1.0544 | 1.0522 | 1.0627 | 1.0526 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 4$ | -0.1601 | -0.1637 | -0.1568 | -0.1587 | **V3** | 1.0347 | 1.0361 | 1.0434 | 1.0335 |
| $\theta 5$ | -0.2507 | -0.2568 | -0.2470 | -0.2520 | **V4** | 1.0257 | 1.0266 | 1.0348 | 1.0250 |
| $\theta 6$ | -0.1890 | -0.1927 | -0.1852 | -0.1876 | **V5** | 1.0256 | 1.0210 | 1.0339 | 1.0235 |
| $\theta 7$ | -0.2185 | -0.2212 | -0.2150 | -0.2189 | **V6** | 1.0226 | 1.0211 | 1.0315 | 1.0217 |
| $\theta 8$ | -0.2006 | -0.2062 | -0.1972 | -0.2006 | **V7** | 1.0160 | 1.0119 | 1.0246 | 1.0142 |
| $\theta 9$ | -0.2583 | -0.2514 | -0.2467 | -0.2428 | **V8** | 1.0212 | 1.0191 | 1.0298 | 1.0195 |
| $\theta 10$ | -0.3049 | -0.2891 | -0.2827 | -0.2749 | **V9** | 1.0289 | 1.0241 | 1.0403 | 1.0329 |
| $\theta 11$ | -0.2564 | -0.2523 | -0.2504 | -0.2471 | **V10** | 1.0199 | 1.0195 | 1.0322 | 1.0252 |
| $\theta 12$ | -0.2783 | -0.2729 | -0.2642 | -0.2609 | **V11** | 1.0606 | 1.0498 | 1.0716 | 1.0649 |
| $\theta 13$ | -0.2779 | -0.2708 | -0.2635 | -0.2606 | **V12** | 1.0115 | 1.0142 | 1.0230 | 1.0141 |
| $\theta 14$ | -0.2887 | -0.2931 | -0.2731 | -0.2695 | **V13** | 1.0240 | 1.0272 | 1.0358 | 1.0270 |
| $\theta 15$ | -0.3041 | -0.2945 | -0.2859 | -0.2802 | **V14** | 1.0040 | 1.0064 | 1.0162 | 1.0068 |
| $\theta 16$ | -0.2915 | -0.2824 | -0.2772 | -0.2722 | **V15** | 0.9894 | 0.9954 | 1.0032 | 0.9955 |
| $\theta 17$ | -0.2990 | -0.2891 | -0.2818 | -0.2751 | **V16** | 1.0031 | 1.0012 | 1.0136 | 1.0058 |
| $\theta 18$ | -0.3020 | -0.2966 | -0.2887 | -0.2842 | **V17** | 1.0094 | 1.0046 | 1.0200 | 1.0127 |
| $\theta 19$ | -0.3031 | -0.2975 | -0.2910 | -0.2865 | **V18** | 0.9925 | 0.9897 | 1.0026 | 0.9943 |
| $\theta 20$ | -0.2995 | -0.2916 | -0.2869 | -0.2819 | **V19** | 0.9916 | 0.9857 | 1.0004 | 0.9922 |
| $\theta 21$ | -0.3359 | -0.3017 | -0.2966 | -0.2843 | **V20** | 0.9968 | 0.9904 | 1.0056 | 0.9976 |
| $\theta 22$ | -0.3368 | -0.3011 | -0.2974 | -0.2838 | **V21** | 0.9992 | 1.0102 | 1.0194 | 1.0146 |
| $\theta 23$ | -0.3471 | -0.3116 | -0.3149 | -0.2983 | **V22** | 0.9987 | 1.0106 | 1.0190 | 1.0147 |
| $\theta 24$ | -0.3686 | -0.3099 | -0.3349 | -0.3000 | **V23** | 0.9730 | 0.9907 | 0.9927 | 0.9900 |
| $\theta 25$ | -0.3641 | -0.2759 | -0.3941 | -0.2714 | **V24** | 0.9719 | 0.9988 | 0.9881 | 0.9955 |
| $\theta 26$ | -0.4639 | -0.2635 | -0.7375 | -0.2611 | **V25** | 0.9706 | 1.0112 | 0.9402 | 0.9996 |
| $\theta 27$ | -0.2675 | -0.2662 | -0.2754 | -0.2625 | **V26** | 0.9206 | 1.0258 | 0.7477 | 1.0105 |
| $\theta 28$ | -0.1975 | -0.2025 | -0.1947 | -0.1969 | **V27** | 1.0274 | 1.0201 | 1.0244 | 1.0071 |
| $\theta 29$ | -0.2292 | -0.2795 | -0.2290 | -0.2747 | **V28** | 1.0219 | 1.0193 | 1.0295 | 1.0190 |
| $\theta 30$ | -0.2331 | -0.2838 | -0.2324 | -0.2810 | **V29** | 1.0511 | 1.0092 | 1.0618 | 0.9883 |
| | | | | | **V30** | 1.0392 | 1.0018 | 1.0507 | 0.9738 |

Table 5.271: Estimation Results and Errors of case 2, non-conforming, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0697 | 1.0690 | 1.0777 | 1.0679 | 0.0097 | 0.0090 | 0.0177 | 0.0079 |
| V2 | 1.0450 | N/A | 1.0544 | 1.0522 | 1.0627 | 1.0526 | 0.0094 | 0.0072 | 0.0177 | 0.0076 |
| V3 | 1.0210 | N/A | 1.0347 | 1.0361 | 1.0434 | 1.0335 | 0.0137 | 0.0151 | 0.0224 | 0.0125 |
| V4 | 1.0120 | N/A | 1.0257 | 1.0266 | 1.0348 | 1.0250 | 0.0137 | 0.0146 | 0.0228 | 0.0130 |
| V5 | 1.0100 | N/A | 1.0256 | 1.0210 | 1.0339 | 1.0235 | 0.0156 | 0.0110 | 0.0239 | 0.0135 |
| V6 | 1.0110 | N/A | 1.0226 | 1.0211 | 1.0315 | 1.0217 | 0.0116 | 0.0101 | 0.0205 | 0.0107 |
| V7 | 1.0030 | N/A | 1.0160 | 1.0119 | 1.0246 | 1.0142 | 0.0130 | 0.0089 | 0.0216 | 0.0112 |
| V8 | 1.0100 | N/A | 1.0212 | 1.0191 | 1.0298 | 1.0195 | 0.0112 | 0.0091 | 0.0198 | 0.0095 |
| V9 | 1.0510 | N/A | 1.0289 | 1.0241 | 1.0403 | 1.0329 | 0.0221 | 0.0269 | 0.0107 | 0.0181 |
| V10 | 1.0450 | N/A | 1.0199 | 1.0195 | 1.0322 | 1.0252 | 0.0251 | 0.0255 | 0.0128 | 0.0198 |
| V11 | 1.0820 | N/A | 1.0606 | 1.0498 | 1.0716 | 1.0649 | 0.0214 | 0.0322 | 0.0104 | 0.0171 |
| V12 | 1.0570 | N/A | 1.0115 | 1.0142 | 1.0230 | 1.0141 | 0.0455 | 0.0428 | 0.0340 | 0.0429 |
| V13 | 1.0710 | N/A | 1.0240 | 1.0272 | 1.0358 | 1.0270 | 0.0470 | 0.0438 | 0.0352 | 0.0440 |
| V14 | 1.0430 | N/A | 1.0040 | 1.0064 | 1.0162 | 1.0068 | 0.0390 | 0.0366 | 0.0268 | 0.0362 |
| V15 | 1.0380 | N/A | 0.9894 | 0.9954 | 1.0032 | 0.9955 | 0.0486 | 0.0426 | 0.0348 | 0.0425 |
| V16 | 1.0450 | N/A | 1.0031 | 1.0012 | 1.0136 | 1.0058 | 0.0419 | 0.0438 | 0.0314 | 0.0392 |
| V17 | 1.0400 | N/A | 1.0094 | 1.0046 | 1.0200 | 1.0127 | 0.0306 | 0.0354 | 0.0200 | 0.0273 |
| V18 | 1.0280 | N/A | 0.9925 | 0.9897 | 1.0026 | 0.9943 | 0.0355 | 0.0383 | 0.0254 | 0.0337 |
| V19 | 1.0260 | N/A | 0.9916 | 0.9857 | 1.0004 | 0.9922 | 0.0344 | 0.0403 | 0.0256 | 0.0338 |
| V20 | 1.0300 | N/A | 0.9968 | 0.9904 | 1.0056 | 0.9976 | 0.0332 | 0.0396 | 0.0244 | 0.0324 |
| V21 | 1.0330 | N/A | 0.9992 | 1.0102 | 1.0194 | 1.0146 | 0.0338 | 0.0228 | 0.0136 | 0.0184 |
| V22 | 1.0340 | N/A | 0.9987 | 1.0106 | 1.0190 | 1.0147 | 0.0353 | 0.0234 | 0.0150 | 0.0193 |
| V23 | 1.0270 | N/A | 0.9730 | 0.9907 | 0.9927 | 0.9900 | 0.0540 | 0.0363 | 0.0343 | 0.0370 |
| V24 | 1.0220 | N/A | 0.9719 | 0.9988 | 0.9881 | 0.9955 | 0.0501 | 0.0232 | 0.0339 | 0.0265 |
| V25 | 1.0180 | N/A | 0.9706 | 1.0112 | 0.9402 | 0.9996 | 0.0474 | 0.0068 | 0.0778 | 0.0184 |
| V26 | 1.0000 | N/A | 0.9206 | 1.0258 | 0.7477 | 1.0105 | 0.0794 | 0.0258 | 0.2523 | 0.0105 |
| V27 | 1.0240 | N/A | 1.0274 | 1.0201 | 1.0244 | 1.0071 | 0.0034 | 0.0039 | 0.0004 | 0.0169 |
| V28 | 1.0070 | N/A | 1.0219 | 1.0193 | 1.0295 | 1.0190 | 0.0149 | 0.0123 | 0.0225 | 0.0120 |
| V29 | 1.0040 | N/A | 1.0511 | 1.0092 | 1.0618 | 0.9883 | 0.0471 | 0.0052 | 0.0578 | 0.0157 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V30 | 0.9920 | N/A | 1.0392 | 1.0018 | 1.0507 | 0.9738 | 0.0472 | 0.0098 | 0.0587 | 0.0182 |
| P1 | 2.6096 | 2.6540 | 2.7438 | 2.8361 | 2.7379 | 2.7322 | 0.1342 | 0.2265 | 0.1283 | 0.1226 |
| **P2** | **0.1830** | **-0.1256** | **-0.0422** | **-0.1256** | **-0.0506** | **-0.0592** | **0.2252** | **0.3086** | **0.2336** | **0.2422** |
| P3 | -0.0240 | -0.0774 | 0.0348 | 0.0604 | 0.0281 | 0.0225 | 0.0588 | 0.0844 | 0.0521 | 0.0465 |
| P4 | -0.0760 | -0.1165 | -0.0749 | -0.1165 | -0.0909 | -0.1064 | 0.0011 | 0.0405 | 0.0149 | 0.0304 |
| P5 | -0.9420 | -1.0892 | -1.0139 | -1.0462 | -1.0192 | -1.0242 | 0.0719 | 0.1042 | 0.0772 | 0.0822 |
| P6 | 0.0000 | 0.0719 | 0.1189 | 0.0779 | 0.1040 | 0.0842 | 0.1189 | 0.0779 | 0.1040 | 0.0842 |
| P7 | -0.2280 | -0.2117 | -0.1381 | -0.1075 | -0.1445 | -0.1525 | 0.0899 | 0.1205 | 0.0835 | 0.0755 |
| P8 | -0.3000 | -0.3377 | -0.2915 | -0.3377 | -0.3034 | -0.3267 | 0.0085 | 0.0377 | 0.0034 | 0.0267 |
| P9 | 0.0000 | 0.0685 | 0.0838 | 0.0685 | 0.0538 | 0.0510 | 0.0838 | 0.0685 | 0.0538 | 0.0510 |
| P10 | -0.0580 | -0.1436 | 0.0280 | -0.1338 | -0.0847 | -0.1007 | 0.0860 | 0.0758 | 0.0267 | 0.0427 |
| P11 | 0.0000 | -0.0051 | 0.0100 | -0.0051 | -0.0196 | -0.0223 | 0.0100 | 0.0051 | 0.0196 | 0.0223 |
| P12 | -0.1120 | -0.1241 | -0.1367 | -0.1241 | -0.1349 | -0.1372 | 0.0247 | 0.0121 | 0.0229 | 0.0252 |
| P13 | 0.0000 | 0.0160 | 0.0034 | 0.0160 | 0.0052 | 0.0029 | 0.0034 | 0.0160 | 0.0052 | 0.0029 |
| P14 | -0.0620 | -0.0464 | 0.0263 | -0.0464 | 0.0231 | 0.0138 | 0.0883 | 0.0156 | 0.0851 | 0.0758 |
| P15 | -0.0820 | -0.1252 | -0.1081 | -0.1252 | -0.1115 | -0.1286 | 0.0261 | 0.0432 | 0.0295 | 0.0466 |
| P16 | -0.0350 | -0.0365 | -0.0431 | -0.0365 | -0.0591 | -0.0575 | 0.0081 | 0.0015 | 0.0241 | 0.0225 |
| P17 | -0.0900 | -0.0982 | -0.0076 | -0.0867 | -0.0532 | -0.0573 | 0.0824 | 0.0033 | 0.0368 | 0.0327 |
| P18 | -0.0320 | -0.0006 | 0.0225 | -0.0006 | 0.0102 | 0.0038 | 0.0545 | 0.0314 | 0.0422 | 0.0358 |
| P19 | -0.0950 | -0.0403 | -0.0814 | -0.1122 | -0.1013 | -0.1056 | 0.0136 | 0.0172 | 0.0063 | 0.0106 |
| P20 | -0.0220 | 0.0335 | 0.0532 | 0.0335 | 0.0154 | 0.0077 | 0.0752 | 0.0555 | 0.0374 | 0.0297 |
| P21 | -0.1750 | -0.2182 | -0.4084 | -0.2182 | -0.1933 | -0.1784 | 0.2334 | 0.0432 | 0.0183 | 0.0034 |
| P22 | 0.0000 | 0.0039 | -0.0788 | 0.0039 | 0.0797 | 0.0539 | 0.0788 | 0.0039 | 0.0797 | 0.0539 |
| P23 | -0.0320 | -0.0927 | -0.1324 | -0.0927 | -0.0687 | -0.0840 | 0.1004 | 0.0607 | 0.0367 | 0.0520 |
| P24 | -0.0870 | -0.1427 | -0.2565 | -0.1427 | -0.1001 | -0.1801 | 0.1695 | 0.0557 | 0.0131 | 0.0931 |
| P25 | 0.0000 | -0.0003 | -0.2270 | -0.0003 | -0.0540 | -0.0097 | 0.2270 | 0.0003 | 0.0540 | 0.0097 |
| P26 | -0.0350 | 0.0416 | -0.2123 | 0.0416 | -0.5552 | 0.0324 | 0.1773 | 0.0766 | 0.5202 | 0.0674 |
| P27 | 0.0000 | -0.0385 | 0.1439 | -0.0385 | 0.1873 | -0.0322 | 0.1439 | 0.0385 | 0.1873 | 0.0322 |
| P28 | 0.0000 | 0.0186 | 0.0600 | 0.0186 | 0.0635 | 0.0240 | 0.0600 | 0.0186 | 0.0635 | 0.0240 |
| P29 | -0.0240 | -0.0353 | 0.1219 | -0.0223 | 0.1531 | -0.0180 | 0.1459 | 0.0017 | 0.1771 | 0.0060 |
| P30 | -0.1060 | -0.0501 | 0.0379 | -0.0501 | 0.0624 | -0.0689 | 0.1439 | 0.0559 | 0.1684 | 0.0371 |
| Q1 | -0.2042 | -0.2587 | -0.2530 | -0.2587 | -0.2604 | -0.2560 | 0.0488 | 0.0545 | 0.0562 | 0.0518 |
| Q2 | 0.4337 | 0.4353 | 0.4364 | 0.4353 | 0.4310 | 0.4322 | 0.0027 | 0.0016 | 0.0027 | 0.0015 |
| Q3 | -0.0120 | 0.0156 | -0.0063 | 0.0156 | -0.0103 | -0.0102 | 0.0057 | 0.0276 | 0.0017 | 0.0018 |
| Q4 | -0.0160 | 0.0390 | -0.0156 | 0.0390 | -0.0225 | -0.0247 | 0.0004 | 0.0550 | 0.0065 | 0.0087 |
| Q5 | 0.1666 | 0.2438 | 0.2465 | 0.2438 | 0.2440 | 0.2456 | 0.0799 | 0.0772 | 0.0774 | 0.0790 |
| Q6 | 0.0000 | 0.0043 | 0.0001 | 0.0043 | -0.0031 | -0.0057 | 0.0001 | 0.0043 | 0.0031 | 0.0057 |
| Q7 | -0.1090 | -0.1836 | -0.1462 | -0.1836 | -0.1481 | -0.1480 | 0.0372 | 0.0746 | 0.0391 | 0.0390 |
| Q8 | 0.0611 | 0.0240 | 0.0211 | 0.0240 | 0.0200 | 0.0179 | 0.0400 | 0.0371 | 0.0411 | 0.0432 |
| Q9 | 0.0000 | -0.0531 | -0.0190 | -0.0531 | -0.0199 | -0.0173 | 0.0190 | 0.0531 | 0.0199 | 0.0173 |
| Q10 | -0.0200 | 0.0975 | 0.0851 | 0.0975 | 0.0798 | 0.0843 | 0.1051 | 0.1175 | 0.0998 | 0.1043 |
| Q11 | 0.1606 | 0.1298 | 0.1619 | 0.1298 | 0.1611 | 0.1635 | 0.0013 | 0.0308 | 0.0005 | 0.0029 |
| Q12 | -0.0750 | -0.0376 | -0.0415 | -0.0376 | -0.0383 | -0.0376 | 0.0335 | 0.0374 | 0.0367 | 0.0374 |
| Q13 | 0.1045 | 0.0949 | 0.0911 | 0.0949 | 0.0942 | 0.0949 | 0.0134 | 0.0096 | 0.0103 | 0.0096 |
| Q14 | -0.0160 | 0.0284 | -0.0127 | 0.0284 | -0.0101 | -0.0106 | 0.0033 | 0.0444 | 0.0059 | 0.0054 |
| Q15 | -0.0250 | -0.0632 | -0.0662 | -0.0632 | -0.0640 | -0.0654 | 0.0412 | 0.0382 | 0.0390 | 0.0404 |
| Q16 | -0.0180 | -0.0881 | -0.0490 | -0.0599 | -0.0499 | -0.0487 | 0.0310 | 0.0419 | 0.0319 | 0.0307 |
| Q17 | -0.0580 | -0.1291 | -0.0913 | -0.1291 | -0.0948 | -0.0927 | 0.0333 | 0.0711 | 0.0368 | 0.0347 |
| Q18 | -0.0090 | 0.0154 | 0.0098 | 0.0050 | 0.0092 | 0.0092 | 0.0188 | 0.0140 | 0.0182 | 0.0182 |
| Q19 | -0.0340 | -0.0429 | -0.0419 | -0.0429 | -0.0428 | -0.0423 | 0.0079 | 0.0089 | 0.0088 | 0.0083 |
| Q20 | -0.0070 | -0.0168 | -0.0617 | -0.0884 | -0.0618 | -0.0604 | 0.0547 | 0.0814 | 0.0548 | 0.0534 |
| Q21 | -0.1120 | -0.0410 | -0.0597 | -0.0410 | -0.0667 | -0.0658 | 0.0523 | 0.0710 | 0.0453 | 0.0462 |
| Q22 | 0.0000 | 0.0146 | 0.0026 | 0.0146 | -0.0013 | 0.0001 | 0.0026 | 0.0146 | 0.0013 | 0.0001 |
| Q23 | -0.0160 | -0.0061 | -0.0045 | -0.0061 | 0.0027 | -0.0049 | 0.0115 | 0.0099 | 0.0187 | 0.0111 |
| Q24 | -0.0670 | 0.0124 | -0.0280 | -0.0275 | -0.0084 | -0.0281 | 0.0390 | 0.0395 | 0.0586 | 0.0389 |
| Q25 | 0.0000 | -0.0402 | -0.0208 | -0.0402 | 0.0402 | -0.0446 | 0.0208 | 0.0402 | 0.0402 | 0.0446 |
| Q26 | -0.0230 | 0.0118 | 0.0327 | 0.0118 | 0.1010 | 0.0075 | 0.0557 | 0.0348 | 0.1240 | 0.0305 |
| Q27 | 0.0000 | 0.0418 | 0.0757 | 0.0418 | 0.0995 | 0.0412 | 0.0757 | 0.0418 | 0.0995 | 0.0412 |
| Q28 | 0.0000 | -0.0122 | -0.0083 | -0.0122 | 0.0003 | 0.0012 | 0.0083 | 0.0122 | 0.0003 | 0.0012 |
| Q29 | -0.0090 | 0.0018 | 0.0249 | 0.0018 | 0.0436 | -0.0035 | 0.0339 | 0.0108 | 0.0526 | 0.0055 |
| Q30 | -0.0190 | -0.0773 | -0.0262 | -0.0199 | -0.0115 | -0.0481 | 0.0072 | 0.0009 | 0.0075 | 0.0291 |
| P1-2 | 1.7331 | N/A | 1.8742 | 1.9544 | 1.8723 | 1.8711 | 0.1411 | 0.2213 | 0.1392 | 0.1380 |
| P1-3 | 0.8765 | 0.8817 | 0.8695 | 0.8817 | 0.8655 | 0.8611 | 0.0070 | 0.0052 | 0.0110 | 0.0154 |
| P2-4 | 0.4365 | 0.4726 | 0.3936 | 0.3837 | 0.3879 | 0.3818 | 0.0429 | 0.0528 | 0.0486 | 0.0547 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P3-4 | 0.8214 | 0.9507 | 0.8744 | 0.9113 | 0.8644 | 0.8541 | 0.0530 | 0.0899 | 0.0430 | 0.0327 |
| P2-5 | 0.8236 | 0.7903 | 0.8238 | 0.8312 | 0.8253 | 0.8271 | 0.0002 | 0.0076 | 0.0017 | 0.0035 |
| P2-6 | 0.6038 | N/A | 0.5547 | 0.5488 | 0.5497 | 0.5431 | 0.0491 | 0.0550 | 0.0541 | 0.0607 |
| P4-6 | 0.7213 | 0.7172 | 0.6977 | 0.7172 | 0.7013 | 0.6986 | 0.0236 | 0.0041 | 0.0200 | 0.0227 |
| P5-7 | -0.1478 | -0.2445 | -0.2189 | -0.2445 | -0.2225 | -0.2263 | 0.0711 | 0.0967 | 0.0747 | 0.0785 |
| P6-7 | 0.3813 | 0.3594 | 0.3639 | 0.3594 | 0.3740 | 0.3863 | 0.0174 | 0.0219 | 0.0073 | 0.0050 |
| P6-8 | 0.2956 | N/A | 0.2768 | 0.3215 | 0.2923 | 0.3116 | 0.0188 | 0.0259 | 0.0033 | 0.0160 |
| P6-9 | 0.2772 | 0.3192 | 0.3505 | 0.2947 | 0.3171 | 0.2802 | 0.0733 | 0.0175 | 0.0399 | 0.0030 |
| P6-10 | 0.1584 | N/A | 0.2170 | 0.1802 | 0.1864 | 0.1643 | 0.0586 | 0.0218 | 0.0280 | 0.0059 |
| P9-11 | 0.0000 | N/A | -0.0100 | 0.0051 | 0.0196 | 0.0223 | 0.0100 | 0.0051 | 0.0196 | 0.0223 |
| P9-10 | 0.2772 | 0.2500 | 0.4444 | 0.3581 | 0.3514 | 0.3088 | 0.1672 | 0.0809 | 0.0742 | 0.0316 |
| P4-12 | 0.4419 | N/A | 0.4778 | 0.4434 | 0.4434 | 0.4142 | 0.0359 | 0.0015 | 0.0015 | 0.0277 |
| P12-13 | 0.0000 | N/A | -0.0034 | -0.0160 | -0.0052 | -0.0029 | 0.0034 | 0.0160 | 0.0052 | 0.0029 |
| **P12-14** | **0.0786** | **-0.1031** | **0.0451** | **0.0777** | **0.0399** | **0.0391** | **0.0335** | **0.0009** | **0.0387** | **0.0395** |
| P12-15 | 0.1789 | 0.2159 | 0.2274 | 0.1924 | 0.1995 | 0.1775 | 0.0485 | 0.0135 | 0.0206 | 0.0014 |
| P12-16 | 0.0724 | N/A | 0.0720 | 0.0652 | 0.0741 | 0.0633 | 0.0004 | 0.0072 | 0.0017 | 0.0091 |
| P14-15 | 0.0158 | N/A | 0.0711 | 0.0306 | 0.0628 | 0.0526 | 0.0553 | 0.0148 | 0.0470 | 0.0368 |
| P16-17 | 0.0369 | -0.0700 | 0.0284 | 0.0282 | 0.0145 | 0.0054 | 0.0085 | 0.0087 | 0.0224 | 0.0315 |
| P15-18 | 0.0602 | 0.0182 | -0.0130 | 0.0182 | 0.0112 | 0.0168 | 0.0732 | 0.0420 | 0.0490 | 0.0434 |
| P18-19 | 0.0278 | 0.0955 | 0.0094 | 0.0175 | 0.0214 | 0.0206 | 0.0184 | 0.0103 | 0.0064 | 0.0072 |
| P19-20 | -0.0673 | -0.1209 | -0.0720 | -0.0947 | -0.0799 | -0.0851 | 0.0047 | 0.0274 | 0.0126 | 0.0178 |
| P10-20 | 0.0903 | 0.1383 | 0.0200 | 0.0631 | 0.0661 | 0.0791 | 0.0703 | 0.0272 | 0.0242 | 0.0112 |
| P10-17 | 0.0533 | 0.0595 | -0.0201 | 0.0595 | 0.0394 | 0.0526 | 0.0734 | 0.0062 | 0.0139 | 0.0007 |
| **P10-21** | **0.1579** | **0.9297** | **0.4561** | **0.1913** | **0.2285** | **0.1623** | **0.2982** | **0.0334** | **0.0706** | **0.0044** |
| P10-22 | 0.0762 | N/A | 0.2333 | 0.0907 | 0.1192 | 0.0783 | 0.1571 | 0.0145 | 0.0430 | 0.0021 |
| P21-22 | -0.0183 | -0.0282 | 0.0406 | -0.0282 | 0.0333 | -0.0171 | 0.0589 | 0.0099 | 0.0516 | 0.0012 |
| P15-23 | 0.0504 | N/A | 0.1988 | 0.0767 | 0.1361 | 0.0819 | 0.1484 | 0.0263 | 0.0857 | 0.0315 |
| P22-24 | 0.0574 | 0.2028 | 0.1912 | 0.0657 | 0.2311 | 0.1146 | 0.1338 | 0.0083 | 0.1737 | 0.0572 |
| P23-24 | 0.0180 | 0.0593 | 0.0623 | -0.0166 | 0.0656 | -0.0028 | 0.0443 | 0.0346 | 0.0476 | 0.0208 |
| P24-25 | -0.0121 | N/A | -0.0079 | -0.0942 | 0.1900 | -0.0700 | 0.0042 | 0.0821 | 0.2021 | 0.0579 |
| P25-26 | 0.0354 | N/A | 0.2262 | -0.0411 | 0.7001 | -0.0322 | 0.1908 | 0.0765 | 0.6647 | 0.0676 |
| **P25-27** | **-0.0476** | **-0.9710** | **-0.4612** | **-0.0551** | **-0.5714** | **-0.0486** | **0.4136** | **0.0075** | **0.5238** | **0.0010** |
| P28-27 | 0.1807 | 0.1671 | 0.1854 | 0.1671 | 0.2150 | 0.1697 | 0.0047 | 0.0136 | 0.0343 | 0.0110 |
| P27-29 | 0.0619 | N/A | -0.1011 | 0.0369 | -0.1320 | 0.0419 | 0.1630 | 0.0250 | 0.1939 | 0.0200 |
| P27-30 | 0.0709 | 0.0570 | -0.0554 | 0.0363 | -0.0778 | 0.0467 | 0.1263 | 0.0346 | 0.1487 | 0.0242 |
| P29-30 | 0.0370 | 0.0721 | 0.0187 | 0.0144 | 0.0174 | 0.0234 | 0.0183 | 0.0226 | 0.0196 | 0.0136 |
| P8-28 | -0.0054 | N/A | -0.0157 | -0.0174 | -0.0120 | -0.0163 | 0.0103 | 0.0120 | 0.0066 | 0.0109 |
| P6-28 | 0.1867 | 0.1690 | 0.1413 | 0.1664 | 0.1639 | 0.1625 | 0.0454 | 0.0203 | 0.0228 | 0.0242 |
| P2-1 | -1.6809 | N/A | -1.8143 | -1.8893 | -1.8134 | -1.8112 | 0.1334 | 0.2084 | 0.1325 | 0.1303 |
| P3-1 | -0.8454 | N/A | -0.8396 | -0.8509 | -0.8363 | -0.8316 | 0.0058 | 0.0055 | 0.0091 | 0.0138 |
| P4-2 | -0.4263 | N/A | -0.3855 | -0.3760 | -0.3801 | -0.3742 | 0.0408 | 0.0503 | 0.0462 | 0.0521 |
| P4-3 | -0.8129 | N/A | -0.8650 | -0.9011 | -0.8554 | -0.8450 | 0.0521 | 0.0882 | 0.0425 | 0.0321 |
| P5-2 | -0.7942 | N/A | -0.7950 | -0.8017 | -0.7968 | -0.7979 | 0.0008 | 0.0075 | 0.0026 | 0.0037 |
| P6-2 | -0.5843 | N/A | -0.5386 | -0.5329 | -0.5341 | -0.5276 | 0.0457 | 0.0514 | 0.0502 | 0.0567 |
| P6-4 | -0.7150 | N/A | -0.6921 | -0.7114 | -0.6957 | -0.6929 | 0.0229 | 0.0036 | 0.0193 | 0.0221 |
| P7-5 | 0.1495 | N/A | 0.2224 | 0.2486 | 0.2259 | 0.2299 | 0.0729 | 0.0991 | 0.0764 | 0.0804 |
| P7-6 | -0.3775 | N/A | -0.3605 | -0.3561 | -0.3704 | -0.3824 | 0.0170 | 0.0214 | 0.0071 | 0.0049 |
| P8-6 | -0.2946 | N/A | -0.2759 | -0.3203 | -0.2913 | -0.3104 | 0.0187 | 0.0257 | 0.0033 | 0.0158 |
| P9-6 | -0.2772 | N/A | -0.3505 | -0.2947 | -0.3171 | -0.2802 | 0.0733 | 0.0175 | 0.0399 | 0.0030 |
| P10-6 | -0.1584 | N/A | -0.2170 | -0.1802 | -0.1864 | -0.1643 | 0.0586 | 0.0218 | 0.0280 | 0.0059 |
| P11-9 | 0.0000 | N/A | 0.0100 | -0.0051 | -0.0196 | -0.0223 | 0.0100 | 0.0051 | 0.0196 | 0.0223 |
| P10-9 | -0.2772 | N/A | -0.4444 | -0.3581 | -0.3514 | -0.3088 | 0.1672 | 0.0809 | 0.0742 | 0.0316 |
| P12-4 | -0.4419 | N/A | -0.4778 | -0.4434 | -0.4434 | -0.4142 | 0.0359 | 0.0015 | 0.0015 | 0.0277 |
| P13-12 | 0.0000 | N/A | 0.0034 | 0.0160 | 0.0052 | 0.0029 | 0.0034 | 0.0160 | 0.0052 | 0.0029 |
| P14-12 | -0.0778 | N/A | -0.0448 | -0.0770 | -0.0398 | -0.0389 | 0.0330 | 0.0008 | 0.0380 | 0.0389 |
| P15-12 | -0.1767 | N/A | -0.2238 | -0.1898 | -0.1968 | -0.1752 | 0.0471 | 0.0131 | 0.0201 | 0.0015 |
| P16-12 | -0.0719 | N/A | -0.0715 | -0.0647 | -0.0736 | -0.0630 | 0.0004 | 0.0072 | 0.0017 | 0.0089 |
| P15-14 | -0.0158 | N/A | -0.0700 | -0.0303 | -0.0620 | -0.0520 | 0.0542 | 0.0145 | 0.0462 | 0.0362 |
| P17-16 | -0.0368 | N/A | -0.0283 | -0.0281 | -0.0144 | -0.0053 | 0.0085 | 0.0087 | 0.0224 | 0.0315 |
| P18-15 | -0.0598 | N/A | 0.0130 | -0.0181 | -0.0112 | -0.0168 | 0.0728 | 0.0417 | 0.0486 | 0.0430 |
| P19-18 | -0.0277 | N/A | -0.0094 | -0.0175 | -0.0214 | -0.0205 | 0.0183 | 0.0102 | 0.0063 | 0.0072 |
| P20-19 | 0.0674 | N/A | 0.0723 | 0.0951 | 0.0802 | 0.0853 | 0.0049 | 0.0277 | 0.0128 | 0.0179 |
| P20-10 | -0.0894 | N/A | -0.0190 | -0.0616 | -0.0648 | -0.0777 | 0.0704 | 0.0278 | 0.0246 | 0.0117 |
| P17-10 | -0.0532 | N/A | 0.0207 | -0.0586 | -0.0388 | -0.0520 | 0.0739 | 0.0054 | 0.0144 | 0.0012 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P21-10 | -0.1567 | N/A | -0.4490 | -0.1900 | -0.2266 | -0.1613 | 0.2923 | 0.0333 | 0.0699 | 0.0046 |
| P22-10 | -0.0757 | N/A | -0.2294 | -0.0901 | -0.1181 | -0.0778 | 0.1537 | 0.0144 | 0.0424 | 0.0021 |
| P22-21 | 0.0183 | N/A | -0.0406 | 0.0282 | -0.0333 | 0.0171 | 0.0589 | 0.0099 | 0.0516 | 0.0012 |
| P23-15 | -0.0500 | N/A | -0.1947 | -0.0761 | -0.1342 | -0.0812 | 0.1447 | 0.0261 | 0.0842 | 0.0312 |
| P24-22 | -0.0569 | N/A | -0.1869 | -0.0652 | -0.2251 | -0.1130 | 0.1300 | 0.0083 | 0.1682 | 0.0561 |
| P24-23 | -0.0180 | N/A | -0.0617 | 0.0167 | -0.0650 | 0.0029 | 0.0437 | 0.0347 | 0.0470 | 0.0209 |
| P25-24 | 0.0122 | N/A | 0.0080 | 0.0959 | -0.1827 | 0.0710 | 0.0042 | 0.0837 | 0.1949 | 0.0588 |
| P26-25 | -0.0350 | N/A | -0.2123 | 0.0416 | -0.5552 | 0.0324 | 0.1773 | 0.0766 | 0.5202 | 0.0674 |
| P27-25 | 0.0479 | N/A | 0.4858 | 0.0554 | 0.6120 | 0.0489 | 0.4379 | 0.0075 | 0.5641 | 0.0010 |
| P27-28 | -0.1807 | N/A | -0.1854 | -0.1671 | -0.2150 | -0.1697 | 0.0047 | 0.0136 | 0.0343 | 0.0110 |
| P29-27 | -0.0610 | N/A | 0.1033 | -0.0366 | 0.1358 | -0.0414 | 0.1643 | 0.0244 | 0.1968 | 0.0196 |
| P30-27 | -0.0693 | N/A | 0.0564 | -0.0358 | 0.0796 | -0.0457 | 0.1257 | 0.0335 | 0.1489 | 0.0236 |
| P30-29 | -0.0367 | N/A | -0.0185 | -0.0143 | -0.0172 | -0.0232 | 0.0182 | 0.0224 | 0.0195 | 0.0135 |
| P28-8 | 0.0055 | N/A | 0.0157 | 0.0174 | 0.0120 | 0.0163 | 0.0102 | 0.0119 | 0.0065 | 0.0108 |
| P28-6 | -0.1862 | N/A | -0.1410 | -0.1659 | -0.1635 | -0.1621 | 0.0452 | 0.0203 | 0.0227 | 0.0241 |
| Q1-2 | -0.2470 | N/A | -0.2737 | -0.2643 | -0.2775 | -0.2736 | 0.0267 | 0.0173 | 0.0305 | 0.0266 |
| Q1-3 | 0.0428 | 0.0595 | 0.0207 | 0.0056 | 0.0171 | 0.0175 | 0.0221 | 0.0372 | 0.0257 | 0.0253 |
| Q2-4 | 0.0475 | 0.0671 | 0.0360 | 0.0201 | 0.0334 | 0.0325 | 0.0115 | 0.0274 | 0.0141 | 0.0150 |
| Q3-4 | -0.0385 | -0.0159 | -0.0498 | -0.0461 | -0.0541 | -0.0552 | 0.0113 | 0.0076 | 0.0156 | 0.0167 |
| Q2-5 | 0.0278 | 0.0213 | -0.0045 | 0.0074 | -0.0048 | -0.0026 | 0.0323 | 0.0204 | 0.0326 | 0.0304 |
| Q2-6 | 0.0137 | N/A | 0.0111 | 0.0080 | 0.0089 | 0.0085 | 0.0026 | 0.0057 | 0.0048 | 0.0052 |
| Q4-6 | -0.1591 | -0.1829 | -0.1174 | -0.0646 | -0.1148 | -0.1130 | 0.0417 | 0.0945 | 0.0443 | 0.0461 |
| Q5-7 | 0.1149 | 0.1580 | 0.1662 | 0.1722 | 0.1655 | 0.1655 | 0.0513 | 0.0573 | 0.0506 | 0.0506 |
| Q6-7 | -0.0278 | -0.0959 | -0.0398 | -0.0067 | -0.0373 | -0.0355 | 0.0120 | 0.0211 | 0.0095 | 0.0077 |
| Q6-8 | -0.0720 | N/A | -0.0485 | -0.0468 | -0.0438 | -0.0381 | 0.0235 | 0.0252 | 0.0282 | 0.0339 |
| Q6-9 | -0.0809 | -0.1233 | -0.0190 | -0.0062 | -0.0337 | -0.0476 | 0.0619 | 0.0747 | 0.0472 | 0.0333 |
| Q6-10 | 0.0019 | N/A | 0.0175 | 0.0117 | 0.0078 | 0.0006 | 0.0156 | 0.0098 | 0.0059 | 0.0013 |
| Q9-11 | -0.1560 | N/A | -0.1570 | -0.1266 | -0.1564 | -0.1585 | 0.0010 | 0.0294 | 0.0004 | 0.0025 |
| Q9-10 | 0.0588 | 0.1108 | 0.0946 | 0.0500 | 0.0829 | 0.0774 | 0.0358 | 0.0088 | 0.0241 | 0.0186 |
| Q4-12 | 0.1441 | N/A | 0.0849 | 0.0736 | 0.0714 | 0.0649 | 0.0592 | 0.0705 | 0.0727 | 0.0792 |
| Q12-13 | -0.1032 | N/A | -0.0900 | -0.0937 | -0.0931 | -0.0937 | 0.0132 | 0.0095 | 0.0101 | 0.0095 |
| Q12-14 | 0.0240 | 0.0570 | 0.0083 | -0.0055 | 0.0080 | 0.0101 | 0.0157 | 0.0295 | 0.0160 | 0.0139 |
| Q12-15 | 0.0679 | 0.0645 | 0.0586 | 0.0503 | 0.0564 | 0.0558 | 0.0093 | 0.0176 | 0.0115 | 0.0121 |
| Q12-16 | 0.0335 | N/A | 0.0093 | 0.0358 | 0.0135 | 0.0122 | 0.0242 | 0.0023 | 0.0200 | 0.0213 |
| Q14-15 | 0.0065 | N/A | -0.0049 | 0.0214 | -0.0025 | -0.0009 | 0.0114 | 0.0149 | 0.0090 | 0.0074 |
| Q16-17 | 0.0144 | -0.0008 | -0.0408 | -0.0251 | -0.0374 | -0.0373 | 0.0552 | 0.0395 | 0.0518 | 0.0517 |
| Q15-18 | 0.0160 | 0.0172 | -0.0074 | 0.0172 | -0.0029 | -0.0029 | 0.0234 | 0.0012 | 0.0189 | 0.0189 |
| Q18-19 | 0.0062 | 0.0088 | 0.0024 | 0.0221 | 0.0064 | 0.0062 | 0.0038 | 0.0159 | 0.0002 | 0.0000 |
| Q19-20 | -0.0279 | 0.0134 | -0.0395 | -0.0209 | -0.0366 | -0.0361 | 0.0116 | 0.0070 | 0.0087 | 0.0082 |
| Q10-20 | 0.0371 | 0.1134 | 0.1039 | 0.1134 | 0.1018 | 0.1003 | 0.0668 | 0.0763 | 0.0647 | 0.0632 |
| Q10-17 | 0.0443 | 0.0676 | 0.1340 | 0.1568 | 0.1341 | 0.1319 | 0.0897 | 0.1125 | 0.0898 | 0.0876 |
| Q10-21 | 0.1001 | 0.0896 | 0.0759 | 0.0386 | 0.0717 | 0.0708 | 0.0242 | 0.0615 | 0.0284 | 0.0293 |
| Q10-22 | 0.0460 | N/A | 0.0342 | 0.0167 | 0.0339 | 0.0342 | 0.0118 | 0.0293 | 0.0121 | 0.0118 |
| Q21-22 | -0.0143 | -0.0051 | 0.0008 | -0.0051 | 0.0010 | 0.0028 | 0.0151 | 0.0092 | 0.0153 | 0.0171 |
| Q15-23 | 0.0291 | N/A | -0.0132 | -0.0140 | -0.0133 | -0.0126 | 0.0423 | 0.0431 | 0.0424 | 0.0417 |
| Q22-24 | 0.0306 | 0.0781 | 0.0296 | 0.0249 | 0.0314 | 0.0359 | 0.0010 | 0.0057 | 0.0008 | 0.0053 |
| Q23-24 | 0.0124 | 0.0278 | -0.0259 | -0.0213 | -0.0144 | -0.0189 | 0.0383 | 0.0337 | 0.0268 | 0.0313 |
| Q24-25 | 0.0201 | N/A | 0.0084 | 0.0180 | 0.0400 | 0.0290 | 0.0117 | 0.0021 | 0.0199 | 0.0089 |
| Q25-26 | 0.0237 | N/A | -0.0120 | -0.0111 | 0.1154 | -0.0071 | 0.0357 | 0.0348 | 0.0917 | 0.0308 |
| Q25-27 | -0.0037 | 0.0094 | -0.0005 | -0.0142 | -0.0478 | -0.0104 | 0.0032 | 0.0105 | 0.0441 | 0.0067 |
| Q28-27 | 0.0504 | 0.0033 | -0.0077 | 0.0033 | 0.0221 | 0.0361 | 0.0581 | 0.0471 | 0.0283 | 0.0143 |
| Q27-29 | 0.0167 | N/A | -0.0031 | 0.0076 | -0.0196 | 0.0237 | 0.0198 | 0.0091 | 0.0363 | 0.0070 |
| Q27-30 | 0.0166 | 0.0093 | 0.0105 | 0.0120 | -0.0017 | 0.0312 | 0.0061 | 0.0046 | 0.0183 | 0.0146 |
| Q29-30 | 0.0061 | -0.0205 | 0.0178 | 0.0088 | 0.0169 | 0.0192 | 0.0117 | 0.0027 | 0.0108 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0211 | -0.0177 | -0.0176 | -0.0147 | 0.0157 | 0.0123 | 0.0122 | 0.0093 |
| Q6-28 | 0.0011 | -0.0427 | -0.0352 | -0.0231 | -0.0182 | -0.0067 | 0.0363 | 0.0242 | 0.0193 | 0.0078 |
| Q2-1 | 0.3447 | N/A | 0.3937 | 0.3999 | 0.3936 | 0.3937 | 0.0490 | 0.0552 | 0.0489 | 0.0490 |
| Q3-1 | 0.0265 | N/A | 0.0435 | 0.0617 | 0.0438 | 0.0450 | 0.0170 | 0.0352 | 0.0173 | 0.0185 |
| Q4-2 | -0.0554 | N/A | -0.0511 | -0.0365 | -0.0502 | -0.0490 | 0.0043 | 0.0189 | 0.0052 | 0.0064 |
| Q4-3 | 0.0544 | N/A | 0.0680 | 0.0665 | 0.0711 | 0.0723 | 0.0136 | 0.0121 | 0.0167 | 0.0179 |
| Q5-2 | 0.0517 | N/A | 0.0804 | 0.0716 | 0.0786 | 0.0800 | 0.0287 | 0.0199 | 0.0269 | 0.0283 |
| Q6-2 | 0.0058 | N/A | -0.0025 | -0.0001 | -0.0026 | -0.0017 | 0.0083 | 0.0059 | 0.0084 | 0.0075 |
| Q6-4 | 0.1719 | N/A | 0.1276 | 0.0756 | 0.1246 | 0.1232 | 0.0443 | 0.0963 | 0.0473 | 0.0487 |
| Q7-5 | -0.1313 | N/A | -0.1787 | -0.1829 | -0.1783 | -0.1776 | 0.0474 | 0.0516 | 0.0470 | 0.0463 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q7-6 | 0.0223 | N/A | 0.0325 | -0.0007 | 0.0302 | 0.0296 | 0.0102 | 0.0230 | 0.0079 | 0.0073 |
| Q8-6 | 0.0666 | N/A | 0.0423 | 0.0417 | 0.0376 | 0.0327 | 0.0243 | 0.0249 | 0.0290 | 0.0339 |
| Q9-6 | 0.0972 | N/A | 0.0435 | 0.0235 | 0.0536 | 0.0637 | 0.0537 | 0.0737 | 0.0436 | 0.0335 |
| Q10-6 | 0.0110 | N/A | 0.0077 | 0.0057 | 0.0104 | 0.0137 | 0.0033 | 0.0053 | 0.0006 | 0.0027 |
| Q11-9 | 0.1606 | N/A | 0.1619 | 0.1298 | 0.1611 | 0.1635 | 0.0013 | 0.0308 | 0.0005 | 0.0029 |
| Q10-9 | -0.0508 | N/A | -0.0731 | -0.0363 | -0.0696 | -0.0669 | 0.0223 | 0.0145 | 0.0188 | 0.0161 |
| Q12-4 | -0.0972 | N/A | -0.0276 | -0.0245 | -0.0232 | -0.0221 | 0.0696 | 0.0727 | 0.0740 | 0.0751 |
| Q13-12 | 0.1045 | N/A | 0.0911 | 0.0949 | 0.0942 | 0.0949 | 0.0134 | 0.0096 | 0.0103 | 0.0096 |
| Q14-12 | -0.0225 | N/A | -0.0078 | 0.0070 | -0.0076 | -0.0097 | 0.0147 | 0.0295 | 0.0149 | 0.0128 |
| Q15-12 | -0.0636 | N/A | -0.0515 | -0.0453 | -0.0510 | -0.0514 | 0.0121 | 0.0183 | 0.0126 | 0.0122 |
| Q16-12 | -0.0324 | N/A | -0.0083 | -0.0347 | -0.0124 | -0.0114 | 0.0241 | 0.0023 | 0.0200 | 0.0210 |
| Q15-14 | -0.0064 | N/A | 0.0059 | -0.0211 | 0.0032 | 0.0015 | 0.0123 | 0.0147 | 0.0096 | 0.0079 |
| Q17-16 | -0.0141 | N/A | 0.0412 | 0.0254 | 0.0377 | 0.0376 | 0.0553 | 0.0395 | 0.0518 | 0.0517 |
| Q18-15 | -0.0152 | N/A | 0.0075 | -0.0171 | 0.0029 | 0.0029 | 0.0227 | 0.0019 | 0.0181 | 0.0181 |
| Q19-18 | -0.0061 | N/A | -0.0024 | -0.0220 | -0.0063 | -0.0062 | 0.0037 | 0.0159 | 0.0002 | 0.0001 |
| Q20-19 | 0.0283 | N/A | 0.0400 | 0.0216 | 0.0371 | 0.0367 | 0.0117 | 0.0067 | 0.0088 | 0.0084 |
| Q20-10 | -0.0353 | N/A | -0.1017 | -0.1100 | -0.0989 | -0.0971 | 0.0664 | 0.0747 | 0.0636 | 0.0618 |
| Q17-10 | -0.0439 | N/A | -0.1325 | -0.1545 | -0.1325 | -0.1303 | 0.0886 | 0.1106 | 0.0886 | 0.0864 |
| Q21-10 | -0.0977 | N/A | -0.0605 | -0.0359 | -0.0677 | -0.0686 | 0.0372 | 0.0618 | 0.0300 | 0.0291 |
| Q22-10 | -0.0449 | N/A | -0.0262 | -0.0155 | -0.0318 | -0.0331 | 0.0187 | 0.0294 | 0.0131 | 0.0118 |
| Q22-21 | 0.0143 | N/A | -0.0008 | 0.0051 | -0.0010 | -0.0028 | 0.0151 | 0.0092 | 0.0153 | 0.0171 |
| Q23-15 | -0.0284 | N/A | 0.0214 | 0.0152 | 0.0171 | 0.0140 | 0.0498 | 0.0436 | 0.0455 | 0.0424 |
| Q24-22 | -0.0299 | N/A | -0.0229 | -0.0241 | -0.0221 | -0.0334 | 0.0070 | 0.0058 | 0.0078 | 0.0035 |
| Q24-23 | -0.0123 | N/A | 0.0272 | 0.0215 | 0.0157 | 0.0190 | 0.0395 | 0.0338 | 0.0280 | 0.0313 |
| Q25-24 | -0.0200 | N/A | -0.0084 | -0.0149 | -0.0273 | -0.0271 | 0.0116 | 0.0051 | 0.0073 | 0.0071 |
| Q26-25 | -0.0230 | N/A | 0.0327 | 0.0118 | 0.1010 | 0.0075 | 0.0557 | 0.0348 | 0.1240 | 0.0305 |
| Q27-25 | 0.0042 | N/A | 0.0476 | 0.0148 | 0.1255 | 0.0110 | 0.0434 | 0.0106 | 0.1213 | 0.0068 |
| Q27-28 | -0.0375 | N/A | 0.0207 | 0.0073 | -0.0046 | -0.0247 | 0.0582 | 0.0448 | 0.0329 | 0.0128 |
| Q29-27 | -0.0151 | N/A | 0.0071 | -0.0070 | 0.0267 | -0.0227 | 0.0222 | 0.0081 | 0.0418 | 0.0076 |
| Q30-27 | -0.0136 | N/A | -0.0087 | -0.0112 | 0.0052 | -0.0293 | 0.0049 | 0.0024 | 0.0188 | 0.0157 |
| Q30-29 | -0.0054 | N/A | -0.0175 | -0.0087 | -0.0167 | -0.0188 | 0.0121 | 0.0033 | 0.0113 | 0.0134 |
| Q28-8 | -0.0380 | N/A | -0.0235 | -0.0267 | -0.0277 | -0.0297 | 0.0145 | 0.0113 | 0.0103 | 0.0083 |
| Q28-6 | -0.0123 | N/A | 0.0228 | 0.0112 | 0.0059 | -0.0053 | 0.0351 | 0.0235 | 0.0182 | 0.0070 |
| **SUM** | | | | | | | **12.7030** | **8.4977** | **12.8583** | **6.9738** |

LAV and LTAV generate much better estimation than other two estimators by successfully correct bad data, especially for LTAV, it remove the correct bad data and have the best estimation.

#### 5.3.3.3 Four large bad data

Compared with the previous two cases, $P_2 = 0.9256$, $P_{12-14} = 0.9031$, $P_{10-21} = 0.9297$ and $P_{25-27} = -0.9710$ are introduced to simulate the two large bad data in both full and median redundancy case at the same location. The results are presented below. Table 5.272 gives the state variables, another table shows the estimation results and error with actual values.

Table 5.272: State Variable of case 2, multi large, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0847 | -0.0910 | -0.0846 | -0.0947 | **V1** | 1.0693 | 1.0690 | 1.0691 | 1.0673 |
| $\theta 3$ | -0.1461 | -0.1414 | -0.1452 | -0.1339 | **V2** | 1.0586 | 1.0563 | 1.0584 | 1.0537 |
| $\theta 4$ | -0.1796 | -0.1721 | -0.1780 | -0.1656 | **V3** | 1.0291 | 1.0321 | 1.0291 | 1.0311 |
| $\theta 5$ | -0.2629 | -0.2628 | -0.2632 | -0.2566 | **V4** | 1.0196 | 1.0235 | 1.0197 | 1.0224 |
| $\theta 6$ | -0.2100 | -0.2014 | -0.2084 | -0.1952 | **V5** | 1.0230 | 1.0209 | 1.0226 | 1.0219 |
| $\theta 7$ | -0.2381 | -0.2343 | -0.2378 | -0.2261 | **V6** | 1.0166 | 1.0186 | 1.0170 | 1.0190 |
| $\theta 8$ | -0.2236 | -0.2150 | -0.2227 | -0.2089 | **V7** | 1.0110 | 1.0105 | 1.0108 | 1.0118 |
| $\theta 9$ | -0.2981 | -0.2663 | -0.2906 | -0.2567 | **V8** | 1.0152 | 1.0166 | 1.0153 | 1.0168 |
| $\theta 10$ | -0.3502 | -0.3072 | -0.3324 | -0.2906 | **V9** | 1.0254 | 1.0226 | 1.0254 | 1.0311 |
| $\theta 11$ | -0.3002 | -0.2673 | -0.2991 | -0.2624 | **V10** | 1.0176 | 1.0189 | 1.0173 | 1.0237 |
| $\theta 12$ | -0.3426 | -0.2969 | -0.3293 | -0.2804 | **V11** | 1.0583 | 1.0484 | 1.0576 | 1.0634 |
| $\theta 13$ | -0.3361 | -0.2947 | -0.3289 | -0.2797 | **V12** | 1.0101 | 1.0120 | 1.0056 | 1.0131 |
| $\theta 14$ | -0.4146 | -0.3296 | -0.3773 | -0.3004 | **V13** | 1.0262 | 1.0250 | 1.0204 | 1.0268 |
| $\theta 15$ | -0.3801 | -0.3218 | -0.3576 | -0.3022 | **V14** | 0.9757 | 0.9958 | 0.9770 | 1.0009 |
| $\theta 16$ | -0.3434 | -0.3047 | -0.3343 | -0.2895 | **V15** | 0.9835 | 0.9920 | 0.9839 | 0.9935 |
| $\theta 17$ | -0.3459 | -0.3077 | -0.3336 | -0.2912 | **V16** | 1.0040 | 1.0038 | 0.9983 | 1.0050 |
| $\theta 18$ | -0.3663 | -0.3239 | -0.3522 | -0.3041 | **V17** | 1.0080 | 1.0052 | 1.0046 | 1.0113 |
| $\theta 19$ | -0.3618 | -0.3245 | -0.3508 | -0.3053 | **V18** | 0.9864 | 0.9863 | 0.9842 | 0.9922 |
| $\theta 20$ | -0.3555 | -0.3166 | -0.3447 | -0.3001 | **V19** | 0.9861 | 0.9816 | 0.9825 | 0.9901 |
| $\theta 21$ | -0.3822 | -0.3201 | -0.3472 | -0.3004 | **V20** | 0.9919 | 0.9869 | 0.9882 | 0.9956 |
| $\theta 22$ | -0.3834 | -0.3195 | -0.3478 | -0.3001 | **V21** | 0.9965 | 1.0095 | 1.0041 | 1.0128 |
| $\theta 23$ | -0.4101 | -0.3355 | -0.3765 | -0.3180 | **V22** | 0.9959 | 1.0099 | 1.0039 | 1.0129 |
| $\theta 24$ | -0.4180 | -0.3292 | -0.3800 | -0.3170 | **V23** | 0.9671 | 0.9883 | 0.9757 | 0.9876 |
| $\theta 25$ | -0.4002 | -0.2896 | -0.3761 | -0.2853 | **V24** | 0.9676 | 0.9975 | 0.9792 | 0.9932 |
| $\theta 26$ | -0.4958 | -0.2772 | -0.4697 | -0.2750 | **V25** | 0.9672 | 1.0092 | 0.9785 | 0.9972 |
| $\theta 27$ | -0.2994 | -0.2767 | -0.2845 | -0.2744 | **V26** | 0.9183 | 1.0238 | 0.9334 | 1.0077 |
| $\theta 28$ | -0.2209 | -0.2114 | -0.2191 | -0.2054 | **V27** | 1.0244 | 1.0177 | 1.0313 | 1.0053 |
| $\theta 29$ | -0.2621 | -0.2869 | -0.2489 | -0.2868 | **V28** | 1.0160 | 1.0168 | 1.0169 | 1.0163 |
| $\theta 30$ | -0.2660 | -0.2916 | -0.2528 | -0.2930 | **V29** | 1.0482 | 1.0067 | 1.0560 | 0.9865 |
| | | | | | **V30** | 1.0363 | 0.9972 | 1.0441 | 0.9720 |

Table 5.273: Estimation Results and Errors of case 2, multi large, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0693 | 1.0690 | 1.0691 | 1.0673 | 0.0093 | 0.0090 | 0.0091 | 0.0073 |
| V2 | 1.0450 | N/A | 1.0586 | 1.0563 | 1.0584 | 1.0537 | 0.0136 | 0.0113 | 0.0134 | 0.0087 |
| V3 | 1.0210 | N/A | 1.0291 | 1.0321 | 1.0291 | 1.0311 | 0.0081 | 0.0111 | 0.0081 | 0.0101 |
| V4 | 1.0120 | N/A | 1.0196 | 1.0235 | 1.0197 | 1.0224 | 0.0076 | 0.0115 | 0.0077 | 0.0104 |
| V5 | 1.0100 | N/A | 1.0230 | 1.0209 | 1.0226 | 1.0219 | 0.0130 | 0.0109 | 0.0126 | 0.0119 |
| V6 | 1.0110 | N/A | 1.0166 | 1.0186 | 1.0170 | 1.0190 | 0.0056 | 0.0076 | 0.0060 | 0.0080 |
| V7 | 1.0030 | N/A | 1.0110 | 1.0105 | 1.0108 | 1.0118 | 0.0080 | 0.0075 | 0.0078 | 0.0088 |
| V8 | 1.0100 | N/A | 1.0152 | 1.0166 | 1.0153 | 1.0168 | 0.0052 | 0.0066 | 0.0053 | 0.0068 |
| V9 | 1.0510 | N/A | 1.0254 | 1.0226 | 1.0254 | 1.0311 | 0.0256 | 0.0284 | 0.0256 | 0.0199 |
| V10 | 1.0450 | N/A | 1.0176 | 1.0189 | 1.0173 | 1.0237 | 0.0274 | 0.0261 | 0.0277 | 0.0213 |
| V11 | 1.0820 | N/A | 1.0583 | 1.0484 | 1.0576 | 1.0634 | 0.0237 | 0.0336 | 0.0244 | 0.0186 |
| V12 | 1.0570 | N/A | 1.0101 | 1.0120 | 1.0056 | 1.0131 | 0.0469 | 0.0450 | 0.0514 | 0.0439 |
| V13 | 1.0710 | N/A | 1.0262 | 1.0250 | 1.0204 | 1.0268 | 0.0448 | 0.0460 | 0.0506 | 0.0442 |
| V14 | 1.0430 | N/A | 0.9757 | 0.9958 | 0.9770 | 1.0009 | 0.0673 | 0.0472 | 0.0660 | 0.0421 |
| V15 | 1.0380 | N/A | 0.9835 | 0.9920 | 0.9839 | 0.9935 | 0.0545 | 0.0460 | 0.0541 | 0.0445 |
| V16 | 1.0450 | N/A | 1.0040 | 1.0038 | 0.9983 | 1.0050 | 0.0410 | 0.0412 | 0.0467 | 0.0400 |
| V17 | 1.0400 | N/A | 1.0080 | 1.0052 | 1.0046 | 1.0113 | 0.0320 | 0.0348 | 0.0354 | 0.0287 |
| V18 | 1.0280 | N/A | 0.9864 | 0.9863 | 0.9842 | 0.9922 | 0.0416 | 0.0417 | 0.0438 | 0.0358 |
| V19 | 1.0260 | N/A | 0.9861 | 0.9816 | 0.9825 | 0.9901 | 0.0399 | 0.0444 | 0.0435 | 0.0359 |
| V20 | 1.0300 | N/A | 0.9919 | 0.9869 | 0.9882 | 0.9956 | 0.0381 | 0.0431 | 0.0418 | 0.0344 |
| V21 | 1.0330 | N/A | 0.9965 | 1.0095 | 1.0041 | 1.0128 | 0.0365 | 0.0235 | 0.0289 | 0.0202 |
| V22 | 1.0340 | N/A | 0.9959 | 1.0099 | 1.0039 | 1.0129 | 0.0381 | 0.0241 | 0.0301 | 0.0211 |
| V23 | 1.0270 | N/A | 0.9671 | 0.9883 | 0.9757 | 0.9876 | 0.0599 | 0.0387 | 0.0513 | 0.0394 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V24 | 1.0220 | N/A | 0.9676 | 0.9975 | 0.9792 | 0.9932 | 0.0544 | 0.0245 | 0.0428 | 0.0288 |
| V25 | 1.0180 | N/A | 0.9672 | 1.0092 | 0.9785 | 0.9972 | 0.0508 | 0.0088 | 0.0395 | 0.0208 |
| V26 | 1.0000 | N/A | 0.9183 | 1.0238 | 0.9334 | 1.0077 | 0.0817 | 0.0238 | 0.0666 | 0.0077 |
| V27 | 1.0240 | N/A | 1.0244 | 1.0177 | 1.0313 | 1.0053 | 0.0004 | 0.0063 | 0.0073 | 0.0187 |
| V28 | 1.0070 | N/A | 1.0160 | 1.0168 | 1.0169 | 1.0163 | 0.0090 | 0.0098 | 0.0099 | 0.0093 |
| V29 | 1.0040 | N/A | 1.0482 | 1.0067 | 1.0560 | 0.9865 | 0.0442 | 0.0027 | 0.0520 | 0.0175 |
| V30 | 0.9920 | N/A | 1.0363 | 0.9972 | 1.0441 | 0.9720 | 0.0443 | 0.0052 | 0.0521 | 0.0200 |
| P1 | 2.6096 | 2.6540 | 2.5672 | 2.6540 | 2.5575 | 2.6681 | 0.0424 | 0.0444 | 0.0521 | 0.0585 |
| **P2** | **0.1830** | **0.9256** | **0.8277** | **0.4943** | **0.8149** | **0.2493** | **0.6447** | **0.3113** | **0.6319** | **0.0663** |
| P3 | -0.0240 | -0.0774 | -0.0373 | -0.0774 | -0.0479 | -0.0058 | 0.0133 | 0.0534 | 0.0239 | 0.0182 |
| P4 | -0.0760 | -0.1165 | -0.1133 | -0.1165 | -0.1394 | -0.1260 | 0.0373 | 0.0405 | 0.0634 | 0.0500 |
| P5 | -0.9420 | -1.0892 | -1.0911 | -1.0892 | -1.0983 | -1.0527 | 0.1491 | 0.1472 | 0.1563 | 0.1107 |
| P6 | 0.0000 | 0.0719 | 0.0741 | 0.0464 | 0.0506 | 0.0665 | 0.0741 | 0.0464 | 0.0506 | 0.0665 |
| P7 | -0.2280 | -0.2117 | -0.1825 | -0.2117 | -0.1923 | -0.1692 | 0.0455 | 0.0163 | 0.0357 | 0.0588 |
| P8 | -0.3000 | -0.3377 | -0.3314 | -0.3377 | -0.3546 | -0.3426 | 0.0314 | 0.0377 | 0.0546 | 0.0426 |
| P9 | 0.0000 | 0.0685 | 0.0628 | 0.0685 | 0.0287 | 0.0434 | 0.0628 | 0.0685 | 0.0287 | 0.0434 |
| P10 | -0.0580 | -0.1436 | 0.0188 | -0.1436 | -0.1024 | -0.1039 | 0.0768 | 0.0856 | 0.0444 | 0.0459 |
| P11 | 0.0000 | -0.0051 | -0.0107 | -0.0051 | -0.0443 | -0.0298 | 0.0107 | 0.0051 | 0.0443 | 0.0298 |
| P12 | -0.1120 | -0.1241 | -0.0919 | -0.1241 | -0.1372 | -0.1349 | 0.0201 | 0.0121 | 0.0252 | 0.0229 |
| P13 | 0.0000 | 0.0160 | 0.0479 | 0.0160 | 0.0029 | 0.0052 | 0.0479 | 0.0160 | 0.0029 | 0.0052 |
| P14 | -0.0620 | -0.0464 | -0.3641 | -0.1366 | -0.2495 | -0.0601 | 0.3021 | 0.0746 | 0.1875 | 0.0019 |
| P15 | -0.0820 | -0.1252 | -0.1185 | -0.1252 | -0.1065 | -0.1328 | 0.0365 | 0.0432 | 0.0245 | 0.0508 |
| P16 | -0.0350 | -0.0365 | -0.0083 | -0.0365 | -0.0466 | -0.0537 | 0.0267 | 0.0015 | 0.0116 | 0.0187 |
| P17 | -0.0900 | -0.0982 | 0.0003 | -0.0711 | -0.0519 | -0.0566 | 0.0903 | 0.0189 | 0.0381 | 0.0334 |
| P18 | -0.0320 | -0.0006 | 0.0275 | -0.0006 | 0.0166 | 0.0039 | 0.0595 | 0.0314 | 0.0486 | 0.0359 |
| P19 | -0.0950 | -0.0403 | -0.0790 | -0.1384 | -0.1002 | -0.1056 | 0.0160 | 0.0434 | 0.0052 | 0.0106 |
| P20 | -0.0220 | 0.0335 | 0.0396 | 0.0278 | 0.0034 | 0.0042 | 0.0616 | 0.0498 | 0.0254 | 0.0262 |
| P21 | -0.1750 | -0.2182 | -0.4139 | -0.2182 | -0.2124 | -0.1832 | 0.2389 | 0.0432 | 0.0374 | 0.0082 |
| P22 | 0.0000 | 0.0039 | -0.0800 | 0.0039 | 0.0484 | 0.0512 | 0.0800 | 0.0039 | 0.0484 | 0.0512 |
| P23 | -0.0320 | -0.0927 | -0.1225 | -0.0927 | -0.0825 | -0.0840 | 0.0905 | 0.0607 | 0.0505 | 0.0520 |
| P24 | -0.0870 | -0.1427 | -0.2586 | -0.1427 | -0.1983 | -0.1822 | 0.1716 | 0.0557 | 0.1113 | 0.0952 |
| P25 | 0.0000 | -0.0003 | -0.2197 | -0.0003 | -0.2196 | -0.0105 | 0.2197 | 0.0003 | 0.2196 | 0.0105 |
| P26 | -0.0350 | 0.0416 | -0.2037 | 0.0416 | -0.2014 | 0.0317 | 0.1687 | 0.0766 | 0.1664 | 0.0667 |
| P27 | 0.0000 | -0.0385 | 0.1411 | -0.0385 | 0.1394 | -0.0321 | 0.1411 | 0.0385 | 0.1394 | 0.0321 |
| P28 | 0.0000 | 0.0186 | 0.0433 | 0.0186 | 0.0223 | 0.0174 | 0.0433 | 0.0186 | 0.0223 | 0.0174 |
| P29 | -0.0240 | -0.0353 | 0.1197 | -0.0139 | 0.1183 | -0.0179 | 0.1437 | 0.0101 | 0.1423 | 0.0061 |
| P30 | -0.1060 | -0.0501 | 0.0362 | -0.0501 | 0.0351 | -0.0689 | 0.1422 | 0.0559 | 0.1411 | 0.0371 |
| Q1 | -0.2042 | -0.2587 | -0.2488 | -0.2587 | -0.2494 | -0.2556 | 0.0446 | 0.0545 | 0.0452 | 0.0514 |
| Q2 | 0.4337 | 0.4353 | 0.4435 | 0.4353 | 0.4425 | 0.4336 | 0.0098 | 0.0016 | 0.0088 | 0.0001 |
| Q3 | -0.0120 | 0.0156 | 0.0117 | 0.0156 | 0.0117 | -0.0051 | 0.0237 | 0.0276 | 0.0237 | 0.0069 |
| Q4 | -0.0160 | 0.0390 | 0.0051 | 0.0390 | 0.0062 | -0.0196 | 0.0211 | 0.0550 | 0.0222 | 0.0036 |
| Q5 | 0.1666 | 0.2438 | 0.2627 | 0.2491 | 0.2628 | 0.2508 | 0.0961 | 0.0825 | 0.0962 | 0.0842 |
| Q6 | 0.0000 | 0.0043 | 0.0137 | 0.0043 | 0.0110 | -0.0019 | 0.0137 | 0.0043 | 0.0110 | 0.0019 |
| Q7 | -0.1090 | -0.1836 | -0.1366 | -0.1424 | -0.1374 | -0.1451 | 0.0276 | 0.0334 | 0.0284 | 0.0361 |
| Q8 | 0.0611 | 0.0240 | 0.0336 | 0.0240 | 0.0301 | 0.0215 | 0.0275 | 0.0371 | 0.0310 | 0.0396 |
| Q9 | 0.0000 | -0.0531 | -0.0132 | -0.0531 | -0.0165 | -0.0155 | 0.0132 | 0.0531 | 0.0165 | 0.0155 |
| Q10 | -0.0200 | 0.0975 | 0.0867 | 0.0975 | 0.0788 | 0.0849 | 0.1067 | 0.1175 | 0.0988 | 0.1049 |
| Q11 | 0.1606 | 0.1298 | 0.1673 | 0.1298 | 0.1642 | 0.1652 | 0.0067 | 0.0308 | 0.0036 | 0.0046 |
| Q12 | -0.0750 | -0.0376 | -0.0134 | -0.0376 | -0.0239 | -0.0311 | 0.0616 | 0.0374 | 0.0511 | 0.0439 |
| Q13 | 0.1045 | 0.0949 | 0.1183 | 0.0949 | 0.1082 | 0.1012 | 0.0138 | 0.0096 | 0.0037 | 0.0033 |
| Q14 | -0.0160 | 0.0284 | 0.0764 | 0.0284 | 0.0191 | 0.0054 | 0.0924 | 0.0444 | 0.0351 | 0.0214 |
| Q15 | -0.0250 | -0.0632 | -0.0805 | -0.0632 | -0.0678 | -0.0671 | 0.0555 | 0.0382 | 0.0428 | 0.0421 |
| Q16 | -0.0180 | -0.0881 | -0.0461 | -0.0289 | -0.0493 | -0.0483 | 0.0281 | 0.0109 | 0.0313 | 0.0303 |
| Q17 | -0.0580 | -0.1291 | -0.0946 | -0.1291 | -0.0966 | -0.0933 | 0.0366 | 0.0711 | 0.0386 | 0.0353 |
| Q18 | -0.0090 | 0.0154 | 0.0017 | 0.0102 | 0.0059 | 0.0079 | 0.0107 | 0.0192 | 0.0149 | 0.0169 |
| Q19 | -0.0340 | -0.0429 | -0.0456 | -0.0429 | -0.0447 | -0.0428 | 0.0116 | 0.0089 | 0.0107 | 0.0088 |
| Q20 | -0.0070 | -0.0168 | -0.0605 | -0.0922 | -0.0608 | -0.0597 | 0.0535 | 0.0852 | 0.0538 | 0.0527 |
| Q21 | -0.1120 | -0.0410 | -0.0587 | -0.0410 | -0.0693 | -0.0654 | 0.0533 | 0.0710 | 0.0427 | 0.0466 |
| Q22 | 0.0000 | 0.0146 | 0.0032 | 0.0146 | -0.0080 | 0.0005 | 0.0032 | 0.0146 | 0.0080 | 0.0005 |
| Q23 | -0.0160 | -0.0061 | -0.0169 | -0.0061 | -0.0101 | -0.0072 | 0.0009 | 0.0099 | 0.0059 | 0.0088 |
| Q24 | -0.0670 | 0.0124 | -0.0269 | -0.0208 | -0.0329 | -0.0274 | 0.0401 | 0.0462 | 0.0341 | 0.0396 |
| Q25 | 0.0000 | -0.0402 | -0.0244 | -0.0402 | -0.0193 | -0.0455 | 0.0244 | 0.0402 | 0.0193 | 0.0455 |
| Q26 | -0.0230 | 0.0118 | 0.0288 | 0.0118 | 0.0346 | 0.0066 | 0.0518 | 0.0348 | 0.0576 | 0.0296 |
| Q27 | 0.0000 | 0.0418 | 0.0774 | 0.0418 | 0.0813 | 0.0416 | 0.0774 | 0.0418 | 0.0813 | 0.0416 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q28 | 0.0000 | -0.0122 | -0.0024 | -0.0122 | -0.0098 | 0.0028 | 0.0024 | 0.0122 | 0.0098 | 0.0028 |
| Q29 | -0.0090 | 0.0018 | 0.0262 | 0.0018 | 0.0292 | -0.0032 | 0.0352 | 0.0108 | 0.0382 | 0.0058 |
| Q30 | -0.0190 | -0.0773 | -0.0252 | -0.0278 | -0.0229 | -0.0479 | 0.0062 | 0.0088 | 0.0039 | 0.0289 |
| P1-2 | 1.7331 | N/A | 1.5803 | 1.7008 | 1.5771 | 1.7660 | 0.1528 | 0.0323 | 0.1560 | 0.0329 |
| P1-3 | 0.8765 | 0.8817 | 0.9869 | 0.9532 | 0.9804 | 0.9021 | 0.1104 | 0.0767 | 0.1039 | 0.0256 |
| P2-4 | 0.4365 | 0.4726 | 0.6100 | 0.5202 | 0.6012 | 0.4575 | 0.1735 | 0.0837 | 0.1647 | 0.0210 |
| P3-4 | 0.8214 | 0.9507 | 0.9109 | 0.8398 | 0.8943 | 0.8640 | 0.0895 | 0.0184 | 0.0729 | 0.0426 |
| P2-5 | 0.8236 | 0.7903 | 0.9781 | 0.9404 | 0.9800 | 0.8825 | 0.1545 | 0.1168 | 0.1564 | 0.0589 |
| P2-6 | 0.6038 | N/A | 0.7769 | 0.6848 | 0.7679 | 0.6217 | 0.1731 | 0.0810 | 0.1641 | 0.0179 |
| P4-6 | 0.7213 | 0.7172 | 0.7251 | 0.7172 | 0.7239 | 0.7118 | 0.0038 | 0.0041 | 0.0026 | 0.0095 |
| P5-7 | -0.1478 | -0.2445 | -0.1534 | -0.1863 | -0.1588 | -0.2034 | 0.0056 | 0.0385 | 0.0110 | 0.0556 |
| P6-7 | 0.3813 | 0.3594 | 0.3412 | 0.4050 | 0.3569 | 0.3795 | 0.0401 | 0.0237 | 0.0244 | 0.0018 |
| P6-8 | 0.2956 | N/A | 0.3188 | 0.3222 | 0.3365 | 0.3282 | 0.0232 | 0.0266 | 0.0409 | 0.0326 |
| P6-9 | 0.2772 | 0.3192 | 0.4413 | 0.3246 | 0.4114 | 0.3109 | 0.1641 | 0.0474 | 0.1342 | 0.0337 |
| P6-10 | 0.1584 | N/A | 0.2600 | 0.1972 | 0.2300 | 0.1787 | 0.1016 | 0.0388 | 0.0716 | 0.0203 |
| P9-11 | 0.0000 | N/A | 0.0107 | 0.0051 | 0.0443 | 0.0298 | 0.0107 | 0.0051 | 0.0443 | 0.0298 |
| P9-10 | 0.2772 | 0.2500 | 0.4933 | 0.3880 | 0.3959 | 0.3245 | 0.2161 | 0.1108 | 0.1187 | 0.0473 |
| P4-12 | 0.4419 | N/A | 0.6530 | 0.5036 | 0.6036 | 0.4634 | 0.2111 | 0.0617 | 0.1617 | 0.0215 |
| P12-13 | 0.0000 | N/A | -0.0479 | -0.0160 | -0.0029 | -0.0052 | 0.0479 | 0.0160 | 0.0029 | 0.0052 |
| **P12-14** | **0.0786** | **0.9031** | **0.2817** | **0.1305** | **0.1952** | **0.0835** | **0.2031** | **0.0519** | **0.1166** | **0.0049** |
| P12-15 | 0.1789 | 0.2159 | 0.3121 | 0.2159 | 0.2392 | 0.1960 | 0.1332 | 0.0370 | 0.0603 | 0.0171 |
| P12-16 | 0.0724 | N/A | 0.0152 | 0.0491 | 0.0350 | 0.0541 | 0.0572 | 0.0233 | 0.0374 | 0.0183 |
| P14-15 | 0.0158 | N/A | -0.0920 | -0.0081 | -0.0590 | 0.0226 | 0.1078 | 0.0239 | 0.0748 | 0.0068 |
| P16-17 | 0.0369 | -0.0700 | 0.0068 | 0.0124 | -0.0118 | 0.0001 | 0.0301 | 0.0245 | 0.0487 | 0.0368 |
| P15-18 | 0.0602 | 0.0182 | -0.0540 | 0.0182 | -0.0197 | 0.0094 | 0.1142 | 0.0420 | 0.0799 | 0.0508 |
| P18-19 | 0.0278 | 0.0955 | -0.0269 | 0.0175 | -0.0031 | 0.0133 | 0.0547 | 0.0103 | 0.0309 | 0.0145 |
| P19-20 | -0.0673 | -0.1209 | -0.1060 | -0.1209 | -0.1034 | -0.0923 | 0.0387 | 0.0536 | 0.0361 | 0.0250 |
| P10-20 | 0.0903 | 0.1383 | 0.0681 | 0.0956 | 0.1022 | 0.0900 | 0.0222 | 0.0053 | 0.0119 | 0.0003 |
| P10-17 | 0.0533 | 0.0595 | -0.0066 | 0.0595 | 0.0644 | 0.0572 | 0.0599 | 0.0062 | 0.0111 | 0.0039 |
| **P10-21** | **0.1579** | **0.9297** | **0.4695** | **0.1940** | **0.2359** | **0.1697** | **0.3116** | **0.0361** | **0.0780** | **0.0118** |
| P10-22 | 0.0762 | N/A | 0.2411 | 0.0925 | 0.1210 | 0.0824 | 0.1649 | 0.0163 | 0.0448 | 0.0062 |
| P21-22 | -0.0183 | -0.0282 | 0.0480 | -0.0255 | 0.0215 | -0.0145 | 0.0663 | 0.0072 | 0.0398 | 0.0038 |
| P15-23 | 0.0504 | N/A | 0.1461 | 0.0611 | 0.0884 | 0.0735 | 0.0957 | 0.0107 | 0.0380 | 0.0231 |
| P22-24 | 0.0574 | 0.2028 | 0.2049 | 0.0703 | 0.1897 | 0.1185 | 0.1475 | 0.0129 | 0.1323 | 0.0611 |
| P23-24 | 0.0180 | 0.0593 | 0.0214 | -0.0320 | 0.0051 | -0.0110 | 0.0034 | 0.0500 | 0.0129 | 0.0290 |
| P24-25 | -0.0121 | N/A | -0.0373 | -0.1052 | -0.0076 | -0.0765 | 0.0252 | 0.0931 | 0.0045 | 0.0644 |
| P25-26 | 0.0354 | N/A | 0.2165 | -0.0411 | 0.2136 | -0.0314 | 0.1811 | 0.0765 | 0.1782 | 0.0668 |
| **P25-27** | **-0.0476** | **-0.9710** | **-0.4739** | **-0.0666** | **-0.4408** | **-0.0568** | **0.4263** | **0.0190** | **0.3932** | **0.0092** |
| P28-27 | 0.1807 | 0.1671 | 0.2063 | 0.1704 | 0.1731 | 0.1778 | 0.0256 | 0.0103 | 0.0076 | 0.0029 |
| P27-29 | 0.0619 | N/A | -0.0989 | 0.0309 | -0.0975 | 0.0419 | 0.1608 | 0.0310 | 0.1594 | 0.0200 |
| P27-30 | 0.0709 | 0.0570 | -0.0539 | 0.0339 | -0.0529 | 0.0467 | 0.1248 | 0.0370 | 0.1238 | 0.0242 |
| P29-30 | 0.0370 | 0.0721 | 0.0187 | 0.0167 | 0.0188 | 0.0234 | 0.0183 | 0.0203 | 0.0182 | 0.0136 |
| P8-28 | -0.0054 | N/A | -0.0138 | -0.0167 | -0.0195 | -0.0156 | 0.0084 | 0.0113 | 0.0141 | 0.0102 |
| P6-28 | 0.1867 | 0.1690 | 0.1773 | 0.1690 | 0.1708 | 0.1766 | 0.0094 | 0.0177 | 0.0159 | 0.0101 |
| P2-1 | -1.6809 | N/A | -1.5373 | -1.6511 | -1.5342 | -1.7123 | 0.1436 | 0.0298 | 0.1467 | 0.0314 |
| P3-1 | -0.8454 | N/A | -0.9482 | -0.9172 | -0.9422 | -0.8697 | 0.1028 | 0.0718 | 0.0968 | 0.0243 |
| P4-2 | -0.4263 | N/A | -0.5908 | -0.5063 | -0.5826 | -0.4466 | 0.1645 | 0.0800 | 0.1563 | 0.0203 |
| P4-3 | -0.8129 | N/A | -0.9006 | -0.8311 | -0.8843 | -0.8547 | 0.0877 | 0.0182 | 0.0714 | 0.0418 |
| P5-2 | -0.7942 | N/A | -0.9377 | -0.9029 | -0.9394 | -0.8493 | 0.1435 | 0.1087 | 0.1452 | 0.0551 |
| P6-2 | -0.5843 | N/A | -0.7455 | -0.6603 | -0.7373 | -0.6014 | 0.1612 | 0.0760 | 0.1530 | 0.0171 |
| P6-4 | -0.7150 | N/A | -0.7189 | -0.7113 | -0.7177 | -0.7059 | 0.0039 | 0.0037 | 0.0027 | 0.0091 |
| P7-5 | 0.1495 | N/A | 0.1557 | 0.1891 | 0.1612 | 0.2065 | 0.0062 | 0.0396 | 0.0117 | 0.0570 |
| P7-6 | -0.3775 | N/A | -0.3382 | -0.4008 | -0.3535 | -0.3758 | 0.0393 | 0.0233 | 0.0240 | 0.0017 |
| P8-6 | -0.2946 | N/A | -0.3176 | -0.3210 | -0.3351 | -0.3269 | 0.0230 | 0.0264 | 0.0405 | 0.0323 |
| P9-6 | -0.2772 | N/A | -0.4413 | -0.3246 | -0.4114 | -0.3109 | 0.1641 | 0.0474 | 0.1342 | 0.0337 |
| P10-6 | -0.1584 | N/A | -0.2600 | -0.1972 | -0.2300 | -0.1787 | 0.1016 | 0.0388 | 0.0716 | 0.0203 |
| P11-9 | 0.0000 | N/A | -0.0107 | -0.0051 | -0.0443 | -0.0298 | 0.0107 | 0.0051 | 0.0443 | 0.0298 |
| P10-9 | -0.2772 | N/A | -0.4933 | -0.3880 | -0.3959 | -0.3245 | 0.2161 | 0.1108 | 0.1187 | 0.0473 |
| P12-4 | -0.4419 | N/A | -0.6530 | -0.5036 | -0.6036 | -0.4634 | 0.2111 | 0.0617 | 0.1617 | 0.0215 |
| P13-12 | 0.0000 | N/A | 0.0479 | 0.0160 | 0.0029 | 0.0052 | 0.0479 | 0.0160 | 0.0029 | 0.0052 |
| P14-12 | -0.0778 | N/A | -0.2721 | -0.1285 | -0.1905 | -0.0827 | 0.1943 | 0.0507 | 0.1127 | 0.0049 |
| P15-12 | -0.1767 | N/A | -0.3056 | -0.2127 | -0.2353 | -0.1933 | 0.1289 | 0.0360 | 0.0586 | 0.0166 |
| P16-12 | -0.0719 | N/A | -0.0152 | -0.0489 | -0.0349 | -0.0538 | 0.0567 | 0.0230 | 0.0370 | 0.0181 |
| P15-14 | -0.0158 | N/A | 0.0950 | 0.0083 | 0.0601 | -0.0224 | 0.1108 | 0.0241 | 0.0759 | 0.0066 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P17-16 | -0.0368 | N/A | -0.0068 | -0.0123 | 0.0118 | 0.0000 | 0.0300 | 0.0245 | 0.0486 | 0.0368 |
| P18-15 | -0.0598 | N/A | 0.0544 | -0.0181 | 0.0197 | -0.0093 | 0.1142 | 0.0417 | 0.0795 | 0.0505 |
| P19-18 | -0.0277 | N/A | 0.0269 | -0.0175 | 0.0032 | -0.0133 | 0.0546 | 0.0102 | 0.0309 | 0.0144 |
| P20-19 | 0.0674 | N/A | 0.1064 | 0.1214 | 0.1038 | 0.0927 | 0.0390 | 0.0540 | 0.0364 | 0.0253 |
| P20-10 | -0.0894 | N/A | -0.0669 | -0.0936 | -0.1004 | -0.0885 | 0.0225 | 0.0042 | 0.0110 | 0.0009 |
| P17-10 | -0.0532 | N/A | 0.0071 | -0.0588 | -0.0637 | -0.0566 | 0.0603 | 0.0056 | 0.0105 | 0.0034 |
| P21-10 | -0.1567 | N/A | -0.4619 | -0.1927 | -0.2339 | -0.1686 | 0.3052 | 0.0360 | 0.0772 | 0.0119 |
| P22-10 | -0.0757 | N/A | -0.2369 | -0.0918 | -0.1199 | -0.0818 | 0.1612 | 0.0161 | 0.0442 | 0.0061 |
| P22-21 | 0.0183 | N/A | -0.0480 | 0.0255 | -0.0215 | 0.0145 | 0.0663 | 0.0072 | 0.0398 | 0.0038 |
| P23-15 | -0.0500 | N/A | -0.1439 | -0.0607 | -0.0876 | -0.0730 | 0.0939 | 0.0107 | 0.0376 | 0.0230 |
| P24-22 | -0.0569 | N/A | -0.2000 | -0.0696 | -0.1855 | -0.1168 | 0.1431 | 0.0127 | 0.1286 | 0.0599 |
| P24-23 | -0.0180 | N/A | -0.0213 | 0.0322 | -0.0051 | 0.0111 | 0.0033 | 0.0502 | 0.0129 | 0.0291 |
| P25-24 | 0.0122 | N/A | 0.0377 | 0.1075 | 0.0077 | 0.0778 | 0.0255 | 0.0953 | 0.0045 | 0.0656 |
| P26-25 | -0.0350 | N/A | -0.2037 | 0.0416 | -0.2014 | 0.0317 | 0.1687 | 0.0766 | 0.1664 | 0.0667 |
| P27-25 | 0.0479 | N/A | 0.5002 | 0.0671 | 0.4630 | 0.0572 | 0.4523 | 0.0192 | 0.4151 | 0.0093 |
| P27-28 | -0.1807 | N/A | -0.2063 | -0.1704 | -0.1731 | -0.1778 | 0.0256 | 0.0103 | 0.0076 | 0.0029 |
| P29-27 | -0.0610 | N/A | 0.1010 | -0.0306 | 0.0995 | -0.0414 | 0.1620 | 0.0304 | 0.1605 | 0.0196 |
| P30-27 | -0.0693 | N/A | 0.0548 | -0.0335 | 0.0538 | -0.0457 | 0.1241 | 0.0358 | 0.1231 | 0.0236 |
| P30-29 | -0.0367 | N/A | -0.0186 | -0.0166 | -0.0186 | -0.0232 | 0.0181 | 0.0201 | 0.0181 | 0.0135 |
| P28-8 | 0.0055 | N/A | 0.0138 | 0.0168 | 0.0195 | 0.0156 | 0.0083 | 0.0113 | 0.0140 | 0.0101 |
| P28-6 | -0.1862 | N/A | -0.1767 | -0.1685 | -0.1703 | -0.1761 | 0.0095 | 0.0177 | 0.0159 | 0.0101 |
| Q1-2 | -0.2470 | N/A | -0.2867 | -0.2800 | -0.2867 | -0.2791 | 0.0397 | 0.0330 | 0.0397 | 0.0321 |
| Q1-3 | 0.0428 | 0.0595 | 0.0379 | 0.0213 | 0.0373 | 0.0236 | 0.0049 | 0.0215 | 0.0055 | 0.0192 |
| Q2-4 | 0.0475 | 0.0671 | 0.0447 | 0.0285 | 0.0447 | 0.0348 | 0.0028 | 0.0190 | 0.0028 | 0.0127 |
| Q3-4 | -0.0385 | -0.0159 | -0.0467 | -0.0497 | -0.0456 | -0.0550 | 0.0082 | 0.0112 | 0.0071 | 0.0165 |
| Q2-5 | 0.0278 | 0.0213 | 0.0202 | 0.0213 | 0.0210 | 0.0064 | 0.0076 | 0.0065 | 0.0068 | 0.0214 |
| Q2-6 | 0.0137 | N/A | 0.0229 | 0.0164 | 0.0213 | 0.0121 | 0.0092 | 0.0027 | 0.0076 | 0.0016 |
| Q4-6 | -0.1591 | -0.1829 | -0.1276 | -0.0786 | -0.1335 | -0.1152 | 0.0315 | 0.0805 | 0.0256 | 0.0439 |
| Q5-7 | 0.1149 | 0.1580 | 0.1586 | 0.1580 | 0.1587 | 0.1630 | 0.0437 | 0.0431 | 0.0438 | 0.0481 |
| Q6-7 | -0.0278 | -0.0959 | -0.0454 | -0.0341 | -0.0437 | -0.0372 | 0.0176 | 0.0063 | 0.0159 | 0.0094 |
| Q6-8 | -0.0720 | N/A | -0.0601 | -0.0467 | -0.0586 | -0.0412 | 0.0119 | 0.0253 | 0.0134 | 0.0308 |
| Q6-9 | -0.0809 | -0.1233 | -0.0238 | -0.0094 | -0.0243 | -0.0497 | 0.0571 | 0.0715 | 0.0566 | 0.0312 |
| Q6-10 | 0.0019 | N/A | 0.0164 | 0.0099 | 0.0137 | 0.0000 | 0.0145 | 0.0080 | 0.0118 | 0.0019 |
| Q9-11 | -0.1560 | N/A | -0.1621 | -0.1266 | -0.1588 | -0.1600 | 0.0061 | 0.0294 | 0.0028 | 0.0040 |
| Q9-10 | 0.0588 | 0.1108 | 0.0858 | 0.0430 | 0.0839 | 0.0750 | 0.0270 | 0.0158 | 0.0251 | 0.0162 |
| Q4-12 | 0.1441 | N/A | 0.0911 | 0.0774 | 0.1020 | 0.0639 | 0.0530 | 0.0667 | 0.0421 | 0.0802 |
| Q12-13 | -0.1032 | N/A | -0.1162 | -0.0937 | -0.1066 | -0.0998 | 0.0130 | 0.0095 | 0.0034 | 0.0034 |
| Q12-14 | 0.0240 | 0.0570 | 0.0102 | 0.0036 | 0.0231 | 0.0087 | 0.0138 | 0.0204 | 0.0009 | 0.0153 |
| Q12-15 | 0.0679 | 0.0645 | 0.0529 | 0.0478 | 0.0491 | 0.0545 | 0.0150 | 0.0201 | 0.0188 | 0.0134 |
| Q12-16 | 0.0335 | N/A | 0.0237 | 0.0186 | 0.0203 | 0.0157 | 0.0098 | 0.0149 | 0.0132 | 0.0178 |
| Q14-15 | 0.0065 | N/A | 0.0666 | 0.0277 | 0.0324 | 0.0123 | 0.0601 | 0.0212 | 0.0259 | 0.0058 |
| Q16-17 | 0.0144 | -0.0008 | -0.0226 | -0.0108 | -0.0294 | -0.0332 | 0.0370 | 0.0252 | 0.0438 | 0.0476 |
| Q15-18 | 0.0160 | 0.0172 | 0.0141 | 0.0172 | 0.0084 | 0.0013 | 0.0019 | 0.0012 | 0.0076 | 0.0147 |
| Q18-19 | 0.0062 | 0.0088 | 0.0151 | 0.0273 | 0.0143 | 0.0092 | 0.0089 | 0.0211 | 0.0081 | 0.0030 |
| Q19-20 | -0.0279 | 0.0134 | -0.0306 | -0.0158 | -0.0304 | -0.0336 | 0.0027 | 0.0121 | 0.0025 | 0.0057 |
| Q10-20 | 0.0371 | 0.1134 | 0.0947 | 0.1134 | 0.0961 | 0.0975 | 0.0576 | 0.0763 | 0.0590 | 0.0604 |
| Q10-17 | 0.0443 | 0.0676 | 0.1184 | 0.1419 | 0.1279 | 0.1283 | 0.0741 | 0.0976 | 0.0836 | 0.0840 |
| Q10-21 | 0.1001 | 0.0896 | 0.0757 | 0.0387 | 0.0708 | 0.0706 | 0.0244 | 0.0614 | 0.0293 | 0.0295 |
| Q10-22 | 0.0460 | N/A | 0.0340 | 0.0167 | 0.0327 | 0.0340 | 0.0120 | 0.0293 | 0.0133 | 0.0120 |
| Q21-22 | -0.0143 | -0.0051 | 0.0006 | -0.0051 | -0.0029 | 0.0028 | 0.0149 | 0.0092 | 0.0114 | 0.0171 |
| Q15-23 | 0.0291 | N/A | 0.0094 | -0.0113 | -0.0033 | -0.0069 | 0.0197 | 0.0404 | 0.0324 | 0.0360 |
| Q22-24 | 0.0306 | 0.0781 | 0.0292 | 0.0249 | 0.0196 | 0.0361 | 0.0014 | 0.0057 | 0.0110 | 0.0055 |
| Q23-24 | 0.0124 | 0.0278 | -0.0119 | -0.0182 | -0.0150 | -0.0152 | 0.0243 | 0.0306 | 0.0274 | 0.0276 |
| Q24-25 | 0.0201 | N/A | 0.0228 | 0.0273 | 0.0064 | 0.0332 | 0.0027 | 0.0072 | 0.0137 | 0.0131 |
| Q25-26 | 0.0237 | N/A | -0.0097 | -0.0111 | -0.0164 | -0.0062 | 0.0334 | 0.0348 | 0.0401 | 0.0299 |
| Q25-27 | -0.0037 | 0.0094 | 0.0074 | -0.0057 | 0.0035 | -0.0084 | 0.0111 | 0.0020 | 0.0072 | 0.0047 |
| Q28-27 | 0.0504 | 0.0033 | -0.0135 | 0.0033 | -0.0314 | 0.0345 | 0.0639 | 0.0471 | 0.0818 | 0.0159 |
| Q27-29 | 0.0167 | N/A | -0.0046 | 0.0107 | -0.0079 | 0.0234 | 0.0213 | 0.0060 | 0.0246 | 0.0067 |
| Q27-30 | 0.0166 | 0.0093 | 0.0094 | 0.0167 | 0.0071 | 0.0310 | 0.0072 | 0.0001 | 0.0095 | 0.0144 |
| Q29-30 | 0.0061 | -0.0205 | 0.0177 | 0.0121 | 0.0176 | 0.0192 | 0.0116 | 0.0060 | 0.0115 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0215 | -0.0176 | -0.0240 | -0.0148 | 0.0161 | 0.0122 | 0.0186 | 0.0094 |
| Q6-28 | 0.0011 | -0.0427 | -0.0452 | -0.0229 | -0.0534 | -0.0094 | 0.0463 | 0.0240 | 0.0545 | 0.0105 |
| Q2-1 | 0.3447 | N/A | 0.3558 | 0.3691 | 0.3554 | 0.3803 | 0.0111 | 0.0244 | 0.0107 | 0.0356 |
| Q3-1 | 0.0265 | N/A | 0.0584 | 0.0653 | 0.0572 | 0.0499 | 0.0319 | 0.0388 | 0.0307 | 0.0234 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q4-2 | -0.0554 | N/A | -0.0261 | -0.0258 | -0.0278 | -0.0412 | 0.0293 | 0.0296 | 0.0276 | 0.0142 |
| Q4-3 | 0.0544 | N/A | 0.0677 | 0.0660 | 0.0654 | 0.0728 | 0.0133 | 0.0116 | 0.0110 | 0.0184 |
| Q5-2 | 0.0517 | N/A | 0.1041 | 0.0911 | 0.1041 | 0.0878 | 0.0524 | 0.0394 | 0.0524 | 0.0361 |
| Q6-2 | 0.0058 | N/A | 0.0321 | 0.0176 | 0.0315 | 0.0092 | 0.0263 | 0.0118 | 0.0257 | 0.0034 |
| Q6-4 | 0.1719 | N/A | 0.1398 | 0.0897 | 0.1457 | 0.1264 | 0.0321 | 0.0822 | 0.0262 | 0.0455 |
| Q7-5 | -0.1313 | N/A | -0.1739 | -0.1720 | -0.1738 | -0.1762 | 0.0426 | 0.0407 | 0.0425 | 0.0449 |
| Q7-6 | 0.0223 | N/A | 0.0373 | 0.0296 | 0.0365 | 0.0311 | 0.0150 | 0.0073 | 0.0142 | 0.0088 |
| Q8-6 | 0.0666 | N/A | 0.0551 | 0.0416 | 0.0540 | 0.0363 | 0.0115 | 0.0250 | 0.0126 | 0.0303 |
| Q9-6 | 0.0972 | N/A | 0.0631 | 0.0305 | 0.0584 | 0.0695 | 0.0341 | 0.0667 | 0.0388 | 0.0277 |
| Q10-6 | 0.0110 | N/A | 0.0201 | 0.0110 | 0.0148 | 0.0171 | 0.0091 | 0.0000 | 0.0038 | 0.0061 |
| Q11-9 | 0.1606 | N/A | 0.1673 | 0.1298 | 0.1642 | 0.1652 | 0.0067 | 0.0308 | 0.0036 | 0.0046 |
| Q10-9 | -0.0508 | N/A | -0.0596 | -0.0270 | -0.0668 | -0.0635 | 0.0088 | 0.0238 | 0.0160 | 0.0127 |
| Q12-4 | -0.0972 | N/A | 0.0159 | -0.0139 | -0.0097 | -0.0103 | 0.1131 | 0.0833 | 0.0875 | 0.0869 |
| Q13-12 | 0.1045 | N/A | 0.1183 | 0.0949 | 0.1082 | 0.1012 | 0.0138 | 0.0096 | 0.0037 | 0.0033 |
| Q14-12 | -0.0225 | N/A | 0.0098 | 0.0007 | -0.0133 | -0.0070 | 0.0323 | 0.0232 | 0.0092 | 0.0155 |
| Q15-12 | -0.0636 | N/A | -0.0401 | -0.0416 | -0.0414 | -0.0493 | 0.0235 | 0.0220 | 0.0222 | 0.0143 |
| Q16-12 | -0.0324 | N/A | -0.0236 | -0.0181 | -0.0199 | -0.0151 | 0.0088 | 0.0143 | 0.0125 | 0.0173 |
| Q15-14 | -0.0064 | N/A | -0.0639 | -0.0275 | -0.0315 | -0.0122 | 0.0575 | 0.0211 | 0.0251 | 0.0058 |
| Q17-16 | -0.0141 | N/A | 0.0227 | 0.0108 | 0.0296 | 0.0334 | 0.0368 | 0.0249 | 0.0437 | 0.0475 |
| Q18-15 | -0.0152 | N/A | -0.0134 | -0.0171 | -0.0083 | -0.0013 | 0.0018 | 0.0019 | 0.0069 | 0.0139 |
| Q19-18 | -0.0061 | N/A | -0.0150 | -0.0271 | -0.0142 | -0.0092 | 0.0089 | 0.0210 | 0.0081 | 0.0031 |
| Q20-19 | 0.0283 | N/A | 0.0314 | 0.0168 | 0.0312 | 0.0343 | 0.0031 | 0.0115 | 0.0029 | 0.0060 |
| Q20-10 | -0.0353 | N/A | -0.0919 | -0.1090 | -0.0921 | -0.0940 | 0.0566 | 0.0737 | 0.0568 | 0.0587 |
| Q17-10 | -0.0439 | N/A | -0.1173 | -0.1399 | -0.1262 | -0.1267 | 0.0734 | 0.0960 | 0.0823 | 0.0828 |
| Q21-10 | -0.0977 | N/A | -0.0593 | -0.0359 | -0.0664 | -0.0682 | 0.0384 | 0.0618 | 0.0313 | 0.0295 |
| Q22-10 | -0.0449 | N/A | -0.0255 | -0.0154 | -0.0305 | -0.0329 | 0.0194 | 0.0295 | 0.0144 | 0.0120 |
| Q22-21 | 0.0143 | N/A | -0.0005 | 0.0051 | 0.0029 | -0.0028 | 0.0148 | 0.0092 | 0.0114 | 0.0171 |
| Q23-15 | -0.0284 | N/A | -0.0050 | 0.0121 | 0.0049 | 0.0080 | 0.0234 | 0.0405 | 0.0333 | 0.0364 |
| Q24-22 | -0.0299 | N/A | -0.0215 | -0.0239 | -0.0131 | -0.0335 | 0.0084 | 0.0060 | 0.0168 | 0.0036 |
| Q24-23 | -0.0123 | N/A | 0.0121 | 0.0186 | 0.0151 | 0.0153 | 0.0244 | 0.0309 | 0.0274 | 0.0276 |
| Q25-24 | -0.0200 | N/A | -0.0221 | -0.0234 | -0.0064 | -0.0309 | 0.0021 | 0.0034 | 0.0136 | 0.0109 |
| Q26-25 | -0.0230 | N/A | 0.0288 | 0.0118 | 0.0346 | 0.0066 | 0.0518 | 0.0348 | 0.0576 | 0.0296 |
| Q27-25 | 0.0042 | N/A | 0.0427 | 0.0066 | 0.0389 | 0.0091 | 0.0385 | 0.0024 | 0.0347 | 0.0049 |
| Q27-28 | -0.0375 | N/A | 0.0299 | 0.0078 | 0.0432 | -0.0219 | 0.0674 | 0.0453 | 0.0807 | 0.0156 |
| Q29-27 | -0.0151 | N/A | 0.0085 | -0.0103 | 0.0116 | -0.0224 | 0.0236 | 0.0048 | 0.0267 | 0.0073 |
| Q30-27 | -0.0136 | N/A | -0.0077 | -0.0158 | -0.0055 | -0.0291 | 0.0059 | 0.0022 | 0.0081 | 0.0155 |
| Q30-29 | -0.0054 | N/A | -0.0175 | -0.0119 | -0.0174 | -0.0188 | 0.0121 | 0.0065 | 0.0120 | 0.0134 |
| Q28-8 | -0.0380 | N/A | -0.0226 | -0.0266 | -0.0201 | -0.0294 | 0.0154 | 0.0114 | 0.0179 | 0.0086 |
| Q28-6 | -0.0123 | N/A | 0.0337 | 0.0111 | 0.0417 | -0.0023 | 0.0460 | 0.0234 | 0.0540 | 0.0100 |
| **SUM** | | | | | | | **15.8685** | **8.5567** | **12.9863** | **6.4768** |

With WLS and LTS generate unsatisfied estimation, they perform bad on bad data correction. LAV's performance is much better, based on LAV, LTAV generate best estimation both on bad data and overall error summation.

### 5.3.4 Case 3: Multiple interacting bad data

In this case, four bad data interacting with each other are used for performance evaluation. The location of these four bad data are $P_2$, $P_{2-4}$, $P_{4-6}$ and $P_{6-9}$. Three cases are conducted as usual.

#### 5.3.4.1 Four reverse bad data

The two reverse bad data are $P_2 = -0.1256$, $P_{2-4} = -0.4726$, $P_{4-6} = -0.7172$ and $P_{6-9} = -0.3192$ by add a negative sign to original measurements. the system state variables are presented in Table 5.274, Table 5.275 gives the estimation results and error with actual value.

Table 5.274: State Variable of case 3, multi reverse, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0944 | -0.0972 | -0.0932 | -0.0999 | **V1** | 1.0590 | 1.0690 | 1.0619 | 1.0673 |
| $\theta 3$ | -0.1170 | -0.1253 | -0.1274 | -0.1263 | **V2** | 1.0460 | 1.0542 | 1.0496 | 1.0521 |
| $\theta 4$ | -0.1394 | -0.1520 | -0.1543 | -0.1573 | **V3** | 1.0266 | 1.0358 | 1.0270 | 1.0333 |
| $\theta 5$ | -0.2285 | -0.2405 | -0.2224 | -0.2513 | **V4** | 1.0208 | 1.0277 | 1.0200 | 1.0248 |
| $\theta 6$ | -0.1435 | -0.1762 | -0.1423 | -0.1854 | **V5** | 1.0209 | 1.0248 | 1.0257 | 1.0230 |
| $\theta 7$ | -0.1798 | -0.2047 | -0.1756 | -0.2172 | **V6** | 1.0241 | 1.0231 | 1.0283 | 1.0217 |
| $\theta 8$ | -0.1500 | -0.1894 | -0.1473 | -0.1982 | **V7** | 1.0154 | 1.0147 | 1.0201 | 1.0140 |
| $\theta 9$ | -0.1327 | -0.2144 | -0.0968 | -0.2359 | **V8** | 1.0235 | 1.0210 | 1.0279 | 1.0195 |
| $\theta 10$ | -0.1708 | -0.2415 | -0.1233 | -0.2693 | **V9** | 1.0269 | 1.0219 | 1.0346 | 1.0324 |
| $\theta 11$ | -0.0998 | -0.2154 | -0.0579 | -0.2366 | **V10** | 1.0184 | 1.0141 | 1.0265 | 1.0247 |
| $\theta 12$ | -0.2179 | -0.2442 | -0.1433 | -0.2599 | **V11** | 1.0551 | 1.0477 | 1.0642 | 1.0640 |
| $\theta 13$ | -0.2231 | -0.2420 | -0.1288 | -0.2573 | **V12** | 1.0031 | 1.0062 | 1.0061 | 1.0136 |
| $\theta 14$ | -0.2390 | -0.2628 | -0.1509 | -0.2794 | **V13** | 1.0199 | 1.0193 | 1.0192 | 1.0268 |
| $\theta 15$ | -0.2271 | -0.2621 | -0.1489 | -0.2807 | **V14** | 0.9889 | 0.9978 | 0.9980 | 1.0012 |
| $\theta 16$ | -0.1906 | -0.2428 | -0.1242 | -0.2668 | **V15** | 0.9848 | 0.9869 | 0.9928 | 0.9942 |
| $\theta 17$ | -0.1729 | -0.2413 | -0.1198 | -0.2689 | **V16** | 0.9993 | 0.9924 | 1.0080 | 1.0062 |
| $\theta 18$ | -0.2065 | -0.2605 | -0.1353 | -0.2810 | **V17** | 1.0057 | 0.9988 | 1.0152 | 1.0126 |
| $\theta 19$ | -0.1967 | -0.2583 | -0.1312 | -0.2817 | **V18** | 0.9845 | 0.9830 | 0.9957 | 0.9935 |
| $\theta 20$ | -0.1876 | -0.2502 | -0.1255 | -0.2767 | **V19** | 0.9833 | 0.9787 | 0.9951 | 0.9916 |
| $\theta 21$ | -0.1788 | -0.2547 | -0.1297 | -0.2801 | **V20** | 0.9893 | 0.9836 | 1.0007 | 0.9971 |
| $\theta 22$ | -0.1787 | -0.2546 | -0.1294 | -0.2798 | **V21** | 1.0075 | 1.0016 | 1.0171 | 1.0133 |
| $\theta 23$ | -0.2254 | -0.2754 | -0.1561 | -0.2972 | **V22** | 1.0076 | 1.0018 | 1.0174 | 1.0134 |
| $\theta 24$ | -0.2069 | -0.2681 | -0.1552 | -0.2977 | **V23** | 0.9800 | 0.9791 | 0.9907 | 0.9881 |
| $\theta 25$ | -0.1865 | -0.2467 | -0.1600 | -0.2702 | **V24** | 0.9879 | 0.9831 | 0.9986 | 0.9936 |
| $\theta 26$ | -0.1767 | -0.2341 | -0.1571 | -0.2622 | **V25** | 0.9982 | 1.0005 | 1.0049 | 0.9981 |
| $\theta 27$ | -0.1824 | -0.2449 | -0.1640 | -0.2594 | **V26** | 1.0112 | 1.0152 | 1.0170 | 1.0078 |
| $\theta 28$ | -0.1452 | -0.1852 | -0.1416 | -0.1945 | **V27** | 1.0074 | 1.0125 | 1.0121 | 1.0070 |
| $\theta 29$ | -0.1864 | -0.2627 | -0.1681 | -0.2702 | **V28** | 1.0232 | 1.0205 | 1.0279 | 1.0190 |
| $\theta 30$ | -0.1923 | -0.2659 | -0.1739 | -0.2763 | **V29** | 0.9932 | 1.0002 | 0.9994 | 0.9892 |
| | | | | | **V30** | 0.9790 | 0.9946 | 0.9853 | 0.9747 |

Table 5.275: Estimation Results and Errors of case 3, multi reverse, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0590 | 1.0690 | 1.0619 | 1.0673 | 0.0010 | 0.0090 | 0.0019 | 0.0073 |
| V2 | 1.0450 | N/A | 1.0460 | 1.0542 | 1.0496 | 1.0521 | 0.0010 | 0.0092 | 0.0046 | 0.0071 |
| V3 | 1.0210 | N/A | 1.0266 | 1.0358 | 1.0270 | 1.0333 | 0.0056 | 0.0148 | 0.0060 | 0.0123 |
| V4 | 1.0120 | N/A | 1.0208 | 1.0277 | 1.0200 | 1.0248 | 0.0088 | 0.0157 | 0.0080 | 0.0128 |
| V5 | 1.0100 | N/A | 1.0209 | 1.0248 | 1.0257 | 1.0230 | 0.0109 | 0.0148 | 0.0157 | 0.0130 |
| V6 | 1.0110 | N/A | 1.0241 | 1.0231 | 1.0283 | 1.0217 | 0.0131 | 0.0121 | 0.0173 | 0.0107 |
| V7 | 1.0030 | N/A | 1.0154 | 1.0147 | 1.0201 | 1.0140 | 0.0124 | 0.0117 | 0.0171 | 0.0110 |
| V8 | 1.0100 | N/A | 1.0235 | 1.0210 | 1.0279 | 1.0195 | 0.0135 | 0.0110 | 0.0179 | 0.0095 |
| V9 | 1.0510 | N/A | 1.0269 | 1.0219 | 1.0346 | 1.0324 | 0.0241 | 0.0291 | 0.0164 | 0.0186 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V10 | 1.0450 | N/A | 1.0184 | 1.0141 | 1.0265 | 1.0247 | 0.0266 | 0.0309 | 0.0185 | 0.0203 |
| V11 | 1.0820 | N/A | 1.0551 | 1.0477 | 1.0642 | 1.0640 | 0.0269 | 0.0343 | 0.0178 | 0.0180 |
| V12 | 1.0570 | N/A | 1.0031 | 1.0062 | 1.0061 | 1.0136 | 0.0539 | 0.0508 | 0.0509 | 0.0434 |
| V13 | 1.0710 | N/A | 1.0199 | 1.0193 | 1.0192 | 1.0268 | 0.0511 | 0.0517 | 0.0518 | 0.0442 |
| V14 | 1.0430 | N/A | 0.9889 | 0.9978 | 0.9980 | 1.0012 | 0.0541 | 0.0452 | 0.0450 | 0.0418 |
| V15 | 1.0380 | N/A | 0.9848 | 0.9869 | 0.9928 | 0.9942 | 0.0532 | 0.0511 | 0.0452 | 0.0438 |
| V16 | 1.0450 | N/A | 0.9993 | 0.9924 | 1.0080 | 1.0062 | 0.0457 | 0.0526 | 0.0370 | 0.0388 |
| V17 | 1.0400 | N/A | 1.0057 | 0.9988 | 1.0152 | 1.0126 | 0.0343 | 0.0412 | 0.0248 | 0.0274 |
| V18 | 1.0280 | N/A | 0.9845 | 0.9830 | 0.9957 | 0.9935 | 0.0435 | 0.0450 | 0.0323 | 0.0345 |
| V19 | 1.0260 | N/A | 0.9833 | 0.9787 | 0.9951 | 0.9916 | 0.0427 | 0.0473 | 0.0309 | 0.0344 |
| V20 | 1.0300 | N/A | 0.9893 | 0.9836 | 1.0007 | 0.9971 | 0.0407 | 0.0464 | 0.0293 | 0.0329 |
| V21 | 1.0330 | N/A | 1.0075 | 1.0016 | 1.0171 | 1.0133 | 0.0255 | 0.0314 | 0.0159 | 0.0197 |
| V22 | 1.0340 | N/A | 1.0076 | 1.0018 | 1.0174 | 1.0134 | 0.0264 | 0.0322 | 0.0166 | 0.0206 |
| V23 | 1.0270 | N/A | 0.9800 | 0.9791 | 0.9907 | 0.9881 | 0.0470 | 0.0479 | 0.0363 | 0.0389 |
| V24 | 1.0220 | N/A | 0.9879 | 0.9831 | 0.9986 | 0.9936 | 0.0341 | 0.0389 | 0.0234 | 0.0284 |
| V25 | 1.0180 | N/A | 0.9982 | 1.0005 | 1.0049 | 0.9981 | 0.0198 | 0.0175 | 0.0131 | 0.0199 |
| V26 | 1.0000 | N/A | 1.0112 | 1.0152 | 1.0170 | 1.0078 | 0.0112 | 0.0152 | 0.0170 | 0.0078 |
| V27 | 1.0240 | N/A | 1.0074 | 1.0125 | 1.0121 | 1.0070 | 0.0166 | 0.0115 | 0.0119 | 0.0170 |
| V28 | 1.0070 | N/A | 1.0232 | 1.0205 | 1.0279 | 1.0190 | 0.0162 | 0.0135 | 0.0209 | 0.0120 |
| V29 | 1.0040 | N/A | 0.9932 | 1.0002 | 0.9994 | 0.9892 | 0.0108 | 0.0038 | 0.0046 | 0.0148 |
| V30 | 0.9920 | N/A | 0.9790 | 0.9946 | 0.9853 | 0.9747 | 0.0130 | 0.0026 | 0.0067 | 0.0173 |
| P1 | 2.6096 | 2.6540 | 2.5102 | 2.6690 | 2.5685 | 2.7194 | 0.0994 | 0.0594 | 0.0411 | 0.1098 |
| **P2** | **0.1830** | **-0.1256** | **-0.3515** | **-0.1256** | **-0.2594** | **-0.0816** | **0.5345** | **0.3086** | **0.4424** | **0.2646** |
| P3 | -0.0240 | -0.0774 | -0.1506 | -0.0774 | -0.0987 | 0.0232 | 0.1266 | 0.0534 | 0.0747 | 0.0472 |
| P4 | -0.0760 | -0.1165 | -0.5011 | -0.1165 | -1.4822 | -0.1080 | 0.4251 | 0.0405 | 1.4062 | 0.0320 |
| P5 | -0.9420 | -1.0892 | -1.0550 | -1.0025 | -1.0229 | -1.0288 | 0.1130 | 0.0605 | 0.0809 | 0.0868 |
| P6 | 0.0000 | 0.0719 | 0.2516 | 0.0719 | 0.2875 | 0.0872 | 0.2516 | 0.0719 | 0.2875 | 0.0872 |
| P7 | -0.2280 | -0.2117 | -0.0823 | -0.1074 | -0.0625 | -0.1516 | 0.1457 | 0.1206 | 0.1655 | 0.0764 |
| P8 | -0.3000 | -0.3377 | -0.1766 | -0.3377 | -0.1456 | -0.3241 | 0.1234 | 0.0377 | 0.1544 | 0.0241 |
| P9 | 0.0000 | 0.0685 | 0.2458 | 0.0685 | 0.2815 | 0.0695 | 0.2458 | 0.0685 | 0.2815 | 0.0695 |
| P10 | -0.0580 | -0.1436 | 0.0024 | 0.0893 | 0.0286 | -0.0817 | 0.0604 | 0.1473 | 0.0866 | 0.0237 |
| P11 | 0.0000 | -0.0051 | 0.1711 | -0.0051 | 0.2059 | -0.0040 | 0.1711 | 0.0051 | 0.2059 | 0.0040 |
| P12 | -0.1120 | -0.1241 | -0.1786 | -0.1241 | -0.0335 | -0.1205 | 0.0666 | 0.0121 | 0.0785 | 0.0085 |
| P13 | 0.0000 | 0.0160 | -0.0382 | 0.0160 | 0.1065 | 0.0196 | 0.0382 | 0.0160 | 0.1065 | 0.0196 |
| P14 | -0.0620 | -0.0464 | -0.1037 | -0.0464 | -0.0279 | -0.0610 | 0.0417 | 0.0156 | 0.0341 | 0.0010 |
| P15 | -0.0820 | -0.1252 | -0.1641 | -0.1252 | -0.1055 | -0.1275 | 0.0821 | 0.0432 | 0.0235 | 0.0455 |
| P16 | -0.0350 | -0.0365 | 0.0117 | -0.0365 | 0.0521 | -0.0419 | 0.0467 | 0.0015 | 0.0871 | 0.0069 |
| P17 | -0.0900 | -0.0982 | 0.0206 | -0.0431 | 0.0238 | -0.0458 | 0.1106 | 0.0469 | 0.1138 | 0.0442 |
| P18 | -0.0320 | -0.0006 | 0.0180 | -0.0006 | 0.0314 | 0.0080 | 0.0500 | 0.0314 | 0.0634 | 0.0400 |
| P19 | -0.0950 | -0.0403 | -0.0832 | -0.1212 | -0.0766 | -0.1011 | 0.0118 | 0.0262 | 0.0184 | 0.0061 |
| P20 | -0.0220 | 0.0335 | 0.0208 | 0.0335 | 0.0452 | 0.0120 | 0.0428 | 0.0555 | 0.0672 | 0.0340 |
| P21 | -0.1750 | -0.2182 | -0.1515 | -0.2182 | -0.1357 | -0.1943 | 0.0235 | 0.0432 | 0.0393 | 0.0193 |
| P22 | 0.0000 | 0.0039 | 0.0949 | 0.0039 | 0.1069 | 0.0465 | 0.0949 | 0.0039 | 0.1069 | 0.0465 |
| P23 | -0.0320 | -0.0927 | -0.0677 | -0.0927 | -0.0461 | -0.0828 | 0.0357 | 0.0607 | 0.0141 | 0.0508 |
| P24 | -0.0870 | -0.1427 | -0.1531 | -0.1427 | -0.1335 | -0.1821 | 0.0661 | 0.0557 | 0.0465 | 0.0951 |
| P25 | 0.0000 | -0.0003 | -0.0079 | -0.0003 | -0.0217 | -0.0156 | 0.0079 | 0.0003 | 0.0217 | 0.0156 |
| P26 | -0.0350 | 0.0416 | 0.0342 | 0.0416 | 0.0203 | 0.0265 | 0.0692 | 0.0766 | 0.0553 | 0.0615 |
| P27 | 0.0000 | -0.0385 | -0.0090 | -0.0385 | -0.0078 | -0.0277 | 0.0090 | 0.0385 | 0.0078 | 0.0277 |
| P28 | 0.0000 | 0.0186 | 0.0890 | 0.0186 | 0.0963 | 0.0253 | 0.0890 | 0.0186 | 0.0963 | 0.0253 |
| P29 | -0.0240 | -0.0353 | 0.0014 | -0.0353 | 0.0024 | -0.0142 | 0.0254 | 0.0113 | 0.0264 | 0.0098 |
| P30 | -0.1060 | -0.0501 | -0.0544 | -0.0501 | -0.0536 | -0.0661 | 0.0516 | 0.0559 | 0.0524 | 0.0399 |
| Q1 | -0.2042 | -0.2587 | -0.2668 | -0.2587 | -0.2695 | -0.2568 | 0.0626 | 0.0545 | 0.0653 | 0.0526 |
| Q2 | 0.4337 | 0.4353 | 0.4229 | 0.4353 | 0.4195 | 0.4312 | 0.0108 | 0.0016 | 0.0142 | 0.0025 |
| Q3 | -0.0120 | 0.0156 | -0.0212 | 0.0156 | -0.0186 | -0.0107 | 0.0092 | 0.0276 | 0.0066 | 0.0013 |
| Q4 | -0.0160 | 0.0390 | -0.0366 | 0.0390 | -0.0394 | -0.0255 | 0.0206 | 0.0550 | 0.0234 | 0.0095 |
| Q5 | 0.1666 | 0.2438 | 0.2528 | 0.2438 | 0.2465 | 0.2464 | 0.0862 | 0.0772 | 0.0799 | 0.0798 |
| Q6 | 0.0000 | 0.0043 | -0.0130 | 0.0043 | -0.0173 | -0.0064 | 0.0130 | 0.0043 | 0.0173 | 0.0064 |
| Q7 | -0.1090 | -0.1836 | -0.1550 | -0.1836 | -0.1580 | -0.1483 | 0.0460 | 0.0746 | 0.0490 | 0.0393 |
| Q8 | 0.0611 | 0.0240 | 0.0109 | 0.0240 | 0.0067 | 0.0173 | 0.0502 | 0.0371 | 0.0544 | 0.0438 |
| Q9 | 0.0000 | -0.0531 | -0.0363 | -0.0531 | -0.0263 | -0.0193 | 0.0363 | 0.0531 | 0.0263 | 0.0193 |
| Q10 | -0.0200 | 0.0975 | 0.0530 | 0.0975 | 0.0665 | 0.0822 | 0.0730 | 0.1175 | 0.0865 | 0.1022 |
| Q11 | 0.1606 | 0.1298 | 0.1458 | 0.1298 | 0.1552 | 0.1617 | 0.0148 | 0.0308 | 0.0054 | 0.0011 |
| Q12 | -0.0750 | -0.0376 | -0.0091 | -0.0376 | -0.0368 | -0.0352 | 0.0659 | 0.0374 | 0.0382 | 0.0398 |
| Q13 | 0.1045 | 0.0949 | 0.1225 | 0.0949 | 0.0957 | 0.0972 | 0.0180 | 0.0096 | 0.0088 | 0.0073 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q14 | -0.0160 | 0.0284 | 0.0265 | 0.0284 | 0.0023 | 0.0042 | 0.0425 | 0.0444 | 0.0183 | 0.0202 |
| Q15 | -0.0250 | -0.0632 | -0.0587 | -0.0632 | -0.0719 | -0.0693 | 0.0337 | 0.0382 | 0.0469 | 0.0443 |
| Q16 | -0.0180 | -0.0881 | -0.0744 | -0.0881 | -0.0584 | -0.0504 | 0.0564 | 0.0701 | 0.0404 | 0.0324 |
| Q17 | -0.0580 | -0.1291 | -0.1147 | -0.1291 | -0.1033 | -0.0947 | 0.0567 | 0.0711 | 0.0453 | 0.0367 |
| Q18 | -0.0090 | 0.0154 | 0.0002 | 0.0154 | 0.0029 | 0.0070 | 0.0092 | 0.0244 | 0.0119 | 0.0160 |
| Q19 | -0.0340 | -0.0429 | -0.0523 | -0.0429 | -0.0479 | -0.0436 | 0.0183 | 0.0089 | 0.0139 | 0.0096 |
| Q20 | -0.0070 | -0.0168 | -0.0664 | -0.0924 | -0.0664 | -0.0611 | 0.0594 | 0.0854 | 0.0594 | 0.0541 |
| Q21 | -0.1120 | -0.0410 | -0.0830 | -0.0734 | -0.0763 | -0.0667 | 0.0290 | 0.0386 | 0.0357 | 0.0453 |
| Q22 | 0.0000 | 0.0146 | -0.0232 | 0.0146 | -0.0171 | -0.0014 | 0.0232 | 0.0146 | 0.0171 | 0.0014 |
| Q23 | -0.0160 | -0.0061 | -0.0187 | -0.0061 | -0.0160 | -0.0087 | 0.0027 | 0.0099 | 0.0000 | 0.0073 |
| Q24 | -0.0670 | 0.0124 | -0.0452 | -0.0897 | -0.0478 | -0.0300 | 0.0218 | 0.0227 | 0.0192 | 0.0370 |
| Q25 | 0.0000 | -0.0402 | -0.0401 | -0.0402 | -0.0333 | -0.0441 | 0.0401 | 0.0402 | 0.0333 | 0.0441 |
| Q26 | -0.0230 | 0.0118 | 0.0119 | 0.0118 | 0.0188 | 0.0080 | 0.0349 | 0.0348 | 0.0418 | 0.0310 |
| Q27 | 0.0000 | 0.0418 | 0.0415 | 0.0418 | 0.0446 | 0.0415 | 0.0415 | 0.0418 | 0.0446 | 0.0415 |
| Q28 | 0.0000 | -0.0122 | -0.0016 | -0.0122 | -0.0046 | 0.0008 | 0.0016 | 0.0122 | 0.0046 | 0.0008 |
| Q29 | -0.0090 | 0.0018 | -0.0033 | 0.0018 | -0.0007 | -0.0032 | 0.0057 | 0.0108 | 0.0083 | 0.0058 |
| Q30 | -0.0190 | -0.0773 | -0.0480 | -0.0150 | -0.0461 | -0.0479 | 0.0290 | 0.0040 | 0.0271 | 0.0289 |
| P1-2 | 1.7331 | N/A | 1.7316 | 1.8217 | 1.7172 | 1.8673 | 0.0015 | 0.0886 | 0.0159 | 0.1342 |
| P1-3 | 0.8765 | 0.8817 | 0.7786 | 0.8473 | 0.8512 | 0.8521 | 0.0979 | 0.0292 | 0.0253 | 0.0244 |
| **P2-4** | **0.4365** | **-0.4726** | **0.2960** | **0.3589** | **0.3957** | **0.3730** | **0.1405** | **0.0776** | **0.0408** | **0.0635** |
| P3-4 | 0.8214 | 0.9507 | 0.6035 | 0.7415 | 0.7234 | 0.8464 | 0.2179 | 0.0799 | 0.0980 | 0.0250 |
| P2-5 | 0.8236 | 0.7903 | 0.7219 | 0.7841 | 0.7001 | 0.8233 | 0.1017 | 0.0395 | 0.1235 | 0.0003 |
| P2-6 | 0.6038 | N/A | 0.3097 | 0.4962 | 0.3106 | 0.5297 | 0.2941 | 0.1076 | 0.2932 | 0.0741 |
| **P4-6** | **0.7213** | **-0.7172** | **0.0754** | **0.5987** | **-0.3340** | **0.6794** | **0.6459** | **0.1226** | **1.0553** | **0.0419** |
| P5-7 | -0.1478 | -0.2445 | -0.3556 | -0.2445 | -0.3438 | -0.2345 | 0.2078 | 0.0967 | 0.1960 | 0.0867 |
| P6-7 | 0.3813 | 0.3594 | 0.4505 | 0.3594 | 0.4176 | 0.3938 | 0.0692 | 0.0219 | 0.0363 | 0.0125 |
| P6-8 | 0.2956 | N/A | 0.1541 | 0.3194 | 0.1184 | 0.3086 | 0.1415 | 0.0238 | 0.1772 | 0.0130 |
| **P6-9** | **0.2772** | **-0.3192** | **-0.0549** | **0.1921** | **-0.2324** | **0.2561** | **0.3321** | **0.0851** | **0.5096** | **0.0211** |
| P6-10 | 0.1584 | N/A | 0.0511 | 0.1218 | -0.0361 | 0.1579 | 0.1073 | 0.0366 | 0.1945 | 0.0005 |
| P9-11 | 0.0000 | N/A | -0.1711 | 0.0051 | -0.2059 | 0.0040 | 0.1711 | 0.0051 | 0.2059 | 0.0040 |
| P9-10 | 0.2772 | 0.2500 | 0.3620 | 0.2555 | 0.2550 | 0.3215 | 0.0848 | 0.0217 | 0.0222 | 0.0443 |
| P4-12 | 0.4419 | N/A | 0.3137 | 0.3717 | -0.0439 | 0.4157 | 0.1282 | 0.0702 | 0.4858 | 0.0262 |
| P12-13 | 0.0000 | N/A | 0.0382 | -0.0160 | -0.1065 | -0.0196 | 0.0382 | 0.0160 | 0.1065 | 0.0196 |
| P12-14 | 0.0786 | 0.1031 | 0.0887 | 0.0726 | 0.0366 | 0.0822 | 0.0101 | 0.0060 | 0.0420 | 0.0036 |
| P12-15 | 0.1789 | 0.2159 | 0.1123 | 0.1695 | 0.0755 | 0.1890 | 0.0666 | 0.0094 | 0.1034 | 0.0101 |
| P12-16 | 0.0724 | N/A | -0.1041 | 0.0215 | -0.0829 | 0.0437 | 0.1765 | 0.0509 | 0.1553 | 0.0287 |
| P14-15 | 0.0158 | N/A | -0.0160 | 0.0256 | 0.0085 | 0.0204 | 0.0318 | 0.0098 | 0.0073 | 0.0046 |
| P16-17 | 0.0369 | -0.0700 | -0.0939 | -0.0154 | -0.0315 | 0.0017 | 0.1308 | 0.0523 | 0.0684 | 0.0352 |
| P15-18 | 0.0602 | 0.0182 | -0.0725 | 0.0010 | -0.0544 | 0.0024 | 0.1327 | 0.0592 | 0.1146 | 0.0578 |
| P18-19 | 0.0278 | 0.0955 | -0.0552 | 0.0004 | -0.0234 | 0.0104 | 0.0830 | 0.0274 | 0.0512 | 0.0174 |
| P19-20 | -0.0673 | -0.1209 | -0.1388 | -0.1209 | -0.1001 | -0.0907 | 0.0715 | 0.0536 | 0.0328 | 0.0234 |
| P10-20 | 0.0903 | 0.1383 | 0.1207 | 0.0897 | 0.0564 | 0.0805 | 0.0304 | 0.0006 | 0.0339 | 0.0098 |
| P10-17 | 0.0533 | 0.0595 | 0.0745 | 0.0595 | 0.0084 | 0.0448 | 0.0212 | 0.0062 | 0.0449 | 0.0085 |
| P10-21 | 0.1579 | 0.2297 | 0.1474 | 0.2126 | 0.1231 | 0.1830 | 0.0105 | 0.0547 | 0.0348 | 0.0251 |
| P10-22 | 0.0762 | N/A | 0.0729 | 0.1048 | 0.0595 | 0.0893 | 0.0033 | 0.0286 | 0.0167 | 0.0131 |
| P21-22 | -0.0183 | -0.0282 | -0.0050 | -0.0073 | -0.0132 | -0.0125 | 0.0133 | 0.0110 | 0.0051 | 0.0058 |
| P15-23 | 0.0504 | N/A | 0.0030 | 0.0664 | 0.0322 | 0.0769 | 0.0474 | 0.0160 | 0.0182 | 0.0265 |
| P22-24 | 0.0574 | 0.2028 | 0.1624 | 0.1005 | 0.1529 | 0.1227 | 0.1050 | 0.0431 | 0.0955 | 0.0653 |
| P23-24 | 0.0180 | 0.0593 | -0.0647 | -0.0267 | -0.0140 | -0.0065 | 0.0827 | 0.0447 | 0.0320 | 0.0245 |
| P24-25 | -0.0121 | N/A | -0.0590 | -0.0704 | 0.0028 | -0.0678 | 0.0469 | 0.0583 | 0.0149 | 0.0557 |
| P25-26 | 0.0354 | N/A | -0.0338 | -0.0411 | -0.0201 | -0.0264 | 0.0692 | 0.0765 | 0.0555 | 0.0618 |
| P25-27 | -0.0476 | -0.0710 | -0.0337 | -0.0305 | 0.0011 | -0.0581 | 0.0139 | 0.0171 | 0.0487 | 0.0105 |
| P28-27 | 0.1807 | 0.1671 | 0.0969 | 0.1557 | 0.0589 | 0.1681 | 0.0838 | 0.0250 | 0.1218 | 0.0126 |
| P27-29 | 0.0619 | N/A | 0.0217 | 0.0464 | 0.0206 | 0.0380 | 0.0402 | 0.0155 | 0.0413 | 0.0239 |
| P27-30 | 0.0709 | 0.0570 | 0.0324 | 0.0400 | 0.0315 | 0.0439 | 0.0385 | 0.0309 | 0.0394 | 0.0270 |
| P29-30 | 0.0370 | 0.0721 | 0.0229 | 0.0107 | 0.0228 | 0.0233 | 0.0141 | 0.0263 | 0.0142 | 0.0137 |
| P8-28 | -0.0054 | N/A | -0.0227 | -0.0195 | -0.0273 | -0.0165 | 0.0173 | 0.0141 | 0.0219 | 0.0111 |
| P6-28 | 0.1867 | 0.1690 | 0.0306 | 0.1571 | -0.0100 | 0.1597 | 0.1561 | 0.0296 | 0.1967 | 0.0270 |
| P2-1 | -1.6809 | N/A | -1.6792 | -1.7649 | -1.6659 | -1.8075 | 0.0017 | 0.0840 | 0.0150 | 0.1266 |
| P3-1 | -0.8454 | N/A | -0.7541 | -0.8189 | -0.8221 | -0.8232 | 0.0913 | 0.0265 | 0.0233 | 0.0222 |
| P4-2 | -0.4263 | N/A | -0.2913 | -0.3522 | -0.3874 | -0.3657 | 0.1350 | 0.0741 | 0.0389 | 0.0606 |
| P4-3 | -0.8129 | N/A | -0.5990 | -0.7347 | -0.7168 | -0.8375 | 0.2139 | 0.0782 | 0.0961 | 0.0246 |
| P5-2 | -0.7942 | N/A | -0.6994 | -0.7580 | -0.6791 | -0.7944 | 0.0948 | 0.0362 | 0.1151 | 0.0002 |
| P6-2 | -0.5843 | N/A | -0.3046 | -0.4833 | -0.3055 | -0.5149 | 0.2797 | 0.1010 | 0.2788 | 0.0694 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P6-4 | -0.7150 | N/A | -0.0753 | -0.5946 | 0.3354 | -0.6740 | 0.6397 | 0.1204 | 1.0504 | 0.0410 |
| P7-5 | 0.1495 | N/A | 0.3630 | 0.2487 | 0.3506 | 0.2383 | 0.2135 | 0.0992 | 0.2011 | 0.0888 |
| P7-6 | -0.3775 | N/A | -0.4453 | -0.3561 | -0.4132 | -0.3899 | 0.0678 | 0.0214 | 0.0357 | 0.0124 |
| P8-6 | -0.2946 | N/A | -0.1539 | -0.3182 | -0.1183 | -0.3075 | 0.1407 | 0.0236 | 0.1763 | 0.0129 |
| P9-6 | -0.2772 | N/A | 0.0549 | -0.1921 | 0.2324 | -0.2561 | 0.3321 | 0.0851 | 0.5096 | 0.0211 |
| P10-6 | -0.1584 | N/A | -0.0511 | -0.1218 | 0.0361 | -0.1579 | 0.1073 | 0.0366 | 0.1945 | 0.0005 |
| P11-9 | 0.0000 | N/A | 0.1711 | -0.0051 | 0.2059 | -0.0040 | 0.1711 | 0.0051 | 0.2059 | 0.0040 |
| P10-9 | -0.2772 | N/A | -0.3620 | -0.2555 | -0.2550 | -0.3215 | 0.0848 | 0.0217 | 0.0222 | 0.0443 |
| P12-4 | -0.4419 | N/A | -0.3137 | -0.3717 | 0.0439 | -0.4157 | 0.1282 | 0.0702 | 0.4858 | 0.0262 |
| P13-12 | 0.0000 | N/A | -0.0382 | 0.0160 | 0.1065 | 0.0196 | 0.0382 | 0.0160 | 0.1065 | 0.0196 |
| P14-12 | -0.0778 | N/A | -0.0877 | -0.0720 | -0.0364 | -0.0813 | 0.0099 | 0.0058 | 0.0414 | 0.0035 |
| P15-12 | -0.1767 | N/A | -0.1110 | -0.1674 | -0.0748 | -0.1865 | 0.0657 | 0.0093 | 0.1019 | 0.0098 |
| P16-12 | -0.0719 | N/A | 0.1056 | -0.0211 | 0.0837 | -0.0435 | 0.1775 | 0.0508 | 0.1556 | 0.0284 |
| P15-14 | -0.0158 | N/A | 0.0164 | -0.0253 | -0.0084 | -0.0203 | 0.0322 | 0.0095 | 0.0074 | 0.0045 |
| P17-16 | -0.0368 | N/A | 0.0944 | 0.0154 | 0.0316 | -0.0016 | 0.1312 | 0.0522 | 0.0684 | 0.0352 |
| P18-15 | -0.0598 | N/A | 0.0733 | -0.0010 | 0.0548 | -0.0024 | 0.1331 | 0.0588 | 0.1146 | 0.0574 |
| P19-18 | -0.0277 | N/A | 0.0555 | -0.0003 | 0.0234 | -0.0104 | 0.0832 | 0.0274 | 0.0511 | 0.0173 |
| P20-19 | 0.0674 | N/A | 0.1395 | 0.1214 | 0.1004 | 0.0911 | 0.0721 | 0.0540 | 0.0330 | 0.0237 |
| P20-10 | -0.0894 | N/A | -0.1187 | -0.0879 | -0.0552 | -0.0791 | 0.0293 | 0.0015 | 0.0342 | 0.0103 |
| P17-10 | -0.0532 | N/A | -0.0738 | -0.0586 | -0.0078 | -0.0442 | 0.0206 | 0.0054 | 0.0454 | 0.0090 |
| P21-10 | -0.1567 | N/A | -0.1465 | -0.2109 | -0.1224 | -0.1818 | 0.0102 | 0.0542 | 0.0343 | 0.0251 |
| P22-10 | -0.0757 | N/A | -0.0725 | -0.1039 | -0.0592 | -0.0887 | 0.0032 | 0.0282 | 0.0165 | 0.0130 |
| P22-21 | 0.0183 | N/A | 0.0050 | 0.0073 | 0.0132 | 0.0125 | 0.0133 | 0.0110 | 0.0051 | 0.0058 |
| P23-15 | -0.0500 | N/A | -0.0030 | -0.0660 | -0.0321 | -0.0763 | 0.0470 | 0.0160 | 0.0179 | 0.0263 |
| P24-22 | -0.0569 | N/A | -0.1594 | -0.0992 | -0.1503 | -0.1209 | 0.1025 | 0.0423 | 0.0934 | 0.0640 |
| P24-23 | -0.0180 | N/A | 0.0653 | 0.0268 | 0.0141 | 0.0066 | 0.0833 | 0.0448 | 0.0321 | 0.0246 |
| P25-24 | 0.0122 | N/A | 0.0596 | 0.0714 | -0.0027 | 0.0688 | 0.0474 | 0.0592 | 0.0149 | 0.0566 |
| P26-25 | -0.0350 | N/A | 0.0342 | 0.0416 | 0.0203 | 0.0265 | 0.0692 | 0.0766 | 0.0553 | 0.0615 |
| P27-25 | 0.0479 | N/A | 0.0339 | 0.0308 | -0.0010 | 0.0584 | 0.0140 | 0.0171 | 0.0489 | 0.0105 |
| P27-28 | -0.1807 | N/A | -0.0969 | -0.1557 | -0.0589 | -0.1681 | 0.0838 | 0.0250 | 0.1218 | 0.0126 |
| P29-27 | -0.0610 | N/A | -0.0215 | -0.0460 | -0.0204 | -0.0376 | 0.0395 | 0.0150 | 0.0406 | 0.0234 |
| P30-27 | -0.0693 | N/A | -0.0317 | -0.0395 | -0.0310 | -0.0430 | 0.0376 | 0.0298 | 0.0383 | 0.0263 |
| P30-29 | -0.0367 | N/A | -0.0226 | -0.0106 | -0.0226 | -0.0231 | 0.0141 | 0.0261 | 0.0141 | 0.0136 |
| P28-8 | 0.0055 | N/A | 0.0227 | 0.0195 | 0.0274 | 0.0165 | 0.0172 | 0.0140 | 0.0219 | 0.0110 |
| P28-6 | -0.1862 | N/A | -0.0306 | -0.1567 | 0.0100 | -0.1593 | 0.1556 | 0.0295 | 0.1962 | 0.0269 |
| Q1-2 | -0.2470 | N/A | -0.2831 | -0.2707 | -0.2915 | -0.2735 | 0.0361 | 0.0237 | 0.0445 | 0.0265 |
| Q1-3 | 0.0428 | 0.0595 | 0.0164 | 0.0120 | 0.0221 | 0.0166 | 0.0264 | 0.0308 | 0.0207 | 0.0262 |
| Q2-4 | 0.0475 | 0.0671 | 0.0407 | 0.0317 | 0.0398 | 0.0327 | 0.0068 | 0.0158 | 0.0077 | 0.0148 |
| Q3-4 | -0.0385 | -0.0159 | -0.0500 | -0.0312 | -0.0584 | -0.0545 | 0.0115 | 0.0073 | 0.0199 | 0.0160 |
| Q2-5 | 0.0278 | 0.0213 | -0.0139 | 0.0021 | -0.0182 | -0.0030 | 0.0417 | 0.0257 | 0.0460 | 0.0308 |
| Q2-6 | 0.0137 | N/A | 0.0145 | 0.0205 | 0.0113 | 0.0084 | 0.0008 | 0.0068 | 0.0024 | 0.0053 |
| Q4-6 | -0.1591 | -0.1829 | -0.1084 | -0.0551 | -0.1095 | -0.1128 | 0.0507 | 0.1040 | 0.0496 | 0.0463 |
| Q5-7 | 0.1149 | 0.1580 | 0.1891 | 0.1813 | 0.1851 | 0.1669 | 0.0742 | 0.0664 | 0.0702 | 0.0520 |
| Q6-7 | -0.0278 | -0.0959 | -0.0384 | -0.0157 | -0.0354 | -0.0356 | 0.0106 | 0.0121 | 0.0076 | 0.0078 |
| Q6-8 | -0.0720 | N/A | -0.0331 | -0.0427 | -0.0296 | -0.0375 | 0.0389 | 0.0293 | 0.0424 | 0.0345 |
| Q6-9 | -0.0809 | -0.1233 | -0.0132 | 0.0097 | -0.0262 | -0.0464 | 0.0677 | 0.0906 | 0.0547 | 0.0345 |
| Q6-10 | 0.0019 | N/A | 0.0113 | 0.0205 | 0.0036 | 0.0010 | 0.0094 | 0.0186 | 0.0017 | 0.0009 |
| Q9-11 | -0.1560 | N/A | -0.1363 | -0.1266 | -0.1430 | -0.1568 | 0.0197 | 0.0294 | 0.0130 | 0.0008 |
| Q9-10 | 0.0588 | 0.1108 | 0.0862 | 0.0758 | 0.0796 | 0.0777 | 0.0274 | 0.0170 | 0.0208 | 0.0189 |
| Q4-12 | 0.1441 | N/A | 0.0829 | 0.1034 | 0.0556 | 0.0662 | 0.0612 | 0.0407 | 0.0885 | 0.0779 |
| Q12-13 | -0.1032 | N/A | -0.1203 | -0.0937 | -0.0929 | -0.0959 | 0.0171 | 0.0095 | 0.0103 | 0.0073 |
| Q12-14 | 0.0240 | 0.0570 | 0.0138 | -0.0010 | 0.0146 | 0.0102 | 0.0102 | 0.0250 | 0.0094 | 0.0138 |
| Q12-15 | 0.0679 | 0.0645 | 0.0838 | 0.0645 | 0.0649 | 0.0565 | 0.0159 | 0.0034 | 0.0030 | 0.0114 |
| Q12-16 | 0.0335 | N/A | 0.0707 | 0.0599 | 0.0309 | 0.0170 | 0.0372 | 0.0264 | 0.0026 | 0.0165 |
| Q14-15 | 0.0065 | N/A | 0.0382 | 0.0261 | 0.0166 | 0.0127 | 0.0317 | 0.0196 | 0.0101 | 0.0062 |
| Q16-17 | 0.0144 | -0.0008 | -0.0068 | -0.0290 | -0.0290 | -0.0338 | 0.0212 | 0.0434 | 0.0434 | 0.0482 |
| Q15-18 | 0.0160 | 0.0172 | 0.0381 | 0.0172 | 0.0140 | 0.0021 | 0.0221 | 0.0012 | 0.0020 | 0.0139 |
| Q18-19 | 0.0062 | 0.0088 | 0.0367 | 0.0325 | 0.0162 | 0.0091 | 0.0305 | 0.0263 | 0.0100 | 0.0029 |
| Q19-20 | -0.0279 | 0.0134 | -0.0162 | -0.0105 | -0.0318 | -0.0345 | 0.0117 | 0.0174 | 0.0039 | 0.0066 |
| Q10-20 | 0.0371 | 0.1134 | 0.0885 | 0.1080 | 0.1016 | 0.0995 | 0.0514 | 0.0709 | 0.0645 | 0.0624 |
| Q10-17 | 0.0443 | 0.0676 | 0.1249 | 0.1607 | 0.1342 | 0.1302 | 0.0806 | 0.1164 | 0.0899 | 0.0859 |
| Q10-21 | 0.1001 | 0.0896 | 0.0804 | 0.0719 | 0.0722 | 0.0718 | 0.0197 | 0.0282 | 0.0279 | 0.0283 |
| Q10-22 | 0.0460 | N/A | 0.0379 | 0.0331 | 0.0339 | 0.0344 | 0.0081 | 0.0129 | 0.0121 | 0.0116 |
| Q21-22 | -0.0143 | -0.0051 | -0.0046 | -0.0051 | -0.0056 | 0.0023 | 0.0097 | 0.0092 | 0.0087 | 0.0166 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q15-23 | 0.0291 | N/A | 0.0223 | 0.0057 | -0.0058 | -0.0073 | 0.0068 | 0.0234 | 0.0349 | 0.0364 |
| Q22-24 | 0.0306 | 0.0781 | 0.0091 | 0.0408 | 0.0105 | 0.0340 | 0.0215 | 0.0102 | 0.0201 | 0.0034 |
| Q23-24 | 0.0124 | 0.0278 | 0.0035 | -0.0013 | -0.0219 | -0.0172 | 0.0089 | 0.0137 | 0.0343 | 0.0296 |
| Q24-25 | 0.0201 | N/A | 0.0035 | -0.0110 | -0.0206 | 0.0264 | 0.0166 | 0.0311 | 0.0407 | 0.0063 |
| Q25-26 | 0.0237 | N/A | -0.0114 | -0.0111 | -0.0186 | -0.0077 | 0.0351 | 0.0348 | 0.0423 | 0.0314 |
| Q25-27 | -0.0037 | 0.0094 | -0.0264 | -0.0418 | -0.0355 | -0.0118 | 0.0227 | 0.0381 | 0.0318 | 0.0081 |
| Q28-27 | 0.0504 | 0.0033 | 0.0428 | 0.0252 | 0.0417 | 0.0364 | 0.0076 | 0.0252 | 0.0087 | 0.0140 |
| Q27-29 | 0.0167 | N/A | 0.0229 | 0.0059 | 0.0202 | 0.0233 | 0.0062 | 0.0108 | 0.0035 | 0.0066 |
| Q27-30 | 0.0166 | 0.0093 | 0.0304 | 0.0093 | 0.0284 | 0.0308 | 0.0138 | 0.0073 | 0.0118 | 0.0142 |
| Q29-30 | 0.0061 | -0.0205 | 0.0192 | 0.0068 | 0.0191 | 0.0192 | 0.0131 | 0.0007 | 0.0130 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0138 | -0.0134 | -0.0140 | -0.0147 | 0.0084 | 0.0080 | 0.0086 | 0.0093 |
| Q6-28 | 0.0011 | -0.0427 | -0.0002 | -0.0058 | 0.0015 | -0.0062 | 0.0013 | 0.0069 | 0.0004 | 0.0073 |
| Q2-1 | 0.3447 | N/A | 0.3817 | 0.3810 | 0.3865 | 0.3932 | 0.0370 | 0.0363 | 0.0418 | 0.0485 |
| Q3-1 | 0.0265 | N/A | 0.0288 | 0.0468 | 0.0399 | 0.0439 | 0.0023 | 0.0203 | 0.0134 | 0.0174 |
| Q4-2 | -0.0554 | N/A | -0.0655 | -0.0510 | -0.0540 | -0.0501 | 0.0101 | 0.0044 | 0.0014 | 0.0053 |
| Q4-3 | 0.0544 | N/A | 0.0543 | 0.0417 | 0.0686 | 0.0712 | 0.0001 | 0.0127 | 0.0142 | 0.0168 |
| Q5-2 | 0.0517 | N/A | 0.0637 | 0.0625 | 0.0614 | 0.0795 | 0.0120 | 0.0108 | 0.0097 | 0.0278 |
| Q6-2 | 0.0058 | N/A | -0.0389 | -0.0215 | -0.0360 | -0.0038 | 0.0447 | 0.0273 | 0.0418 | 0.0096 |
| Q6-4 | 0.1719 | N/A | 0.0996 | 0.0598 | 0.1049 | 0.1220 | 0.0723 | 0.1121 | 0.0670 | 0.0499 |
| Q7-5 | -0.1313 | N/A | -0.1917 | -0.1918 | -0.1892 | -0.1785 | 0.0604 | 0.0605 | 0.0579 | 0.0472 |
| Q7-6 | 0.0223 | N/A | 0.0367 | 0.0082 | 0.0311 | 0.0302 | 0.0144 | 0.0141 | 0.0088 | 0.0079 |
| Q8-6 | 0.0666 | N/A | 0.0247 | 0.0374 | 0.0207 | 0.0320 | 0.0419 | 0.0292 | 0.0459 | 0.0346 |
| Q9-6 | 0.0972 | N/A | 0.0139 | -0.0023 | 0.0370 | 0.0599 | 0.0833 | 0.0995 | 0.0602 | 0.0373 |
| Q10-6 | 0.0110 | N/A | -0.0098 | -0.0124 | -0.0029 | 0.0123 | 0.0208 | 0.0234 | 0.0139 | 0.0013 |
| Q11-9 | 0.1606 | N/A | 0.1458 | 0.1298 | 0.1552 | 0.1617 | 0.0148 | 0.0308 | 0.0054 | 0.0011 |
| Q10-9 | -0.0508 | N/A | -0.0717 | -0.0683 | -0.0723 | -0.0664 | 0.0209 | 0.0175 | 0.0215 | 0.0156 |
| Q12-4 | -0.0972 | N/A | -0.0570 | -0.0674 | -0.0543 | -0.0230 | 0.0402 | 0.0298 | 0.0429 | 0.0742 |
| Q13-12 | 0.1045 | N/A | 0.1225 | 0.0949 | 0.0957 | 0.0972 | 0.0180 | 0.0096 | 0.0088 | 0.0073 |
| Q14-12 | -0.0225 | N/A | -0.0118 | 0.0023 | -0.0143 | -0.0085 | 0.0107 | 0.0248 | 0.0082 | 0.0140 |
| Q15-12 | -0.0636 | N/A | -0.0812 | -0.0603 | -0.0637 | -0.0515 | 0.0176 | 0.0033 | 0.0001 | 0.0121 |
| Q16-12 | -0.0324 | N/A | -0.0676 | -0.0591 | -0.0293 | -0.0166 | 0.0352 | 0.0267 | 0.0031 | 0.0158 |
| Q15-14 | -0.0064 | N/A | -0.0379 | -0.0258 | -0.0165 | -0.0126 | 0.0315 | 0.0194 | 0.0101 | 0.0062 |
| Q17-16 | -0.0141 | N/A | 0.0085 | 0.0292 | 0.0294 | 0.0340 | 0.0226 | 0.0433 | 0.0435 | 0.0481 |
| Q18-15 | -0.0152 | N/A | -0.0366 | -0.0171 | -0.0133 | -0.0021 | 0.0214 | 0.0019 | 0.0019 | 0.0131 |
| Q19-18 | -0.0061 | N/A | -0.0361 | -0.0324 | -0.0161 | -0.0091 | 0.0300 | 0.0263 | 0.0100 | 0.0030 |
| Q20-19 | 0.0283 | N/A | 0.0176 | 0.0116 | 0.0325 | 0.0351 | 0.0107 | 0.0167 | 0.0042 | 0.0068 |
| Q20-10 | -0.0353 | N/A | -0.0840 | -0.1040 | -0.0989 | -0.0962 | 0.0487 | 0.0687 | 0.0636 | 0.0609 |
| Q17-10 | -0.0439 | N/A | -0.1232 | -0.1583 | -0.1327 | -0.1287 | 0.0793 | 0.1144 | 0.0888 | 0.0848 |
| Q21-10 | -0.0977 | N/A | -0.0783 | -0.0683 | -0.0708 | -0.0690 | 0.0194 | 0.0294 | 0.0269 | 0.0287 |
| Q22-10 | -0.0449 | N/A | -0.0369 | -0.0313 | -0.0332 | -0.0331 | 0.0080 | 0.0136 | 0.0117 | 0.0118 |
| Q22-21 | 0.0143 | N/A | 0.0047 | 0.0051 | 0.0056 | -0.0023 | 0.0096 | 0.0092 | 0.0087 | 0.0166 |
| Q23-15 | -0.0284 | N/A | -0.0222 | -0.0048 | 0.0060 | 0.0085 | 0.0062 | 0.0236 | 0.0344 | 0.0369 |
| Q24-22 | -0.0299 | N/A | -0.0044 | -0.0387 | -0.0065 | -0.0312 | 0.0255 | 0.0088 | 0.0234 | 0.0013 |
| Q24-23 | -0.0123 | N/A | -0.0023 | 0.0015 | 0.0221 | 0.0173 | 0.0100 | 0.0138 | 0.0344 | 0.0296 |
| Q25-24 | -0.0200 | N/A | -0.0023 | 0.0127 | 0.0208 | -0.0246 | 0.0177 | 0.0327 | 0.0408 | 0.0046 |
| Q26-25 | -0.0230 | N/A | 0.0119 | 0.0118 | 0.0188 | 0.0080 | 0.0349 | 0.0348 | 0.0418 | 0.0310 |
| Q27-25 | 0.0042 | N/A | 0.0267 | 0.0424 | 0.0357 | 0.0126 | 0.0225 | 0.0382 | 0.0315 | 0.0084 |
| Q27-28 | -0.0375 | N/A | -0.0385 | -0.0158 | -0.0398 | -0.0251 | 0.0010 | 0.0217 | 0.0023 | 0.0124 |
| Q29-27 | -0.0151 | N/A | -0.0225 | -0.0050 | -0.0198 | -0.0224 | 0.0074 | 0.0101 | 0.0047 | 0.0073 |
| Q30-27 | -0.0136 | N/A | -0.0292 | -0.0083 | -0.0274 | -0.0291 | 0.0156 | 0.0053 | 0.0138 | 0.0155 |
| Q30-29 | -0.0054 | N/A | -0.0188 | -0.0067 | -0.0187 | -0.0188 | 0.0134 | 0.0013 | 0.0133 | 0.0134 |
| Q28-8 | -0.0380 | N/A | -0.0310 | -0.0311 | -0.0311 | -0.0297 | 0.0070 | 0.0069 | 0.0069 | 0.0083 |
| Q28-6 | -0.0123 | N/A | -0.0134 | -0.0063 | -0.0152 | -0.0059 | 0.0011 | 0.0060 | 0.0029 | 0.0064 |
| **SUM** | | | | | | | **15.5771** | **9.2069** | **17.8626** | **7.1332** |

LTS eliminate some bad data and keep the bad data, making it generate worst estimation. LTAV successfully eliminated some bad data based on LAV techniques and generate best estimation.

#### 5.3.4.2 Four non-conforming bad data

Two large and two reverse bad data consist the non-conforming case: $P_2 = -0.1256$, $P_{2-4} = -0.4726$, $P_{4-6} = 1.2172$ and $P_{6-9} = 0.9192$, all results and comparison are shown in the following two tables.

Table 5.276: State Variable of case 3, non-conforming, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.1023 | -0.0991 | -0.1004 | -0.1005 | **V1** | 1.0689 | 1.0690 | 1.0642 | 1.0665 |
| $\theta 3$ | -0.1161 | -0.1173 | -0.1147 | -0.1209 | **V2** | 1.0525 | 1.0532 | 1.0488 | 1.0509 |
| $\theta 4$ | -0.1498 | -0.1513 | -0.1445 | -0.1555 | **V3** | 1.0381 | 1.0385 | 1.0338 | 1.0338 |
| $\theta 5$ | -0.2560 | -0.2474 | -0.2505 | -0.2531 | **V4** | 1.0282 | 1.0278 | 1.0255 | 1.0240 |
| $\theta 6$ | -0.1884 | -0.1830 | -0.1736 | -0.1860 | **V5** | 1.0234 | 1.0225 | 1.0201 | 1.0216 |
| $\theta 7$ | -0.2207 | -0.2116 | -0.2101 | -0.2186 | **V6** | 1.0222 | 1.0213 | 1.0218 | 1.0203 |
| $\theta 8$ | -0.2003 | -0.1965 | -0.1863 | -0.1992 | **V7** | 1.0147 | 1.0126 | 1.0128 | 1.0126 |
| $\theta 9$ | -0.2854 | -0.2465 | -0.2114 | -0.2405 | **V8** | 1.0205 | 1.0193 | 1.0195 | 1.0180 |
| $\theta 10$ | -0.3139 | -0.2869 | -0.2426 | -0.2744 | **V9** | 1.0359 | 1.0213 | 1.0317 | 1.0313 |
| $\theta 11$ | -0.3124 | -0.2474 | -0.2082 | -0.2428 | **V10** | 1.0272 | 1.0153 | 1.0236 | 1.0237 |
| $\theta 12$ | -0.2649 | -0.2680 | -0.2292 | -0.2640 | **V11** | 1.0712 | 1.0471 | 1.0629 | 1.0632 |
| $\theta 13$ | -0.2560 | -0.2659 | -0.2229 | -0.2621 | **V12** | 1.0152 | 1.0106 | 1.0118 | 1.0129 |
| $\theta 14$ | -0.2825 | -0.2895 | -0.2460 | -0.2842 | **V13** | 1.0246 | 1.0235 | 1.0239 | 1.0263 |
| $\theta 15$ | -0.2930 | -0.2919 | -0.2489 | -0.2855 | **V14** | 1.0032 | 1.0023 | 1.0000 | 1.0004 |
| $\theta 16$ | -0.2954 | -0.2776 | -0.2367 | -0.2720 | **V15** | 0.9948 | 0.9905 | 0.9929 | 0.9933 |
| $\theta 17$ | -0.3114 | -0.2864 | -0.2409 | -0.2742 | **V16** | 1.0064 | 0.9937 | 1.0050 | 1.0052 |
| $\theta 18$ | -0.3080 | -0.2988 | -0.2498 | -0.2863 | **V17** | 1.0148 | 0.9992 | 1.0116 | 1.0115 |
| $\theta 19$ | -0.3163 | -0.3027 | -0.2513 | -0.2872 | **V18** | 0.9946 | 0.9825 | 0.9926 | 0.9924 |
| $\theta 20$ | -0.3142 | -0.2951 | -0.2469 | -0.2821 | **V19** | 0.9931 | 0.9777 | 0.9908 | 0.9905 |
| $\theta 21$ | -0.3260 | -0.2996 | -0.2529 | -0.2854 | **V20** | 0.9988 | 0.9837 | 0.9963 | 0.9960 |
| $\theta 22$ | -0.3254 | -0.2990 | -0.2526 | -0.2851 | **V21** | 1.0155 | 1.0059 | 1.0125 | 1.0122 |
| $\theta 23$ | -0.3209 | -0.3093 | -0.2670 | -0.3024 | **V22** | 1.0155 | 1.0063 | 1.0126 | 1.0123 |
| $\theta 24$ | -0.3326 | -0.3078 | -0.2710 | -0.3028 | **V23** | 0.9890 | 0.9860 | 0.9872 | 0.9870 |
| $\theta 25$ | -0.2879 | -0.2698 | -0.2517 | -0.2739 | **V24** | 0.9953 | 0.9943 | 0.9930 | 0.9924 |
| $\theta 26$ | -0.2760 | -0.2575 | -0.2459 | -0.2657 | **V25** | 0.9991 | 1.0096 | 0.9977 | 0.9968 |
| $\theta 27$ | -0.2704 | -0.2579 | -0.2436 | -0.2625 | **V26** | 1.0083 | 1.0242 | 1.0074 | 1.0064 |
| $\theta 28$ | -0.1974 | -0.1930 | -0.1824 | -0.1957 | **V27** | 1.0085 | 1.0204 | 1.0064 | 1.0058 |
| $\theta 29$ | -0.2797 | -0.2687 | -0.2546 | -0.2736 | **V28** | 1.0198 | 1.0195 | 1.0191 | 1.0176 |
| $\theta 30$ | -0.2858 | -0.2744 | -0.2608 | -0.2798 | **V29** | 0.9899 | 1.0113 | 0.9889 | 0.9877 |
| | | | | | **V30** | 0.9755 | 1.0038 | 0.9744 | 0.9733 |

Table 5.277: Estimation Results and Errors of case 3, non-conforming, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0689 | 1.0690 | 1.0642 | 1.0665 | 0.0089 | 0.0090 | 0.0042 | 0.0065 |
| V2 | 1.0450 | N/A | 1.0525 | 1.0532 | 1.0488 | 1.0509 | 0.0075 | 0.0082 | 0.0038 | 0.0059 |
| V3 | 1.0210 | N/A | 1.0381 | 1.0385 | 1.0338 | 1.0338 | 0.0171 | 0.0175 | 0.0128 | 0.0128 |
| V4 | 1.0120 | N/A | 1.0282 | 1.0278 | 1.0255 | 1.0240 | 0.0162 | 0.0158 | 0.0135 | 0.0120 |
| V5 | 1.0100 | N/A | 1.0234 | 1.0225 | 1.0201 | 1.0216 | 0.0134 | 0.0125 | 0.0101 | 0.0116 |
| V6 | 1.0110 | N/A | 1.0222 | 1.0213 | 1.0218 | 1.0203 | 0.0112 | 0.0103 | 0.0108 | 0.0093 |
| V7 | 1.0030 | N/A | 1.0147 | 1.0126 | 1.0128 | 1.0126 | 0.0117 | 0.0096 | 0.0098 | 0.0096 |
| V8 | 1.0100 | N/A | 1.0205 | 1.0193 | 1.0195 | 1.0180 | 0.0105 | 0.0093 | 0.0095 | 0.0080 |
| V9 | 1.0510 | N/A | 1.0359 | 1.0213 | 1.0317 | 1.0313 | 0.0151 | 0.0297 | 0.0193 | 0.0197 |
| V10 | 1.0450 | N/A | 1.0272 | 1.0153 | 1.0236 | 1.0237 | 0.0178 | 0.0297 | 0.0214 | 0.0213 |
| V11 | 1.0820 | N/A | 1.0712 | 1.0471 | 1.0629 | 1.0632 | 0.0108 | 0.0349 | 0.0191 | 0.0188 |
| V12 | 1.0570 | N/A | 1.0152 | 1.0106 | 1.0118 | 1.0129 | 0.0418 | 0.0464 | 0.0452 | 0.0441 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V13 | 1.0710 | N/A | 1.0246 | 1.0235 | 1.0239 | 1.0263 | 0.0464 | 0.0475 | 0.0471 | 0.0447 |
| V14 | 1.0430 | N/A | 1.0032 | 1.0023 | 1.0000 | 1.0004 | 0.0398 | 0.0407 | 0.0430 | 0.0426 |
| V15 | 1.0380 | N/A | 0.9948 | 0.9905 | 0.9929 | 0.9933 | 0.0432 | 0.0475 | 0.0451 | 0.0447 |
| V16 | 1.0450 | N/A | 1.0064 | 0.9937 | 1.0050 | 1.0052 | 0.0386 | 0.0513 | 0.0400 | 0.0398 |
| V17 | 1.0400 | N/A | 1.0148 | 0.9992 | 1.0116 | 1.0115 | 0.0252 | 0.0408 | 0.0284 | 0.0285 |
| V18 | 1.0280 | N/A | 0.9946 | 0.9825 | 0.9926 | 0.9924 | 0.0334 | 0.0455 | 0.0354 | 0.0356 |
| V19 | 1.0260 | N/A | 0.9931 | 0.9777 | 0.9908 | 0.9905 | 0.0329 | 0.0483 | 0.0352 | 0.0355 |
| V20 | 1.0300 | N/A | 0.9988 | 0.9837 | 0.9963 | 0.9960 | 0.0312 | 0.0463 | 0.0337 | 0.0340 |
| V21 | 1.0330 | N/A | 1.0155 | 1.0059 | 1.0125 | 1.0122 | 0.0175 | 0.0271 | 0.0205 | 0.0208 |
| V22 | 1.0340 | N/A | 1.0155 | 1.0063 | 1.0126 | 1.0123 | 0.0185 | 0.0277 | 0.0214 | 0.0217 |
| V23 | 1.0270 | N/A | 0.9890 | 0.9860 | 0.9872 | 0.9870 | 0.0380 | 0.0410 | 0.0398 | 0.0400 |
| V24 | 1.0220 | N/A | 0.9953 | 0.9943 | 0.9930 | 0.9924 | 0.0267 | 0.0277 | 0.0290 | 0.0296 |
| V25 | 1.0180 | N/A | 0.9991 | 1.0096 | 0.9977 | 0.9968 | 0.0189 | 0.0084 | 0.0203 | 0.0212 |
| V26 | 1.0000 | N/A | 1.0083 | 1.0242 | 1.0074 | 1.0064 | 0.0083 | 0.0242 | 0.0074 | 0.0064 |
| V27 | 1.0240 | N/A | 1.0085 | 1.0204 | 1.0064 | 1.0058 | 0.0155 | 0.0036 | 0.0176 | 0.0182 |
| V28 | 1.0070 | N/A | 1.0198 | 1.0195 | 1.0191 | 1.0176 | 0.0128 | 0.0125 | 0.0121 | 0.0106 |
| V29 | 1.0040 | N/A | 0.9899 | 1.0113 | 0.9889 | 0.9877 | 0.0141 | 0.0073 | 0.0151 | 0.0163 |
| V30 | 0.9920 | N/A | 0.9755 | 1.0038 | 0.9744 | 0.9733 | 0.0165 | 0.0118 | 0.0176 | 0.0187 |
| P1 | 2.6096 | 2.6540 | 2.7073 | 2.6540 | 2.6359 | 2.6915 | 0.0977 | 0.0444 | 0.0263 | 0.0819 |
| **P2** | **0.1830** | **-0.1256** | **-0.1760** | **-0.1256** | **-0.2509** | **-0.1001** | **0.3590** | **0.3086** | **0.4339** | **0.2831** |
| P3 | -0.0240 | -0.0774 | 0.1720 | 0.1819 | 0.0695 | 0.1611 | 0.1960 | 0.2059 | 0.0935 | 0.1851 |
| P4 | -0.0760 | -0.1165 | 0.1866 | -0.0155 | -0.0411 | -0.1126 | 0.2626 | 0.0605 | 0.0349 | 0.0366 |
| P5 | -0.9420 | -1.0892 | -1.0509 | -1.0262 | -1.0699 | -1.0353 | 0.1089 | 0.0842 | 0.1279 | 0.0933 |
| P6 | 0.0000 | 0.0719 | 0.1143 | 0.0719 | 0.0883 | 0.0787 | 0.1143 | 0.0719 | 0.0883 | 0.0787 |
| P7 | -0.2280 | -0.2117 | -0.1460 | -0.1074 | -0.1550 | -0.1561 | 0.0820 | 0.1206 | 0.0730 | 0.0719 |
| P8 | -0.3000 | -0.3377 | -0.2979 | -0.3377 | -0.3237 | -0.3317 | 0.0021 | 0.0377 | 0.0237 | 0.0317 |
| P9 | 0.0000 | 0.0685 | -0.0733 | 0.0685 | 0.0910 | 0.0615 | 0.0733 | 0.0685 | 0.0910 | 0.0615 |
| P10 | -0.0580 | -0.1436 | -0.1384 | -0.1436 | -0.0706 | -0.0860 | 0.0804 | 0.0856 | 0.0126 | 0.0280 |
| P11 | 0.0000 | -0.0051 | -0.1439 | -0.0051 | 0.0172 | -0.0119 | 0.1439 | 0.0051 | 0.0172 | 0.0119 |
| P12 | -0.1120 | -0.1241 | -0.0744 | -0.1241 | -0.0933 | -0.1257 | 0.0376 | 0.0121 | 0.0187 | 0.0137 |
| P13 | 0.0000 | 0.0160 | 0.0662 | 0.0160 | 0.0468 | 0.0144 | 0.0662 | 0.0160 | 0.0468 | 0.0144 |
| P14 | -0.0620 | -0.0464 | -0.0300 | -0.0464 | -0.0471 | -0.0632 | 0.0320 | 0.0156 | 0.0149 | 0.0012 |
| P15 | -0.0820 | -0.1252 | -0.1023 | -0.1252 | -0.1165 | -0.1290 | 0.0203 | 0.0432 | 0.0345 | 0.0470 |
| P16 | -0.0350 | -0.0365 | -0.0762 | -0.0365 | -0.0320 | -0.0461 | 0.0412 | 0.0015 | 0.0030 | 0.0111 |
| P17 | -0.0900 | -0.0982 | -0.0908 | -0.0938 | -0.0420 | -0.0485 | 0.0008 | 0.0038 | 0.0480 | 0.0415 |
| P18 | -0.0320 | -0.0006 | 0.0008 | -0.0006 | 0.0112 | 0.0069 | 0.0328 | 0.0314 | 0.0432 | 0.0389 |
| P19 | -0.0950 | -0.0403 | -0.1133 | -0.1590 | -0.0987 | -0.1021 | 0.0183 | 0.0640 | 0.0037 | 0.0071 |
| P20 | -0.0220 | 0.0335 | 0.0063 | 0.0335 | 0.0184 | 0.0105 | 0.0283 | 0.0555 | 0.0404 | 0.0325 |
| P21 | -0.1750 | -0.2182 | -0.2231 | -0.2182 | -0.1880 | -0.1966 | 0.0481 | 0.0432 | 0.0130 | 0.0216 |
| P22 | 0.0000 | 0.0039 | 0.0096 | 0.0039 | 0.0525 | 0.0442 | 0.0096 | 0.0039 | 0.0525 | 0.0442 |
| P23 | -0.0320 | -0.0927 | -0.0942 | -0.0927 | -0.0777 | -0.0844 | 0.0622 | 0.0607 | 0.0457 | 0.0524 |
| P24 | -0.0870 | -0.1427 | -0.2107 | -0.1547 | -0.1752 | -0.1840 | 0.1237 | 0.0677 | 0.0882 | 0.0970 |
| P25 | 0.0000 | -0.0003 | -0.0092 | -0.0003 | -0.0197 | -0.0154 | 0.0092 | 0.0003 | 0.0197 | 0.0154 |
| P26 | -0.0350 | 0.0416 | 0.0331 | 0.0416 | 0.0224 | 0.0268 | 0.0681 | 0.0766 | 0.0574 | 0.0618 |
| P27 | 0.0000 | -0.0385 | -0.0254 | -0.0385 | -0.0280 | -0.0286 | 0.0254 | 0.0385 | 0.0280 | 0.0286 |
| P28 | 0.0000 | 0.0186 | 0.0454 | 0.0186 | 0.0230 | 0.0222 | 0.0454 | 0.0186 | 0.0230 | 0.0222 |
| P29 | -0.0240 | -0.0353 | -0.0124 | -0.0131 | -0.0145 | -0.0149 | 0.0116 | 0.0109 | 0.0095 | 0.0091 |
| P30 | -0.1060 | -0.0501 | -0.0647 | -0.0501 | -0.0663 | -0.0666 | 0.0413 | 0.0559 | 0.0397 | 0.0394 |
| Q1 | -0.2042 | -0.2587 | -0.2582 | -0.2587 | -0.2633 | -0.2554 | 0.0540 | 0.0545 | 0.0591 | 0.0512 |
| Q2 | 0.4337 | 0.4353 | 0.4291 | 0.4353 | 0.4238 | 0.4331 | 0.0046 | 0.0016 | 0.0099 | 0.0006 |
| Q3 | -0.0120 | 0.0156 | -0.0061 | 0.0156 | -0.0137 | -0.0137 | 0.0059 | 0.0276 | 0.0017 | 0.0017 |
| Q4 | -0.0160 | 0.0390 | -0.0183 | 0.0390 | -0.0309 | -0.0243 | 0.0023 | 0.0550 | 0.0149 | 0.0083 |
| Q5 | 0.1666 | 0.2438 | 0.2522 | 0.2438 | 0.2523 | 0.2481 | 0.0856 | 0.0772 | 0.0857 | 0.0815 |
| Q6 | 0.0000 | 0.0043 | 0.0002 | 0.0043 | -0.0104 | -0.0055 | 0.0002 | 0.0043 | 0.0104 | 0.0055 |
| Q7 | -0.1090 | -0.1836 | -0.1454 | -0.1836 | -0.1496 | -0.1475 | 0.0364 | 0.0746 | 0.0406 | 0.0385 |
| Q8 | 0.0611 | 0.0240 | 0.0233 | 0.0240 | 0.0137 | 0.0182 | 0.0378 | 0.0371 | 0.0474 | 0.0429 |
| Q9 | 0.0000 | -0.0531 | 0.0045 | -0.0531 | -0.0217 | -0.0182 | 0.0045 | 0.0531 | 0.0217 | 0.0182 |
| Q10 | -0.0200 | 0.0975 | 0.1065 | 0.0975 | 0.0815 | 0.0832 | 0.1265 | 0.1175 | 0.1015 | 0.1032 |
| Q11 | 0.1606 | 0.1298 | 0.1837 | 0.1298 | 0.1594 | 0.1627 | 0.0231 | 0.0308 | 0.0012 | 0.0021 |
| Q12 | -0.0750 | -0.0376 | -0.0634 | -0.0376 | -0.0441 | -0.0344 | 0.0116 | 0.0374 | 0.0309 | 0.0406 |
| Q13 | 0.1045 | 0.0949 | 0.0695 | 0.0949 | 0.0885 | 0.0980 | 0.0350 | 0.0096 | 0.0160 | 0.0065 |
| Q14 | -0.0160 | 0.0284 | -0.0171 | 0.0284 | -0.0020 | 0.0048 | 0.0011 | 0.0444 | 0.0140 | 0.0208 |
| Q15 | -0.0250 | -0.0632 | -0.0813 | -0.0632 | -0.0732 | -0.0689 | 0.0563 | 0.0382 | 0.0482 | 0.0439 |
| Q16 | -0.0180 | -0.0881 | -0.0348 | -0.0881 | -0.0510 | -0.0498 | 0.0168 | 0.0701 | 0.0330 | 0.0318 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q17 | -0.0580 | -0.1291 | -0.0767 | -0.1291 | -0.0945 | -0.0942 | 0.0187 | 0.0711 | 0.0365 | 0.0362 |
| Q18 | -0.0090 | 0.0154 | 0.0109 | 0.0009 | 0.0065 | 0.0073 | 0.0199 | 0.0099 | 0.0155 | 0.0163 |
| Q19 | -0.0340 | -0.0429 | -0.0372 | -0.0429 | -0.0436 | -0.0433 | 0.0032 | 0.0089 | 0.0096 | 0.0093 |
| Q20 | -0.0070 | -0.0168 | -0.0590 | -0.0826 | -0.0625 | -0.0607 | 0.0520 | 0.0756 | 0.0555 | 0.0537 |
| Q21 | -0.1120 | -0.0410 | -0.0535 | -0.0426 | -0.0673 | -0.0661 | 0.0585 | 0.0694 | 0.0447 | 0.0459 |
| Q22 | 0.0000 | 0.0146 | 0.0186 | 0.0146 | -0.0026 | -0.0006 | 0.0186 | 0.0146 | 0.0026 | 0.0006 |
| Q23 | -0.0160 | -0.0061 | -0.0026 | -0.0061 | -0.0099 | -0.0083 | 0.0134 | 0.0099 | 0.0061 | 0.0077 |
| Q24 | -0.0670 | 0.0124 | -0.0156 | -0.0282 | -0.0332 | -0.0292 | 0.0514 | 0.0388 | 0.0338 | 0.0378 |
| Q25 | 0.0000 | -0.0402 | -0.0495 | -0.0402 | -0.0414 | -0.0445 | 0.0495 | 0.0402 | 0.0414 | 0.0445 |
| Q26 | -0.0230 | 0.0118 | 0.0025 | 0.0118 | 0.0108 | 0.0075 | 0.0255 | 0.0348 | 0.0338 | 0.0305 |
| Q27 | 0.0000 | 0.0418 | 0.0384 | 0.0418 | 0.0425 | 0.0415 | 0.0384 | 0.0418 | 0.0425 | 0.0415 |
| Q28 | 0.0000 | -0.0122 | 0.0044 | -0.0122 | -0.0010 | 0.0013 | 0.0044 | 0.0122 | 0.0010 | 0.0013 |
| Q29 | -0.0090 | 0.0018 | -0.0059 | 0.0018 | -0.0024 | -0.0032 | 0.0031 | 0.0108 | 0.0066 | 0.0058 |
| Q30 | -0.0190 | -0.0773 | -0.0500 | -0.0173 | -0.0473 | -0.0479 | 0.0310 | 0.0017 | 0.0283 | 0.0289 |
| P1-2 | 1.7331 | N/A | 1.9208 | 1.8603 | 1.8657 | 1.8763 | 0.1877 | 0.1272 | 0.1326 | 0.1432 |
| P1-3 | 0.8765 | 0.8817 | 0.7865 | 0.7937 | 0.7702 | 0.8151 | 0.0900 | 0.0828 | 0.1063 | 0.0614 |
| **P2-4** | **0.4365** | **-0.4726** | **0.3125** | **0.3419** | **0.2896** | **0.3585** | **0.1240** | **0.0946** | **0.1469** | **0.0780** |
| P3-4 | 0.8214 | 0.9507 | 0.9339 | 0.9507 | 0.8160 | 0.9497 | 0.1125 | 0.1293 | 0.0054 | 0.1283 |
| P2-5 | 0.8236 | 0.7903 | 0.8359 | 0.8096 | 0.8116 | 0.8279 | 0.0123 | 0.0140 | 0.0120 | 0.0043 |
| P2-6 | 0.6038 | N/A | 0.5335 | 0.5242 | 0.4536 | 0.5295 | 0.0703 | 0.0796 | 0.1502 | 0.0743 |
| **P4-6** | **0.7213** | **1.2172** | **0.9489** | **0.7875** | **0.7086** | **0.7389** | **0.2276** | **0.0662** | **0.0127** | **0.0176** |
| P5-7 | -0.1478 | -0.2445 | -0.2448 | -0.2445 | -0.2866 | -0.2368 | 0.0970 | 0.0967 | 0.1388 | 0.0890 |
| P6-7 | 0.3813 | 0.3594 | 0.3989 | 0.3594 | 0.4520 | 0.4009 | 0.0176 | 0.0219 | 0.0707 | 0.0196 |
| P6-8 | 0.2956 | N/A | 0.2861 | 0.3222 | 0.3068 | 0.3168 | 0.0095 | 0.0266 | 0.0112 | 0.0212 |
| **P6-9** | **0.2772** | **0.9192** | **0.4931** | **0.3180** | **0.1915** | **0.2755** | **0.2159** | **0.0408** | **0.0857** | **0.0017** |
| P6-10 | 0.1584 | N/A | 0.2365 | 0.1935 | 0.1297 | 0.1658 | 0.0781 | 0.0351 | 0.0287 | 0.0074 |
| P9-11 | 0.0000 | N/A | 0.1439 | 0.0051 | -0.0172 | 0.0119 | 0.1439 | 0.0051 | 0.0172 | 0.0119 |
| P9-10 | 0.2772 | 0.2500 | 0.2759 | 0.3814 | 0.2997 | 0.3251 | 0.0013 | 0.1042 | 0.0225 | 0.0479 |
| P4-12 | 0.4419 | N/A | 0.4682 | 0.4723 | 0.3432 | 0.4387 | 0.0263 | 0.0304 | 0.0987 | 0.0032 |
| P12-13 | 0.0000 | N/A | -0.0662 | -0.0160 | -0.0468 | -0.0144 | 0.0662 | 0.0160 | 0.0468 | 0.0144 |
| P12-14 | 0.0786 | 0.1031 | 0.0755 | 0.0821 | 0.0721 | 0.0846 | 0.0031 | 0.0035 | 0.0065 | 0.0060 |
| P12-15 | 0.1789 | 0.2159 | 0.2382 | 0.2095 | 0.1802 | 0.1941 | 0.0593 | 0.0306 | 0.0013 | 0.0152 |
| P12-16 | 0.0724 | N/A | 0.1463 | 0.0727 | 0.0445 | 0.0487 | 0.0739 | 0.0003 | 0.0279 | 0.0237 |
| P14-15 | 0.0158 | N/A | 0.0449 | 0.0349 | 0.0244 | 0.0206 | 0.0291 | 0.0191 | 0.0086 | 0.0048 |
| P16-17 | 0.0369 | -0.0700 | 0.0681 | 0.0354 | 0.0122 | 0.0024 | 0.0312 | 0.0015 | 0.0247 | 0.0345 |
| P15-18 | 0.0602 | 0.0182 | 0.0553 | 0.0390 | 0.0037 | 0.0044 | 0.0049 | 0.0212 | 0.0565 | 0.0558 |
| P18-19 | 0.0278 | 0.0955 | 0.0557 | 0.0382 | 0.0149 | 0.0114 | 0.0279 | 0.0104 | 0.0129 | 0.0164 |
| P19-20 | -0.0673 | -0.1209 | -0.0579 | -0.1209 | -0.0838 | -0.0907 | 0.0094 | 0.0536 | 0.0165 | 0.0234 |
| P10-20 | 0.0903 | 0.1383 | 0.0533 | 0.0898 | 0.0671 | 0.0820 | 0.0370 | 0.0005 | 0.0232 | 0.0083 |
| P10-17 | 0.0533 | 0.0595 | 0.0238 | 0.0595 | 0.0305 | 0.0468 | 0.0295 | 0.0062 | 0.0228 | 0.0065 |
| P10-21 | 0.1579 | 0.2297 | 0.2007 | 0.1913 | 0.1757 | 0.1855 | 0.0428 | 0.0334 | 0.0178 | 0.0276 |
| P10-22 | 0.0762 | N/A | 0.0962 | 0.0907 | 0.0855 | 0.0906 | 0.0200 | 0.0145 | 0.0093 | 0.0144 |
| P21-22 | -0.0183 | -0.0282 | -0.0239 | -0.0282 | -0.0135 | -0.0123 | 0.0056 | 0.0099 | 0.0048 | 0.0060 |
| P15-23 | 0.0504 | N/A | 0.1212 | 0.0768 | 0.0819 | 0.0785 | 0.0708 | 0.0264 | 0.0315 | 0.0281 |
| P22-24 | 0.0574 | 0.2028 | 0.0812 | 0.0658 | 0.1240 | 0.1218 | 0.0238 | 0.0084 | 0.0666 | 0.0644 |
| P23-24 | 0.0180 | 0.0593 | 0.0255 | -0.0165 | 0.0035 | -0.0066 | 0.0075 | 0.0345 | 0.0145 | 0.0246 |
| P24-25 | -0.0121 | N/A | -0.1054 | -0.1062 | -0.0497 | -0.0706 | 0.0933 | 0.0941 | 0.0376 | 0.0585 |
| P25-26 | 0.0354 | N/A | -0.0328 | -0.0411 | -0.0223 | -0.0266 | 0.0682 | 0.0765 | 0.0577 | 0.0620 |
| P25-27 | -0.0476 | -0.0710 | -0.0844 | -0.0675 | -0.0476 | -0.0605 | 0.0368 | 0.0199 | 0.0000 | 0.0129 |
| P28-27 | 0.1807 | 0.1671 | 0.1893 | 0.1704 | 0.1583 | 0.1726 | 0.0086 | 0.0103 | 0.0224 | 0.0081 |
| P27-29 | 0.0619 | N/A | 0.0361 | 0.0302 | 0.0383 | 0.0387 | 0.0258 | 0.0317 | 0.0236 | 0.0232 |
| P27-30 | 0.0709 | 0.0570 | 0.0426 | 0.0336 | 0.0441 | 0.0444 | 0.0283 | 0.0373 | 0.0268 | 0.0265 |
| P29-30 | 0.0370 | 0.0721 | 0.0233 | 0.0169 | 0.0233 | 0.0234 | 0.0137 | 0.0201 | 0.0137 | 0.0136 |
| P8-28 | -0.0054 | N/A | -0.0127 | -0.0167 | -0.0179 | -0.0161 | 0.0073 | 0.0113 | 0.0125 | 0.0107 |
| P6-28 | 0.1867 | 0.1690 | 0.1570 | 0.1690 | 0.1537 | 0.1669 | 0.0297 | 0.0177 | 0.0330 | 0.0198 |
| P2-1 | -1.6809 | N/A | -1.8579 | -1.8012 | -1.8057 | -1.8160 | 0.1770 | 0.1203 | 0.1248 | 0.1351 |
| P3-1 | -0.8454 | N/A | -0.7620 | -0.7688 | -0.7465 | -0.7887 | 0.0834 | 0.0766 | 0.0989 | 0.0567 |
| P4-2 | -0.4263 | N/A | -0.3074 | -0.3357 | -0.2851 | -0.3517 | 0.1189 | 0.0906 | 0.1412 | 0.0746 |
| P4-3 | -0.8129 | N/A | -0.9232 | -0.9396 | -0.8078 | -0.9386 | 0.1103 | 0.1267 | 0.0051 | 0.1257 |
| P5-2 | -0.7942 | N/A | -0.8061 | -0.7817 | -0.7833 | -0.7986 | 0.0119 | 0.0125 | 0.0109 | 0.0044 |
| P6-2 | -0.5843 | N/A | -0.5185 | -0.5097 | -0.4427 | -0.5147 | 0.0658 | 0.0746 | 0.1416 | 0.0696 |
| P6-4 | -0.7150 | N/A | -0.9387 | -0.7805 | -0.7028 | -0.7326 | 0.2237 | 0.0655 | 0.0122 | 0.0176 |
| P7-5 | 0.1495 | N/A | 0.2488 | 0.2487 | 0.2917 | 0.2406 | 0.0993 | 0.0992 | 0.1422 | 0.0911 |
| P7-6 | -0.3775 | N/A | -0.3948 | -0.3561 | -0.4468 | -0.3968 | 0.0173 | 0.0214 | 0.0693 | 0.0193 |

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|---|---|---|---|---|---|---|---|---|---|---|
| P8-6 | -0.2946 | N/A | -0.2852 | -0.3210 | -0.3057 | -0.3156 | 0.0094 | 0.0264 | 0.0111 | 0.0210 |
| P9-6 | -0.2772 | N/A | -0.4931 | -0.3180 | -0.1915 | -0.2755 | 0.2159 | 0.0408 | 0.0857 | 0.0017 |
| P10-6 | -0.1584 | N/A | -0.2365 | -0.1935 | -0.1297 | -0.1658 | 0.0781 | 0.0351 | 0.0287 | 0.0074 |
| P11-9 | 0.0000 | N/A | -0.1439 | -0.0051 | 0.0172 | -0.0119 | 0.1439 | 0.0051 | 0.0172 | 0.0119 |
| P10-9 | -0.2772 | N/A | -0.2759 | -0.3814 | -0.2997 | -0.3251 | 0.0013 | 0.1042 | 0.0225 | 0.0479 |
| P12-4 | -0.4419 | N/A | -0.4682 | -0.4723 | -0.3432 | -0.4387 | 0.0263 | 0.0304 | 0.0987 | 0.0032 |
| P13-12 | 0.0000 | N/A | 0.0662 | 0.0160 | 0.0468 | 0.0144 | 0.0662 | 0.0160 | 0.0468 | 0.0144 |
| P14-12 | -0.0778 | N/A | -0.0748 | -0.0813 | -0.0715 | -0.0837 | 0.0030 | 0.0035 | 0.0063 | 0.0059 |
| P15-12 | -0.1767 | N/A | -0.2345 | -0.2064 | -0.1779 | -0.1915 | 0.0578 | 0.0297 | 0.0012 | 0.0148 |
| P16-12 | -0.0719 | N/A | -0.1443 | -0.0719 | -0.0443 | -0.0485 | 0.0724 | 0.0000 | 0.0276 | 0.0234 |
| P15-14 | -0.0158 | N/A | -0.0444 | -0.0345 | -0.0242 | -0.0205 | 0.0286 | 0.0187 | 0.0084 | 0.0047 |
| P17-16 | -0.0368 | N/A | -0.0677 | -0.0353 | -0.0121 | -0.0023 | 0.0309 | 0.0015 | 0.0247 | 0.0345 |
| P18-15 | -0.0598 | N/A | -0.0549 | -0.0388 | -0.0037 | -0.0044 | 0.0049 | 0.0210 | 0.0561 | 0.0554 |
| P19-18 | -0.0277 | N/A | -0.0554 | -0.0381 | -0.0149 | -0.0114 | 0.0277 | 0.0104 | 0.0128 | 0.0163 |
| P20-19 | 0.0674 | N/A | 0.0581 | 0.1214 | 0.0841 | 0.0910 | 0.0093 | 0.0540 | 0.0167 | 0.0236 |
| P20-10 | -0.0894 | N/A | -0.0518 | -0.0879 | -0.0657 | -0.0805 | 0.0376 | 0.0015 | 0.0237 | 0.0089 |
| P17-10 | -0.0532 | N/A | -0.0232 | -0.0585 | -0.0299 | -0.0462 | 0.0300 | 0.0053 | 0.0233 | 0.0070 |
| P21-10 | -0.1567 | N/A | -0.1992 | -0.1900 | -0.1745 | -0.1842 | 0.0425 | 0.0333 | 0.0178 | 0.0275 |
| P22-10 | -0.0757 | N/A | -0.0955 | -0.0901 | -0.0849 | -0.0899 | 0.0198 | 0.0144 | 0.0092 | 0.0142 |
| P22-21 | 0.0183 | N/A | 0.0239 | 0.0282 | 0.0135 | 0.0123 | 0.0056 | 0.0099 | 0.0048 | 0.0060 |
| P23-15 | -0.0500 | N/A | -0.1197 | -0.0762 | -0.0812 | -0.0778 | 0.0697 | 0.0262 | 0.0312 | 0.0278 |
| P24-22 | -0.0569 | N/A | -0.0801 | -0.0652 | -0.1221 | -0.1200 | 0.0232 | 0.0083 | 0.0652 | 0.0631 |
| P24-23 | -0.0180 | N/A | -0.0252 | 0.0166 | -0.0034 | 0.0066 | 0.0072 | 0.0346 | 0.0146 | 0.0246 |
| P25-24 | 0.0122 | N/A | 0.1080 | 0.1084 | 0.0502 | 0.0717 | 0.0958 | 0.0962 | 0.0380 | 0.0595 |
| P26-25 | -0.0350 | N/A | 0.0331 | 0.0416 | 0.0224 | 0.0268 | 0.0681 | 0.0766 | 0.0574 | 0.0618 |
| P27-25 | 0.0479 | N/A | 0.0852 | 0.0680 | 0.0479 | 0.0609 | 0.0373 | 0.0201 | 0.0000 | 0.0130 |
| P27-28 | -0.1807 | N/A | -0.1893 | -0.1704 | -0.1583 | -0.1726 | 0.0086 | 0.0103 | 0.0224 | 0.0081 |
| P29-27 | -0.0610 | N/A | -0.0357 | -0.0300 | -0.0378 | -0.0383 | 0.0253 | 0.0310 | 0.0232 | 0.0227 |
| P30-27 | -0.0693 | N/A | -0.0417 | -0.0332 | -0.0432 | -0.0435 | 0.0276 | 0.0361 | 0.0261 | 0.0258 |
| P30-29 | -0.0367 | N/A | -0.0231 | -0.0169 | -0.0231 | -0.0231 | 0.0136 | 0.0198 | 0.0136 | 0.0136 |
| P28-8 | 0.0055 | N/A | 0.0127 | 0.0168 | 0.0180 | 0.0161 | 0.0072 | 0.0113 | 0.0125 | 0.0106 |
| P28-6 | -0.1862 | N/A | -0.1566 | -0.1685 | -0.1533 | -0.1665 | 0.0296 | 0.0177 | 0.0329 | 0.0197 |
| Q1-2 | -0.2470 | N/A | -0.2646 | -0.2619 | -0.2692 | -0.2692 | 0.0176 | 0.0149 | 0.0222 | 0.0222 |
| Q1-3 | 0.0428 | 0.0595 | 0.0064 | 0.0032 | 0.0059 | 0.0138 | 0.0364 | 0.0396 | 0.0369 | 0.0290 |
| Q2-4 | 0.0475 | 0.0671 | 0.0314 | 0.0300 | 0.0313 | 0.0344 | 0.0161 | 0.0175 | 0.0162 | 0.0131 |
| Q3-4 | -0.0385 | -0.0159 | -0.0440 | -0.0270 | -0.0496 | -0.0516 | 0.0055 | 0.0115 | 0.0111 | 0.0131 |
| Q2-5 | 0.0278 | 0.0213 | -0.0035 | 0.0071 | -0.0037 | -0.0018 | 0.0313 | 0.0207 | 0.0315 | 0.0296 |
| Q2-6 | 0.0137 | N/A | 0.0074 | 0.0188 | 0.0064 | 0.0097 | 0.0063 | 0.0051 | 0.0073 | 0.0040 |
| Q4-6 | -0.1591 | -0.1829 | -0.1080 | -0.0565 | -0.1075 | -0.1139 | 0.0511 | 0.1026 | 0.0516 | 0.0452 |
| Q5-7 | 0.1149 | 0.1580 | 0.1687 | 0.1786 | 0.1745 | 0.1680 | 0.0538 | 0.0637 | 0.0596 | 0.0531 |
| Q6-7 | -0.0278 | -0.0959 | -0.0393 | -0.0130 | -0.0345 | -0.0367 | 0.0115 | 0.0148 | 0.0067 | 0.0089 |
| Q6-8 | -0.0720 | N/A | -0.0441 | -0.0469 | -0.0339 | -0.0381 | 0.0279 | 0.0251 | 0.0381 | 0.0339 |
| Q6-9 | -0.0809 | -0.1233 | -0.0437 | 0.0099 | -0.0447 | -0.0467 | 0.0372 | 0.0908 | 0.0362 | 0.0342 |
| Q6-10 | 0.0019 | N/A | 0.0055 | 0.0210 | 0.0012 | 0.0011 | 0.0036 | 0.0191 | 0.0007 | 0.0008 |
| Q9-11 | -0.1560 | N/A | -0.1739 | -0.1266 | -0.1546 | -0.1578 | 0.0179 | 0.0294 | 0.0014 | 0.0018 |
| Q9-10 | 0.0588 | 0.1108 | 0.0858 | 0.0632 | 0.0805 | 0.0773 | 0.0270 | 0.0044 | 0.0217 | 0.0185 |
| Q4-12 | 0.1441 | N/A | 0.0794 | 0.0968 | 0.0695 | 0.0681 | 0.0647 | 0.0473 | 0.0746 | 0.0760 |
| Q12-13 | -0.1032 | N/A | -0.0683 | -0.0937 | -0.0872 | -0.0967 | 0.0349 | 0.0095 | 0.0160 | 0.0065 |
| Q12-14 | 0.0240 | 0.0570 | 0.0116 | -0.0061 | 0.0123 | 0.0098 | 0.0124 | 0.0301 | 0.0117 | 0.0142 |
| Q12-15 | 0.0679 | 0.0645 | 0.0407 | 0.0511 | 0.0567 | 0.0559 | 0.0272 | 0.0168 | 0.0112 | 0.0120 |
| Q12-16 | 0.0335 | N/A | -0.0226 | 0.0515 | 0.0136 | 0.0165 | 0.0561 | 0.0180 | 0.0199 | 0.0170 |
| Q14-15 | 0.0065 | N/A | -0.0070 | 0.0206 | 0.0090 | 0.0128 | 0.0135 | 0.0141 | 0.0025 | 0.0063 |
| Q16-17 | 0.0144 | -0.0008 | -0.0617 | -0.0382 | -0.0378 | -0.0338 | 0.0761 | 0.0526 | 0.0522 | 0.0482 |
| Q15-18 | 0.0160 | 0.0172 | -0.0259 | 0.0172 | -0.0004 | 0.0017 | 0.0419 | 0.0012 | 0.0164 | 0.0143 |
| Q18-19 | 0.0062 | 0.0088 | -0.0157 | 0.0177 | 0.0060 | 0.0090 | 0.0219 | 0.0115 | 0.0002 | 0.0028 |
| Q19-20 | -0.0279 | 0.0134 | -0.0534 | -0.0255 | -0.0377 | -0.0344 | 0.0255 | 0.0024 | 0.0098 | 0.0065 |
| Q10-20 | 0.0371 | 0.1134 | 0.1160 | 0.1134 | 0.1037 | 0.0990 | 0.0789 | 0.0763 | 0.0666 | 0.0619 |
| Q10-17 | 0.0443 | 0.0676 | 0.1417 | 0.1705 | 0.1340 | 0.1297 | 0.0974 | 0.1262 | 0.0897 | 0.0854 |
| Q10-21 | 0.1001 | 0.0896 | 0.0683 | 0.0403 | 0.0714 | 0.0715 | 0.0318 | 0.0598 | 0.0287 | 0.0286 |
| Q10-22 | 0.0460 | N/A | 0.0340 | 0.0175 | 0.0342 | 0.0343 | 0.0120 | 0.0285 | 0.0118 | 0.0117 |
| Q21-22 | -0.0143 | -0.0051 | 0.0116 | -0.0051 | 0.0015 | 0.0025 | 0.0259 | 0.0092 | 0.0158 | 0.0168 |
| Q15-23 | 0.0291 | N/A | -0.0296 | -0.0149 | -0.0118 | -0.0072 | 0.0587 | 0.0440 | 0.0409 | 0.0363 |
| Q22-24 | 0.0306 | 0.0781 | 0.0627 | 0.0257 | 0.0319 | 0.0349 | 0.0321 | 0.0049 | 0.0013 | 0.0043 |
| Q23-24 | 0.0124 | 0.0278 | -0.0353 | -0.0223 | -0.0231 | -0.0168 | 0.0477 | 0.0347 | 0.0355 | 0.0292 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q24-25 | 0.0201 | N/A | 0.0520 | 0.0167 | 0.0150 | 0.0285 | 0.0319 | 0.0034 | 0.0051 | 0.0084 |
| Q25-26 | 0.0237 | N/A | -0.0021 | -0.0111 | -0.0105 | -0.0072 | 0.0258 | 0.0348 | 0.0342 | 0.0309 |
| Q25-27 | -0.0037 | 0.0094 | 0.0000 | -0.0163 | -0.0167 | -0.0108 | 0.0037 | 0.0126 | 0.0130 | 0.0071 |
| Q28-27 | 0.0504 | 0.0033 | 0.0361 | 0.0033 | 0.0376 | 0.0361 | 0.0143 | 0.0471 | 0.0128 | 0.0143 |
| Q27-29 | 0.0167 | N/A | 0.0260 | 0.0064 | 0.0225 | 0.0233 | 0.0093 | 0.0103 | 0.0058 | 0.0066 |
| Q27-30 | 0.0166 | 0.0093 | 0.0328 | 0.0104 | 0.0303 | 0.0309 | 0.0162 | 0.0062 | 0.0137 | 0.0143 |
| Q29-30 | 0.0061 | -0.0205 | 0.0193 | 0.0078 | 0.0192 | 0.0192 | 0.0132 | 0.0017 | 0.0131 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0147 | -0.0177 | -0.0146 | -0.0147 | 0.0093 | 0.0123 | 0.0092 | 0.0093 |
| Q6-28 | 0.0011 | -0.0427 | -0.0102 | -0.0231 | -0.0034 | -0.0067 | 0.0113 | 0.0242 | 0.0045 | 0.0078 |
| Q2-1 | 0.3447 | N/A | 0.3936 | 0.3793 | 0.3899 | 0.3908 | 0.0489 | 0.0346 | 0.0452 | 0.0461 |
| Q3-1 | 0.0265 | N/A | 0.0379 | 0.0426 | 0.0359 | 0.0379 | 0.0114 | 0.0161 | 0.0094 | 0.0114 |
| Q4-2 | -0.0554 | N/A | -0.0555 | -0.0512 | -0.0572 | -0.0533 | 0.0001 | 0.0042 | 0.0018 | 0.0021 |
| Q4-3 | 0.0544 | N/A | 0.0657 | 0.0498 | 0.0644 | 0.0748 | 0.0113 | 0.0046 | 0.0100 | 0.0204 |
| Q5-2 | 0.0517 | N/A | 0.0836 | 0.0652 | 0.0778 | 0.0801 | 0.0319 | 0.0135 | 0.0261 | 0.0284 |
| Q6-2 | 0.0058 | N/A | -0.0023 | -0.0151 | -0.0134 | -0.0049 | 0.0081 | 0.0209 | 0.0192 | 0.0107 |
| Q6-4 | 0.1719 | N/A | 0.1342 | 0.0714 | 0.1183 | 0.1265 | 0.0377 | 0.1005 | 0.0536 | 0.0454 |
| Q7-5 | -0.1313 | N/A | -0.1796 | -0.1892 | -0.1826 | -0.1793 | 0.0483 | 0.0579 | 0.0513 | 0.0480 |
| Q7-6 | 0.0223 | N/A | 0.0342 | 0.0056 | 0.0330 | 0.0319 | 0.0119 | 0.0167 | 0.0107 | 0.0096 |
| Q8-6 | 0.0666 | N/A | 0.0380 | 0.0417 | 0.0283 | 0.0329 | 0.0286 | 0.0249 | 0.0383 | 0.0337 |
| Q9-6 | 0.0972 | N/A | 0.0925 | 0.0103 | 0.0524 | 0.0623 | 0.0047 | 0.0869 | 0.0448 | 0.0349 |
| Q10-6 | 0.0110 | N/A | 0.0242 | -0.0008 | 0.0077 | 0.0136 | 0.0132 | 0.0118 | 0.0033 | 0.0026 |
| Q11-9 | 0.1606 | N/A | 0.1837 | 0.1298 | 0.1594 | 0.1627 | 0.0231 | 0.0308 | 0.0012 | 0.0021 |
| Q10-9 | -0.0508 | N/A | -0.0773 | -0.0475 | -0.0705 | -0.0657 | 0.0265 | 0.0033 | 0.0197 | 0.0149 |
| Q12-4 | -0.0972 | N/A | -0.0248 | -0.0405 | -0.0396 | -0.0200 | 0.0724 | 0.0567 | 0.0576 | 0.0772 |
| Q13-12 | 0.1045 | N/A | 0.0695 | 0.0949 | 0.0885 | 0.0980 | 0.0350 | 0.0096 | 0.0160 | 0.0065 |
| Q14-12 | -0.0225 | N/A | -0.0101 | 0.0078 | -0.0109 | -0.0080 | 0.0124 | 0.0303 | 0.0116 | 0.0145 |
| Q15-12 | -0.0636 | N/A | -0.0333 | -0.0452 | -0.0522 | -0.0507 | 0.0303 | 0.0184 | 0.0114 | 0.0129 |
| Q16-12 | -0.0324 | N/A | 0.0269 | -0.0499 | -0.0132 | -0.0160 | 0.0593 | 0.0175 | 0.0192 | 0.0164 |
| Q15-14 | -0.0064 | N/A | 0.0074 | -0.0203 | -0.0088 | -0.0127 | 0.0138 | 0.0139 | 0.0024 | 0.0063 |
| Q17-16 | -0.0141 | N/A | 0.0633 | 0.0387 | 0.0381 | 0.0340 | 0.0774 | 0.0528 | 0.0522 | 0.0481 |
| Q18-15 | -0.0152 | N/A | 0.0267 | -0.0168 | 0.0004 | -0.0017 | 0.0419 | 0.0016 | 0.0156 | 0.0135 |
| Q19-18 | -0.0061 | N/A | 0.0162 | -0.0174 | -0.0060 | -0.0089 | 0.0223 | 0.0113 | 0.0001 | 0.0028 |
| Q20-19 | 0.0283 | N/A | 0.0538 | 0.0266 | 0.0382 | 0.0350 | 0.0255 | 0.0017 | 0.0099 | 0.0067 |
| Q20-10 | -0.0353 | N/A | -0.1128 | -0.1092 | -0.1007 | -0.0957 | 0.0775 | 0.0739 | 0.0654 | 0.0604 |
| Q17-10 | -0.0439 | N/A | -0.1400 | -0.1678 | -0.1325 | -0.1281 | 0.0961 | 0.1239 | 0.0886 | 0.0842 |
| Q21-10 | -0.0977 | N/A | -0.0651 | -0.0375 | -0.0689 | -0.0687 | 0.0326 | 0.0602 | 0.0288 | 0.0290 |
| Q22-10 | -0.0449 | N/A | -0.0325 | -0.0163 | -0.0330 | -0.0329 | 0.0124 | 0.0286 | 0.0119 | 0.0120 |
| Q22-21 | 0.0143 | N/A | -0.0116 | 0.0051 | -0.0015 | -0.0025 | 0.0259 | 0.0092 | 0.0158 | 0.0168 |
| Q23-15 | -0.0284 | N/A | 0.0327 | 0.0162 | 0.0132 | 0.0085 | 0.0611 | 0.0446 | 0.0416 | 0.0369 |
| Q24-22 | -0.0299 | N/A | -0.0608 | -0.0249 | -0.0290 | -0.0321 | 0.0309 | 0.0050 | 0.0009 | 0.0022 |
| Q24-23 | -0.0123 | N/A | 0.0358 | 0.0225 | 0.0232 | 0.0168 | 0.0481 | 0.0348 | 0.0355 | 0.0291 |
| Q25-24 | -0.0200 | N/A | -0.0474 | -0.0128 | -0.0141 | -0.0265 | 0.0274 | 0.0072 | 0.0059 | 0.0065 |
| Q26-25 | -0.0230 | N/A | 0.0025 | 0.0118 | 0.0108 | 0.0075 | 0.0255 | 0.0348 | 0.0338 | 0.0305 |
| Q27-25 | 0.0042 | N/A | 0.0015 | 0.0172 | 0.0172 | 0.0116 | 0.0027 | 0.0130 | 0.0130 | 0.0074 |
| Q27-28 | -0.0375 | N/A | -0.0220 | 0.0078 | -0.0275 | -0.0242 | 0.0155 | 0.0453 | 0.0100 | 0.0133 |
| Q29-27 | -0.0151 | N/A | -0.0252 | -0.0060 | -0.0216 | -0.0225 | 0.0101 | 0.0091 | 0.0065 | 0.0074 |
| Q30-27 | -0.0136 | N/A | -0.0311 | -0.0097 | -0.0286 | -0.0291 | 0.0175 | 0.0039 | 0.0150 | 0.0155 |
| Q30-29 | -0.0054 | N/A | -0.0189 | -0.0077 | -0.0188 | -0.0188 | 0.0135 | 0.0023 | 0.0134 | 0.0134 |
| Q28-8 | -0.0380 | N/A | -0.0298 | -0.0267 | -0.0298 | -0.0296 | 0.0082 | 0.0113 | 0.0082 | 0.0084 |
| Q28-6 | -0.0123 | N/A | -0.0019 | 0.0112 | -0.0088 | -0.0052 | 0.0104 | 0.0235 | 0.0035 | 0.0071 |
| **SUM** | | | | | | | **10.7386** | **9.1471** | **8.4063** | **7.5137** |

Conpared with four reverse bad data case, it's easier for LTS to correctly detect the bad data and generate relative good estimation. LTAV still has best results.

#### 5.3.4.3 Four large bad data

The four large bad data are simulated by replacing the original data with $P_2 = 0.9256$, $P_{2-4} = 0.0726$, $P_{4-6} = 1.2172$ and $P_{6-9} = 0.9192$. Table 5.278 gives system state variables, Table 5.279 presents the estimation results and the comparison between results with actual value.

Table 5.278: State variable of case 3, multi large, 30-bus AC system, median redundancy

| | **WLS** | **LAV** | **LTS** | **LTAV** | | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|
| $\theta 2$ | -0.0867 | -0.0881 | -0.0869 | -0.0850 | **V1** | 1.0712 | 1.0690 | 1.0676 | 1.0678 |
| $\theta 3$ | -0.1398 | -0.1500 | -0.1379 | -0.1469 | **V2** | 1.0594 | 1.0573 | 1.0558 | 1.0570 |
| $\theta 4$ | -0.1750 | -0.1828 | -0.1719 | -0.1798 | **V3** | 1.0334 | 1.0301 | 1.0307 | 1.0280 |
| $\theta 5$ | -0.2688 | -0.2686 | -0.2717 | -0.2652 | **V4** | 1.0232 | 1.0212 | 1.0211 | 1.0190 |
| $\theta 6$ | -0.2154 | -0.2155 | -0.2119 | -0.2115 | **V5** | 1.0229 | 1.0214 | 1.0185 | 1.0208 |
| $\theta 7$ | -0.2445 | -0.2451 | -0.2453 | -0.2408 | **V6** | 1.0171 | 1.0155 | 1.0150 | 1.0153 |
| $\theta 8$ | -0.2302 | -0.2294 | -0.2297 | -0.2267 | **V7** | 1.0111 | 1.0090 | 1.0074 | 1.0090 |
| $\theta 9$ | -0.3325 | -0.3130 | -0.2876 | -0.2914 | **V8** | 1.0152 | 1.0135 | 1.0120 | 1.0131 |
| $\theta 10$ | -0.3655 | -0.3476 | -0.3211 | -0.3282 | **V9** | 1.0350 | 1.0263 | 1.0305 | 1.0301 |
| $\theta 11$ | -0.3645 | -0.3140 | -0.3001 | -0.3028 | **V10** | 1.0277 | 1.0215 | 1.0233 | 1.0234 |
| $\theta 12$ | -0.3227 | -0.3196 | -0.2924 | -0.3166 | **V11** | 1.0721 | 1.0520 | 1.0646 | 1.0638 |
| $\theta 13$ | -0.3202 | -0.3175 | -0.2884 | -0.3187 | **V12** | 1.0170 | 1.0179 | 1.0132 | 1.0140 |
| $\theta 14$ | -0.3452 | -0.3460 | -0.3126 | -0.3406 | **V13** | 1.0290 | 1.0308 | 1.0258 | 1.0291 |
| $\theta 15$ | -0.3531 | -0.3471 | -0.3182 | -0.3412 | **V14** | 1.0044 | 1.0071 | 1.0012 | 1.0012 |
| $\theta 16$ | -0.3529 | -0.3408 | -0.3118 | -0.3281 | **V15** | 0.9961 | 0.9967 | 0.9931 | 0.9938 |
| $\theta 17$ | -0.3652 | -0.3483 | -0.3202 | -0.3295 | **V16** | 1.0076 | 1.0105 | 1.0045 | 1.0052 |
| $\theta 18$ | -0.3673 | -0.3568 | -0.3257 | -0.3437 | **V17** | 1.0153 | 1.0087 | 1.0112 | 1.0112 |
| $\theta 19$ | -0.3742 | -0.3629 | -0.3300 | -0.3448 | **V18** | 0.9954 | 0.9879 | 0.9923 | 0.9924 |
| $\theta 20$ | -0.3711 | -0.3556 | -0.3262 | -0.3394 | **V19** | 0.9937 | 0.9835 | 0.9904 | 0.9902 |
| $\theta 21$ | -0.3788 | -0.3600 | -0.3334 | -0.3405 | **V20** | 0.9993 | 0.9901 | 0.9958 | 0.9957 |
| $\theta 22$ | -0.3783 | -0.3594 | -0.3331 | -0.3402 | **V21** | 1.0157 | 1.0119 | 1.0117 | 1.0116 |
| $\theta 23$ | -0.3794 | -0.3665 | -0.3416 | -0.3585 | **V22** | 1.0157 | 1.0123 | 1.0118 | 1.0116 |
| $\theta 24$ | -0.3864 | -0.3679 | -0.3489 | -0.3566 | **V23** | 0.9895 | 0.9920 | 0.9865 | 0.9867 |
| $\theta 25$ | -0.3338 | -0.3222 | -0.3217 | -0.3172 | **V24** | 0.9948 | 1.0000 | 0.9916 | 0.9913 |
| $\theta 26$ | -0.3207 | -0.3097 | -0.3162 | -0.3069 | **V25** | 0.9966 | 1.0085 | 0.9942 | 0.9943 |
| $\theta 27$ | -0.3127 | -0.3054 | -0.3085 | -0.3014 | **V26** | 1.0051 | 1.0232 | 1.0026 | 1.0033 |
| $\theta 28$ | -0.2278 | -0.2271 | -0.2266 | -0.2236 | **V27** | 1.0059 | 1.0151 | 1.0030 | 1.0032 |
| $\theta 29$ | -0.3246 | -0.3212 | -0.3247 | -0.3141 | **V28** | 1.0145 | 1.0133 | 1.0115 | 1.0125 |
| $\theta 30$ | -0.3309 | -0.3223 | -0.3311 | -0.3204 | **V29** | 0.9865 | 0.9990 | 0.9837 | 0.9844 |
| | | | | | **V30** | 0.9719 | 0.9893 | 0.9691 | 0.9699 |

Table 5.279: Estimation Results and Errors of case 3, multi large, 30-bus AC system, median redundancy

| | **Actual** | **Meas.** | **WLS** | **LAV** | **LTS** | **LTAV** | **WLS** | **LAV** | **LTS** | **LTAV** |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | 1.0600 | 1.0690 | 1.0712 | 1.0690 | 1.0676 | 1.0678 | 0.0112 | 0.0090 | 0.0076 | 0.0078 |
| V2 | 1.0450 | N/A | 1.0594 | 1.0573 | 1.0558 | 1.0570 | 0.0144 | 0.0123 | 0.0108 | 0.0120 |
| V3 | 1.0210 | N/A | 1.0334 | 1.0301 | 1.0307 | 1.0280 | 0.0124 | 0.0091 | 0.0097 | 0.0070 |
| V4 | 1.0120 | N/A | 1.0232 | 1.0212 | 1.0211 | 1.0190 | 0.0112 | 0.0092 | 0.0091 | 0.0070 |
| V5 | 1.0100 | N/A | 1.0229 | 1.0214 | 1.0185 | 1.0208 | 0.0129 | 0.0114 | 0.0085 | 0.0108 |
| V6 | 1.0110 | N/A | 1.0171 | 1.0155 | 1.0150 | 1.0153 | 0.0061 | 0.0045 | 0.0040 | 0.0043 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| V7 | 1.0030 | N/A | 1.0111 | 1.0090 | 1.0074 | 1.0090 | 0.0081 | 0.0060 | 0.0044 | 0.0060 |
| V8 | 1.0100 | N/A | 1.0152 | 1.0135 | 1.0120 | 1.0131 | 0.0052 | 0.0035 | 0.0020 | 0.0031 |
| V9 | 1.0510 | N/A | 1.0350 | 1.0263 | 1.0305 | 1.0301 | 0.0160 | 0.0247 | 0.0205 | 0.0209 |
| V10 | 1.0450 | N/A | 1.0277 | 1.0215 | 1.0233 | 1.0234 | 0.0173 | 0.0235 | 0.0217 | 0.0216 |
| V11 | 1.0820 | N/A | 1.0721 | 1.0520 | 1.0646 | 1.0638 | 0.0099 | 0.0300 | 0.0174 | 0.0182 |
| V12 | 1.0570 | N/A | 1.0170 | 1.0179 | 1.0132 | 1.0140 | 0.0400 | 0.0391 | 0.0438 | 0.0430 |
| V13 | 1.0710 | N/A | 1.0290 | 1.0308 | 1.0258 | 1.0291 | 0.0420 | 0.0402 | 0.0452 | 0.0419 |
| V14 | 1.0430 | N/A | 1.0044 | 1.0071 | 1.0012 | 1.0012 | 0.0386 | 0.0359 | 0.0418 | 0.0418 |
| V15 | 1.0380 | N/A | 0.9961 | 0.9967 | 0.9931 | 0.9938 | 0.0419 | 0.0413 | 0.0449 | 0.0442 |
| V16 | 1.0450 | N/A | 1.0076 | 1.0105 | 1.0045 | 1.0052 | 0.0374 | 0.0345 | 0.0405 | 0.0398 |
| V17 | 1.0400 | N/A | 1.0153 | 1.0087 | 1.0112 | 1.0112 | 0.0247 | 0.0313 | 0.0288 | 0.0288 |
| V18 | 1.0280 | N/A | 0.9954 | 0.9879 | 0.9923 | 0.9924 | 0.0326 | 0.0401 | 0.0357 | 0.0356 |
| V19 | 1.0260 | N/A | 0.9937 | 0.9835 | 0.9904 | 0.9902 | 0.0323 | 0.0425 | 0.0356 | 0.0358 |
| V20 | 1.0300 | N/A | 0.9993 | 0.9901 | 0.9958 | 0.9957 | 0.0307 | 0.0399 | 0.0342 | 0.0343 |
| V21 | 1.0330 | N/A | 1.0157 | 1.0119 | 1.0117 | 1.0116 | 0.0173 | 0.0211 | 0.0213 | 0.0214 |
| V22 | 1.0340 | N/A | 1.0157 | 1.0123 | 1.0118 | 1.0116 | 0.0183 | 0.0217 | 0.0222 | 0.0224 |
| V23 | 1.0270 | N/A | 0.9895 | 0.9920 | 0.9865 | 0.9867 | 0.0375 | 0.0350 | 0.0405 | 0.0403 |
| V24 | 1.0220 | N/A | 0.9948 | 1.0000 | 0.9916 | 0.9913 | 0.0272 | 0.0220 | 0.0304 | 0.0307 |
| V25 | 1.0180 | N/A | 0.9966 | 1.0085 | 0.9942 | 0.9943 | 0.0214 | 0.0095 | 0.0238 | 0.0237 |
| V26 | 1.0000 | N/A | 1.0051 | 1.0232 | 1.0026 | 1.0033 | 0.0051 | 0.0232 | 0.0026 | 0.0033 |
| V27 | 1.0240 | N/A | 1.0059 | 1.0151 | 1.0030 | 1.0032 | 0.0181 | 0.0089 | 0.0210 | 0.0208 |
| V28 | 1.0070 | N/A | 1.0145 | 1.0133 | 1.0115 | 1.0125 | 0.0075 | 0.0063 | 0.0045 | 0.0055 |
| V29 | 1.0040 | N/A | 0.9865 | 0.9990 | 0.9837 | 0.9844 | 0.0175 | 0.0050 | 0.0203 | 0.0196 |
| V30 | 0.9920 | N/A | 0.9719 | 0.9893 | 0.9691 | 0.9699 | 0.0201 | 0.0027 | 0.0229 | 0.0221 |
| P1 | 2.6096 | 2.6540 | 2.5734 | 2.6540 | 2.5474 | 2.5694 | 0.0362 | 0.0444 | 0.0622 | 0.0402 |
| **P2** | **0.1830** | **0.9256** | **0.7880** | **0.7821** | **0.7507** | **0.8372** | **0.6050** | **0.5991** | **0.5677** | **0.6542** |
| P3 | -0.0240 | -0.0774 | 0.0566 | -0.0774 | 0.0325 | -0.0582 | 0.0806 | 0.0534 | 0.0565 | 0.0342 |
| P4 | -0.0760 | -0.1165 | 0.0710 | -0.1165 | 0.0090 | -0.1605 | 0.1470 | 0.0405 | 0.0850 | 0.0845 |
| P5 | -0.9420 | -1.0892 | -1.1093 | -1.0892 | -1.1326 | -1.0961 | 0.1673 | 0.1472 | 0.1906 | 0.1541 |
| P6 | 0.0000 | 0.0719 | 0.0475 | 0.0719 | -0.0354 | 0.0354 | 0.0475 | 0.0719 | 0.0354 | 0.0354 |
| P7 | -0.2280 | -0.2117 | -0.1972 | -0.2117 | -0.2317 | -0.1983 | 0.0308 | 0.0163 | 0.0037 | 0.0297 |
| P8 | -0.3000 | -0.3377 | -0.3576 | -0.3377 | -0.4349 | -0.3700 | 0.0576 | 0.0377 | 0.1349 | 0.0700 |
| P9 | 0.0000 | 0.0685 | -0.1014 | -0.1533 | 0.0066 | 0.0124 | 0.1014 | 0.1533 | 0.0066 | 0.0124 |
| P10 | -0.0580 | -0.1436 | -0.1570 | -0.1436 | -0.1151 | -0.1117 | 0.0990 | 0.0856 | 0.0571 | 0.0537 |
| P11 | 0.0000 | -0.0051 | -0.1707 | -0.0051 | -0.0659 | -0.0602 | 0.1707 | 0.0051 | 0.0659 | 0.0602 |
| P12 | -0.1120 | -0.1241 | -0.1214 | -0.1241 | -0.1103 | -0.1557 | 0.0094 | 0.0121 | 0.0017 | 0.0437 |
| P13 | 0.0000 | 0.0160 | 0.0187 | 0.0160 | 0.0298 | -0.0155 | 0.0187 | 0.0160 | 0.0298 | 0.0155 |
| P14 | -0.0620 | -0.0464 | -0.0534 | -0.0733 | -0.0506 | -0.0767 | 0.0086 | 0.0113 | 0.0114 | 0.0147 |
| P15 | -0.0820 | -0.1252 | -0.1201 | -0.1252 | -0.1179 | -0.1394 | 0.0381 | 0.0432 | 0.0359 | 0.0574 |
| P16 | -0.0350 | -0.0365 | -0.0938 | -0.0642 | -0.0654 | -0.0661 | 0.0588 | 0.0292 | 0.0304 | 0.0311 |
| P17 | -0.0900 | -0.0982 | -0.0975 | -0.0982 | -0.0709 | -0.0618 | 0.0075 | 0.0082 | 0.0191 | 0.0282 |
| P18 | -0.0320 | -0.0006 | -0.0044 | -0.0006 | 0.0033 | 0.0014 | 0.0276 | 0.0314 | 0.0353 | 0.0334 |
| P19 | -0.0950 | -0.0403 | -0.1172 | -0.1710 | -0.1082 | -0.1075 | 0.0222 | 0.0760 | 0.0132 | 0.0125 |
| P20 | -0.0220 | 0.0335 | -0.0042 | 0.0335 | 0.0068 | 0.0006 | 0.0178 | 0.0555 | 0.0288 | 0.0226 |
| P21 | -0.1750 | -0.2182 | -0.2334 | -0.2182 | -0.2114 | -0.2104 | 0.0584 | 0.0432 | 0.0364 | 0.0354 |
| P22 | 0.0000 | 0.0039 | 0.0018 | 0.0039 | 0.0294 | 0.0308 | 0.0018 | 0.0039 | 0.0294 | 0.0308 |
| P23 | -0.0320 | -0.0927 | -0.1021 | -0.0927 | -0.0896 | -0.0930 | 0.0701 | 0.0607 | 0.0576 | 0.0610 |
| P24 | -0.0870 | -0.1427 | -0.2178 | -0.1728 | -0.1911 | -0.1955 | 0.1308 | 0.0858 | 0.1041 | 0.1085 |
| P25 | 0.0000 | -0.0003 | -0.0080 | -0.0003 | -0.0219 | -0.0125 | 0.0080 | 0.0003 | 0.0219 | 0.0125 |
| P26 | -0.0350 | 0.0416 | 0.0343 | 0.0416 | 0.0203 | 0.0298 | 0.0693 | 0.0766 | 0.0553 | 0.0648 |
| P27 | 0.0000 | -0.0385 | -0.0319 | -0.0385 | -0.0408 | -0.0327 | 0.0319 | 0.0385 | 0.0408 | 0.0327 |
| P28 | 0.0000 | 0.0186 | 0.0197 | 0.0186 | -0.0256 | 0.0078 | 0.0197 | 0.0186 | 0.0256 | 0.0078 |
| P29 | -0.0240 | -0.0353 | -0.0178 | -0.0353 | -0.0252 | -0.0184 | 0.0062 | 0.0113 | 0.0012 | 0.0056 |
| P30 | -0.1060 | -0.0501 | -0.0688 | -0.0501 | -0.0743 | -0.0692 | 0.0372 | 0.0559 | 0.0317 | 0.0368 |
| Q1 | -0.2042 | -0.2587 | -0.2518 | -0.2587 | -0.2541 | -0.2522 | 0.0476 | 0.0545 | 0.0499 | 0.0480 |
| Q2 | 0.4337 | 0.4353 | 0.4380 | 0.4353 | 0.4340 | 0.4391 | 0.0043 | 0.0016 | 0.0003 | 0.0054 |
| Q3 | -0.0120 | 0.0156 | 0.0095 | 0.0156 | 0.0061 | 0.0065 | 0.0215 | 0.0276 | 0.0181 | 0.0185 |
| Q4 | -0.0160 | 0.0390 | -0.0022 | 0.0390 | -0.0090 | -0.0075 | 0.0138 | 0.0550 | 0.0070 | 0.0085 |
| Q5 | 0.1666 | 0.2438 | 0.2642 | 0.2652 | 0.2650 | 0.2604 | 0.0976 | 0.0986 | 0.0984 | 0.0938 |
| Q6 | 0.0000 | 0.0043 | 0.0145 | 0.0043 | 0.0092 | 0.0077 | 0.0145 | 0.0043 | 0.0092 | 0.0077 |
| Q7 | -0.1090 | -0.1836 | -0.1356 | -0.1419 | -0.1360 | -0.1385 | 0.0266 | 0.0329 | 0.0270 | 0.0295 |
| Q8 | 0.0611 | 0.0240 | 0.0365 | 0.0240 | 0.0321 | 0.0303 | 0.0246 | 0.0371 | 0.0290 | 0.0308 |
| Q9 | 0.0000 | -0.0531 | 0.0158 | 0.0008 | -0.0048 | -0.0077 | 0.0158 | 0.0008 | 0.0048 | 0.0077 |
| Q10 | -0.0200 | 0.0975 | 0.1117 | 0.0975 | 0.0967 | 0.0906 | 0.1317 | 0.1175 | 0.1167 | 0.1106 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q11 | 0.1606 | 0.1298 | 0.1941 | 0.1298 | 0.1751 | 0.1724 | 0.0335 | 0.0308 | 0.0145 | 0.0118 |
| Q12 | -0.0750 | -0.0376 | -0.0442 | -0.0376 | -0.0405 | -0.0214 | 0.0308 | 0.0374 | 0.0345 | 0.0536 |
| Q13 | 0.1045 | 0.0949 | 0.0884 | 0.0949 | 0.0921 | 0.1106 | 0.0161 | 0.0096 | 0.0124 | 0.0061 |
| Q14 | -0.0160 | 0.0284 | -0.0048 | 0.0284 | -0.0014 | 0.0125 | 0.0112 | 0.0444 | 0.0146 | 0.0285 |
| Q15 | -0.0250 | -0.0632 | -0.0733 | -0.0632 | -0.0722 | -0.0635 | 0.0483 | 0.0382 | 0.0472 | 0.0385 |
| Q16 | -0.0180 | -0.0881 | -0.0307 | 0.0122 | -0.0404 | -0.0441 | 0.0127 | 0.0302 | 0.0224 | 0.0261 |
| Q17 | -0.0580 | -0.1291 | -0.0752 | -0.1291 | -0.0853 | -0.0906 | 0.0172 | 0.0711 | 0.0273 | 0.0326 |
| Q18 | -0.0090 | 0.0154 | 0.0128 | -0.0060 | 0.0097 | 0.0093 | 0.0218 | 0.0030 | 0.0187 | 0.0183 |
| Q19 | -0.0340 | -0.0429 | -0.0362 | -0.0429 | -0.0399 | -0.0416 | 0.0022 | 0.0089 | 0.0059 | 0.0076 |
| Q20 | -0.0070 | -0.0168 | -0.0549 | -0.0737 | -0.0582 | -0.0571 | 0.0479 | 0.0667 | 0.0512 | 0.0501 |
| Q21 | -0.1120 | -0.0410 | -0.0503 | -0.0457 | -0.0588 | -0.0618 | 0.0617 | 0.0663 | 0.0532 | 0.0502 |
| Q22 | 0.0000 | 0.0146 | 0.0217 | 0.0146 | 0.0076 | 0.0049 | 0.0217 | 0.0146 | 0.0076 | 0.0049 |
| Q23 | -0.0160 | -0.0061 | 0.0005 | -0.0061 | -0.0049 | -0.0048 | 0.0165 | 0.0099 | 0.0111 | 0.0112 |
| Q24 | -0.0670 | 0.0124 | -0.0106 | 0.0002 | -0.0248 | -0.0226 | 0.0564 | 0.0672 | 0.0422 | 0.0444 |
| Q25 | 0.0000 | -0.0402 | -0.0523 | -0.0402 | -0.0435 | -0.0481 | 0.0523 | 0.0402 | 0.0435 | 0.0481 |
| Q26 | -0.0230 | 0.0118 | -0.0004 | 0.0118 | 0.0086 | 0.0040 | 0.0226 | 0.0348 | 0.0316 | 0.0270 |
| Q27 | 0.0000 | 0.0418 | 0.0395 | 0.0418 | 0.0447 | 0.0417 | 0.0395 | 0.0418 | 0.0447 | 0.0417 |
| Q28 | 0.0000 | -0.0122 | 0.0109 | -0.0122 | 0.0082 | 0.0073 | 0.0109 | 0.0122 | 0.0082 | 0.0073 |
| Q29 | -0.0090 | 0.0018 | -0.0050 | 0.0018 | -0.0005 | -0.0031 | 0.0040 | 0.0108 | 0.0085 | 0.0059 |
| Q30 | -0.0190 | -0.0773 | -0.0492 | -0.0368 | -0.0458 | -0.0478 | 0.0302 | 0.0178 | 0.0268 | 0.0288 |
| P1-2 | 1.7331 | N/A | 1.6259 | 1.6442 | 1.6184 | 1.5807 | 0.1072 | 0.0889 | 0.1147 | 0.1524 |
| P1-3 | 0.8765 | 0.8817 | 0.9475 | 1.0098 | 0.9290 | 0.9887 | 0.0710 | 0.1333 | 0.0525 | 0.1122 |
| **P2-4** | **0.4365** | **0.0726** | **0.5695** | **0.6041** | **0.5451** | **0.6069** | **0.1330** | **0.1676** | **0.1086** | **0.1704** |
| P3-4 | 0.8214 | 0.9507 | 0.9686 | 0.8920 | 0.9271 | 0.8916 | 0.1472 | 0.0706 | 0.1057 | 0.0702 |
| P2-5 | 0.8236 | 0.7903 | 1.0009 | 0.9883 | 1.0085 | 0.9862 | 0.1773 | 0.1647 | 0.1849 | 0.1626 |
| P2-6 | 0.6038 | N/A | 0.7982 | 0.7873 | 0.7705 | 0.7816 | 0.1944 | 0.1835 | 0.1667 | 0.1778 |
| **P4-6** | **0.7213** | **1.2172** | **0.9829** | **0.7972** | **0.9695** | **0.7587** | **0.2616** | **0.0759** | **0.2482** | **0.0374** |
| P5-7 | -0.1478 | -0.2445 | -0.1507 | -0.1422 | -0.1673 | -0.1511 | 0.0029 | 0.0056 | 0.0195 | 0.0033 |
| P6-7 | 0.3813 | 0.3594 | 0.3534 | 0.3594 | 0.4058 | 0.3549 | 0.0279 | 0.0219 | 0.0245 | 0.0264 |
| P6-8 | 0.2956 | N/A | 0.3490 | 0.3287 | 0.4233 | 0.3582 | 0.0534 | 0.0331 | 0.1277 | 0.0626 |
| **P6-9** | **0.2772** | **0.9192** | **0.5912** | **0.4877** | **0.3802** | **0.4010** | **0.3140** | **0.2105** | **0.1030** | **0.1238** |
| P6-10 | 0.1584 | N/A | 0.2811 | 0.2456 | 0.2035 | 0.2176 | 0.1227 | 0.0872 | 0.0451 | 0.0592 |
| P9-11 | 0.0000 | N/A | 0.1707 | 0.0051 | 0.0659 | 0.0602 | 0.1707 | 0.0051 | 0.0659 | 0.0602 |
| P9-10 | 0.2772 | 0.2500 | 0.3191 | 0.3293 | 0.3209 | 0.3532 | 0.0419 | 0.0521 | 0.0437 | 0.0760 |
| P4-12 | 0.4419 | N/A | 0.5980 | 0.5537 | 0.4857 | 0.5503 | 0.1561 | 0.1118 | 0.0438 | 0.1084 |
| P12-13 | 0.0000 | N/A | -0.0187 | -0.0160 | -0.0298 | 0.0155 | 0.0187 | 0.0160 | 0.0298 | 0.0155 |
| P12-14 | 0.0786 | 0.1031 | 0.0931 | 0.1031 | 0.0841 | 0.0977 | 0.0145 | 0.0245 | 0.0055 | 0.0191 |
| P12-15 | 0.1789 | 0.2159 | 0.2555 | 0.2379 | 0.2224 | 0.2159 | 0.0766 | 0.0590 | 0.0435 | 0.0370 |
| P12-16 | 0.0724 | N/A | 0.1467 | 0.1046 | 0.0987 | 0.0656 | 0.0743 | 0.0322 | 0.0263 | 0.0068 |
| P14-15 | 0.0158 | N/A | 0.0387 | 0.0285 | 0.0327 | 0.0199 | 0.0229 | 0.0127 | 0.0169 | 0.0041 |
| P16-17 | 0.0369 | -0.0700 | 0.0509 | 0.0394 | 0.0323 | -0.0009 | 0.0140 | 0.0025 | 0.0046 | 0.0378 |
| P15-18 | 0.0602 | 0.0182 | 0.0530 | 0.0512 | 0.0289 | 0.0116 | 0.0072 | 0.0090 | 0.0313 | 0.0486 |
| P18-19 | 0.0278 | 0.0955 | 0.0482 | 0.0503 | 0.0321 | 0.0129 | 0.0204 | 0.0225 | 0.0043 | 0.0149 |
| P19-20 | -0.0673 | -0.1209 | -0.0692 | -0.1209 | -0.0761 | -0.0946 | 0.0019 | 0.0536 | 0.0088 | 0.0273 |
| P10-20 | 0.0903 | 0.1383 | 0.0751 | 0.0898 | 0.0710 | 0.0960 | 0.0152 | 0.0005 | 0.0193 | 0.0057 |
| P10-17 | 0.0533 | 0.0595 | 0.0475 | 0.0595 | 0.0393 | 0.0633 | 0.0058 | 0.0062 | 0.0140 | 0.0100 |
| P10-21 | 0.1579 | 0.2297 | 0.2161 | 0.1913 | 0.2009 | 0.2012 | 0.0582 | 0.0334 | 0.0430 | 0.0433 |
| P10-22 | 0.0762 | N/A | 0.1046 | 0.0907 | 0.0982 | 0.0986 | 0.0284 | 0.0145 | 0.0220 | 0.0224 |
| P21-22 | -0.0183 | -0.0282 | -0.0190 | -0.0282 | -0.0120 | -0.0107 | 0.0007 | 0.0099 | 0.0063 | 0.0076 |
| P15-23 | 0.0504 | N/A | 0.1165 | 0.0860 | 0.1047 | 0.0815 | 0.0661 | 0.0356 | 0.0543 | 0.0311 |
| P22-24 | 0.0574 | 0.2028 | 0.0865 | 0.0658 | 0.1149 | 0.1178 | 0.0291 | 0.0084 | 0.0575 | 0.0604 |
| P23-24 | 0.0180 | 0.0593 | 0.0129 | -0.0075 | 0.0139 | -0.0122 | 0.0051 | 0.0255 | 0.0041 | 0.0302 |
| P24-25 | -0.0121 | N/A | -0.1198 | -0.1152 | -0.0641 | -0.0917 | 0.1077 | 0.1031 | 0.0520 | 0.0796 |
| P25-26 | 0.0354 | N/A | -0.0340 | -0.0411 | -0.0201 | -0.0295 | 0.0694 | 0.0765 | 0.0555 | 0.0649 |
| P25-27 | -0.0476 | -0.0710 | -0.0974 | -0.0772 | -0.0668 | -0.0766 | 0.0498 | 0.0296 | 0.0192 | 0.0290 |
| P28-27 | 0.1807 | 0.1671 | 0.2186 | 0.2031 | 0.2095 | 0.1993 | 0.0379 | 0.0224 | 0.0288 | 0.0186 |
| P27-29 | 0.0619 | N/A | 0.0417 | 0.0466 | 0.0494 | 0.0424 | 0.0202 | 0.0153 | 0.0125 | 0.0195 |
| P27-30 | 0.0709 | 0.0570 | 0.0466 | 0.0401 | 0.0520 | 0.0470 | 0.0243 | 0.0308 | 0.0189 | 0.0239 |
| P29-30 | 0.0370 | 0.0721 | 0.0234 | 0.0107 | 0.0237 | 0.0235 | 0.0136 | 0.0263 | 0.0133 | 0.0135 |
| P8-28 | -0.0054 | N/A | -0.0101 | -0.0103 | -0.0137 | -0.0133 | 0.0047 | 0.0049 | 0.0083 | 0.0079 |
| P6-28 | 0.1867 | 0.1690 | 0.2098 | 0.1954 | 0.2498 | 0.2055 | 0.0231 | 0.0087 | 0.0631 | 0.0188 |
| P2-1 | -1.6809 | N/A | -1.5806 | -1.5977 | -1.5733 | -1.5375 | 0.1003 | 0.0832 | 0.1076 | 0.1434 |
| P3-1 | -0.8454 | N/A | -0.9120 | -0.9694 | -0.8946 | -0.9498 | 0.0666 | 0.1240 | 0.0492 | 0.1044 |
| P4-2 | -0.4263 | N/A | -0.5529 | -0.5854 | -0.5297 | -0.5879 | 0.1266 | 0.1591 | 0.1034 | 0.1616 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| P4-3 | -0.8129 | N/A | -0.9570 | -0.8820 | -0.9164 | -0.8816 | 0.1441 | 0.0691 | 0.1035 | 0.0687 |
| P5-2 | -0.7942 | N/A | -0.9587 | -0.9470 | -0.9653 | -0.9450 | 0.1645 | 0.1528 | 0.1711 | 0.1508 |
| P6-2 | -0.5843 | N/A | -0.7651 | -0.7550 | -0.7395 | -0.7498 | 0.1808 | 0.1707 | 0.1552 | 0.1655 |
| P6-4 | -0.7150 | N/A | -0.9718 | -0.7899 | -0.9586 | -0.7519 | 0.2568 | 0.0749 | 0.2436 | 0.0369 |
| P7-5 | 0.1495 | N/A | 0.1529 | 0.1443 | 0.1698 | 0.1533 | 0.0034 | 0.0052 | 0.0203 | 0.0038 |
| P7-6 | -0.3775 | N/A | -0.3501 | -0.3560 | -0.4015 | -0.3516 | 0.0274 | 0.0215 | 0.0240 | 0.0259 |
| P8-6 | -0.2946 | N/A | -0.3475 | -0.3274 | -0.4212 | -0.3567 | 0.0529 | 0.0328 | 0.1266 | 0.0621 |
| P9-6 | -0.2772 | N/A | -0.5912 | -0.4877 | -0.3802 | -0.4010 | 0.3140 | 0.2105 | 0.1030 | 0.1238 |
| P10-6 | -0.1584 | N/A | -0.2811 | -0.2456 | -0.2035 | -0.2176 | 0.1227 | 0.0872 | 0.0451 | 0.0592 |
| P11-9 | 0.0000 | N/A | -0.1707 | -0.0051 | -0.0659 | -0.0602 | 0.1707 | 0.0051 | 0.0659 | 0.0602 |
| P10-9 | -0.2772 | N/A | -0.3191 | -0.3293 | -0.3209 | -0.3532 | 0.0419 | 0.0521 | 0.0437 | 0.0760 |
| P12-4 | -0.4419 | N/A | -0.5980 | -0.5537 | -0.4857 | -0.5503 | 0.1561 | 0.1118 | 0.0438 | 0.1084 |
| P13-12 | 0.0000 | N/A | 0.0187 | 0.0160 | 0.0298 | -0.0155 | 0.0187 | 0.0160 | 0.0298 | 0.0155 |
| P14-12 | -0.0778 | N/A | -0.0921 | -0.1018 | -0.0832 | -0.0966 | 0.0143 | 0.0240 | 0.0054 | 0.0188 |
| P15-12 | -0.1767 | N/A | -0.2512 | -0.2341 | -0.2190 | -0.2127 | 0.0745 | 0.0574 | 0.0423 | 0.0360 |
| P16-12 | -0.0719 | N/A | -0.1447 | -0.1036 | -0.0978 | -0.0652 | 0.0728 | 0.0317 | 0.0259 | 0.0067 |
| P15-14 | -0.0158 | N/A | -0.0384 | -0.0282 | -0.0324 | -0.0197 | 0.0226 | 0.0124 | 0.0166 | 0.0039 |
| P17-16 | -0.0368 | N/A | -0.0506 | -0.0394 | -0.0322 | 0.0010 | 0.0138 | 0.0026 | 0.0046 | 0.0378 |
| P18-15 | -0.0598 | N/A | -0.0526 | -0.0509 | -0.0288 | -0.0116 | 0.0072 | 0.0089 | 0.0310 | 0.0482 |
| P19-18 | -0.0277 | N/A | -0.0480 | -0.0501 | -0.0321 | -0.0129 | 0.0203 | 0.0224 | 0.0044 | 0.0148 |
| P20-19 | 0.0674 | N/A | 0.0694 | 0.1215 | 0.0764 | 0.0949 | 0.0020 | 0.0541 | 0.0090 | 0.0275 |
| P20-10 | -0.0894 | N/A | -0.0736 | -0.0880 | -0.0696 | -0.0944 | 0.0158 | 0.0014 | 0.0198 | 0.0050 |
| P17-10 | -0.0532 | N/A | -0.0469 | -0.0588 | -0.0387 | -0.0627 | 0.0063 | 0.0056 | 0.0145 | 0.0095 |
| P21-10 | -0.1567 | N/A | -0.2144 | -0.1900 | -0.1994 | -0.1997 | 0.0577 | 0.0333 | 0.0427 | 0.0430 |
| P22-10 | -0.0757 | N/A | -0.1037 | -0.0901 | -0.0975 | -0.0978 | 0.0280 | 0.0144 | 0.0218 | 0.0221 |
| P22-21 | 0.0183 | N/A | 0.0191 | 0.0282 | 0.0120 | 0.0107 | 0.0008 | 0.0099 | 0.0063 | 0.0076 |
| P23-15 | -0.0500 | N/A | -0.1150 | -0.0852 | -0.1036 | -0.0808 | 0.0650 | 0.0352 | 0.0536 | 0.0308 |
| P24-22 | -0.0569 | N/A | -0.0852 | -0.0652 | -0.1132 | -0.1161 | 0.0283 | 0.0083 | 0.0563 | 0.0592 |
| P24-23 | -0.0180 | N/A | -0.0128 | 0.0076 | -0.0138 | 0.0122 | 0.0052 | 0.0256 | 0.0042 | 0.0302 |
| P25-24 | 0.0122 | N/A | 0.1234 | 0.1181 | 0.0650 | 0.0937 | 0.1112 | 0.1059 | 0.0528 | 0.0815 |
| P26-25 | -0.0350 | N/A | 0.0343 | 0.0416 | 0.0203 | 0.0298 | 0.0693 | 0.0766 | 0.0553 | 0.0648 |
| P27-25 | 0.0479 | N/A | 0.0985 | 0.0779 | 0.0673 | 0.0773 | 0.0506 | 0.0300 | 0.0194 | 0.0294 |
| P27-28 | -0.1807 | N/A | -0.2186 | -0.2031 | -0.2095 | -0.1993 | 0.0379 | 0.0224 | 0.0288 | 0.0186 |
| P29-27 | -0.0610 | N/A | -0.0412 | -0.0460 | -0.0488 | -0.0419 | 0.0198 | 0.0150 | 0.0122 | 0.0191 |
| P30-27 | -0.0693 | N/A | -0.0455 | -0.0394 | -0.0508 | -0.0460 | 0.0238 | 0.0299 | 0.0185 | 0.0233 |
| P30-29 | -0.0367 | N/A | -0.0232 | -0.0107 | -0.0234 | -0.0232 | 0.0135 | 0.0260 | 0.0133 | 0.0135 |
| P28-8 | 0.0055 | N/A | 0.0101 | 0.0103 | 0.0137 | 0.0133 | 0.0046 | 0.0048 | 0.0082 | 0.0078 |
| P28-6 | -0.1862 | N/A | -0.2091 | -0.1947 | -0.2488 | -0.2048 | 0.0229 | 0.0085 | 0.0626 | 0.0186 |
| Q1-2 | -0.2470 | N/A | -0.2798 | -0.2859 | -0.2785 | -0.2870 | 0.0328 | 0.0389 | 0.0315 | 0.0400 |
| Q1-3 | 0.0428 | 0.0595 | 0.0279 | 0.0272 | 0.0245 | 0.0348 | 0.0149 | 0.0156 | 0.0183 | 0.0080 |
| Q2-4 | 0.0475 | 0.0671 | 0.0378 | 0.0288 | 0.0345 | 0.0393 | 0.0097 | 0.0187 | 0.0130 | 0.0082 |
| Q3-4 | -0.0385 | -0.0159 | -0.0470 | -0.0600 | -0.0499 | -0.0560 | 0.0085 | 0.0215 | 0.0114 | 0.0175 |
| Q2-5 | 0.0278 | 0.0213 | 0.0239 | 0.0213 | 0.0279 | 0.0232 | 0.0039 | 0.0065 | 0.0001 | 0.0046 |
| Q2-6 | 0.0137 | N/A | 0.0209 | 0.0196 | 0.0173 | 0.0199 | 0.0072 | 0.0059 | 0.0036 | 0.0062 |
| Q4-6 | -0.1591 | -0.1829 | -0.1160 | -0.0804 | -0.1140 | -0.1197 | 0.0431 | 0.0787 | 0.0451 | 0.0394 |
| Q5-7 | 0.1149 | 0.1580 | 0.1561 | 0.1580 | 0.1565 | 0.1557 | 0.0412 | 0.0431 | 0.0416 | 0.0408 |
| Q6-7 | -0.0278 | -0.0959 | -0.0435 | -0.0387 | -0.0394 | -0.0399 | 0.0157 | 0.0109 | 0.0116 | 0.0121 |
| Q6-8 | -0.0720 | N/A | -0.0558 | -0.0464 | -0.0490 | -0.0492 | 0.0162 | 0.0256 | 0.0230 | 0.0228 |
| Q6-9 | -0.0809 | -0.1233 | -0.0528 | -0.0287 | -0.0612 | -0.0561 | 0.0281 | 0.0522 | 0.0197 | 0.0248 |
| Q6-10 | 0.0019 | N/A | 0.0018 | 0.0054 | -0.0040 | -0.0020 | 0.0001 | 0.0035 | 0.0059 | 0.0039 |
| Q9-11 | -0.1560 | N/A | -0.1820 | -0.1266 | -0.1687 | -0.1663 | 0.0260 | 0.0294 | 0.0127 | 0.0103 |
| Q9-10 | 0.0588 | 0.1108 | 0.0742 | 0.0506 | 0.0728 | 0.0694 | 0.0154 | 0.0082 | 0.0140 | 0.0106 |
| Q4-12 | 0.1441 | N/A | 0.0692 | 0.0512 | 0.0606 | 0.0576 | 0.0749 | 0.0929 | 0.0835 | 0.0865 |
| Q12-13 | -0.1032 | N/A | -0.0873 | -0.0937 | -0.0908 | -0.1089 | 0.0159 | 0.0095 | 0.0124 | 0.0057 |
| Q12-14 | 0.0240 | 0.0570 | 0.0061 | -0.0051 | 0.0078 | 0.0051 | 0.0179 | 0.0291 | 0.0162 | 0.0189 |
| Q12-15 | 0.0679 | 0.0645 | 0.0368 | 0.0474 | 0.0458 | 0.0505 | 0.0311 | 0.0205 | 0.0221 | 0.0174 |
| Q12-16 | 0.0335 | N/A | -0.0193 | -0.0109 | -0.0015 | 0.0140 | 0.0528 | 0.0444 | 0.0350 | 0.0195 |
| Q14-15 | 0.0065 | N/A | -0.0008 | 0.0206 | 0.0046 | 0.0152 | 0.0073 | 0.0141 | 0.0019 | 0.0087 |
| Q16-17 | 0.0144 | -0.0008 | -0.0541 | -0.0008 | -0.0438 | -0.0310 | 0.0685 | 0.0152 | 0.0582 | 0.0454 |
| Q15-18 | 0.0160 | 0.0172 | -0.0226 | 0.0155 | -0.0104 | 0.0006 | 0.0386 | 0.0005 | 0.0264 | 0.0154 |
| Q18-19 | 0.0062 | 0.0088 | -0.0106 | 0.0088 | -0.0010 | 0.0099 | 0.0168 | 0.0026 | 0.0072 | 0.0037 |
| Q19-20 | -0.0279 | 0.0134 | -0.0471 | -0.0344 | -0.0410 | -0.0317 | 0.0192 | 0.0065 | 0.0131 | 0.0038 |
| Q10-20 | 0.0371 | 0.1134 | 0.1058 | 0.1134 | 0.1028 | 0.0930 | 0.0687 | 0.0763 | 0.0657 | 0.0559 |
| Q10-17 | 0.0443 | 0.0676 | 0.1319 | 0.1319 | 0.1311 | 0.1233 | 0.0876 | 0.0876 | 0.0868 | 0.0790 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q10-21 | 0.1001 | 0.0896 | 0.0650 | 0.0433 | 0.0655 | 0.0687 | 0.0351 | 0.0568 | 0.0346 | 0.0314 |
| Q10-22 | 0.0460 | N/A | 0.0321 | 0.0190 | 0.0313 | 0.0330 | 0.0139 | 0.0270 | 0.0147 | 0.0130 |
| Q21-22 | -0.0143 | -0.0051 | 0.0110 | -0.0051 | 0.0035 | 0.0037 | 0.0253 | 0.0092 | 0.0178 | 0.0180 |
| Q15-23 | 0.0291 | N/A | -0.0234 | -0.0183 | -0.0181 | -0.0048 | 0.0525 | 0.0474 | 0.0472 | 0.0339 |
| Q22-24 | 0.0306 | 0.0781 | 0.0631 | 0.0273 | 0.0409 | 0.0400 | 0.0325 | 0.0033 | 0.0103 | 0.0094 |
| Q23-24 | 0.0124 | 0.0278 | -0.0257 | -0.0259 | -0.0254 | -0.0109 | 0.0381 | 0.0383 | 0.0378 | 0.0233 |
| Q24-25 | 0.0201 | N/A | 0.0671 | 0.0435 | 0.0301 | 0.0459 | 0.0470 | 0.0234 | 0.0100 | 0.0258 |
| Q25-26 | 0.0237 | N/A | 0.0008 | -0.0111 | -0.0084 | -0.0036 | 0.0229 | 0.0348 | 0.0321 | 0.0273 |
| Q25-27 | -0.0037 | 0.0094 | 0.0077 | 0.0094 | -0.0067 | -0.0020 | 0.0114 | 0.0131 | 0.0030 | 0.0017 |
| Q28-27 | 0.0504 | 0.0033 | 0.0311 | 0.0033 | 0.0303 | 0.0315 | 0.0193 | 0.0471 | 0.0201 | 0.0189 |
| Q27-29 | 0.0167 | N/A | 0.0252 | 0.0149 | 0.0208 | 0.0233 | 0.0085 | 0.0018 | 0.0041 | 0.0066 |
| Q27-30 | 0.0166 | 0.0093 | 0.0323 | 0.0224 | 0.0292 | 0.0309 | 0.0157 | 0.0058 | 0.0126 | 0.0143 |
| Q29-30 | 0.0061 | -0.0205 | 0.0193 | 0.0158 | 0.0192 | 0.0192 | 0.0132 | 0.0097 | 0.0131 | 0.0131 |
| Q8-28 | -0.0054 | N/A | -0.0150 | -0.0176 | -0.0151 | -0.0150 | 0.0096 | 0.0122 | 0.0097 | 0.0096 |
| Q6-28 | 0.0011 | -0.0427 | -0.0197 | -0.0220 | -0.0162 | -0.0156 | 0.0208 | 0.0231 | 0.0173 | 0.0167 |
| Q2-1 | 0.3447 | N/A | 0.3554 | 0.3656 | 0.3542 | 0.3567 | 0.0107 | 0.0209 | 0.0095 | 0.0120 |
| Q3-1 | 0.0265 | N/A | 0.0565 | 0.0756 | 0.0560 | 0.0625 | 0.0300 | 0.0491 | 0.0295 | 0.0360 |
| Q4-2 | -0.0554 | N/A | -0.0269 | -0.0115 | -0.0274 | -0.0211 | 0.0285 | 0.0439 | 0.0280 | 0.0343 |
| Q4-3 | 0.0544 | N/A | 0.0715 | 0.0797 | 0.0718 | 0.0758 | 0.0171 | 0.0253 | 0.0174 | 0.0214 |
| Q5-2 | 0.0517 | N/A | 0.1082 | 0.1072 | 0.1085 | 0.1047 | 0.0565 | 0.0555 | 0.0568 | 0.0530 |
| Q6-2 | 0.0058 | N/A | 0.0391 | 0.0382 | 0.0367 | 0.0366 | 0.0333 | 0.0324 | 0.0309 | 0.0308 |
| Q6-4 | 0.1719 | N/A | 0.1453 | 0.0965 | 0.1424 | 0.1339 | 0.0266 | 0.0754 | 0.0295 | 0.0380 |
| Q7-5 | -0.1313 | N/A | -0.1716 | -0.1736 | -0.1712 | -0.1711 | 0.0403 | 0.0423 | 0.0399 | 0.0398 |
| Q7-6 | 0.0223 | N/A | 0.0360 | 0.0317 | 0.0352 | 0.0326 | 0.0137 | 0.0094 | 0.0129 | 0.0103 |
| Q8-6 | 0.0666 | N/A | 0.0515 | 0.0416 | 0.0472 | 0.0453 | 0.0151 | 0.0250 | 0.0194 | 0.0213 |
| Q9-6 | 0.0972 | N/A | 0.1236 | 0.0768 | 0.0912 | 0.0892 | 0.0264 | 0.0204 | 0.0060 | 0.0080 |
| Q10-6 | 0.0110 | N/A | 0.0406 | 0.0271 | 0.0264 | 0.0275 | 0.0296 | 0.0161 | 0.0154 | 0.0165 |
| Q11-9 | 0.1606 | N/A | 0.1941 | 0.1298 | 0.1751 | 0.1724 | 0.0335 | 0.0308 | 0.0145 | 0.0118 |
| Q10-9 | -0.0508 | N/A | -0.0632 | -0.0390 | -0.0616 | -0.0560 | 0.0124 | 0.0118 | 0.0108 | 0.0052 |
| Q12-4 | -0.0972 | N/A | 0.0194 | 0.0247 | -0.0018 | 0.0179 | 0.1166 | 0.1219 | 0.0954 | 0.1151 |
| Q13-12 | 0.1045 | N/A | 0.0884 | 0.0949 | 0.0921 | 0.1106 | 0.0161 | 0.0096 | 0.0124 | 0.0061 |
| Q14-12 | -0.0225 | N/A | -0.0040 | 0.0078 | -0.0060 | -0.0027 | 0.0185 | 0.0303 | 0.0165 | 0.0198 |
| Q15-12 | -0.0636 | N/A | -0.0284 | -0.0400 | -0.0392 | -0.0442 | 0.0352 | 0.0236 | 0.0244 | 0.0194 |
| Q16-12 | -0.0324 | N/A | 0.0235 | 0.0130 | 0.0034 | -0.0131 | 0.0559 | 0.0454 | 0.0358 | 0.0193 |
| Q15-14 | -0.0064 | N/A | 0.0011 | -0.0204 | -0.0044 | -0.0151 | 0.0075 | 0.0140 | 0.0020 | 0.0087 |
| Q17-16 | -0.0141 | N/A | 0.0552 | 0.0011 | 0.0444 | 0.0312 | 0.0693 | 0.0152 | 0.0585 | 0.0453 |
| Q18-15 | -0.0152 | N/A | 0.0233 | -0.0148 | 0.0106 | -0.0006 | 0.0385 | 0.0004 | 0.0258 | 0.0146 |
| Q19-18 | -0.0061 | N/A | 0.0109 | -0.0085 | 0.0011 | -0.0099 | 0.0170 | 0.0024 | 0.0072 | 0.0038 |
| Q20-19 | 0.0283 | N/A | 0.0475 | 0.0356 | 0.0415 | 0.0324 | 0.0192 | 0.0073 | 0.0132 | 0.0041 |
| Q20-10 | -0.0353 | N/A | -0.1025 | -0.1092 | -0.0997 | -0.0895 | 0.0672 | 0.0739 | 0.0644 | 0.0542 |
| Q17-10 | -0.0439 | N/A | -0.1304 | -0.1302 | -0.1296 | -0.1218 | 0.0865 | 0.0863 | 0.0857 | 0.0779 |
| Q21-10 | -0.0977 | N/A | -0.0614 | -0.0406 | -0.0623 | -0.0655 | 0.0363 | 0.0571 | 0.0354 | 0.0322 |
| Q22-10 | -0.0449 | N/A | -0.0304 | -0.0178 | -0.0298 | -0.0314 | 0.0145 | 0.0271 | 0.0151 | 0.0135 |
| Q22-21 | 0.0143 | N/A | -0.0110 | 0.0051 | -0.0035 | -0.0037 | 0.0253 | 0.0092 | 0.0178 | 0.0180 |
| Q23-15 | -0.0284 | N/A | 0.0262 | 0.0198 | 0.0205 | 0.0062 | 0.0546 | 0.0482 | 0.0489 | 0.0346 |
| Q24-22 | -0.0299 | N/A | -0.0611 | -0.0264 | -0.0383 | -0.0373 | 0.0312 | 0.0035 | 0.0084 | 0.0074 |
| Q24-23 | -0.0123 | N/A | 0.0260 | 0.0261 | 0.0256 | 0.0110 | 0.0383 | 0.0384 | 0.0379 | 0.0233 |
| Q25-24 | -0.0200 | N/A | -0.0608 | -0.0385 | -0.0284 | -0.0424 | 0.0408 | 0.0185 | 0.0084 | 0.0224 |
| Q26-25 | -0.0230 | N/A | -0.0004 | 0.0118 | 0.0086 | 0.0040 | 0.0226 | 0.0348 | 0.0316 | 0.0270 |
| Q27-25 | 0.0042 | N/A | -0.0057 | -0.0082 | 0.0076 | 0.0033 | 0.0099 | 0.0124 | 0.0034 | 0.0009 |
| Q27-28 | -0.0375 | N/A | -0.0123 | 0.0126 | -0.0130 | -0.0158 | 0.0252 | 0.0501 | 0.0245 | 0.0217 |
| Q29-27 | -0.0151 | N/A | -0.0242 | -0.0140 | -0.0196 | -0.0223 | 0.0091 | 0.0011 | 0.0045 | 0.0072 |
| Q30-27 | -0.0136 | N/A | -0.0304 | -0.0212 | -0.0271 | -0.0290 | 0.0168 | 0.0076 | 0.0135 | 0.0154 |
| Q30-29 | -0.0054 | N/A | -0.0189 | -0.0156 | -0.0187 | -0.0188 | 0.0135 | 0.0102 | 0.0133 | 0.0134 |
| Q28-8 | -0.0380 | N/A | -0.0290 | -0.0263 | -0.0287 | -0.0289 | 0.0090 | 0.0117 | 0.0093 | 0.0091 |
| Q28-6 | -0.0123 | N/A | 0.0088 | 0.0108 | 0.0065 | 0.0047 | 0.0211 | 0.0231 | 0.0188 | 0.0170 |
| **SUM** | | | | | | | **11.7185** | **10.1516** | **9.6367** | **9.4660** |

For this case with all interacting bad data have large error, data connected with these bad data are interfered, thus it's hard for the estimators to detect and

correct the bad data.

#### 5.3.4.4 Summary

Table 5.280 presents the comparison between each estimator for median redundancy case only.

Table 5.280: Comparison of median redundancy 30-bus estimation between estimators

| N.o | Type | WLS | LAV | LTS | LTAV |
|---|---|---|---|---|---|
| **Case 0** | **Random** | **6.5405** | 7.8971 | **6.5405** | **6.5405** |
| **Case 1** | **Reverse** | **7.0127** | 8.4977 | **7.0127** | **7.0127** |
| | **Large** | 9.0223 | 8.0533 | 9.0223 | **6.6611** |
| **Case 2** | **Reverse** | 7.9108 | 8.3359 | 7.9108 | **7.1024** |
| | **Non-conf** | 12.7030 | 8.4977 | 12.8583 | **6.9738** |
| | **Large** | 15.8685 | 8.5567 | 12.9863 | **6.4768** |
| **Case 3** | **Reverse** | 15.5771 | 9.2069 | 17.8626 | **7.1332** |
| | **Non-conf** | 10.7386 | 9.1471 | 8.4063 | **7.5137** |
| | **Large** | 11.7185 | 10.1516 | 9.6367 | **9.4660** |

In all the nine cases, LAV and LTAV generate better overall estimation than others, LTAV has all best estimation. When there are large bad data, LAV and LTAV have much better results since they can correctly detect and correct them. For single and multiple non-interacting large bad data case, estimation of LTAV are very close to (or even better than) the estimation of base case, this further verify that LTAV is very good at detect large bad data.

For cases with multiple interacting large bad data, they offset each other and interfere the estimation of buses connected with them, making detection a hard job, so the estimation are not satisfied for all estimators.

## 5.4 Chapter Summary

In this chapter, 6-bus and IEEE 14-bus power system are conducted with both DC and AC state estimation, full and median redundancy are also conducted separately, IEEE 30-bus power system is only used in AC median redundancy state estimation. For DC estimation, WLS, LAV, LMS, LMR, LTS and LTAV are the six estimators for performance evaluation, while for AC state estimation, since it takes a long time (more than half an hour for 6-bus system) to converge, LMS and LMR are not considered, LTS and LTAV are conducted with the general conventional steps rather than mixed linear programming.

Among all 89 cases, WLS generate worst estimator in almost all of them, especially for cases with large errors. Overall performance of LTAV is best compared with others though in the 14-bus median redundancy multiple interacting cases it generate worse estimation, which has been verified to be specialized locations that lead to the problem. With other locations that are also interacting with each other, LTAV still generates very satisfiable estimation.

Regarding full and median redundancy cases, full redundancy cases generate better estimation for almost all cases, especially for AC state estimation. For 6-bus DC estimation, ascendancy of full redundancy cases are relatively small, since it doesn't provide too much data compared with median counterpart, but for 14-bus DC estimation, the advantage become larger as more available data not only increase local redundancy but also making the system less able to unobservable after bad data elimination. As for AC state estimation, the overall redundancy

of full cases are almost three times of median counterparts, making it have huge advantage over median redundancy cases for all estimators.

# CHAPTER 6

# CONCLUSIONS AND FUTURE WORK

## 6.1 Conclusion

Robust estimator plays a key role in power system state estimation, various estimators are proposed to improve the efficiency in detecting different type of bad data and estimating actual values. As a good estimator, it must be able to solve both well-conditioned and ill-conditioned problems.

Least Trimmed Absolute Value, inspired by LAV and LTS, is proposed to solve practical problems by combining the merits by both of them, both general steps and mixed integer linear programming implementation are provided. 6-bus, IEEE 14-bus and 30-bus system are used to evaluate the performance, both DC and AC state estimation are performed with various bad data scenarios in full and median redundancy cases. LTAV estimator is shown to be robust and efficient in many

scenarios both in bad-data correction and real value estimation.

Regarding system data redundancy, it plays an very important role in state estimation. If overall and local redundancy are essential large, even WLS estimator can generate better estimation than other robust estimators in median redundancy cases. The more larger the redundancy are, the estimation are more accurate. If redundancy can't be guarantee or large redundancy are not realistic, robust estimator helps a lot in state estimation with available data.

## 6.2 Future Work

Though LTAV combines the advantages of LAV and LTS and shows good performance, it also inherits some defects of them. LTAV can not converge in some kinds of bad data, which is from LAV; And in cases with extreme low or even zero local redundancy after bad data elimination, LTAV cannot estimate the real value, just like all other estimators.

New implementation method and new estimator based on more advanced statistic algorithms may be investigated in the future to improve the robustness and accuracy.

# APPENDIX: SYSTEM RAW DATA

For each case, two tables are presented to demonstrate the system data. The first table shows the line data of power system, including branch resistance, branch reactance, shunt capacitance and Tap ratio of transformers. The second table shows the bus data of the system, including generator data, load data, capacitor bank and some others. All data are in p.u. system.

6 bus system, IEEE 14 bus system are illustrated with both DC and AC system, IEEE 30 bus system only shows the data for AC system.

## 6 Bus System

### 6 bus DC system

Table 1: Line data of 6-bus DC system

| No. | From Bus | To Bus | R (p.u) | X (p.u) | B/2(p.u) | Tap |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0 | 1 | 0 | 1 |
| 2 | 1 | 4 | 0 | 1 | 0 | 1 |
| 3 | 1 | 5 | 0 | 1 | 0 | 1 |
| 4 | 2 | 3 | 0 | 1 | 0 | 1 |
| 5 | 2 | 4 | 0 | 1 | 0 | 1 |
| 6 | 2 | 5 | 0 | 1 | 0 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 | 2 | 6 | 0 | 1 | 0 | 1 |
| 8 | 3 | 5 | 0 | 1 | 0 | 1 |
| 9 | 3 | 6 | 0 | 1 | 0 | 1 |
| 10 | 4 | 5 | 0 | 1 | 0 | 1 |
| 11 | 5 | 6 | 0 | 1 | 0 | 1 |

Table 2: Bus data of 6-bus DC system

| Bus | Type | Voltage | | Load | | Generator | | Q min | Q max | Q inj |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Mag. | Ang. | Real | Reac. | Real | Reac. | | | |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 1 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 |
| 3 | 2 | 1 | 0 | 0 | 0 | 0.6 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0.7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 0.7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 1 | 0 | 0.7 | 0 | 0 | 0 | 0 | 0 | 0 |

## 6 bus AC system

Table 3: Line data of 6-bus AC system

| No. | From Bus | To Bus | R (p.u) | X (p.u) | B/2(p.u) | Tap |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0.1 | 0.2 | 0.02 | 1 |
| 2 | 1 | 4 | 0.05 | 0.2 | 0.02 | 1 |
| 3 | 1 | 5 | 0.08 | 0.3 | 0.03 | 1 |
| 4 | 2 | 3 | 0.05 | 0.25 | 0.03 | 1 |
| 5 | 2 | 4 | 0.05 | 0.1 | 0.01 | 1 |
| 6 | 2 | 5 | 0.1 | 0.3 | 0.02 | 1 |
| 7 | 2 | 6 | 0.07 | 0.2 | 0.025 | 1 |
| 8 | 3 | 5 | 0.12 | 0.26 | 0.025 | 1 |
| 9 | 3 | 6 | 0.02 | 0.1 | 0.01 | 1 |
| 10 | 4 | 5 | 0.2 | 0.4 | 0.04 | 1 |
| 11 | 5 | 6 | 0.1 | 0.3 | 0.03 | 1 |

Table 4: Bus data of 6-bus AC system

| Bus | Type | Voltage | | Load | | Generator | | Q min | Q max | Q inj |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Mag. | Ang. | Real | Reac. | Real | Reac. | | | |
| 1 | 1 | 1.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 1.05 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 |
| 3 | 2 | 1.07 | 0 | 0 | 0 | 0.6 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0.7 | 0.7 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 0.7 | 0.7 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 1 | 0 | 0.7 | 0.7 | 0 | 0 | 0 | 0 | 0 |

# 14 Bus System

## 14 bus DC system

Table 5: Line data of 14-bus DC system

| No. | From Bus | To Bus | R (p.u) | X (p.u) | B/2(p.u) | Tap |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0 | 1 | 0 | 1 |
| 2 | 1 | 5 | 0 | 1 | 0 | 1 |
| 3 | 2 | 3 | 0 | 1 | 0 | 1 |
| 4 | 2 | 4 | 0 | 1 | 0 | 1 |
| 5 | 2 | 5 | 0 | 1 | 0 | 1 |
| 6 | 3 | 4 | 0 | 1 | 0 | 1 |
| 7 | 4 | 5 | 0 | 1 | 0 | 1 |
| 8 | 4 | 7 | 0 | 1 | 0 | 1 |
| 9 | 4 | 9 | 0 | 1 | 0 | 1 |
| 10 | 5 | 6 | 0 | 1 | 0 | 1 |
| 11 | 6 | 11 | 0 | 1 | 0 | 1 |
| 12 | 6 | 12 | 0 | 1 | 0 | 1 |
| 13 | 6 | 13 | 0 | 1 | 0 | 1 |
| 14 | 7 | 8 | 0 | 1 | 0 | 1 |
| 15 | 7 | 9 | 0 | 1 | 0 | 1 |
| 16 | 9 | 10 | 0 | 1 | 0 | 1 |
| 17 | 9 | 14 | 0 | 1 | 0 | 1 |
| 18 | 10 | 11 | 0 | 1 | 0 | 1 |
| 19 | 12 | 13 | 0 | 1 | 0 | 1 |
| 20 | 13 | 14 | 0 | 1 | 0 | 1 |

Table 6: Bus data of 14-bus DC system

| Bus | Type | Voltage | | Load | | Generator | | Q min | Q max | Q inj |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Mag. | Ang. | Real | Reac. | Real | Reac. | | | |
| 1 | 1 | 1 | 0 | 0.0000 | 0 | 0.2500 | 0 | 0 | 0 | 0 |
| 2 | 2 | 1 | 0 | 0.1500 | 0 | 0.3000 | 0 | 0 | 0 | 0 |
| 3 | 2 | 1 | 0 | 0.0750 | 0 | 0.1500 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0.0600 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 0.0750 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 6 | 2 | 1 | 0 | 0.0750 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 7 | 0 | 1 | 0 | 0.0150 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 8 | 2 | 1 | 0 | 0.0000 | 0 | 0.9500 | 0 | 0 | 0 | 0 |
| 9 | 0 | 1 | 0 | 0.0450 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1 | 0 | 0.0550 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 11 | 0 | 1 | 0 | 0.0800 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 12 | 0 | 1 | 0 | 0.0700 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 13 | 0 | 1 | 0 | 0.0900 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| 14 | 0 | 1 | 0 | 0.1250 | 0 | 0.0000 | 0 | 0 | 0 | 0 |

## 14 bus AC system

Table 7: Line data of 14-bus AC system

| No. | From Bus | To Bus | R (p.u) | X (p.u) | B/2(p.u) | Tap |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0.01938 | 0.05917 | 0.0264 | 1 |
| 2 | 1 | 5 | 0.05403 | 0.22304 | 0.0246 | 1 |
| 3 | 2 | 3 | 0.04699 | 0.19797 | 0.0219 | 1 |
| 4 | 2 | 4 | 0.05811 | 0.17632 | 0.017 | 1 |
| 5 | 2 | 5 | 0.05695 | 0.17388 | 0.0173 | 1 |
| 6 | 3 | 4 | 0.06701 | 0.17103 | 0.0064 | 1 |
| 7 | 4 | 5 | 0.01335 | 0.04211 | 0 | 1 |
| 8 | 4 | 7 | 0 | 0.20912 | 0 | 0.978 |
| 9 | 4 | 9 | 0 | 0.55618 | 0 | 0.969 |
| 10 | 5 | 6 | 0 | 0.25202 | 0 | 0.932 |
| 11 | 6 | 11 | 0.09498 | 0.1989 | 0 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 6 | 12 | 0.12291 | 0.25581 | 0 | 1 |
| 13 | 6 | 13 | 0.06615 | 0.13027 | 0 | 1 |
| 14 | 7 | 8 | 0 | 0.17615 | 0 | 1 |
| 15 | 7 | 9 | 0 | 0.11001 | 0 | 1 |
| 16 | 9 | 10 | 0.03181 | 0.0845 | 0 | 1 |
| 17 | 9 | 14 | 0.12711 | 0.27038 | 0 | 1 |
| 18 | 10 | 11 | 0.08205 | 0.19207 | 0 | 1 |
| 19 | 12 | 13 | 0.22092 | 0.19988 | 0 | 1 |
| 20 | 13 | 14 | 0.17093 | 0.34802 | 0 | 1 |

Table 8: Bus data of 14-bus AC system

| **Bus** | **Type** | **Voltage** | | **Load** | | **Generator** | | **Q min** | **Q max** | **Q inj** |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **Mag.** | **Ang.** | **Real** | **Reac.** | **Real** | **Reac.** | | | |
| 1 | 1 | 1.06 | 0 | 0 | 0 | 2.3239 | -0.1655 | 0 | 0 | 0 |
| 2 | 2 | 1.045 | -4.983 | 0.217 | 0.127 | 0.4 | 0.4356 | -0.4 | 0.5 | 0 |
| 3 | 2 | 1.01 | -12.725 | 0.942 | 0.19 | 0 | 0.2508 | 0 | 0.4 | 0 |
| 4 | 0 | 1.018 | -10.313 | 0.478 | -0.039 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 1.02 | -8.774 | 0.076 | 0.016 | 0 | 0 | 0 | 0 | 0 |
| 6 | 2 | 1.07 | -14.221 | 0.112 | 0.075 | 0 | 0.1273 | -0.06 | 0.24 | 0 |
| 7 | 0 | 1.062 | -13.36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 2 | 1.09 | -13.36 | 0 | 0 | 0 | 0.1762 | -0.06 | 0.24 | 0 |
| 9 | 0 | 1.056 | -14.939 | 0.295 | 0.166 | 0 | 0 | 0 | 0 | 0.19 |
| 10 | 0 | 1.051 | -15.097 | 0.09 | 0.058 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 1.057 | -14.791 | 0.035 | 0.018 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 1.055 | -15.076 | 0.061 | 0.016 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 1.05 | -15.156 | 0.135 | 0.058 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 1.036 | -16.034 | 0.149 | 0.05 | 0 | 0 | 0 | 0 | 0 |

# 30 Bus System

## 30 bus AC system

Table 9: Line data of 30-bus AC system

| **No.** | **From Bus** | **To Bus** | **R (p.u)** | **X (p.u)** | **B/2(p.u)** | **Tap** |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0.0192 | 0.0575 | 0.0264 | 1 |
| 2 | 1 | 3 | 0.0452 | 0.1652 | 0.0204 | 1 |
| 3 | 2 | 4 | 0.057 | 0.1737 | 0.0184 | 1 |
| 4 | 3 | 4 | 0.0132 | 0.0379 | 0.0042 | 1 |
| 5 | 2 | 5 | 0.0472 | 0.1983 | 0.0209 | 1 |
| 6 | 2 | 6 | 0.0581 | 0.1763 | 0.0187 | 1 |
| 7 | 4 | 6 | 0.0119 | 0.0414 | 0.0045 | 1 |
| 8 | 5 | 7 | 0.046 | 0.116 | 0.0102 | 1 |
| 9 | 6 | 7 | 0.0267 | 0.082 | 0.0085 | 1 |
| 10 | 6 | 8 | 0.012 | 0.042 | 0.0045 | 1 |
| 11 | 6 | 9 | 0 | 0.208 | 0 | 0.978 |
| 12 | 6 | 10 | 0 | 0.556 | 0 | 0.969 |
| 13 | 9 | 11 | 0 | 0.208 | 0 | 1 |
| 14 | 9 | 10 | 0 | 0.11 | 0 | 1 |
| 15 | 4 | 12 | 0 | 0.256 | 0 | 0.932 |
| 16 | 12 | 13 | 0 | 0.14 | 0 | 1 |
| 17 | 12 | 14 | 0.1231 | 0.2559 | 0 | 1 |
| 18 | 12 | 15 | 0.0662 | 0.1304 | 0 | 1 |
| 19 | 12 | 16 | 0.0945 | 0.1987 | 0 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 20 | 14 | 15 | 0.221 | 0.1997 | 0 | 1 |
| 21 | 16 | 17 | 0.0524 | 0.1923 | 0 | 1 |
| 22 | 15 | 18 | 0.1073 | 0.2185 | 0 | 1 |
| 23 | 18 | 19 | 0.0639 | 0.1292 | 0 | 1 |
| 24 | 19 | 20 | 0.034 | 0.068 | 0 | 1 |
| 25 | 10 | 20 | 0.0936 | 0.209 | 0 | 1 |
| 26 | 10 | 17 | 0.0324 | 0.0845 | 0 | 1 |
| 27 | 10 | 21 | 0.0348 | 0.0749 | 0 | 1 |
| 28 | 10 | 22 | 0.0727 | 0.1499 | 0 | 1 |
| 29 | 21 | 22 | 0.0116 | 0.0236 | 0 | 1 |
| 30 | 15 | 23 | 0.1 | 0.202 | 0 | 1 |
| 31 | 22 | 24 | 0.115 | 0.179 | 0 | 1 |
| 32 | 23 | 24 | 0.132 | 0.27 | 0 | 1 |
| 33 | 24 | 25 | 0.1885 | 0.3292 | 0 | 1 |
| 34 | 25 | 26 | 0.2544 | 0.38 | 0 | 1 |
| 35 | 25 | 27 | 0.1093 | 0.2087 | 0 | 1 |
| 36 | 28 | 27 | 0 | 0.396 | 0 | 0.968 |
| 37 | 27 | 29 | 0.2198 | 0.4153 | 0 | 1 |
| 38 | 27 | 30 | 0.3202 | 0.6027 | 0 | 1 |
| 39 | 29 | 30 | 0.2399 | 0.4533 | 0 | 1 |
| 40 | 8 | 28 | 0.0636 | 0.2 | 0.0214 | 1 |
| 41 | 6 | 28 | 0.0169 | 0.0599 | 0.0065 | 1 |

Table 10: Bus data of 30-bus AC system

| **Bus** | **Type** | **Voltage** | | **Load** | | **Generator** | | **Q min** | **Q max** | **Q inj** |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **Mag.** | **Ang.** | **Real** | **Reac.** | **Real** | **Reac.** | | | |
| 1 | 1 | 1.06 | 0 | 0 | 0 | 2.6096 | -0.2042 | 0 | 0 | 0 |
| 2 | 2 | 1.045 | -5.378 | 0.217 | 0.127 | 0.4 | 0.5607 | -0.4 | 0.5 | 0 |
| 3 | 0 | 1 | -7.529 | 0.024 | 0.012 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1.012 | -9.279 | 0.076 | 0.016 | 0 | 0 | 0 | 0 | 0 |
| 5 | 2 | 1.01 | -14.149 | 0.942 | 0.19 | 0 | 0.3566 | -0.4 | 0.4 | 0 |
| 6 | 0 | 1.011 | -11.055 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 1 | -12.852 | 0.228 | 0.109 | 0 | 0 | 0 | 0 | 0 |
| 8 | 2 | 1.01 | -11.797 | 0.3 | 0.3 | 0 | 0.3611 | -0.1 | 0.4 | 0 |
| 9 | 0 | 1.051 | -14.098 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1.045 | -15.688 | 0.058 | 0.02 | 0 | 0 | 0 | 0 | 0.19 |
| 11 | 2 | 1.082 | -14.098 | 0 | 0 | 0 | 0.1606 | -0.06 | 0.24 | 0 |
| 12 | 0 | 1.057 | -14.933 | 0.112 | 0.075 | 0 | 0 | 0 | 0 | 0 |
| 13 | 2 | 1.071 | -14.933 | 0 | 0 | 0 | 0.1045 | -0.06 | 0.24 | 0 |
| 14 | 0 | 1.043 | -15.825 | 0.062 | 0.016 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 1.038 | -15.916 | 0.082 | 0.025 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 1.045 | -15.515 | 0.035 | 0.018 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 1.04 | -15.85 | 0.09 | 0.058 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 1.028 | -16.53 | 0.032 | 0.009 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 1.026 | -16.704 | 0.095 | 0.034 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 1.03 | -16.507 | 0.022 | 0.007 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 1.033 | -16.131 | 0.175 | 0.112 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 1.034 | -16.116 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 1.027 | -16.307 | 0.032 | 0.016 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 1.022 | -16.483 | 0.087 | 0.067 | 0 | 0 | 0 | 0 | 0.043 |
| 25 | 0 | 1.018 | -16.055 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 1 | -16.474 | 0.035 | 0.023 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 1.024 | -15.53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 1.007 | -11.677 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 1.004 | -16.759 | 0.024 | 0.009 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0.992 | -17.642 | 0.106 | 0.019 | 0 | 0 | 0 | 0 | 0 |

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